Quantum Agravity

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Windows on Quantum Gravity: Season 2

Based on

- Salvio, Strumia, arXiv:1403.4226 (JHEP)
- Kannike, Hütsi, Pizza, Racioppi, Raidal, Salvio, Strumia, arXiv:1502.01334 (JHEP)

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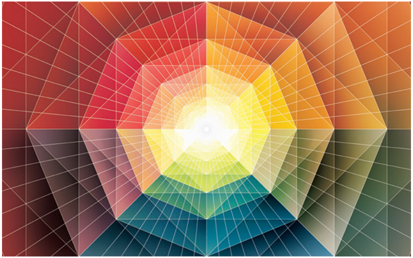

Andy Gilmore for Quanta Magazine

For an outreach article on agravity see
[Natalie Wolchover for Quanta Magazine (2014)]

Some motivations for models without fundamental masses (agravity)

Motivation 1: inflation

The potential of a scalar s in agravity is

$$U(s) = \frac{\lambda_S s^4}{(2\xi_S s^2)^2} \bar{M}_{\rm Pl}^4 = \frac{\lambda_S}{4\xi_S^2} \bar{M}_{\rm Pl}^4$$

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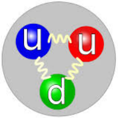
Motivation 2: origin of mass and EW symmetry breaking

Most of the mass of the matter we see has a dynamical origin

Example: the proton mass $\longrightarrow$



Is it possible to generate all the mass dynamically? If yes, with $m \ll M_{\rm Pl}$?

Agravity scenario

The *most general* agravity Lagrangian:

$$\mathscr{L} = \sqrt{-g}\left(\frac{R^2}{6f_0^2} + \frac{\frac{1}{3}R^2 - R_{\mu\nu}^2}{f_2^2} - \sum_i \xi_i \frac{\varphi_i^2}{2} R + \mathscr{L}_{\text{matter}}\right)$$

The corresponding action is

$$\mathscr{S} = \int d^4x \, \mathscr{L}(x)$$

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- There must be a scalar s whose quantum $\langle s \rangle$ generates $\bar{M}_{\rm Pl} \approx 2.4 \times 10^{18}$ GeV:

$$-\xi_S s^2 R \to \bar{M}_{\rm Pl}^2 = 2\xi_S \langle s \rangle^2$$

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- *Agravity is renormalizable*, however, looking at the spectrum:

  (i) massless graviton

  (ii) scalar z with mass $M_0^2 \sim \frac{1}{2} f_0^2 \bar{M}_{\rm Pl}^2$

  (iii) massive graviton with mass $M_2^2 = \frac{1}{2} f_2^2 \bar{M}_{\rm Pl}^2$ and negative norm, but with energy bounded from below

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$$\mathscr{L}_{\rm SM}^{\rm adim} = -\frac{F_{\mu\nu}^2}{4} + \bar{\psi} i \not{D} \psi + |D_\mu H|^2 - (yH\psi\psi + \text{h.c.}) - \lambda_H |H|^4$$

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- $\mathscr{L}_{\rm BSM}^{\rm adim}$ describes physics beyond the SM (BSM).
  It contains a term that generates the weak scale:

$$\lambda_{HS} s^2 |H|^2 \to m^2 = 2\lambda_{HS} \langle s \rangle^2$$

  $m \sim M_h$ and $\langle s \rangle \sim \bar{M}_{\rm Pl}$ so λ_{HS} is tiny (the s-sector is a hidden sector)

Quantum corrections

They are mostly encoded in the RGEs *and are important to generate the scales*

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- We computed the 1-loop RGEs for all couplings

$$\frac{dp}{d\ln\mu} = \beta_p \qquad \text{(of all parameters } p\text{)}$$

of the most general agravity:

$$\frac{R^2}{6f_0^2} + \frac{\frac{1}{3}R^2 - R_{\mu\nu}^2}{f_2^2} - \frac{\left(F_{\mu\nu}^A\right)^2}{4} + \frac{(D_\mu\phi_a)^2}{2} - \frac{\xi_{ab}}{2}\phi_a\phi_b R - \frac{\lambda_{abcd}}{4!}\phi_a\phi_b\phi_c\phi_d + \bar{\psi}_j i\not{D}\psi_j - Y_{ij}^a\psi_i\psi_j\phi_a$$

Without gravity this was done before
[Machacek and Vaughn (1983,1984,1985)]

Results for RGEs

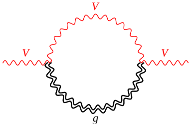
Gauge couplings

Their contributions to the RGEs cancel!

This was previously noticed in
[Narain, Anishetty (2013)]

Possible explanation:
the graviton is not charged

Possible new gravity contributions



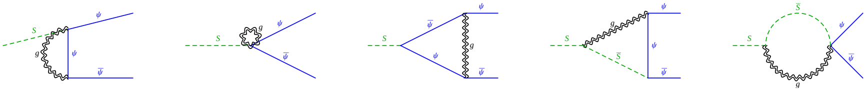
(Rainbow)

(Seagull)

Yukawa couplings

We find the one-loop RGE (where $C_{2F} \equiv t^A t^A$ and $t^A \equiv$ "fermion gauge generators"):

$$(4\pi)^2 \frac{dY^a}{d\ln\mu} = \frac{1}{2}(Y^{\dagger b} Y^b Y^a + Y^a Y^{\dagger b} Y^b) + 2 Y^b Y^{\dagger a} Y^b + Y^b \mathrm{Tr}(Y^{\dagger b} Y^a) - 3\{C_{2F}, Y^a\} + \frac{15}{8} f_2^2 Y^a$$



All remaining RGEs

We also computed all remaining RGEs: for λ_{abcd} ξ_{ab} f_0 and f_2

Dynamical generation of $\langle s \rangle$

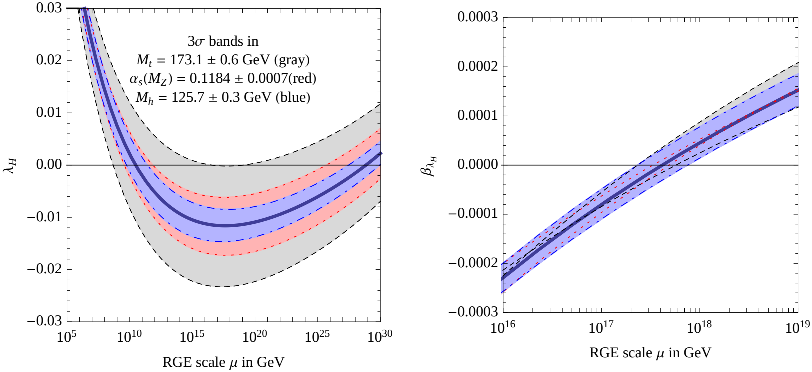
Agravity successfully generates $\langle s \rangle$ when

$$\begin{cases} \lambda_S(\langle s \rangle) & \approx & 0 & \leftrightarrow & \text{nearly vanishing cosmological constant} \\ \frac{d\lambda_S}{ds}(\langle s \rangle) & = & 0 & \leftrightarrow & \text{minimum condition} \\ \xi_S(\langle s \rangle)\langle s \rangle^2 & = & \bar{M}_{\rm Pl}^2 & \leftrightarrow & \text{observed Planck mass} \end{cases}$$

Is the dynamical generation of $\langle s \rangle$ possible?

Yes:

RGE running of the $\overline{\text{MS}}$ quartic Higgs coupling in the SM



Tricks to bring the theory in a more standard form

$$\frac{\mathscr{L}}{\sqrt{-g}} = \frac{R^2}{6f_0^2} - \xi \frac{\varphi^2}{2} R + \mathscr{L}_{\rm matter}$$

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First, we can trade the R^2 term with a scalar field: consider an auxiliary field χ

$$\frac{\mathscr{L}}{\sqrt{-g}} = \frac{R^2}{6f_0^2} - \xi\frac{\varphi^2}{2}R + \mathscr{L}_{\rm matter} \underbrace{-\frac{(R+3f_0^2\chi/2)^2}{6f_0^2}}_{\text{zero on-shell}} = -\frac{f}{2}R - \frac{3f_0^2}{8}\chi^2 + \mathscr{L}_{\rm matter}$$

where $\quad f = \chi + \xi\varphi^2 \quad$ and $\quad \mathscr{L}_{\rm matter} = \dfrac{(D_\mu\varphi)^2}{2} - \dfrac{1}{4}F_{\mu\nu}^2 + \bar\psi i\not{D}\psi + (y\,\varphi\psi\psi + \text{h.c.}) - V(\varphi)$

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Second, perform a conformal transformation of the metric and the other fields:

$$g_{\mu\nu}^E = g_{\mu\nu}\times f/\bar{M}_{\rm Pl}^2 \qquad \varphi_E = \varphi\times(\bar{M}_{\rm Pl}^2/f)^{1/2}, \qquad \psi_E = \psi\times(\bar{M}_{\rm Pl}^2/f)^{3/4}, \qquad A_{\mu E} = A_\mu$$

$$\frac{\mathscr{L}}{\sqrt{-g_E}} = -\frac{1}{4}F_{E\mu\nu}^2 + \bar{\psi}_E i\not{D}\psi_E + (y\varphi_E\psi_E\psi_E + \text{h.c.}) - \frac{\bar{M}_{\rm Pl}^2}{2}R_E + \mathscr{L}_\varphi - V_E$$

where $\quad \mathscr{L}_\varphi = \bar{M}_{\rm Pl}^2\left[\frac{(D_\mu\varphi)^2}{2f} + \frac{3(\partial_\mu f)^2}{4f^2}\right] \quad$ and $\quad V_E = \frac{\bar{M}_{\rm Pl}^4}{f^2}\left[V(\varphi) + \frac{3f_0^2}{8}\chi^2\right]$

By redefining $z = \sqrt{6f}$,

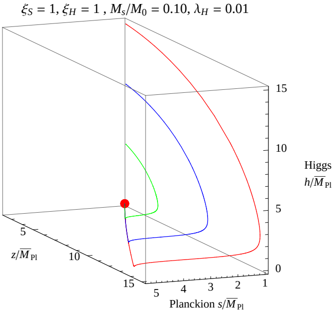
$$\mathscr{L}_\varphi = \frac{6\bar{M}_{\rm Pl}^2}{z^2}\frac{(D_\mu\varphi)^2 + (\partial_\mu z)^2}{2}, \qquad V_E(z,\varphi) = \frac{36\bar{M}_{\rm Pl}^4}{z^4}\left[V(\varphi) + \frac{3f_0^2}{8}\left(\frac{z^2}{6} - \xi_\varphi\varphi^2\right)^2\right]$$

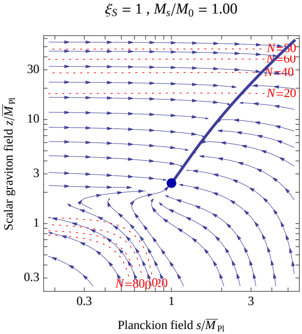
Predictions for inflation (generically a multifield inflation)

The minimal realistic model has at least 3 scalars:

the SM scalar h
the Planckion s
the graviscalar z

$M_s \sim$ mass of s
$M_0 \sim$ mass of z





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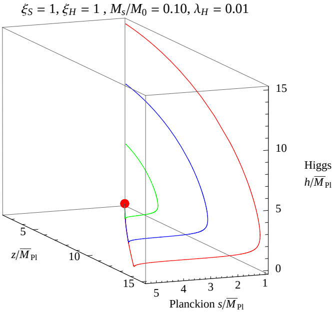
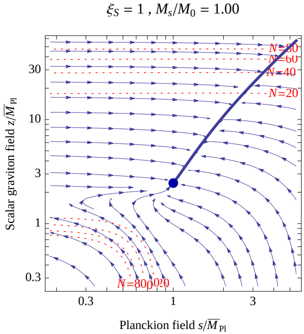
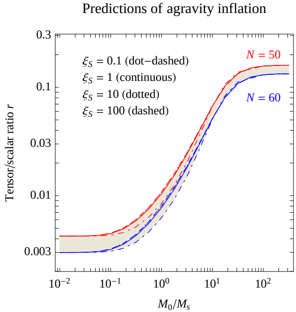
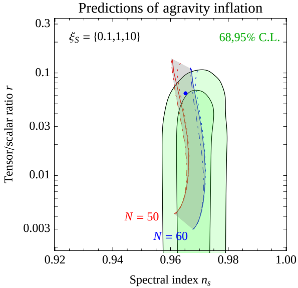




- **left:** when $M_s \ll (\gg) M_0$, the inflaton is s (z)
- **right:** comparison with a global fit of PLANCK and BICEP2/KECK





Matching the scalar amplitude

1. **Planckion inflation** $(M_0 \gg M_s)$

$$n_s \approx 1 - \frac{2}{N} \overset{N\approx 60}{\approx} 0.967, \qquad r \approx \frac{8}{N} \overset{N\approx 60}{\approx} 0.13$$

The scalar amplitude $P_R = M_s^2 N^2 / 6\pi^2 \bar{M}_{\rm Pl}^2$ is reproduced for $M_s \approx 1.4 \times 10^{13}$GeV

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So generically we have

$$f_0 \gtrsim 10^{-5}$$

Other virtues

This scenario also

- leads to successful reaheating
- has natural dark matter candidates and can account for neutrino masses

Natural dynamical generation of the weak scale

1) *Low energies* ($\mu < M_{0,2}$): agravity can be neglected and we recover the SM
$\rightarrow$ *m* is the only mass parameter and we do not see any large corrections to it

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$$(4\pi)^2 \frac{d}{d\ln\mu} \frac{m^2}{\bar{M}_{\rm Pl}^2} \quad = \quad -\xi_H[5f_2^4 + f_0^4(1+6\xi_H)] + \dots$$

The red term is a non-multiplicative potentially dangerous correction to m

$$m^2 \sim \bar{M}_{\rm Pl}^2 g^2, \quad \text{naturalness} \ \to \ f_2, f_0(1+6\xi_H)^{1/4} \sim \sqrt{\frac{4\pi m}{M_{\rm Pl}}} \sim 10^{-8}$$

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3) *Large energies* ($\mu > \bar{M}_{\rm Pl}$):

$$\lambda_{HS}|H|^2 s^2 \quad \rightarrow \quad m^2 = 2\lambda_{HS}\langle s\rangle^2$$

$$\rightarrow \quad \lambda_{HS} \sim 10^{-32}$$

λ_{HS} can be naturally this small (looking at the RGE of λ_{HS})

Conclusions

1. *A rationale for*

   - *inflation*
   - $M_h \ll \bar{M}_{\rm Pl}$
   - *...*

   is achieved in no-scale theories of all interactions (including gravity): agravity

2. *Inflation: the minimal realistic model predicts* $n_s \approx 0.967, \quad 0.003 < r < 0.13$, *in agreement with* PLANCK *and* BICEP2/KECK.

Thank you very much for your attention!

Extra slides

RGEs for the quartic couplings

Tens of Feynman diagrams contribute to these RGEs ... we obtain

$$(4\pi)^2 \frac{d\lambda_{abcd}}{d\ln\mu} = \sum_{\text{perms}} \Big[\frac{1}{8}\lambda_{abef}\lambda_{efcd} + \frac{3}{8}\{\theta^A,\theta^B\}_{ab}\{\theta^A,\theta^B\}_{cd} - \mathrm{Tr}\, Y^a Y^{\dagger b} Y^c Y^{\dagger d} +$$

$$+\frac{5}{8} f_2^4 \xi_{ab}\xi_{cd} + \frac{f_0^4}{8}\xi_{ae}\xi_{cf}(\delta_{eb}+6\xi_{eb})(\delta_{fd}+6\xi_{fd})$$

$$+\frac{f_0^2}{4!}(\delta_{ae}+6\xi_{ae})(\delta_{bf}+6\xi_{bf})\lambda_{efcd}\Big] + \lambda_{abcd}\Big[\sum_k (Y_2^k - 3C_{2S}^k) + 5f_2^2\Big],$$

where the first sum runs over the 4! permutations of *abcd* and the second sum over $k = \{a,b,c,d\}$, with Y_2^k and C_2^k defined by

$$\mathrm{Tr}(Y^{\dagger a} Y^b) = Y_2^a \delta^{ab}, \quad \theta^A_{ac}\theta^A_{cb} = C_{2S}^a \delta_{ab}$$

(θ^A are the scalar gauge generators)

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RGEs for the quartic couplings: SM case

For the SM H plus the complex scalar singlet S the RGEs become:

$$
\begin{aligned}
(4\pi)^2 \frac{d\lambda_S}{d\ln\mu} &= 20\lambda_S^2 + 2\lambda_{HS}^2 + \frac{\xi_S^2}{2}\left[5f_2^4 + f_0^4(1+6\xi_S)^2\right] + \lambda_S\left[5f_2^2 + f_0^2(1+6\xi_S)^2\right] \\
(4\pi)^2 \frac{d\lambda_{HS}}{d\ln\mu} &= -\xi_H\xi_S\left[5f_2^4 + f_0^4(6\xi_S+1)(6\xi_H+1)\right] - 4\lambda_{HS}^2 + \lambda_{HS}\left\{8\lambda_S + 12\lambda_H + 6y_t^2\right. \\
&\quad \left. +5f_2^2 + \frac{f_0^2}{6}\left[(6\xi_S+1)^2 + (6\xi_H+1)^2 + 4(6\xi_S+1)(6\xi_H+1)\right]\right\} \\
(4\pi)^2 \frac{d\lambda_H}{d\ln\mu} &= \frac{9}{8}g_2^4 + \frac{9}{20}g_1^2g_2^2 + \frac{27}{200}g_1^4 - 6y_t^4 + 24\lambda_H^2 + \lambda_{HS}^2 + \frac{\xi_H^2}{2}\left[5f_2^4 + f_0^4(1+6\xi_H)^2\right] \\
&\quad +\lambda_H\left(5f_2^2 + f_0^2(1+6\xi_H)^2 + 12y_t^2 - 9g_2^2 - \frac{9}{5}g_1^2\right).
\end{aligned}
$$

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RGEs for the scalar/graviton couplings

Complicated calculation (but computer algebra helps!)

$$(4\pi)^2 \frac{d\xi_{ab}}{d\ln\mu} = \frac{1}{6}\lambda_{abcd}(6\xi_{cd}+\delta_{cd}) + (6\xi_{ab}+\delta_{ab})\sum_k \left[\frac{Y_2^k}{3} - \frac{C_{2S}^k}{2}\right] + -\frac{5f_2^4}{3f_0^2}\xi_{ab} + f_0^2\xi_{ac}\left(\xi_{cd}+\frac{2}{3}\delta_{cd}\right)(6\xi_{db}+\delta_{db})$$

For the SM H plus the complex scalar singlet S the RGEs become:

$$(4\pi)^2 \frac{d\xi_S}{d\ln\mu} = (1+6\xi_S)\frac{4}{3}\lambda_S - \frac{2\lambda_{HS}}{3}(1+6\xi_H) + \frac{f_0^2}{3}\xi_S(1+6\xi_S)(2+3\xi_S) - \frac{5}{3}\frac{f_2^4}{f_0^2}\xi_S$$

$$(4\pi)^2 \frac{d\xi_H}{d\ln\mu} = (1+6\xi_H)(2y_t^2 - \frac{3}{4}g_2^2 - \frac{3}{20}g_1^2 + 2\lambda_H) - \frac{\lambda_{HS}}{3}(1+6\xi_S) + + \frac{f_0^2}{3}\xi_H(1+6\xi_H)(2+3\xi_H) - \frac{5}{3}\frac{f_2^4}{f_0^2}\xi_H$$

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RGE for the gravitational couplings

Huge calculation ... (computer algebra practically needed!!)

$$
\begin{aligned}
(4\pi)^2 \frac{df_2^2}{d\ln\mu} &= -f_2^4\left(\frac{133}{10}+\frac{N_V}{5}+\frac{N_f}{20}+\frac{N_s}{60}\right)\\
(4\pi)^2 \frac{df_0^2}{d\ln\mu} &= \frac{5}{3}f_2^4+5f_2^2f_0^2+\frac{5}{6}f_0^4+\frac{f_0^4}{12}(\delta_{ab}+6\xi_{ab})(\delta_{ab}+6\xi_{ab})
\end{aligned}
$$

Here N_V, N_f, N_s are the number of vectors, Weyl fermions and real scalars.

In the SM $N_V = 12$, $N_f = 45$, $N_s = 4$.

We confirmed the calculations of *[Avramidi (1995)]*
rather than those of *[Fradkin and Tseytlin (1981,1982)]*

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Agravity inflation

All scalar fields in agravity are inflaton candidates

Agravity inflation: a simple single field case

We identify inflaton = s by taking the other scalar fields heavy ...

Then we can easily convert s into a scalar s_E with canonical kinetic term and find

$$\epsilon \equiv \frac{\bar{M}_{\rm Pl}^2}{2}\left(\frac{1}{U}\frac{\partial U}{\partial s_E}\right)^2 = \frac{1}{2}\frac{\xi_S}{1+6\xi_S}\left(\frac{\beta_{\lambda_S}}{\lambda_S} - 2\frac{\beta_{\xi_S}}{\xi_S}\right)^2$$

$$\eta \equiv \bar{M}_{\rm Pl}^2\frac{1}{U}\frac{\partial^2 U}{\partial s_E^2} = \frac{\xi_S}{1+6\xi_S}\left(\frac{\beta(\beta_{\lambda_S})}{\lambda_S} - 2\frac{\beta(\beta_{\xi_S})}{\xi_S} + \frac{5+36\xi_S}{1+6\xi_S}\frac{\beta_{\xi_S}^2}{\xi_S^2} - \frac{7+48\xi_S}{1+6\xi_S}\frac{\beta_{\lambda_S}\beta_{\xi_S}}{2\lambda_S\xi_S}\right)$$

The slow-roll parameters are given by the β-functions ...

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The slow-roll parameters are given by the β-functions ...

We can insert them in the formulae for the observable parameters A_s, n_s and $r = \frac{A_t}{A_s}$:

$$n_s = 1 - 6\epsilon + 2\eta, \qquad A_s = \frac{U/\epsilon}{24\pi^2 \bar{M}_{\rm Pl}^4}, \qquad r = 16\epsilon$$

where everything is evaluated at about $N \approx 60$ e-foldings when the inflaton $s_E(N)$ was

$$N = \frac{1}{\bar{M}_{\rm Pl}^2}\int_0^{s_E(N)} \frac{U(s_E)}{U'(s_E)}\, ds_E$$

Reheating

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The decay of I with mass M_I and width Γ_I reheats the universe up to a temperature

$$T_{\rm RH} = \left[\frac{90}{\pi^2 g_*}\,\Gamma_I^2 \bar{M}_{\rm Pl}^2\right]^{1/4},$$

where $g_* \sim 100$ is the number of relativistic degrees of freedom at $T_{\rm RH}$.

The inflaton I is in the hidden sector. How can it reheat the universe?

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The Planckion s and the graviscalar z respectively couple to

$$\frac{\partial_\mu \mathscr{D}_\mu}{\bar{M}_{\rm Pl}/\sqrt{\xi_S}} \qquad \text{and} \qquad \frac{T_{\mu\mu}}{\bar{M}_{\rm Pl}},$$

($T_{\mu\mu}$ is the trace of the energy-momentum tensor and $\mathscr{D}_\mu$ is the dilatation current)

The theory is classically scale-invariant $\rightarrow$ a non-zero $\partial_\mu \mathscr{D}_\mu$ arises only at loop level:

$$\partial_\mu \mathscr{D}_\mu = \frac{\beta_{g_1}}{2g_1} Y_{\mu\nu}^2 + \frac{\beta_{g_2}}{2g_2} W_{\mu\nu}^2 + \frac{\beta_{g_3}}{2g_3} G_{\mu\nu}^2 + \beta_{y_t} H Q_3 U_3 + \beta_{\lambda_H} |H|^4 + \ldots,$$

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leading decay from $\partial_\mu \mathscr{D}_\mu$: $\Gamma(I \to gg) \approx \frac{|\xi_S| g_3^4 M_s^3}{(4\pi)^5 \bar{M}_{\rm Pl}^2} \quad \to \quad T_{\rm RH} \overset{\xi_S \sim 1}{\sim} 10^7\,{\rm GeV} \left(\frac{M_s}{10^{13}\,{\rm GeV}}\right)^{3/2}$

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leading decay from $T_{\mu\mu}$: $\Gamma(I \to h_E h_E) \approx \frac{(1+6\xi_H)^2 |\xi_S|}{|1+6\xi_S|} \frac{M_I^3}{64\pi \bar{M}_{\rm Pl}^2} \quad \rightarrow \quad T_{\rm RH} \overset{\xi_S \sim 1}{\sim} 10^9\,{\rm GeV}$

Dark matter and neutrino masses

There should be fermions in the s-sector. Two types of candidates come to mind

Dark matter and neutrino masses

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Fermions in the s-sector with no gauge interactions

- They can couple to the SM behaving as right-handed neutrinos N and generate the observed neutrino masses via NLH couplings. The right-handed neutrino masses can be generated by sN^2 terms. *[Alexander-Nunneley, Pilaftsis (2010)]*
- They can provide baryogenesis via leptogenesis.
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Fermions in the s-sector which cannot couple to the SM

The lightest fermion in the s-sector is a stable DM candidate if it cannot couple to the SM sector (for example because it has gauge interactions under the inflaton sector).

Dark Matter: a Concrete example

A predictive model (no extra parameters)

Take a 2^{nd} copy of the SM and impose a Z_2 symmetry, spontaneously broken by the fact that the mirror Higgs field (S) has

$$\langle S \rangle \sim \bar{M}_{\mathrm{Pl}} \quad \text{while} \quad \langle H \rangle \sim M_W$$

Mirror SM particles (e.g. a mirror neutrino or electron) may be DM ...

Interactions between these candidates and the SM are suppressed by λ_{HS} ...

Dark matter abundance

Terms in $\partial_\mu \mathscr{D}_\mu$ and $T_{\mu\mu}$ lead to decays of the inflaton I into DM
(along the lines of the reheating calculation)

More in general the DM fermions can also get a mass M from another source.
Then such fermion masses would contribute to $\partial_\mu \mathscr{D}_\mu$ and to $T_{\mu\mu}$ as $M\bar{\Psi}\Psi$
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By identifying the fermion Ψ with DM, its abundance is

$$\Omega_{\rm DM} \equiv \frac{\rho_{\rm DM}}{\rho_{\rm cr}} \approx \frac{0.110}{h^2} \times \frac{M}{0.40\text{eV}} \frac{\Gamma(I \to \rm DM)}{\Gamma(I \to \rm SM)}$$

having inserted the present Hubble constant $H_0 = h \times 100\,\mathrm{km/sec\ Mpc}$

Dark matter abundance: result

The observed DM abundance is reproduced for

$$M \approx (10 - 200)\text{TeV} \left(\frac{M_I}{10^{13}\text{GeV}}\right)^{2/3}$$

where the lower (higher) estimate applies if $\Gamma(I \to gg)$ ($\Gamma(I \to h_E h_E)$) dominates

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