Semiclassical S-matrix for black holes

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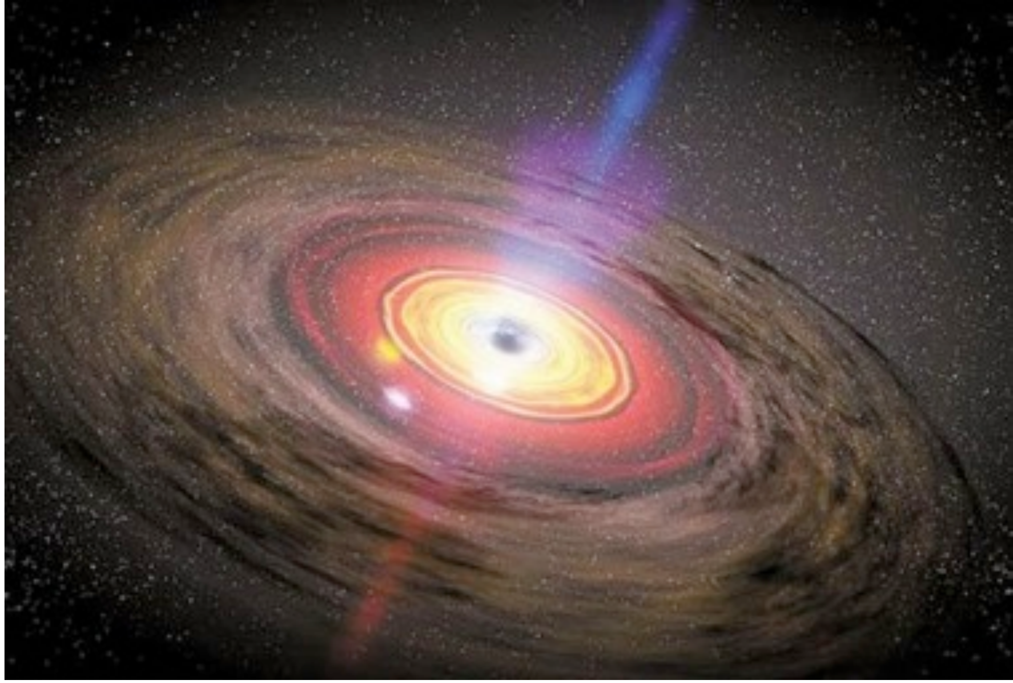
1503.07181 with F. Bezkurov, D. Levkov
work in progress with D.Levkov, M. Fitkevich

Windows on Quantum Gravity, October 28–30, 2015

Black holes in the sky:



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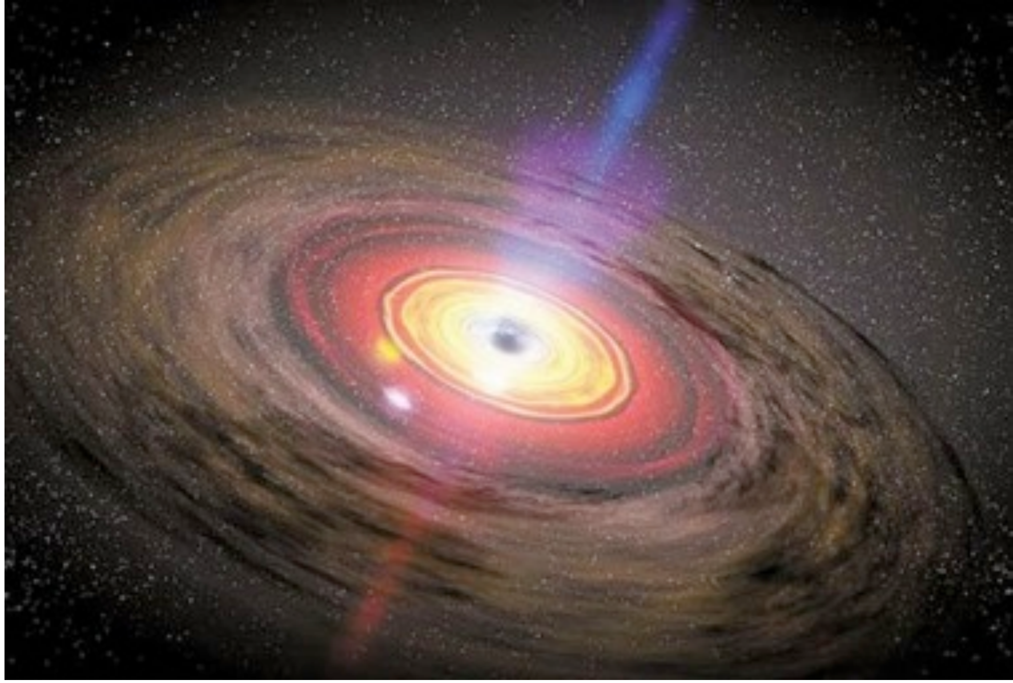
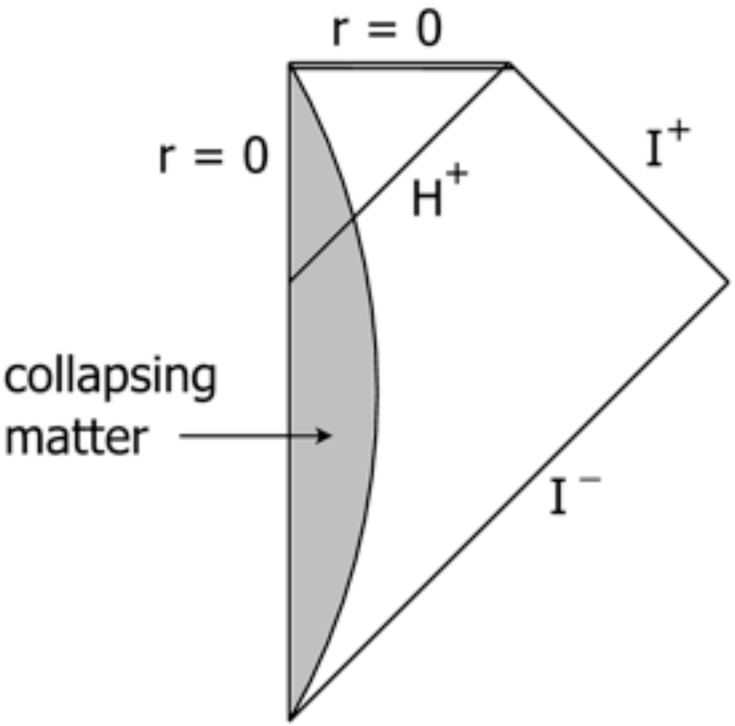


Black holes in theorist's mind:

clean environment to study *quantum gravity*

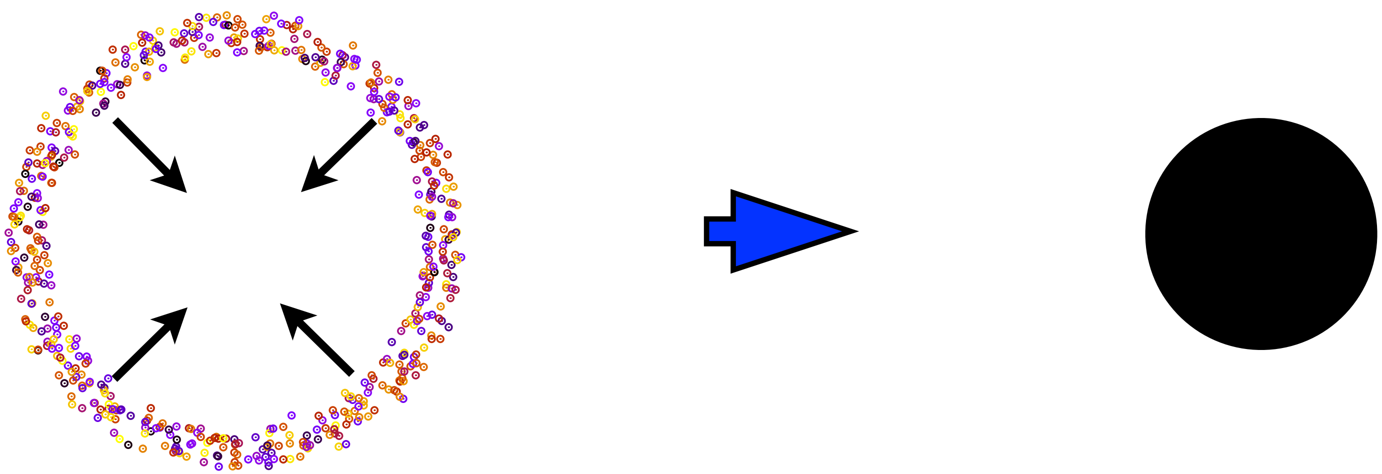


Plan

- motivation
- S-matrix generalities
- what we would like to do
- what we have been able to do
- outlook

Information loss puzzle

In classical GR black holes do not have hair

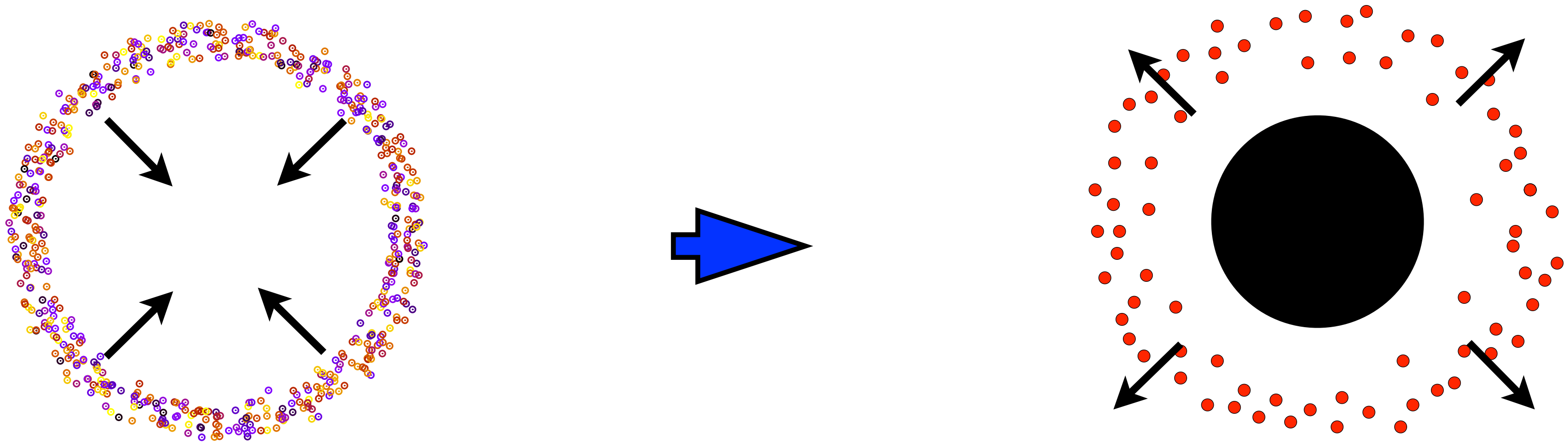


Information about the initial state is buried inside the black hole

Information loss problem

In perturbation theory around classical background
black hole radiates thermally

Hawking (1975)

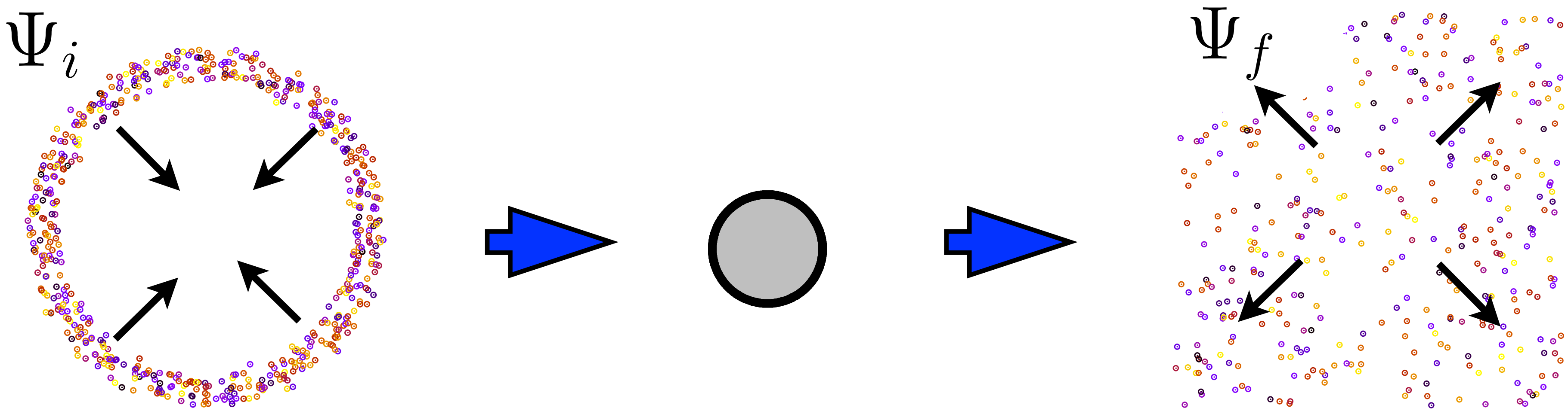


Information about the initial state is lost when the black hole evaporates completely.

Incompatible with unitary evolution.

The weak link: expansion around classical black hole geometry

General belief (or wishful thinking): *The back-reaction of emitted quanta on geometry restores correlations between them*



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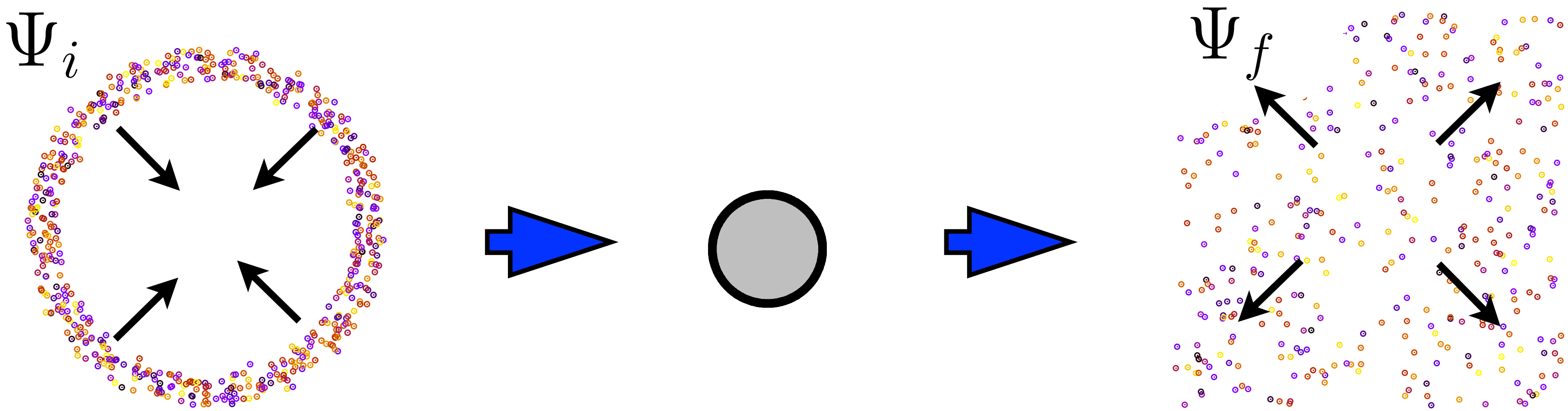

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How can we verify this conjecture ?

Unitarity and AdS / CFT

gravity in AdS_{d+1} = CFT_d

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(see e.g. the *firewall proposal* by *Almheiri et al. (2012)*)

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To clear up the confusion ask well-posed questions

➡ Calculate S-matrix elements

S-matrix: generalities

Universal formula for transition amplitudes:

$$\mathcal{A}_{fi} = \int \mathcal{D}\Phi_i \mathcal{D}\Phi_f \Psi_f^*[\Phi_f]\Psi_i[\Phi_i] \int \mathcal{D}\Phi \exp(iS[\Phi]/\hbar)$$

initial / final -state wavefunctions

fields

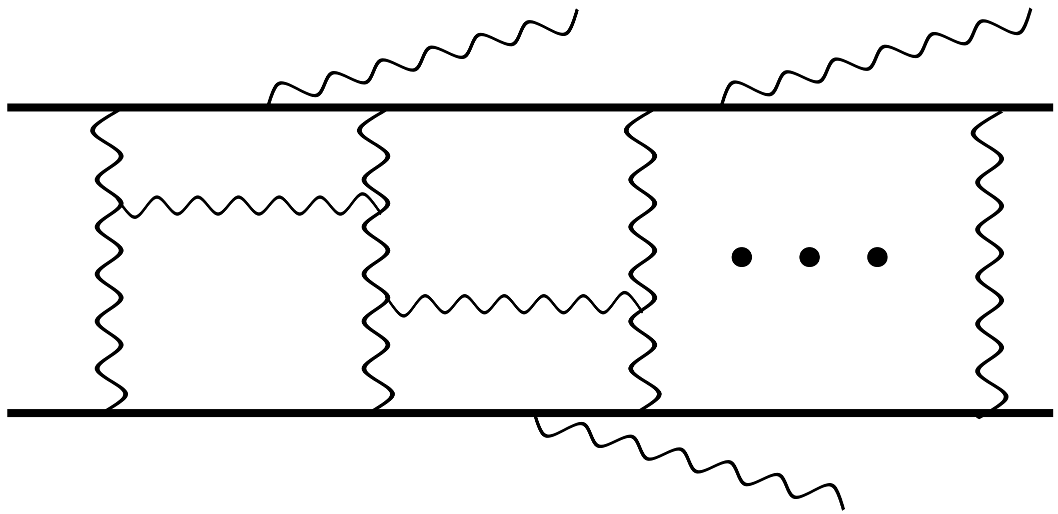
action

Tractable regimes:

i) Perturbative.

Works for trans-Planckian scattering with large impact parameter b

Amati, Ciafaloni, Veneziano (1987)
+ many follow-ups



But breaks down when b approaches E/M_P^2

Reason: high multiplicity

$$\left.\begin{array}{l}\alpha \sim 1/(bM_P)^2 \ll 1 \\ N \sim Eb\end{array}\right\} \Rightarrow \alpha N \sim E/(bM_P^2) \to 1$$

NB. The exchange gravitons are still soft and interact weakly

Tractable regimes:

ii) Semiclassical: weak coupling, many particles at the beginning and at the end

Focus on a scattering when both initial and final states are coherent (~classical) wavepackets made of large number of soft particles

$$|\Psi_i\rangle = |\alpha\rangle \equiv \exp\left[\int dk\, \alpha_k a_k^+\right]|0\rangle$$

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Caution: the dominant - Hawking - final state may not be semiclassical due to small occupation numbers in each mode

Still, the S-matrix in coherent-state subsector can already cotain interesting information (see below)

Semiclassical approximation for the amplitude

$$\mathcal{A}_{fi} = \int \mathcal{D}\Phi_i \mathcal{D}\Phi_f \Psi_f^*[\Phi_f]\Psi_i[\Phi_i] \int \mathcal{D}\Phi \exp(iS[\Phi]/\hbar)$$

Take all functional integrals by the saddle-point

- $\frac{\delta S}{\delta \Phi} = 0$ ➡ solution of the classical e.o.m.'s
- $\frac{\delta}{\delta \Phi_i} = \frac{\delta}{\delta \Phi_f} = 0$ ➡ impose boundary conditions determined by $\Psi_{i,f}$

Relate the amplitude to the action of the saddle-point solution

$$\mathcal{A}_{fi} \sim \mathrm{e}^{iS_{tot}}$$

$$iS[\Phi_{cl}] + B_i[\Phi_{cl}, \alpha] + B_f[\Phi_{cl}, \beta^*]$$

Works in flat space e.g. instanton transitions in the Standard Model, multiparticle scattering, etc.

Tinyakov (1993)

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Challenge I: solution may be not unique (in general complex-valued solutions have many branches)

Challenge II: solution may not exist

What we would like to do

Consider spherically symmetric gravity + a scalar field

$$ds^2 = g_{ab}(y)dy^a dy^b + r^2(y)d\Omega^2$$

$$\phi(y)\ , \quad a, b = 0, 1$$

The action is obtained by spherical reduction from 4d

Process: collapse of a contracting spherical wavepacket with subsequent decay of the black hole into an expanding wavepacket

Technical task: solve partial differential equations of 2d dilaton gravity coupled to a scalar field in complex domain

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Difference from the traditional approach: Hawking evaporation is a "one-loop" effect. We want to take into account back-reaction in the leading order

Modified semiclassical method

Physical idea: restrict integration to configurations with matter spending only a finite time T_{int} within a given radius. Integrate over T_{int} at the end.

Practical implementation:

- add an imaginary term to the action
- solve the resulting e.o.m.'s
- compute the action on the solution

  $\Rightarrow$ scattering amplitude

- send the coefficient in front of the imaginary term to zero

The correct branch of solutions is chosen for free: regularization continuously connects physical solutions with the same topology at different energies

Bezrukov, Levkov (2003)
Levkov, Panin, S.S. (2007)

Strategy: start from real classical solutions at low energy and increase energy by small steps

Wish list:

$$\mathcal{A}_{\alpha\beta} \sim e^{iS_{tot}[\alpha,\beta]}$$

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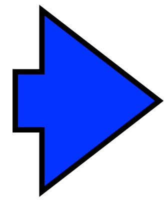

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- Test of unitarity

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Compare with $\langle\beta|\mathbf{1}|\alpha\rangle = e^{\int \beta^* \alpha}$

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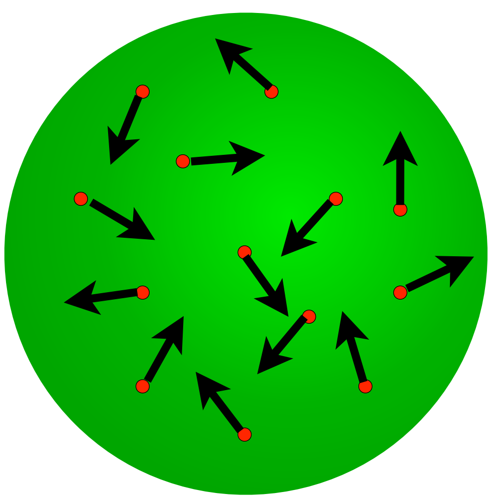

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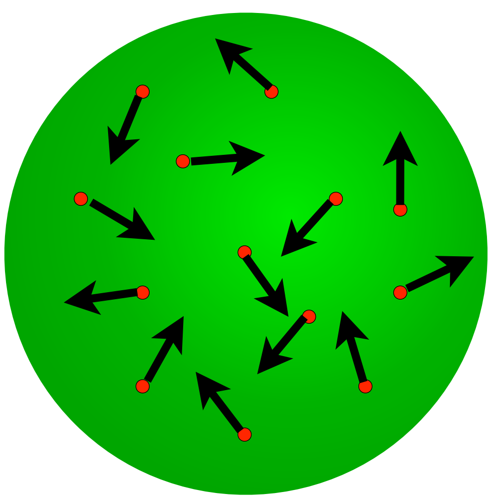

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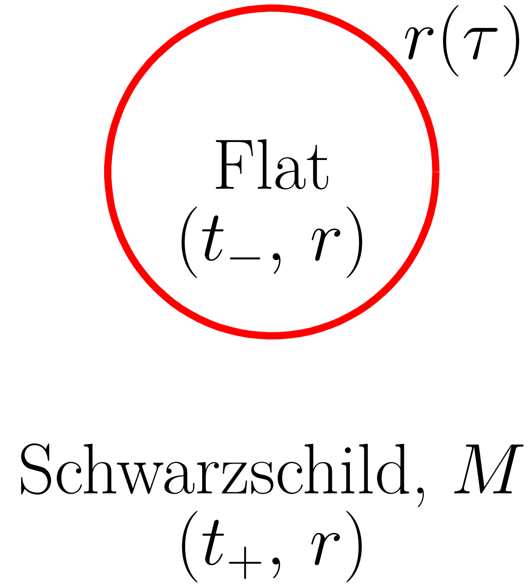

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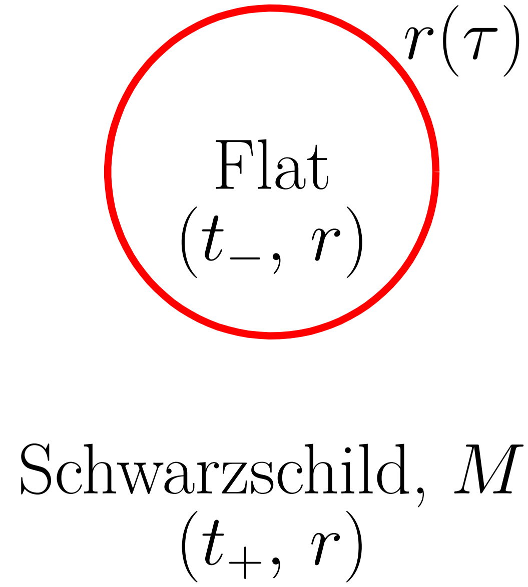

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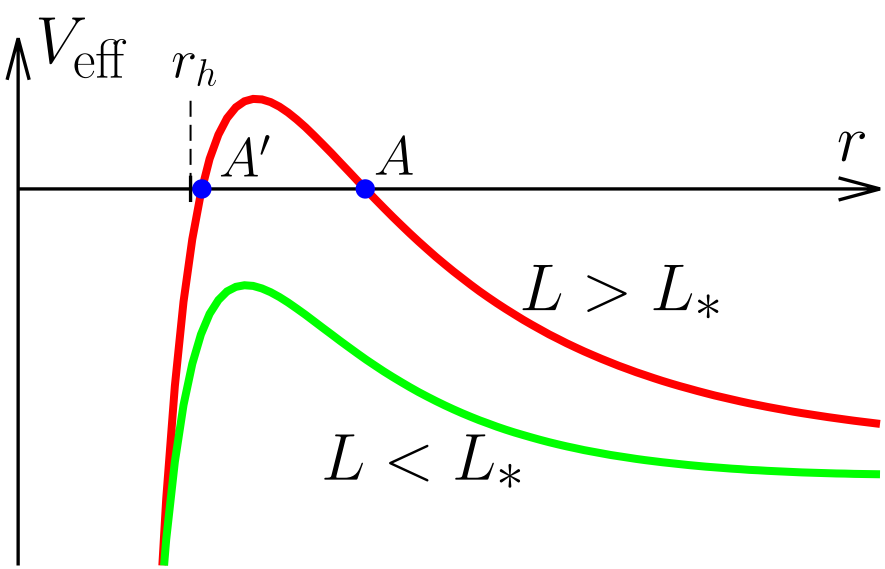

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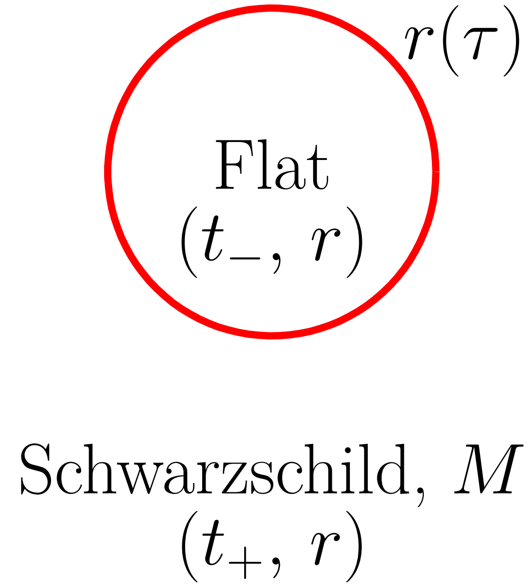

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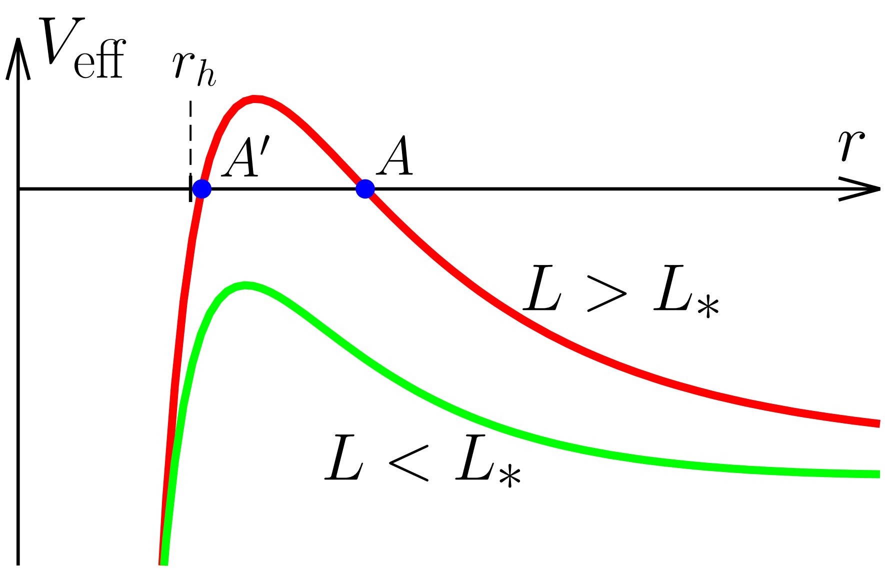

$L > L_*$: classical reflection from the centrifugal barrier

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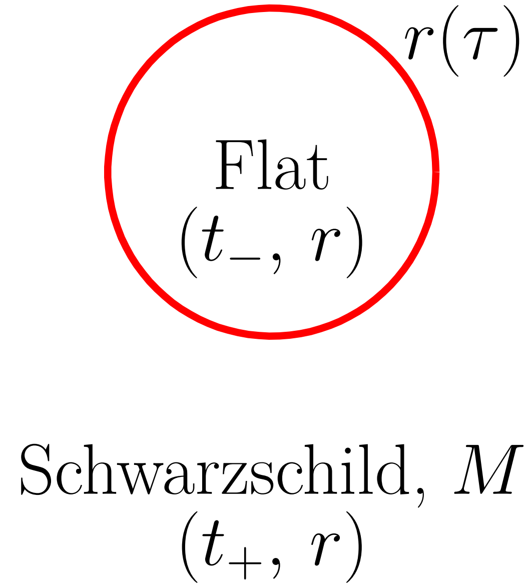

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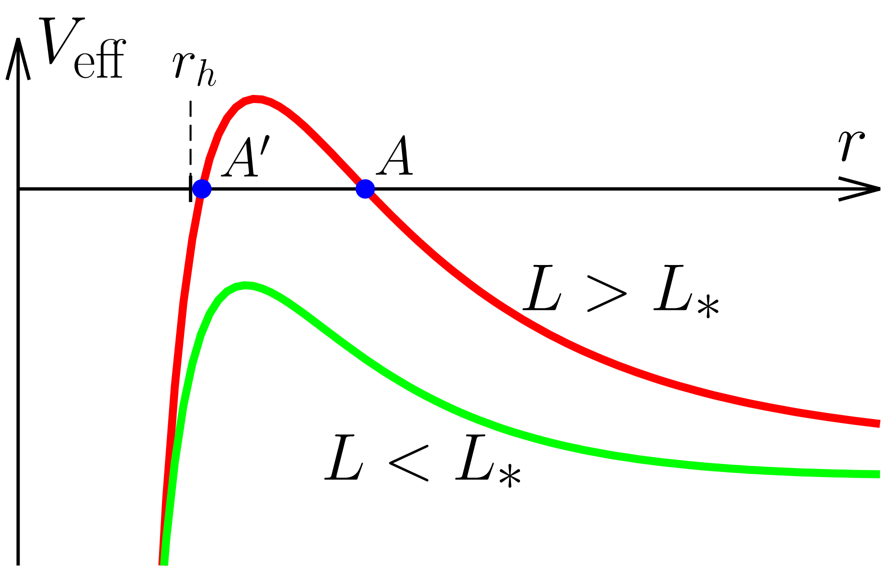

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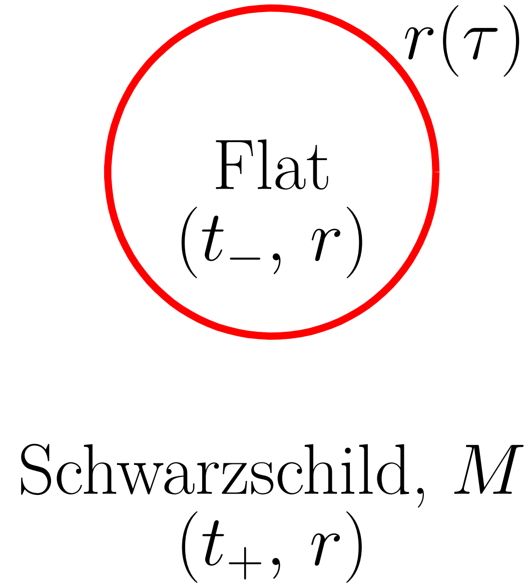

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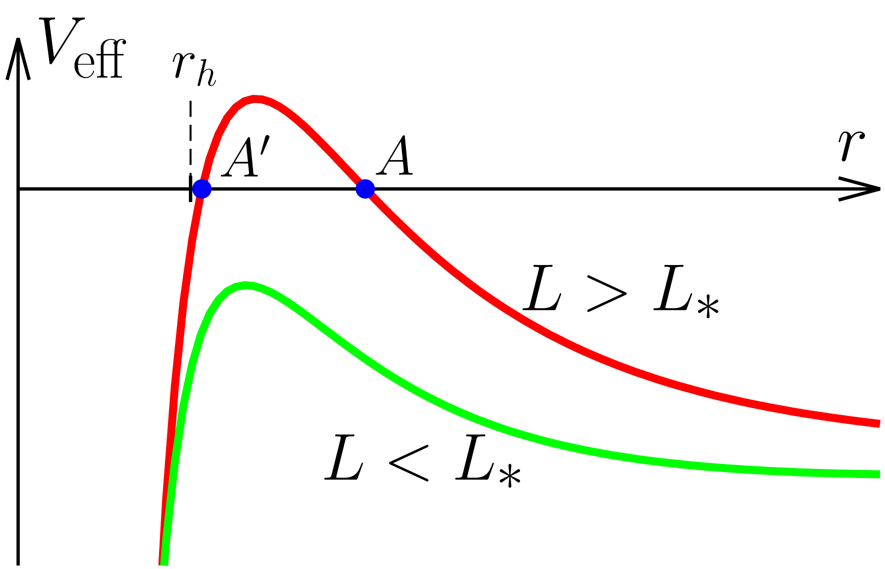

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$L \to 0$: BH formation + decay

The transition amplitude

At $L \to 0$ the trajectory goes almost along the real axis, bouncing close to the singularity, but does not hit it

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Bekenstein-Hawking entropy of the intermediate black hole

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NB. The full structure of the gravitational action is important (Gibbons - Hawking term)

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We can also compute the phase of the amplitude.
For a massless shell:

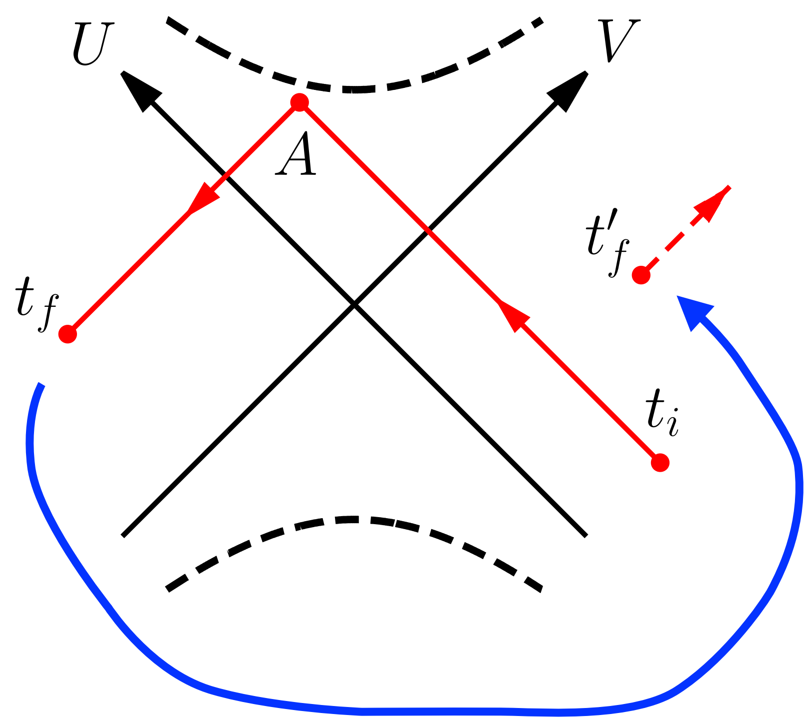
$$\mathrm{Re}\, S_{tot} = 2Mr_0 + 2M^2 \log(r_0/2M) + M^2$$

parameter in the shell wavefunction

Space-time picture

Caution: the solution is complex-valued (though $r(\tau)$ is real, $t(\tau)$ is not), so it does not, strictly speaking describe any physical geometry.

Still, we can try to embed the shell trajectory into the Kruskal extension, to get a hint what an outside observer will see



$$t \mapsto t - i4\pi M$$

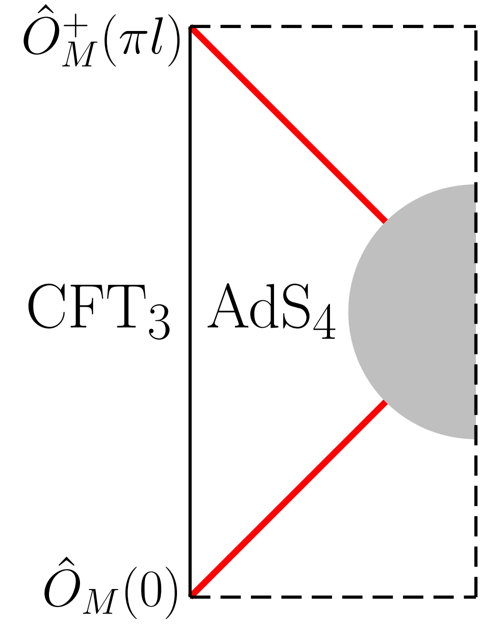
More shell models: AdS

$|\mathcal{A}_{fi}|^2 \sim \mathrm{e}^{-\pi r_h^2}$

where r_h is the root of $f(r) = 1 - \frac{2M}{r} + \frac{r^2}{l^2}$

AdS radius

NB. Admits AdS / CFT interpretation as exponential suppression of certain correlators in CFT_3



More shell models: elementary charged shell

$$|\mathcal{A}_{fi}|^2 \sim \mathrm{e}^{-\pi\left((r_h^+)^2-(r_h^-)^2\right)}$$

where $r_h^{\pm} = M \pm \sqrt{M^2 - Q^2}$ are outer and inner horizons of the Reissner -- Nordstrom metric:

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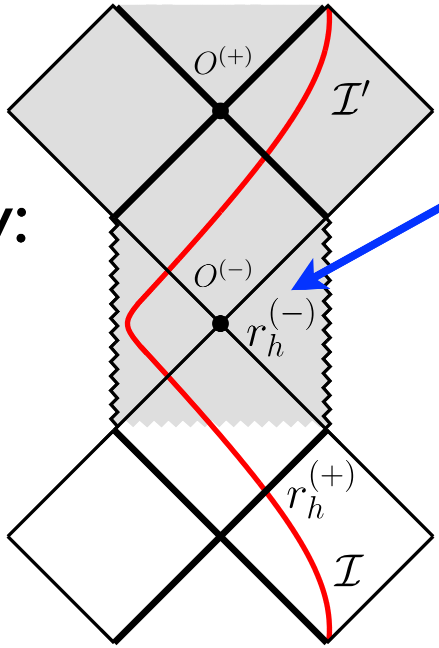
Look at the shell trajectory:



grey region does not exist in field theory due to the instability of the Cauchy horizon

More shell models: shell with discharge

$$S_{EM} = -\frac{1}{4}\int d^2y \frac{F_{ab}^2}{e^2(r/Q)} - Q\int_{shell} A_a dy^a$$

electromagnetic "formfactor"

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Recover the entropic suppression $|\mathcal{A}_{fi}|^2 \sim \mathrm{e}^{-\pi r_h^2}$

<u>independently of the choice of</u> $e(x)$

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- gives reasonable results for shell models, if interpreted as narrow wavepackets of a field theory: entropic suppression, the phase of the amplitude; consistent with unitarity

Summary

- semiclassical method is a promising approach to trans-Planckian gravitational scattering, in particular black hole-mediated precesses
- gives reasonable results for shell models, if interpreted as narrow wavepackets of a field theory: entropic suppression, the phase of the amplitude; consistent with unitarity
- singularity is avoided by semiclassical solutions implying that Planck-scale physics is irrelevant

Outlook

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- study S-matrix in field theory: tests of "thermality", "chaos", unitarity
  - minimally coupled spherically symmetric scalar
  - 2d dilaton gravity (*Callan et. al (1992)*)

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- describe measurements of an infalling observer. Requires the *in-in* formalism with the corresponding modification of the path integral. Semiclassical solutions can be different

➡ test of black hole complementarity (*Susskind et al. (1993)*)