Quantum Gravity Comes of Age

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Windows on Quantum Gravity

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What Is Quantum Gravity?

Quantum Gravity =

- Gravity +
- Relativity +
- Quantum Mechanics

General relativity = Gravity (G) + Relativity (c)

- Works great IF we ignore fluctuations
- Eg., GPS accounts for slower time on Earth

Quantum Mechanics ($\hbar$)

- Each degree of freedom MUST fluctuate a little
- There are a LOT of degrees of freedom!

The Problem of Quantum Gravity

For frequency ω QG theory predicts

$$\left(\frac{\hbar G\omega^2}{c^5}\right)^N \left\{1+\alpha_1\left(\frac{\hbar G\omega^2}{c^5}\right)+\alpha_2\left(\frac{\hbar G\omega^2}{c^5}\right)^2+\cdots\right\}$$

Two problems:

1. For ω's we can reach $\left(\frac{\hbar G\omega^2}{c^5}\right) \ll 1$

   Eg. FM radio $\omega \sim 10^8$Hz ➔ $\left(\frac{\hbar G\omega^2}{c^5}\right) \sim 10^{-71}$

2. Theory predicts divergences in $\alpha_1, \alpha_2, \cdots$

Mathematical Consistency Isn't Enough

Gertrude Stein: "The trouble with Oakland is that, when you get there, there isn't any there there."

String Theorist (on being told the 2011 Nobel Prize-winning result that the universe is accelerating): "I'm sure the data is wrong because string theory predicts a negative cosmological constant. And if it's right, I'll quit doing physics." (He's still doing string theory.)

WE NEED DATA!

Big questions in M-theory

Recently there has been a lot of discussion about questions in physics and mathematics that we should confront in the new millenium; see The top 10 physics problems for the next millenium and The Clay Mathematics Institute millenium prize problems. In this spirit, and to stimulate discussion, I've begun collecting outstanding questions in M-theory. Please submit your favorite question to *giddings@physics.ucsb.edu*; questions added will be credited.

Fundamental principles

What are the fundamental degrees of freedom of M-theory?

(Does this question apply, or is it time to abandon the reductionist paradigm?)

What are the dynamical laws governing their interaction?

What are the observables of M-theory? *(R. Bousso)*

Cosmology

Flat space: $ds^2 = -c^2 dt^2 + d\vec{x} \cdot d\vec{x}$

Cosmology: $ds^2 = -c^2 dt^2 + a^2(t) d\vec{x} \cdot d\vec{x}$

- $3c^2 H^2 = 8\pi G\, \rho$ E.g. $\rho_{mat} \sim \left(\frac{a_0}{a(t)}\right)^3$
- $(-3 + 2\epsilon) c^2 H^2 = 8\pi G\, p$ E.g. $p_{rad} = ⅓\, \rho_{rad}$

Two derivatives:

- Hubble parameter: $H(t) \equiv \frac{\dot{a}}{a}$
- 1st slow roll parameter: $\epsilon(t) \equiv -\frac{\dot{H}}{H^2}$

Current values:

- $H_0 = 67.3 \pm 1.2 \frac{km}{s{-}Mpc} \sim 2.2 \times 10^{-18} Hz$
- $\epsilon_0 = 0.47 \pm 0.03$

A Cautionary Tale

Urgent Mission

WE WERE GOING TO USE THE TIME MACHINE TO PREVENT THE ROBOT APOCALYPSE, BUT THE GUY WHO BUILT IT WAS AN ELECTRICAL ENGINEER.

All hate Franklin's signs

Clever theorists tried to avoid this with $\ddot{a}(t)$

- Before 1998 we were CERTAIN $\ddot{a}(t_0) < 0$
- Hence "deceleration parameter" $q(t) \equiv -\frac{a\ddot{a}}{\dot{a}^2}$
- But $q_0 = -0.53 \pm 0.03$

Moral: don't believe before you see the data

Brief History of **ε**(t)

N = # of e-foldings since inflation

- $a(t) = a_i\, e^N$
- NB $e^{60} \cong 10^{26}$
- 6000 Mpc now was 2 m then!

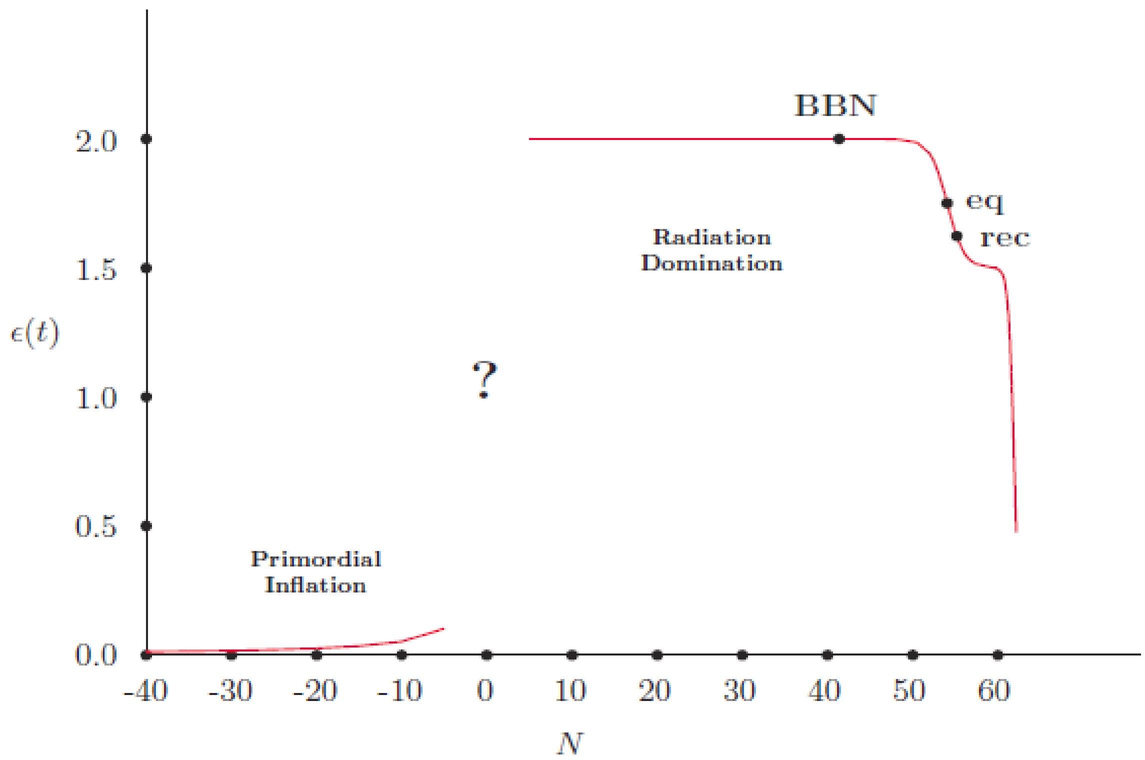
Cosmological Epochs (Roughly)

1. Late acceleration $59 \leq N \leq 60$
2. Matter domination $52 \leq N \leq 59$
3. Radiation dom. $5 \leq N \leq 52$
4. Reheating $-5 \leq N \leq 5$
5. Primordial Inflation $N \leq -5$

We can observe early events

1. Recombination $t \cong 300{,}000\ yrs$
2. Big Bang Nucleosynthesis
   $t \cong 1\ s$

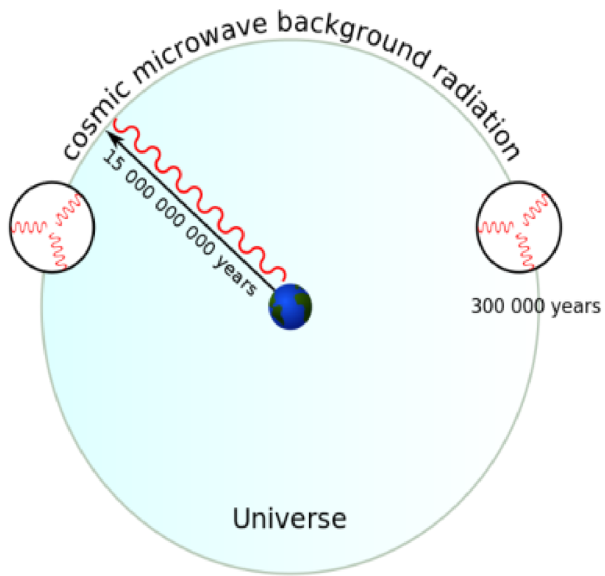
NB $t_0 \cong 13{,}800{,}000{,}000\ yrs$



Horizon Problem

$$0 = -c^2 dt^2 + a^2(t) dr^2 \rightarrow dr = \frac{c\,dt}{a(t)}$$

CMB in thermal equilibrium to
1 part in 10^5 **(better than this room!)**



Past and Future Light-cones

- $P(t) \equiv \int_t^{t_0} dt' \frac{c a_0}{a(t')}$
- $F(t) \equiv \int_{t_i}^{t} dt' \frac{c a_0}{a(t')}$

$q > 0$ ➔ upper limit dominates

- ~5000 causally distinct regions in equilibrium!
- $\sim 10^{17}$ regions at BBN!

$q < 0$ ➔ lower limit dominates

- $a(t_i) \rightarrow 0$ makes $F(t) \rightarrow \infty$

QG Effects from Inflation observable because:

1. $\frac{\hbar G H_{inf}^2}{c^5} \sim 10^{-11}$ is small but observable
   - Compare $\frac{\hbar G H_0^2}{c^5} \sim 10^{-122}$
2. Lots of graviton & MMC scalar production
3. Long λ perturbations “fossilize” so they can survive to late times

Energy-Time Uncertainty Principle in Flat Space

Virtual particles of energy E exist for $\Delta t \cong \frac{\hbar}{E}$

Wave number $k = \frac{2\pi}{\lambda}$ ➔ $E = [m^2c^4 + c^2\hbar^2k^2]^{½}$

This is why $m = 0$ particles give $F(r) \propto \frac{1}{r^2}$ forces

- $m = 0$ ➔ $E = c\hbar k$ ➔ $\Delta t \cong \frac{1}{ck}$
- $r = c\Delta t = \frac{1}{k}$
- $F = \frac{\Delta p}{\Delta t} = \frac{\hbar k}{1/ck} = \frac{\hbar c}{r^2}$

Also why $m \neq 0$ particles give short range forces

- $m \neq 0$ ➔ $E \sim mc^2$ ➔ $\Delta t \cong \frac{\hbar}{mc^2}$
- $r = c\Delta t = \frac{\hbar}{mc}$ fixed

Any inflationary virtual with $ck < H(t)a(t)$ lives forever

$k = \frac{2\pi}{\lambda}$ fixed but $\frac{k}{a(t)}$ physical, hence

$$E(t,k) = \left[m^2c^4 + \left(\frac{c\hbar k}{a(t)}\right)^2\right]^{½}$$

Virtuals can live for $\int_t^{t+\Delta t} dt'\, E(t',k) \cong \hbar$

$m = 0$ virtuals have $c\hbar k \int_t^{t+\Delta t} \frac{dt\prime}{a(t')} \cong \hbar$

- We just saw this integral!
- For inflation can take $\Delta t \to \infty$ ➔ $\frac{c\hbar k}{H(t)a(t)} \cong \hbar$

A Killer Symmetry: Conformal Invariance

- $d\eta \equiv \frac{dt}{a(t)}$ ➔ $\mathrm{d}s^2 = a^2[-d\eta^2 + dx^2]$➔$g_{\mu\nu} = a^2\eta_{\mu\nu}$
- Conformal transformation using $\Omega(\eta, x)$
  - $g_{\mu\nu} \rightarrow \Omega^2 g_{\mu\nu}$, $\psi \rightarrow \Omega^{-3/2}\psi$
  - $A_\mu \rightarrow A_\mu$, $\varphi \rightarrow \Omega^{-1}\varphi$
- For example, $\mathcal{L}_{EM} = -¼F_{\mu\nu}F_{\rho\sigma}g^{\mu\rho}g^{\nu\sigma}\sqrt{-g}$
- Conformal invariance ➔ same $\mathcal{L}$ as flat
- Emission rate $\frac{dN}{d\eta} = \Gamma_{flat}$ ➔ $\frac{dN}{dt} = \frac{\Gamma_{flat}}{a(t)}$
- Any emitted virtual lives forever, but few emitted

Exceptions: MMC scalars & gravitons

- $L = \frac{m}{2}[\dot{q}^2 - \omega^2 q^2]$ ➔ $q(t) = \alpha u(t) + \alpha^\dagger u^*(t)$
  - $\frac{d}{dt}(m\dot{u}) + m\omega^2 u = 0$ ➔ $u(t) = \sqrt{\frac{\hbar}{2m\omega}}$ Exp$\left[-i\int_{t_i}^{t} dt'\,\omega\right]$
  - $E(t) = \left\langle\Omega\middle|\frac{m}{2}[\dot{q}^2(t) + \omega^2 q^2(t)]\middle|\Omega\right\rangle = \frac{m}{2}[|\dot{u}(t)|^2 + \omega^2|u(t)|^2] = ½\hbar\omega$
- $\mathcal{L}_{MMCS} = \frac{a^3}{2c^2}\left[\dot{\varphi}^2 - \frac{c^2\|\vec{\nabla}\varphi\|^2}{a^2}\right]$ ➔ each $\vec{k}$ SHO with $m(t) = \frac{a^3}{c^2}$ & $\omega(t) = \frac{ck}{a}$
  - $u(t,k) = \sqrt{\frac{\hbar c}{2ka^2(t)}}\left[1 + \frac{iHa(t)}{ck}\right]Exp\left[-i\int_{t_i}^{t} dt'\,\frac{ck}{a(t')}\right]$ for $\epsilon = 0$
  - $E(t,k) = \frac{a^3(t)}{2c^2}\left[|\dot{u}(t,k)|^2 + \frac{c^2k^2}{a^2(t)}|u(t,k)|^2\right] = \frac{\hbar ck}{a(t)}\left[½ + \left(\frac{Ha(t)}{2ck}\right)^2\right]$
- Each $\vec{k}$ has $N(t,k) = \left[\frac{Ha(t)}{2ck}\right]^2$ conformal has $N(t,k) = 0$
- NB $|u(t,k)|^2 \to \frac{\hbar H^2}{2ck^3}$ a constant! conformal has $u(t,k) \to 0$

We Have Data!

- Scalar Power spectrum

$$\Delta^2_{\mathcal{R}}(k) = A_s \left(\frac{k}{k_0}\right)^{n_s - 1}$$

$$A_s = 2.196^{+0.051}_{-0.060} \times 10^{-9}$$

$$n_s = 0.9603 \pm 0.0073$$

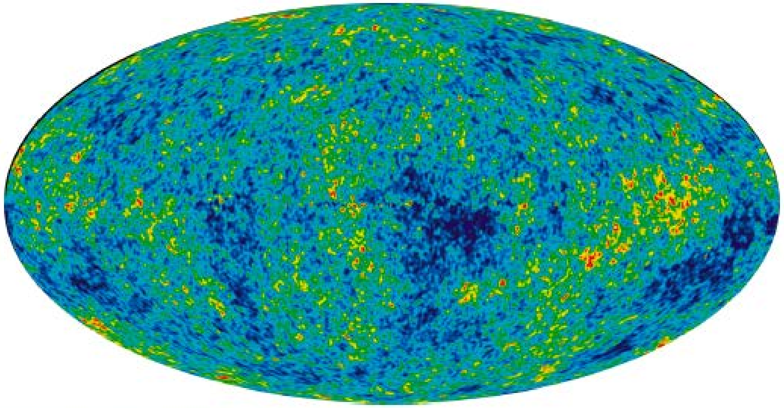
$$k_0 = 0.050\ Mpc^{-1}$$

- Tensor-to-Scalar ratio

$$r(k) \equiv \frac{\Delta^2_h(k)}{\Delta^2_{\mathcal{R}}(k)}$$

$$r(0.002\ Mpc^{-1}) < 0.09$$

- Many new polarization probes
- Already 10^7 data points
- Factor of 10^{14} increase in 21 cm radiation from early structures



$$\Delta^2_{\mathcal{R}}(k) \approx \frac{\hbar G H^2(t_k)}{\pi c^5 \epsilon(t_k)}$$

$$\Delta^2_h(k) \approx \frac{16 \hbar G H^2(t_k)}{\pi c^5}$$

This is quantum gravitational data!

The first ever collected!

1. "It's not QG if it's only tree order"
   - $\Delta_{\mathcal{R}}^2$ & $\Delta_h^2 \sim \hbar G$ ➔ it's quantum gravity
   - Famous tree effects in QFT:

     ➔Bhabha scattering (QED) & beta decay (weak int)
2. "It's not QG if it doesn't involve gravitons"
   - Solar system tests response to classical matter
   - $\Delta_{\mathcal{R}}^2(k)$ tests response to quantum matter
   - More interesting for QG than gravitons
3. "It's not QG if we can't predict it uniquely"
   - Rotation curves & Hubble plots measure gravity
   - Even though we don't understand what drives them

Impact on Quantum Gravity

1. Discretization to solve UV no longer viable
   - Minimum Δx cuts off UV divergences

   But QG only small if $\Delta x^2 \gg \frac{\hbar G}{c^3} \sim (10^{-35} m)^2$

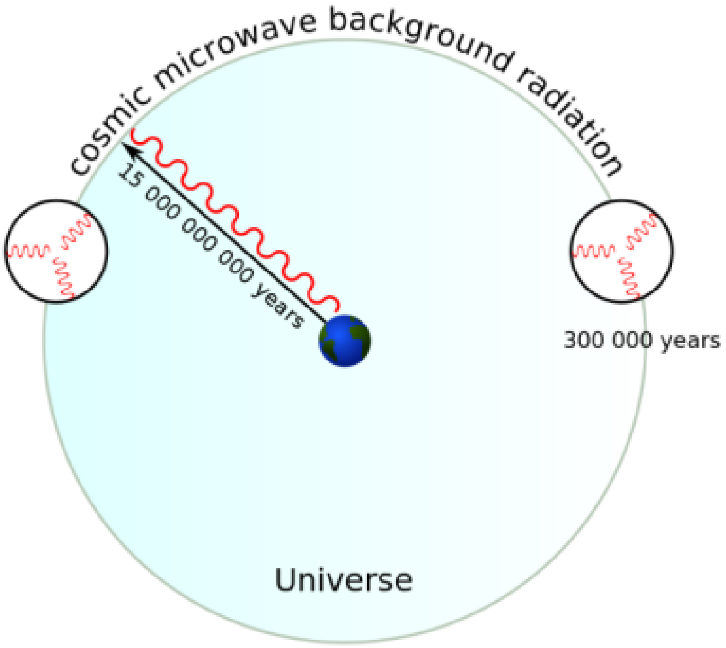
   - QG effects small at t_k with $\frac{a_0}{a(t_k)} \sim e^{110} \sim 10^{48}$

   $a(t_k)\Delta x \sim 10^{-35} m$ ➔ $a_0 \Delta x \sim 10^{13} m$

2. Failing to quantize gravity no longer viable
   - $R_{\mu\nu} - ½ g_{\mu\nu} R = \frac{8\pi G}{c^4} \langle \psi | T_{\mu\nu} | \psi \rangle$
   - But $\langle \psi | T_{\mu\nu} | \psi \rangle$ has no fluctuations!

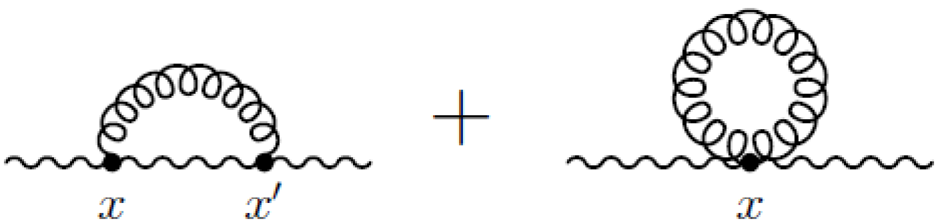
A Tomograph is better than an X-ray!

- CMB from entire path
  - $N \sim \ell_{max}^2$
  - $\ell_{max} \sim 3000$
- 21 cm redshift fixes WHERE it came from
  - $N \sim \ell_{max}^3$
  - $\ell_{max} \sim 10^7$
  - Data to resolve one loop
- Not easy, or quick
  1. Lunar radio telescope
  2. Superb astrophysics
  3. Unique inflation model



Scalars & gravitons alter kinematics of other quanta

- Vacuum polarization $i[^{\mu}\Pi^{\nu}](x;x')$



- Quantum-corrected Maxwell Equation

$$\partial_{\nu}\left[\sqrt{-g}\,g^{\nu\rho}g^{\mu\sigma}F_{\rho\sigma}(x)\right] + \int d^4x'[^{\mu}\Pi^{\nu}](x;x')\,A_{\nu}(x') = J^{\mu}(x)$$

  - $J^{\mu}(x) = 0$ ➔ dynamical photons
  - $J^{\mu}(x) \neq 0$ ➔ electromagnetic forces

Known Secondary Effects

MMC Scalar Effects

1. On themselves
   - Brunier, Kahya, Onemli
2. On fermions
   - Garbrecht, Prokopec
3. On photons
   - Degueldre, Prokopec, Tornkvist
4. On gravitons
   - Leonard, Park, Prokopec

Graviton Effects

1. On scalars
   - Kahya, Park
2. On fermions
   - Miao
3. On photons
   - Glavan, Leonard, Miao, Prokopec
4. On gravitons
   - Mora, Tsamis

Conclusions → Dawning "Golden Age" of QG

- QG from primordial inflation small but observable
  1. $H_{inf} \sim 10^{56} \times H_0$ → $\frac{\hbar G H_{inf}^2}{c^5} \sim 10^{-11}$
  2. $\epsilon_{inf} < 0.0056$ →gravitons & MMC scalars have $N(t, \vec{k}) \sim \left[\frac{Ha}{2ck}\right]^2$
  3. Fossilized perturbations survive to late times
- Primary effects
  - $\Delta_{\mathcal{R}}^2(k) \sim \frac{\hbar G H^2(t_k)}{\pi c^5 \epsilon(t_k)}$ (resolved) $\Delta_h^2(k) \sim \frac{16\hbar G H^2(t_k)}{\pi c^5}$ (not yet)
- Secondary effects → loops & kinematics of other quanta
  - Enough data to eventually resolve
- Crucial questions
  1. What is causing late time acceleration?
  2. What drove primordial inflation?
  3. What are the observables?