Keeping the cosmological constant small at all scales

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Why?

- Framework: GR + SM as an effective field theory below an energy scale E_{C}.

- Renormalization of coupling constants. For the cosmological constant (only one particle with mass m):

$$\Lambda' = \Lambda + \kappa \frac{m^4}{64\pi^2} \ln\left(\frac{m^2}{\mu^2}\right)$$

- Raising or lowering the cutoff μ an order of magnitude leads to

$$|\Delta\Lambda| \sim 10^8 \ \mathrm{GeV}^4$$

- Clear tension with the observed value of the cosmological constant. Solar system observations alone imply (Martin2012)

$$|\Lambda| \leq 10^{-32} \ \mathrm{GeV}^4$$

Motivation

- Unimodular gravity: traceless Einstein equations

$$R_{ab} - \frac{1}{4}Rg_{ab} = \kappa \left(T_{ab} - \frac{1}{4}Tg_{ab} \right)$$

- Equivalent to Einstein field equations under conservation of T_{ab} (Ellis+2010).

- Cosmological constant: integration constant.

- Claims of strong (classical & quantum) equivalence (Padilla+2014).

- Simple way to show inequivalence?

Outline

- A raw calculation
- Interpretation of the results

A raw calculation

Weyl transverse gravity (Alvarez+2006)

$$\mathcal{A} := \frac{1}{2\kappa} \int_{\mathcal{M}} \boldsymbol{\omega}\, R[|\omega|^{1/4}|g|^{-1/4} g_{ab}]$$

- Theory of dynamical conformal structures (Ehlers+1972) on 4−dimensional manifold $\mathcal{M}$; $\boldsymbol{\omega}$ fixed volume element.
- Invariant under transverse diffeomorphisms and Weyl transformations:

$$\delta_{\xi,\varphi} g_{ab} = \mathcal{L}_\xi g_{ab} + \varphi(x) g_{ab} \qquad \nabla_a \xi^a = 0$$

- Dynamical volume element $\sqrt{-g}$ forbidden by symmetries.
- Matter couples to the composite field

$$\hat{g}_{ab} := |\omega|^{1/4}|g|^{-1/4} g_{ab}$$

From gravity to gravitons

- Expand around flat spacetime ω_{ab}, $\det(\omega_{ab}) = |\omega|$:

$$g_{ab} = \omega_{ab} + \lambda h_{ab}$$

- At the lowest order: invariant under

$$h'^{ab} = h^{ab} + \omega^{ac}\nabla^{\omega}_{c}\xi^{b} + \omega^{bc}\nabla^{\omega}_{c}\xi^{a} + \varphi(x)\omega^{ab} \qquad \nabla^{\omega}_{a}\xi^{a} = 0$$

- On-shell equivalence to Fierz-Pauli theory (Izawa1995,Alvarez+2006).

- Higher orders describe the interaction vertices of gravitons.

- Nonlinear theory of a spin-2 particle.

Classical nonlinear theory

- In the gauge $g = \omega$, one recovers the traceless Einstein equations

$$R_{ab} - \frac{1}{4}Rg_{ab} = \kappa\left(T_{ab} - \frac{1}{4}Tg_{ab}\right)$$

- These equations are equivalent to Einstein field equations in the same gauge.
- The cosmological constant Λ is a constant of integration.
- Shift symmetry $\mathcal{L} \to \mathcal{L} + C_0$:

$$T_{ab} \longrightarrow T_{ab} + g_{ab}C_0, \qquad \Lambda \longrightarrow \Lambda - \frac{\kappa}{4}C_0$$

Semiclassical theory

- Classical gravitational fields, quantum matter fields: effective action

$$\mathcal{S}_{g_{ab}} = \frac{1}{2} \ln \det(\mathcal{O}_{g_{ab}})$$

- It is convenient to consider a ficudiary configuration g^0_{ab}, and construct the difference [Visser2002]

$$\mathcal{S}_{g_{ab}} - \mathcal{S}_{g^0_{ab}} = \frac{1}{2} \int \mathrm{d}^4x \ln \left(\mathcal{O}_{g_{ab}} / \mathcal{O}_{g^0_{ab}} \right)$$
$$= \frac{1}{2} \lim_{\epsilon \to 0} \int \mathrm{d}^4x \int_\epsilon^\infty \frac{\mathrm{d}s}{s} \left[\exp(-\mathcal{O}_{g^0_{ab}}) - \exp(-\mathcal{O}_{g_{ab}}) \right]$$

- The last fancy step comes from the properties of the logarithm with $\alpha, \beta \in \mathbb{R}$,

$$\ln(\alpha/\beta) = \lim_{\epsilon \to 0} \int_\epsilon^\infty \frac{\mathrm{d}s}{s} \left[\exp(-s\beta) - \exp(-s\alpha) \right]$$

Heat kernel expansion (I)

- Let us define $\mu^{-2} := \epsilon$; μ is interpreted as the cutoff, singular expressions in the limit $\mu \to \infty$ ($\epsilon \to 0$).

- Seeley-DeWitt expansion:

$$\exp(-s\mathcal{O}_{g_{ab}}) = \frac{\sqrt{|\omega|}}{(4\pi s)^2}\left[a_0(\hat{g}_{ab}) + a_1(\hat{g}_{ab})s + a_2(\hat{g}_{ab})s^2 + \mathcal{O}(s^3)\right]$$

- Recall that

$$\hat{g}_{ab} = |\omega|^{1/4}|g|^{-1/4}g_{ab}$$

- Seeley-DeWitt coefficients:

$$a_0 = 1 \qquad a_1 = k_1 R - m^2 \qquad \ldots$$

Heat kernel expansion (II)

- Coming back to our effective action:

$$\mathcal{S}_{g_{ab}} = \mathcal{S}_{g^0_{ab}} - \frac{1}{32\pi^2}\int_{\mathcal{M}} \boldsymbol{\omega}\, \{\mu^2[a_1(\hat{g}_{ab}) - a_1(\hat{g}^0_{ab})] + \\ + \ln(\mu^2/m^2)[a_2(\hat{g}_{ab}) - a_2(\hat{g}^0_{ab})]\}$$

- No term corresponding to a_0, which in general relativity leads to the renormalization of the cosmological constant.

- Due to the non-dynamical volume form $\boldsymbol{\omega}$, which is the same for all the configurations of the gravitational field.

Renormalization group

- Renormalization of gravitational couplings; e.g., gravitational constant
$$\frac{1}{\kappa'} = \frac{1}{\kappa} + C_1\mu^2 + C_2 \log\left(\frac{\mu}{C_3}\right)$$

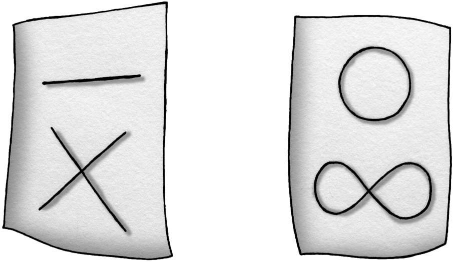
- There is NO renormalization equation for the cosmological constant.

- Intimately related to the shift symmetry on the Lagrangian $\mathcal{L} \to \mathcal{L} + C_0$; C_0 drops off from field equations.

Interpretation of the results

The quantum vacuum

- Scattering amplitudes and vacuum bubbles:



- In GR vacuum bubbles couple to the 'dynamical' volume form (leading to the renormalization of the cosmological constant)

$$\boldsymbol{\epsilon} := \sqrt{|g|}\, \mathrm{d}x^1 \wedge \ldots \wedge \mathrm{d}x^n$$

- In Weyl transverse gravity the volume form is non-dynamical. Vacuum bubbles have no effect, as in flat spacetime.

Effective field theory reminder

- Let us consider an effective field theory description of GR + SM below an energy E_{C}.
- A general Lagrangian density should contain all the terms that are allowed by symmetry.
- One can assign a dimension to each operator and classify them this way.
- Relevant, but non-natural (negative dimension) operator:

$$\int \mathrm{d}^4x \sqrt{|g|}\,\Lambda$$

Weyl transverse gravity

- Scale transformations of the gravitational field:

$$g_{ab} \longrightarrow \zeta^2 g_{ab} \qquad\qquad \zeta \in \mathbb{R}$$

- This symmetry would be enough to protect the cosmological constant sector for getting radiative corrections.

- But scale invariance is generally anomalous. Only a non-anomalous symmetry could guarantee a complete protection.

- This simmetry is non-anomalous in Weyl transverse gravity: reduction of diffeomorphisms to the subgroup of tranverse diffeomorphisms.

Anomalies

- Generic result: not all symmetries can be preserved in the quantization. Path integral:

$$\int [\mathcal{D}\Psi] \exp(iS[\Psi])$$

- A symmetry is not anomalous *per se*, but with respect to other symmetries.

- A necessary condition is that different symmetries act non-trivially on the same fields.

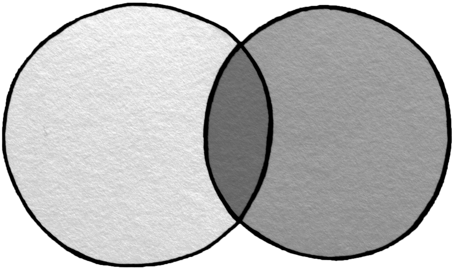
Conformal anomaly

- Diffeomorphisms:

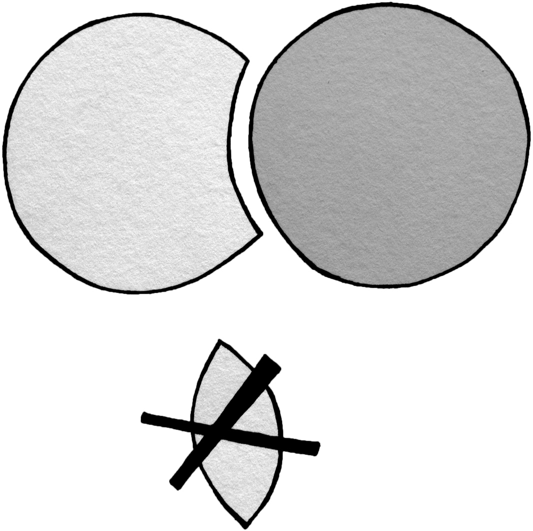
$$\delta\sqrt{|g|} \propto \nabla_a \xi^a$$

- Conformal transformations:

$$\delta\sqrt{|g|} \propto \Omega^4$$



Avoiding the anomalies



Fujikawa's method

- Inner product:

$$\langle \phi, \phi' \rangle := \int_{\mathcal{M}} \mathrm{d}^D x \sqrt{|\hat{g}|}\, \phi(x)\phi'(x) = \int_{\mathcal{M}} \boldsymbol{\omega}\, \phi\, \phi'$$

- Decomposition coefficients:

$$c_n := \langle \phi_n, \phi \rangle = \int_{\mathcal{M}} \boldsymbol{\omega}\, \phi_n\, \phi$$

- Path integral measure:

$$\prod_{n=0}^{\infty} \frac{\mathrm{d}c_n}{\sqrt{2\pi}}$$

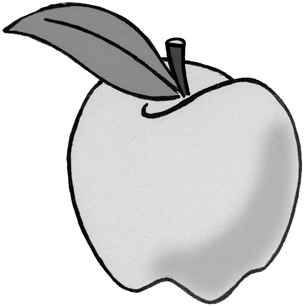
- Absence of anomalies:

$$\delta c_n = \int_{\mathcal{M}} \mathrm{d}^D x\, \phi_n(x)\phi(x)\, \delta\sqrt{|\hat{g}|} = 0$$

Conclusions and future directions

Conclusions and future directions

- Weyl transverse gravity: theory of dynamical conformal structures that uses an auxiliary, non-dynamical volume form.
- Effective description in which the cosmological constant is not renormalized: non-anomalous gravitational scale invariance.
- Cosmological constant as mysterious as (but no more than) any other parameter: gravitational constant, electron charge, ...
- Additional principle to fix a small value (now stable) of the cosmological constant? For instance: (Volovik2003).
- Inclusion of quantum-mechanical properties of the gravitational field (Alvarez+2015).



Thank you for your attention.