

Electrodynamical Effects of Inflationary Gravitons

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FRW Geometry

- Homogeneous, isotropic and spatial flat:

$$dS^2 = -dt^2 + a^2(t)d\vec{x} \cdot d\vec{x}$$

- Physical distance: $a(t)d\vec{x}$

- Physical momentum: $\frac{\vec{k}}{a(t)}$

- $\epsilon \rightarrow 0 \rightarrow$ Locally de Sitter Background

- $H(t) = H = \text{const.} \rightarrow a(t) \sim e^{Ht} \rightarrow$ Max. accelerated

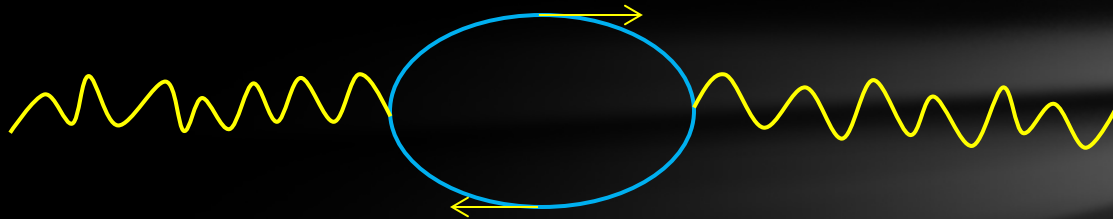
- $H < 10^{14} \text{ GeV} \rightarrow$ Particles almost effectively massless

- $GH^2 < 5 \times 10^{-11}$

- QG still perturbative, not negligible

Spacetime Expansion Strengthens Loop Effects

- How to think about QFT Effects?
 - Classical response to virtual particles



- Maximum IR Enhancements:
 - PERSISTENCE TIME $\rightarrow m = 0 + \text{inflation}$
 - EMERGENCE RATE $\rightarrow \text{no conformal invariance}$
- Realized by
 - Massless, minimally coupled scalars
 - Gravitons

Fossilized quantum fluctuations

- Dynamical gravitons ~ MMC scalar (1946: Lifshitz)

$$\mathcal{L} = -\frac{1}{2} \partial_\mu \varphi \partial_\nu \varphi g^{\mu\nu} \sqrt{-g} \text{ in co-moving coordinates}$$

- $L = \int d^3x \mathcal{L} \rightarrow \int \frac{d^3k}{(2\pi)^3} \frac{a^3}{2} \left[\left| \frac{d\tilde{\varphi}}{dt} \right|^2 - \left(\frac{kc}{a} \right)^2 |\tilde{\varphi}|^2 \right]$

- Each mode k is S.H.O. with $m(t) \sim a^3(t)$ & $\omega(t) = \frac{kc}{a(t)}$

- $L = \frac{m}{2} \dot{q}^2 - \frac{m\omega^2}{2} q^2$

$$\rightarrow \dot{q}^2 + 3H\dot{q} + \omega^2 q^2 = 0$$

$$\rightarrow q(t) = u(t, k)\alpha + u^*(t, k)\alpha^+, \quad [\alpha, \alpha^+] = 1$$

- $u(t, k) = \frac{H}{\sqrt{2k^3}} [1 - ix] e^{ix}, \quad x = \frac{k}{Ha(t)}$

- $k/a(t) \ll H \rightarrow x \ll 1 \rightarrow u(t, k) \sim \text{constant}$

- $\langle \Omega | E(t) | \Omega \rangle = [N + \frac{1}{2}] \hbar \omega, \quad N(t, k) = \left[\frac{1}{2x} \right]^2$

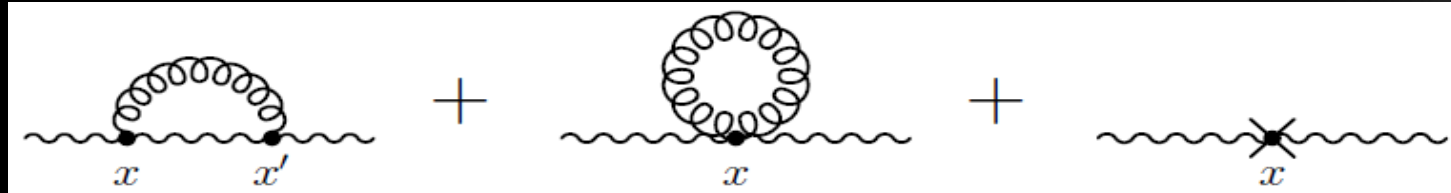
Motivation

- Gravitons + Massless Fermion
 - 1st IR log using dimensional regularization in de Sitter
 - Fermion mode fun. $\sim \#GH^2 \ln[a(t)]$ at 1-loop
- No $\#GH^2 \ln[a(t)]$ in “QG + MMC” (Kahya & Woodard)
- IR gravitons only couple to mmc through red-shift K.E.
- But fermions has extra SPIN in addition (0803.2377)
- secular effects from spin-spin interactions
- simple rules for catching leading log. has derived
- It differs from Starobinsky's IR truncation
- Photon also has spin

What We Did

$$\mathcal{L} = \frac{(R-2\Lambda)\sqrt{-g}}{16\pi G} - \frac{1}{4}F_{\mu\nu}F_{\rho\sigma}g^{\mu\rho}g^{\nu\sigma}\sqrt{-g} + \text{c-terms}$$

1. Compute $i[\mu\Pi^\nu](x; x')$ from gravitons at 1 loop in dim. reg.



2. Renormalize with BPHZ counter terms (cf. LEFT)

$$\Delta\mathcal{L} = C_4 D_\alpha F_{\mu\nu} D_\beta F_{\rho\sigma} g^{\alpha\beta} g^{\mu\rho} g^{\nu\sigma} \sqrt{-g} + \bar{C} H^2 F^{\mu\nu} F_{\rho\sigma} g^{\mu\rho} g^{\nu\sigma} \sqrt{-g} \\ + \Delta C H^2 F_{ij} F_{kl} g^{ik} g^{jl} \sqrt{-g}$$

3. Quantum-correct Maxwell's equations

$$\partial_\nu [\sqrt{-g} g^{\nu\rho} g^{\mu\sigma} F_{\rho\sigma}(x)] + \int d^4 x' [\mu\Pi^\nu](x; x') A_\nu(x') = J^\mu(x)$$

- $J^\mu = 0 \rightarrow$ dynamical photons
- $J^\mu \neq 0 \rightarrow$ electromagnetic forces

Why there should be an effect

Neither photons nor gravitons are charged

- So how can they polarize the vacuum?

But they DO carry momentum

- Dynamical photons have $\vec{p} = \vec{k}$
- Virtual photons carry $\vec{F} = \frac{\Delta\vec{p}}{\Delta t}$

Virtual gravitons can add or subtract to this

- de Sitter gravitons (\vec{k}, λ) have $N(t, k) = \left[\frac{H}{2k/a(t)} \right]^2$
- Gravitons with $k_{phys} \sim H \gg k_{photon} = \frac{k}{a(t)}$

What happens on a flat background

arXiv:1202.5800 by Leonard & Woodard

- $\kappa^2 \equiv 16\pi G$ $\Delta x^2 \equiv \|\vec{x} - \vec{x}'\|^2 - (|t - t'| - i\epsilon)^2$
- $i[\mu\Pi^\nu](x; x') = (\eta^{\mu\nu}\partial' \cdot \partial - \partial'^\mu\partial^\nu) \left[\frac{\#\kappa^2}{\Delta x^{2D-2}} \right]$
- $\Delta\mathcal{L} = C_4 D_\alpha F_{\mu\nu} D_\beta F_{\rho\sigma} g^{\alpha\beta} g^{\mu\rho} g^{\nu\sigma} \sqrt{-g}$
- $i[\mu\Pi_{ren}^\nu](x; x') = (\eta^{\mu\nu}\partial' \cdot \partial - \partial'^\mu\partial^\nu) \partial^4 \left[\frac{\kappa^2}{192\pi^2} \frac{\ln(\mu^2\Delta x^2)}{\Delta x^2} \right]$
- No change in dynamical photons
- Coulomb: $\Phi(r) = \frac{q}{4\pi r} \left[1 + \frac{2G}{3\pi r^2} + O(G^2) \right]$
 - Radkowski (1970)
 - 0th order field distorts virtual gravitons nearby & these add momentum to virtual photons carrying force

$i[\mu\Pi^\nu](x; x')$ for de Sitter

arXiv:1304.7265 by Leonard & Woodard

de Sitter breaking \rightarrow 2 structure functions

- $i[\mu\Pi^\nu] = (\eta^{\mu\nu}\eta^{\rho\sigma} - \eta^{\mu\sigma}\eta^{\nu\rho})\partial_\rho\partial'_\sigma F(x; x')$
 $+ (\bar{\eta}^{\mu\nu}\bar{\eta}^{\rho\sigma} - \bar{\eta}^{\mu\sigma}\bar{\eta}^{\nu\rho})\partial_\rho\partial'_\sigma G(x; x')$
- $\bar{\eta}^{\mu\nu} \equiv \eta^{\mu\nu} + \delta_0^\mu\delta_0^\nu$

de Sitter breaking \rightarrow noninvariant counterterm

- Regulation tech. + gauge fixing
- It's easy to criticize when you never compute anything

But simple EFE in conformal coordinates

- $\partial_\nu F^{\nu\mu}(x) + \partial_\nu \int d^4x' [iF(x; x')F^{\nu\mu}(x') + iG(x; x')\bar{F}^{\nu\mu}(x')] = J^\mu(x)$
- Solve perturbatively using Schwinger-Keldysh

1-loop structure functions

$$F(x; x') = \frac{\kappa^2}{8\pi^2} \left\{ H^2 [\ln(a) + \alpha] + \frac{1}{a^2} \left[-\frac{1}{3} \ln(a) + \beta \right] (\partial^2 + 2Ha\partial_0) + \frac{H}{3a} \partial_0 \right\} i\delta^4(x - x')$$
$$- \frac{\kappa^2}{1536\pi^4} \frac{1}{a} \partial^6 \left\{ \frac{1}{a'} \left[\ln^2 \left(\frac{1}{4} H^2 \Delta x^2 \right) - 2 \ln \left(\frac{1}{4} H^2 \Delta x^2 \right) \right] \right\}$$
$$+ \frac{\kappa^2 H^2}{128\pi^4} \left\{ \left[\frac{1}{4} \partial^4 + \partial^2 \partial_0^2 \right] \ln^2 \left(\frac{1}{4} H^2 \Delta x^2 \right) + \left[-\frac{1}{2} \partial^4 + 2\partial^2 \partial_0^2 \right] \ln \left(\frac{1}{4} H^2 \Delta x^2 \right) \right\},$$
$$G(x; x') = \frac{\kappa^2 H^2}{6\pi^2} [-\ln(a) + \gamma] i\delta^4(x - x')$$
$$- \frac{\kappa^2 H^2}{384\pi^4} \partial^4 \left\{ \ln^2 \left(\frac{1}{4} H^2 \Delta x^2 \right) - 2 \ln \left(\frac{1}{4} H^2 \Delta x^2 \right) \right\}.$$

Quantum Maxwell for de Sitter photons

arXiv:1408.1448 with Wang & Woodard

0th order photons on de Sitter same as flat

- $F_{(0)}^{0i} = ik\varepsilon^i e^{-ik\eta + i\vec{k}\cdot\vec{x}}$, $F_{(0)}^{ij} = i(k^i\varepsilon^j - k^j\varepsilon^i) e^{-ik\eta + i\vec{k}\cdot\vec{x}}$
- $\varepsilon^i(\vec{k}, \lambda)$ same as in flat space

EFE's only reliable at late times

- Missing perturbative initial state corrections
- Counterterms have arbitrary finite parts

1 loop field strengths: E grows and B falls

- $F_{(1)}^{0i} \rightarrow F_{(0)}^{0i} \times \frac{\kappa^2 H^2 \ln(a)}{8\pi^2}$, $F_{(1)}^{ij} \rightarrow F_{(0)}^{ij} \times \frac{\kappa^2 H^2}{8\pi^2} \frac{ik \ln(a)}{Ha}$
- Inflationary gravitons jiggle the photon

Perturbation theory eventually breaks down

- Needs $\ln(a) = Ht \sim \frac{8\pi^2}{\kappa^2 H^2} > 10^{10}$ e-foldings!

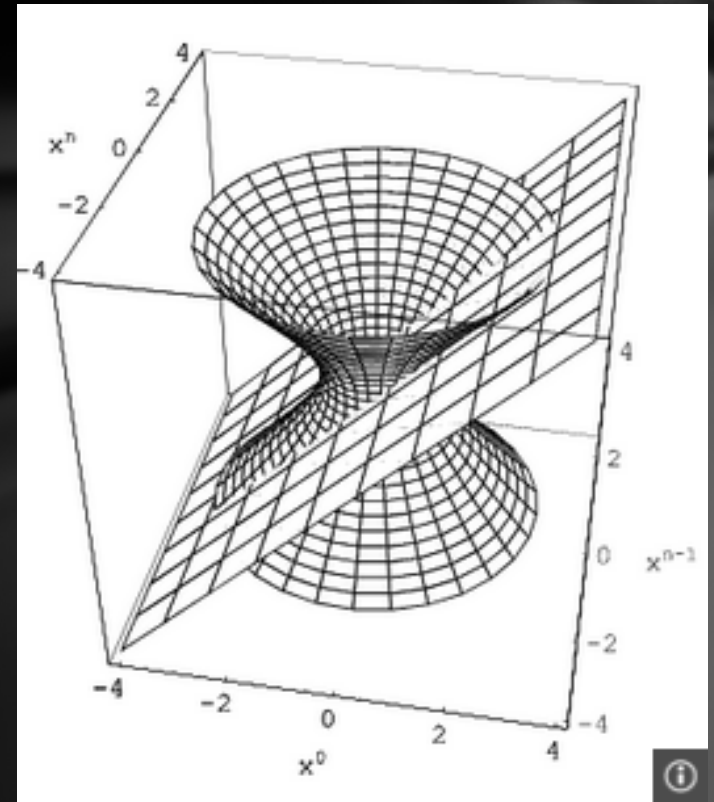
Co-Moving versus Static Observers

FLAT SLICING

- $ds^2 = a^2[-d\eta^2 + d\vec{x} \cdot d\vec{x}]$
- $a = \frac{-1}{H\eta} \quad (-\infty < \eta < 0)$
- $D(0, x; \eta) = ax \quad \text{GROWS}$

STATIC COORDINATES

- $ds^2 = -[1 - H^2 r^2]dt^2 + \frac{dr^2}{1 - H^2 r^2} + r^2 d\Omega^2$
- $0 \leq r < H^{-1} \quad (\text{STUFF LEAVES})$
- $D(0, r; t) = H^{-1} \arcsin(Hr) \quad \text{CONSTANT}$
- $a = e^{Ht} \sqrt{1 - H^2 r^2} \quad , \quad x = \frac{r}{a}$



Co-moving charge & magnetic dipole

arXiv:1308.3453 by Glavan, Miao, Prokopec & Woodard

- $J^\mu(\eta, \vec{x}) = q\delta_0^\mu\delta^3(\vec{x})$
- $\Phi = \frac{q}{4\pi x} \left\{ 1 + \frac{\kappa^2 H^2}{8\pi^2} \left[\frac{1}{3a^2 H^2 x^2} + \ln(aHx) \right] + \dots \right\}$
- Inflationary gravitons have $p \sim H \gg \frac{k}{a} \sim \frac{1}{ax}$
- $J^\mu(\eta, \vec{x}) = \epsilon^{0\mu jk} m_j \partial_k \delta^3(\vec{x})$
- $\vec{A} = \vec{m} \times \vec{\nabla} \left(\frac{1}{4\pi x} \left\{ 1 + \frac{\kappa^2 H^2}{8\pi^2} \left[\frac{1}{3a^2 H^2 x^2} - \frac{2}{3} \ln(Hx) \right] + \dots \right\} \right)$
- Not quite flat space but no secular enhancement

Static charge & magnetic dipole

arXiv:1308.3453 by Glavan, Miao, Prokopec & Woodard

Point charge

- $\tilde{\Phi} = \frac{q}{4\pi r} \left\{ 1 + \frac{\kappa^2 H^2}{8\pi^2} \left[\frac{1}{3H^2 r^2} + \ln(Hr) \right] + \dots \right\}$
- No secular effect but a logarithmic de Sitter tail

Point magnetic dipole

$$\begin{aligned} \tilde{F}_{0i}(\tau, \vec{r}) &= e^{H\tau} \sqrt{1-H^2 r^2} \frac{H \epsilon^{ijk} m^j \hat{r}^k}{4\pi r^2} \left\{ 1 \right. \\ &\quad \left. + \frac{\kappa^2 H^2}{8\pi^2} \left[\frac{5}{H^2 r^2} + \frac{2}{3} \left[2 - \ln\left(\frac{Hr}{a}\right) \right] + \text{Irrelevant} \right] + O(\kappa^4) \right\}, \\ \tilde{F}_{ij}(\tau, \vec{r}) &= \frac{e^{H\tau}}{\sqrt{1-H^2 r^2}} \frac{\epsilon^{ijk}}{4\pi r^3} \left\{ m^k - 3\vec{m} \cdot \hat{r} \hat{r}^k \left[1 - \frac{2}{3} H^2 r^2 \right] + \frac{\kappa^2 H^2}{8\pi^2} \left[\frac{3m^k - 5\vec{m} \cdot \hat{r} \hat{r}^k}{H^2 r^2} \right. \right. \\ &\quad \left. \left. - \frac{2}{3} m^k \left[1 + \ln\left(\frac{Hr}{a}\right) \right] + 2\vec{m} \cdot \hat{r} \hat{r}^k \left[\frac{2}{3} + \frac{2}{3} H^2 r^2 + \left(1 - \frac{2}{3} H^2 r^2 \right) \ln\left(\frac{Hr}{a}\right) \right] + \text{Irrel.} \right] + O(\kappa^4) \right\}. \end{aligned}$$

The Gauge Issue:

Photon & gravitons require fixing!

- EM gauge irrelevant because coupling to $F_{\mu\nu}$
- But flat $\partial^\mu h_{\mu\nu} = \left(\frac{b}{2}\right)\partial_\nu h$ gives

$$i \left[{}^\mu \Pi_{\text{flat}}^\nu \right] (x; x') = \frac{\kappa^2 \Gamma(\frac{D}{2}) \Gamma(\frac{D}{2} - 1)}{16(D-1)\pi^D} \left[C_2 + C_0(b) \right] \left[\eta^{\mu\nu} \partial' \cdot \partial - \partial'^\mu \partial^\nu \right] \frac{1}{\Delta x^{2D-2}}$$

$$C_2 = \frac{(D-4)(D-2)^2(D+1)(D+2)}{4(D-1)}$$

$$C_0(b) = -\frac{1}{4}(D-2)^2 \left[\left(\frac{Db-2}{b-2} \right)^2 - 4 \left(\frac{D-4}{D-2} \right) \left(\frac{Db-2}{b-2} \right) + \frac{2(D-4)^2}{(D-2)(D-1)} \right]$$

- The result of Bjerrum-Bohr (hep-th/0206236)
- Effect in gauge independent S-matrix

Misconceptions about the gauge issue

1. Gauge fixed GF's are complete nonsense

- We construct the flat space S-matrix from them!
- Need to separate physical information from unphysical

2. Gauge invariant GF's are different from gauge fixed

- Every gauge fixed GF represents some invariant

$$A_0(t, x) = 0 = A_1(0, x) \rightarrow A_1(t, x) = \int_0^t ds F_{01}(s, x)$$

- Gauge independent GF's are sums of gauge fixed

E.g., Woodard's thesis work !

3. This is a trivial problem

- No cosmological S-matrix → what is being measured?
- Why is everyone ignoring this problem?

Our Conjecture:

arXiv:1204.1784 by Miao & Woodard

- $i^{[\mu}\Pi^{\nu]}$ on de Sitter must inherit the gauge dependence of $i^{[\mu}\Pi^{\nu]}$ on flat background
- Maybe leading secular effects gauge independent
 - NB these effects aren't present in flat space
 - Cf. poles terms of gauge fixed GF's in flat space QFT
- How to check? → re-compute in other gauges
 - EM gauge doesn't matter
 - General covariant GR gauge: $D^\mu h_{\mu\nu} = \frac{b}{2} \partial_\nu h^\mu_\mu$
 - arxiv:1106.0925 by Miao, Tsamis & Woodard
 - arXiv:1205.4468 by Mora, Tsamis & Woodard

Old gauge noncovariant but SIMPLE

$$g_{\mu\nu} \equiv a^2[\eta_{\mu\nu} + \kappa h_{\mu\nu}]$$

$$i[\mu\nu\Delta_{\rho\sigma}](x; x') = \sum_{I=A,B,C} [\mu\nu T_{\mu\nu}^I] \times i\Delta_I(x; x')$$

- $[\mu\nu T_{\rho\sigma}^I]$ are constants
- $i\Delta_I(x; x')$ are simple functions in $D=4$
 - $y(x; x') \equiv H^2 aa' [\|\vec{x} - \vec{x}'\|^2 - (|\eta - \eta'| - i\epsilon)^2]$
 - $i\Delta_A \sim \frac{1}{y} - \ln(y) + \ln(aa')$, $i\Delta_B \sim i\Delta_C \sim \frac{1}{y}$
- But our gauge does break de Sitter invariance
 - We need a noninvariant counterterm
 - The mathematical physicists don't like this

New gauge covariant but HARD

$$g_{\mu\nu} \equiv a^2 \eta_{\mu\nu} + \kappa h_{\mu\nu}$$

- $i[\mu\nu\Delta_{\rho\sigma}] = i[\mu\nu\Delta_{\rho\sigma}^2] + i[\mu\nu\Delta_{\rho\sigma}^0]$
- $i[\mu\nu\Delta_{\rho\sigma}^2](x; x') = P_{\mu\nu}^{\alpha\beta}(x) P_{\rho\sigma}^{\gamma\delta}(x') [\partial_\alpha \partial'_\gamma y \partial_\beta \partial'_\delta y S^2(x; x')]$
 - $P_{\mu\nu}^{\alpha\beta}$ 4th order, transverse & traceless
 - $S^2(x; x')$ independent of b but breaks de Sitter
- $i[\mu\nu\Delta_{\rho\sigma}^0](x; x') = P_{\mu\nu}(x) P_{\rho\sigma}(x') [S^0(x; x')]$
 - $P_{\mu\nu} = D_\mu D_\nu - \beta^{-1} g_{\mu\nu} [\square - (\beta - D)H^2] \quad \beta \equiv \frac{Db-2}{b-2}$
 - $S^0(x; x')$ depends on b but de Sitter invariant for $b > 2$
- Easier to enthuse over de Sitter invariance without computing!
 - Tensor factors no longer constant
 - Propagator expansions no longer terminate for $D = 4$

STILL need noninvariant counterterm!

$$i \left[{}^\mu \Pi_{3\text{pt}}^\nu \right] (x; x') = \partial_\kappa \partial'_\theta \left\{ i \kappa a^{D-6} V^{\mu\rho\kappa\lambda\alpha\beta} i \left[{}_{\alpha\beta} \Delta_{\gamma\delta} \right] (x; x') \right. \\ \left. \times i \kappa a'^{D-6} V^{\nu\sigma\theta\phi\gamma\delta} \partial_\lambda \partial'_\phi i \left[{}_\rho \Delta_\sigma \right] (x; x') \right\}$$

- Three reasons:
 1. Time-ordered interactions
 2. QG interactions have two derivatives
$$\partial_\mu \partial'_\nu i \Delta(x; x') = \# \delta_\mu^0 \delta_\nu^0 i \delta^D(x - x') + \text{naive}$$
 3. Coincident graviton propagator diverges
- Analytic continuation from Euclidean fails
 - It also misses power law IR divergences
 - And sometimes results in negative-normed states

Spin 2 Structure Functions

$$\mathcal{L}(y) \equiv \text{Li}_2\left(\frac{y}{4}\right) + \ln\left(1 - \frac{y}{4}\right) \ln\left(\frac{y}{4}\right) - \frac{1}{2} \ln^2\left(\frac{y}{4}\right)$$

$$F_2(x; x') = \frac{85\kappa^2 H^2}{72\pi^2} \ln(a) i\delta^4(x - x') - \frac{\kappa^2 H^2}{16\pi^4} \left[\ln(aa') + \frac{1}{3} - \gamma \right] \nabla^2 \left(\frac{1}{\Delta x^2} \right) \\ + \frac{5\kappa^2 H^2}{144\pi^4} \partial^2 \left(\frac{\ln(\mu^2 \Delta x^2)}{\Delta x^2} \right) - \frac{5\kappa^2 H^6 (aa')^2}{144\pi^4} \left\{ \frac{\mathcal{L}(y)}{2} + \frac{2(2-y) \ln(\frac{y}{4})}{4y-y^2} + \frac{2}{y} \right\}$$

$$G_2(x; x') = -\frac{5\kappa^2 H^2}{4\pi^2} \ln(a) i\delta^4(x - x') + \frac{\kappa^2 H^2}{24\pi^4} \left[\ln(aa') + \frac{1}{3} - \gamma \right] \nabla^2 \left(\frac{1}{\Delta x^2} \right) \\ + \frac{\kappa^2 H^2 aa'}{96\pi^4} (\partial_0^2 + \nabla^2) \left(\frac{1}{\Delta x^2} \right) + \frac{5\kappa^2 H^6 (aa')^2}{72\pi^4} \left\{ \frac{(1-y)\mathcal{L}(y)}{4} + \frac{(y-3) \ln(\frac{y}{4})}{4-y} \right\}$$

Spin 0 Structure Functions

$$F_{0d}^{\text{ren}}(y) = \left(\frac{2b-1}{b-2}\right)^2 \left\{ \frac{\kappa^2}{48\pi^2} \frac{\ln(a)}{aa'} \partial^2 i\delta^4(x-x') - \left(\frac{b-8}{b-2}\right) \frac{\kappa^2 H^2}{72\pi^2} i\delta^4(x-x') \right. \\ \left. - \frac{\kappa^2 H}{48\pi^2 a} \partial_0 i\delta^4(x-x') + \frac{\kappa^2}{384\pi^4} \frac{\partial^4}{aa'} \left[\frac{\ln(\mu^2 \Delta x^2)}{\Delta x^2} \right] \right. \\ \left. + \left(\frac{b-8}{b-2}\right) \frac{\kappa^2 H^2}{576\pi^4} \partial^2 \left[\frac{\ln(\mu^2 \Delta x^2)}{\Delta x^2} \right] - \frac{\kappa^2 H^6}{6\pi^4} (aa')^2 \mathcal{N}_F(y) \right\}$$

$$G_{0d}^{\text{ren}}(y) = \left(\frac{2b-1}{b-2}\right)^2 \left\{ \frac{\kappa^2 H^2}{24\pi^2} [1 - \ln(a)] i\delta^4(x-x') \right. \\ \left. - \frac{\kappa^2 H^2}{192\pi^4} \partial^2 \left[\frac{\ln(\mu^2 \Delta x^2)}{\Delta x^2} \right] + \frac{\kappa^2 H^6}{12\pi^4} (aa')^2 \mathcal{N}_G(y) \right\}$$

Conclusions

- Some terms same but others different
- Must re-solve EFE to check conjecture
- Tedious but straightforward integrations
- Intuition says the effect is real
 - Inflation really does produce huge ensemble of IR
 - Should scatter photons more the further they go
 - This would not even be doubted in flat space
- But proving it IS an important step in cosmo QFT

One more thought...

- $i[\mu\pi^\nu](x; x') = \langle \Omega \left| \frac{i\delta S}{\delta A_\mu(x)} \frac{i\delta S}{\delta A_\nu(x')} + \frac{i\delta^2 S}{\delta A_\mu(x)\delta A_\nu(x')} \right| \Omega \rangle$
- Gauge dependent because x^μ & x'^μ not geometrically fixed
- So fix it !
E. g., geodesic from origin in direction x^μ