Asymptotic Safety (the Exact Renormalization Group approach)

Windows on Quantum Gravity, 29/10/15

Tim Morris,
Physics & Astronomy,
University of Southampton, UK.

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GR:

- $g_{\mu\nu}$ is a field

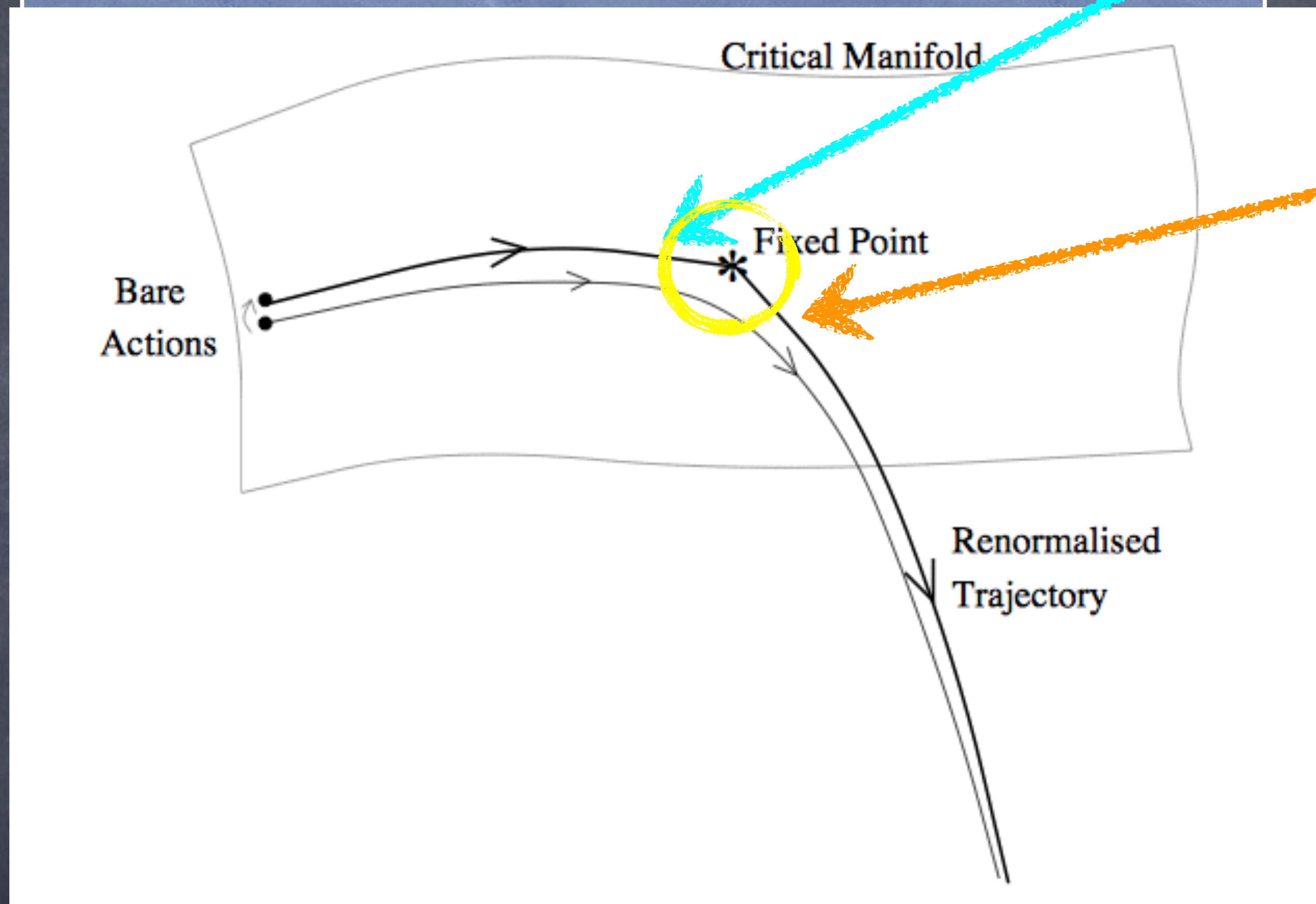
QM:

- which must be consistent with QM

↓

- Effective quantum field theory

Wilsonian RG



TRM, Prog. Theor. Phys. Suppl. **131** (1998) 395.

G is irrelevant about Gaussian FP.

RG step makes no difference ⇨ scale invariant

So what is the UV completion?

⇨ massless continuum limit!

Non-perturbative UV FP

- Higgs? K.G. Wilson & J. Kogut, Phys. Rep. **12C** (1974) 75
- Large N Gross-Neveu model (four-fermi) in <4 dimensions K.G. Wilson, Phys. Rev. D10 (1973) 2911

Gravity: asymptotic Safety

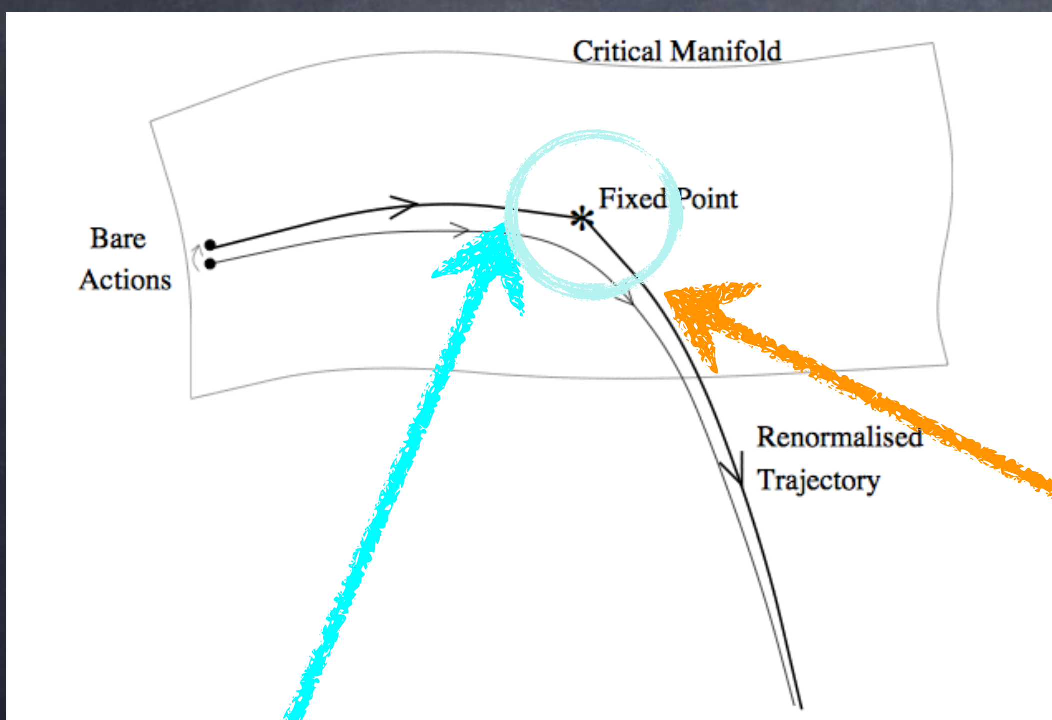
- Gravity D=2+ε S. Weinberg, in *Hawking, S.W., Israel, W.: General Relativity* (1979) 790-831; Proc. Int. School of Subnuclear physics, Erice (1976).
- Large N L. Smolin, Nucl. Phys. **B208** (1982) 439; R. Percacci, Phys. Rev. **D73** (2006) 041501(R)

Theory space...

$$\Gamma_k \sim \int d^4x\,\sqrt{g}\,\{g_k^0 + g_k^1 R + g_k^{2a} R^2 + g_k^{2b} R^{\mu\nu} R_{\mu\nu} + g_k^{3a} R^3 + g_k^{3b} R R^{\mu\nu} R_{\mu\nu} + \cdots$$

$$g_k^0 = \frac{\Lambda_k}{8\pi G_k} \qquad g_k^1 = -\frac{1}{16\pi G_k}$$

$$g_k^i = k^{4-2i} g^i_{\text{phys}}(k)$$



$$k\frac{d}{dk} g_k^i = \beta^i(g_k) = B^i{}_j (g_k^j - g_*^j) + \cdots$$

$$g_k^i = g_*^i + \sum_n \alpha_n v_n^i \left(\frac{k}{\mu}\right)^{\vartheta_n} + \cdots$$

$\vartheta_n < 0$ means $\sim \int d^4x\,\sqrt{g}\sum_i v_n^i R^i$ is a relevant operator

$\alpha_n \mu^{-\vartheta_n}$ is a coupling in the continuum theory

$\vartheta_n > 0$ means $\sim \int d^4x\,\sqrt{g}\sum_i v_n^i R^i$ is an irrelevant operator

Non-trivial questions for asymptotic safety

- Does the UV FP exist?
- Does it have a finite number of (3?) relevant directions (i.e. couplings)?
- Is it unitary?
- What are the phenomenological consequences?

- Is it unitary?

$$\Gamma_k \sim \int d^4x \sqrt{g}\, \{g_k^0 + g_k^1 R + g_k^{2a} R^2 + g_k^{2b} R^{\mu\nu} R_{\mu\nu}\}$$

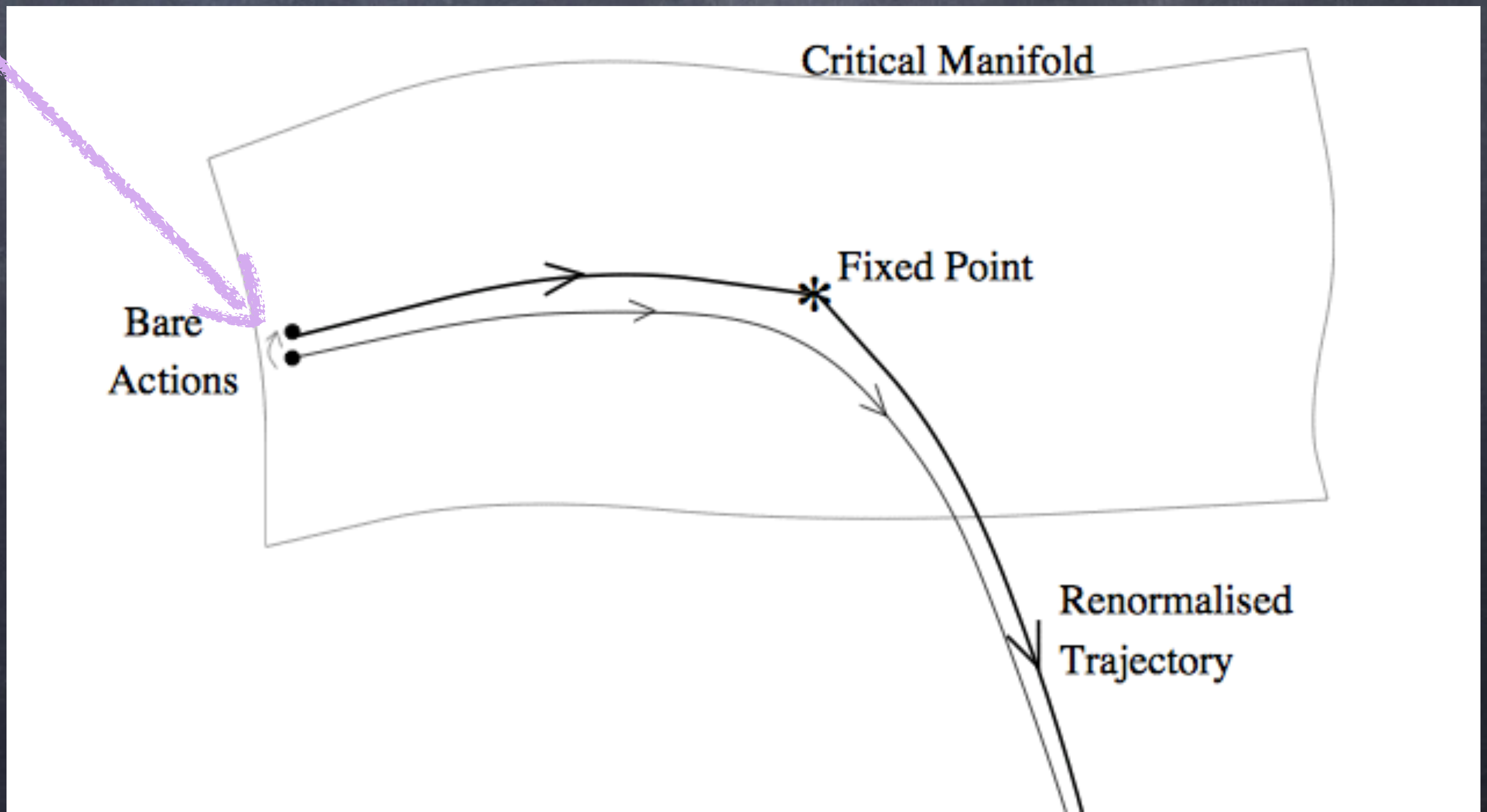
K. S. Stelle, Phys. Rev. **D16** (1977) 953

- How do we tell?

Need bare Hamiltonian

Compute $\langle h_{\mu\nu}\, h_{\alpha\beta}\rangle(p)$?

T. R. Morris & Z. Slade, "Solving the reconstruction problem …" arXiv:1507.08657



Exact RG...

K.G. Wilson & J. Kogut, Phys. Rep. **12C** (1974) 75; F.J. Wegner & A. Houghton, Phys. Rev. **A8** (1973) 401

$$\frac{\partial}{\partial k}\Gamma[\varphi] = \frac{1}{2}\,\mathrm{tr}\,\left[\mathcal{R} + \frac{\delta^2\Gamma}{\delta\varphi\delta\varphi}\right]^{-1}\frac{\partial}{\partial k}\mathcal{R}\,.$$

C. Wetterich, Phys. Lett. **B301** (1993) 90;

T.R. Morris, Int. J. Mod. Phys. **A9** (1994) 2411.

Momentum dependent mass term (IR cutoff):

$$\frac{1}{2}\varphi\cdot\mathcal{R}\cdot\varphi$$

suppresses momenta $p < k$

- k→0 gives full Legendre effective action.
- If k→∞ exists then continuum limit constructed

… applied to QG

M. Reuter, Phys. Rev. **D57** (1998) 971

Reviews…

M. Niedermaier & M. Reuter, Living Rev. Rel. **9** (2006) 5;

R. Percacci, in *Oriti, D (ed.)* arXiv:0709.3851;

D.F. Litim, arXiv:0810.3675;

M. Reuter & F. Saueressig, New J. Phys. **14** (2012) 055022

1) A proper fixed functional for four-dimensional Quantum Einstein Gravity
By Maximilian Demmel, Frank Saueressig, Omar Zanusso.
arXiv:1504.07656 [hep-th].
2) Cosmic fluctuations from quantum effective action
By C. Wetterich.
arXiv:1503.07860 [gr-qc].
3) Black holes in Asymptotically Safe Gravity
By Frank Saueressig, Natalia Alkofer, Giulio D'Odorico, Francesca Vidotto.
arXiv:1503.06472 [hep-th].
4) Critical scaling in quantum gravity from the renormalisation group
By Kevin Falls.
arXiv:1503.06233 [hep-th].
5) Foliation fiber bundle and quantization of Einstein gravity
By I.Y. Park.
arXiv:1503.02015 [hep-th].
6) Universally Finite Gravitational & Gauge Theories
By Leonardo Modesto, Leslaw Rachwal.
arXiv:1503.00261 [hep-th].
7) Global solutions of functional fixed point equations via pseudo-spectral methods
By Julia Borchardt, Benjamin Knorr.
arXiv:1502.07511 [hep-th].
8) Background independent exact renormalization group for conformally reduced gravity
By Juergen A. Dietz, Tim R. Morris.
arXiv:1502.07396 [hep-th].
10.1007/JHEP04(2015)118.
JHEP 1504 (2015) 118.
9) Spin-base invariance of Fermions in arbitrary dimensions
By Stefan Lippoldt.
arXiv:1502.05607 [hep-th].
10) Is there a C-function in 4D Quantum Einstein Gravity?
By Daniel Becker, Martin Reuter.
arXiv:1502.03292 [hep-th].
11) Global surpluses of spin-base invariant fermions
By Holger Gies, Stefan Lippoldt.
arXiv:1502.00918 [hep-th].
10.1016/j.physletb.2015.03.014.
Phys.Lett. B743 (2015) 415-419.
12) The Renormalization Group flow of unimodular f(R) gravity
By Astrid Eichhorn.
arXiv:1501.05848 [gr-qc].
10.1007/JHEP04(2015)096.
JHEP 1504 (2015) 096.
13) On the renormalisation of Newton's constant
By Kevin Falls.
arXiv:1501.05331 [hep-th].
14) Search of scaling solutions in scalar-tensor gravity
By Roberto Percacci, Gian Paolo Vacca.
arXiv:1501.00888 [hep-th].
10.1140/epjc/s10052-015-3410-0.
Eur.Phys.J. C75 (2015) 5, 188.
15) Black Hole Solutions for Scale Dependent Couplings: The de Sitter and the Reissner-Nordstr\"om Case
By Benjamin Koch, Paola Rioseco.
arXiv:1501.00904 [gr-qc].
16) Black Hole Remnants and the Information Loss Paradox

Einstein-Hilbert truncation

$$\Gamma \sim \frac{1}{16\pi G}\int d^4x\,\sqrt{g}\,\{-R+2\Lambda\}$$

$$G := k^2 G_{phys}(k)\,, \quad \Lambda := \Lambda_{phys}/k^2, \quad t := \ln(k/\mu)$$

E.g. using sharp cutoff:

$$\begin{aligned}
\partial_t \Lambda &= -(2-\eta)\Lambda - \frac{G}{\pi}\left[5\ln(1-2\Lambda) - 2\zeta(3) + \frac{5}{2}\eta\right], \\
\partial_t G &= (2+\eta)G\,, \\
\eta &= -\frac{2G}{6\pi+5G}\left[\frac{18}{1-2\Lambda} + 5\ln(1-2\Lambda) - \zeta(2) + 6\right].
\end{aligned}$$

M. Reuter & F. Saueressig, Phys. Rev. **D65** (2002) 065016, New J. Phys. **14** (2012) 055022

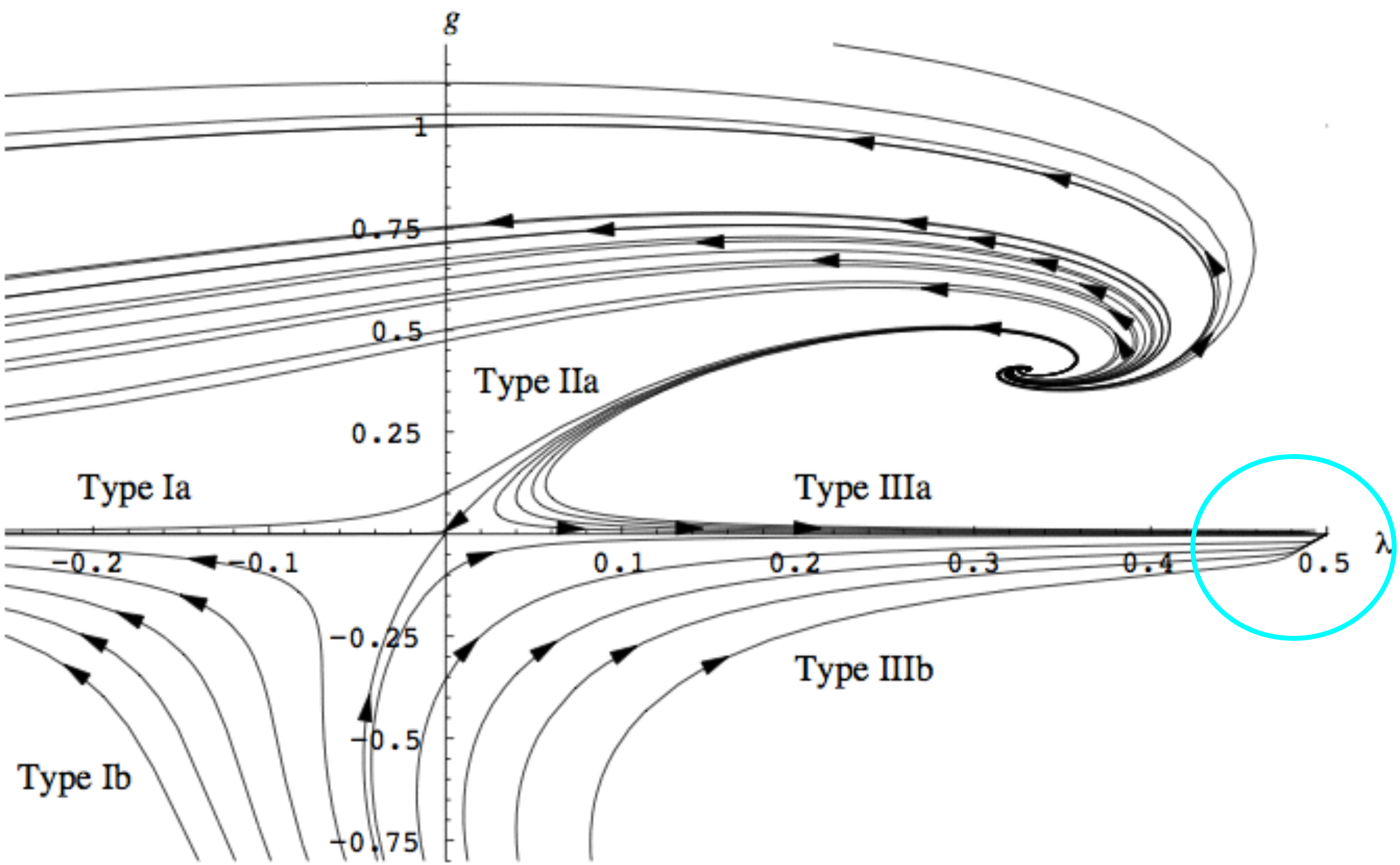


Figure 4: RG flow in the g-λ–plane. The arrows point in the direction of increasing coarse graining, i.e., of decreasing k. (From [14].)

M. Reuter & F. Saueressig, Phys. Rev. **D65** (2002) 065016, New J. Phys. **14** (2012) 055022

Wilsonian RG can break down!

Can be caused by poor approximation...

VOLUME 66, NUMBER 25 PHYSICAL REVIEW LETTERS 24 JUNE 1991

Renormalization Transformations in the Vicinity of First-Order Phase Transitions: What Can and Cannot Go Wrong

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(Received 7 December 1990)

We reconsider the conceptual foundations of the renormalization-group (RG) formalism. We show that the RG map, defined on a suitable space of interactions, is always single valued and Lipschitz continuous on its domain of definition. This rules out a recently proposed scenario for the RG description of first-order phase transitions. On the other hand, we prove in several cases that near a first-order phase transition the renormalized measure is not a Gibbs measure for any reasonable interaction. It follows that the conventional RG description of first-order transitions is not universally valid.

Reparametrise or change cutoff...

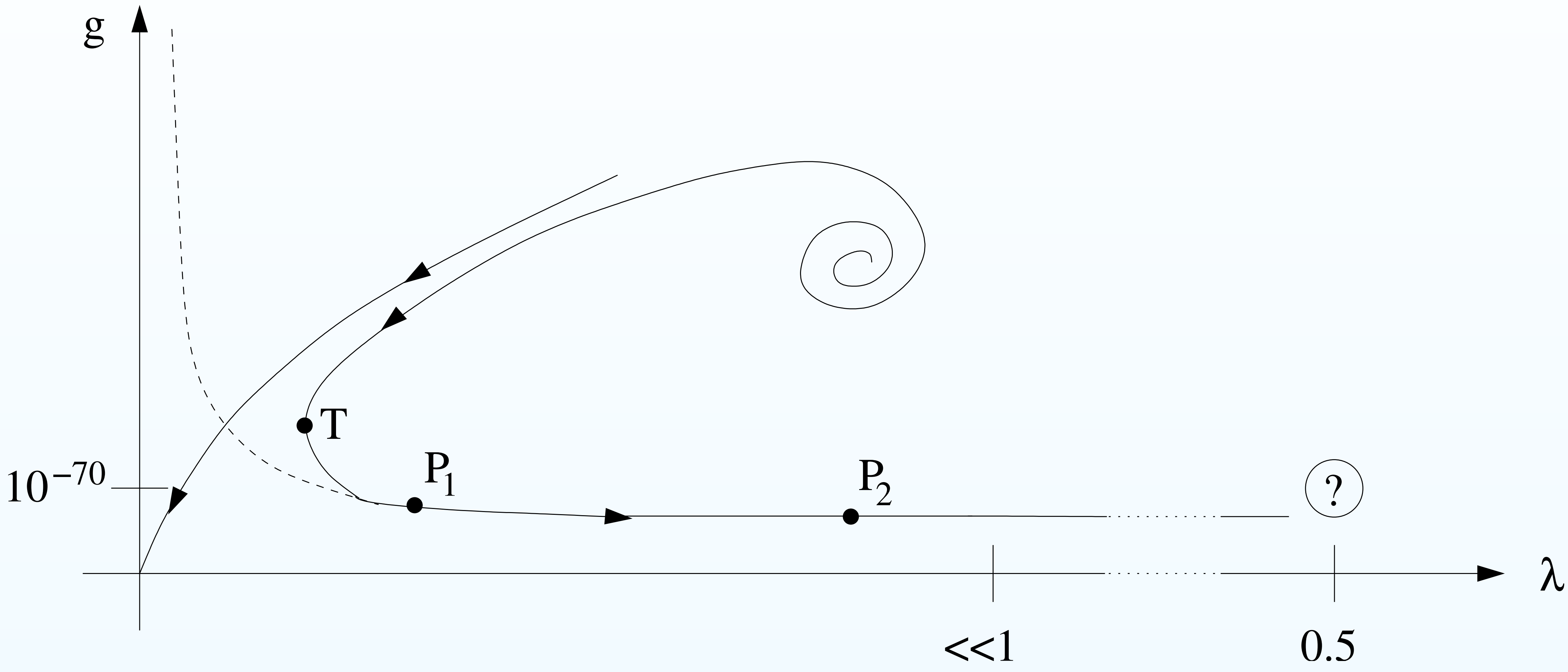
Global Flows in QG, M. Christiansen, B. Knorr, J.M. Pawlowski & A. Rodigast, arXiv:1403.1232;

R. Percacci and G. P. Vacca, Eur. Phys. J. C**75** (2015) 5, 188

The RG trajectory realized in Nature

M. Reuter, H. Weyer, JCAP 0412 (2004) 001, hep-th/0410119

measurement of G_N, Λ in classical regime:



- originates at NGFP (quantum regime: $G(k) = k^{2-d} g_*, \Lambda(k) = k^2 \lambda_*$)
- passing *extremely* close to the GFP
- long classical GR regime (classical regime: $G(k) = \text{const}, \Lambda(k) = \text{const}$)

Cosmological consequences...

P. Binétruy

$a(t)/k$ and $t \lesssim M_{\mathrm{Planck}} \sim 10^{19}$ GeV

Super-Planckian initial conditions

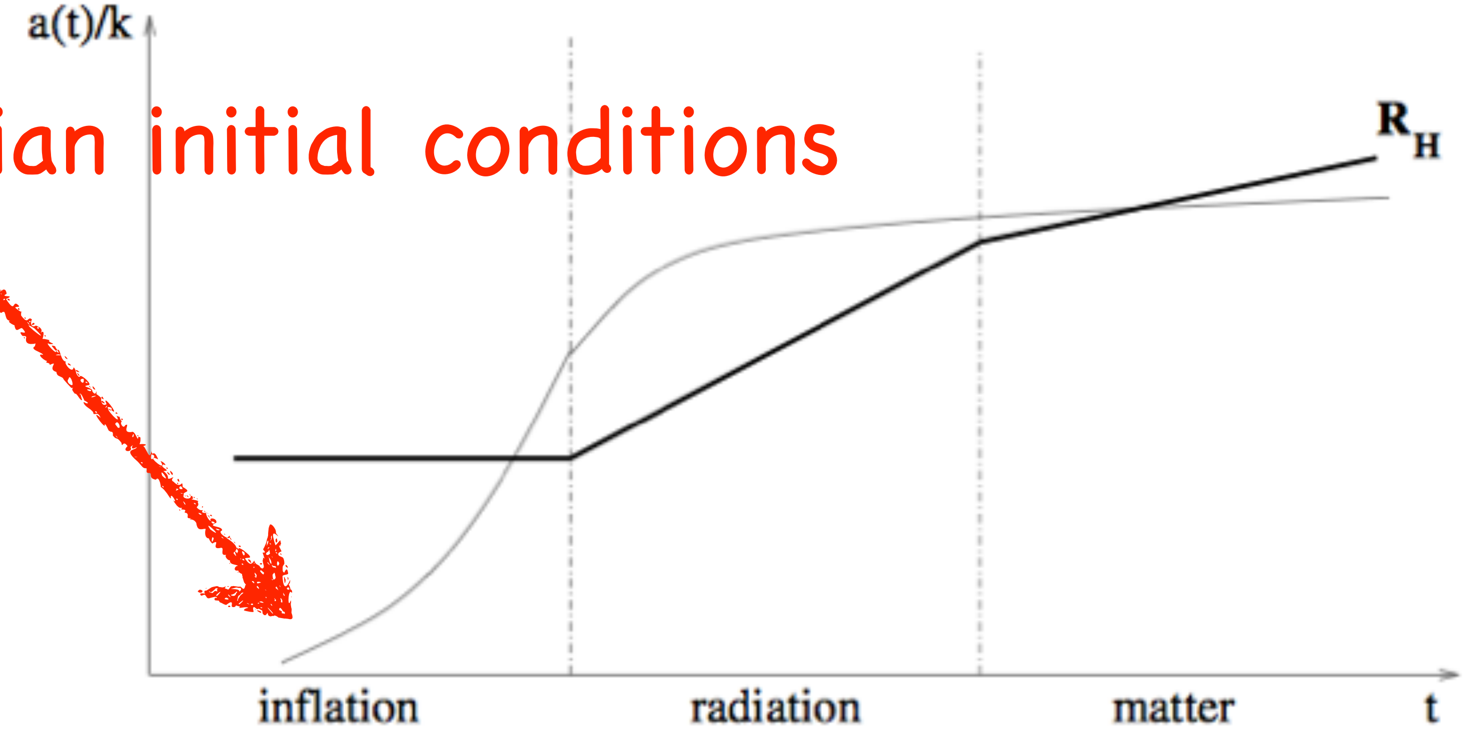
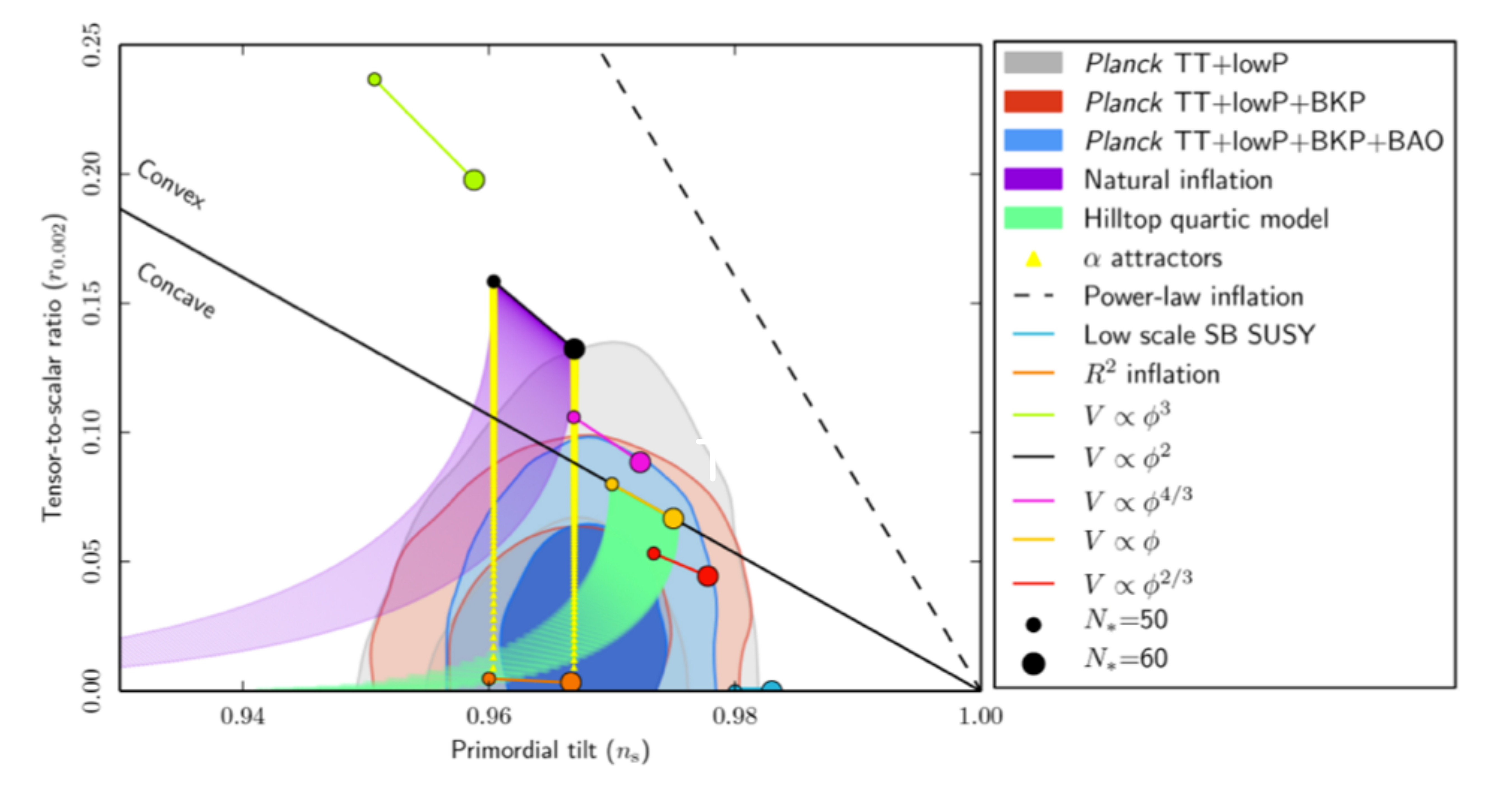


Fig. 9: Evolution of a physical comoving fluctuation scale with respect to the Hubble radius during the inflation phase ($R_H(t) = H_{\mathrm{vac}}^{-1}$), the radiation dominated phase ($R_H(t) = 2t$) and matter dominated phases ($R_H(t) = 3t/2$).

...quantum gravity fueled inflation

A. Bonnano & M. Reuter, 2003, 2007, 2008; Bonanno, Espisota & Rubano, 2003, 2004, 2005; Bonanno, Contillo, Percacci, 2010; Weinberg, 2010; Tye & Xu, 2010; A. Bonanno, 2012.

Precision cosmology...



Black holes in Asymptotically Safe Gravity Frank Saueressig

+Alkofer, D'Odorico & Vidotto
1503.06472

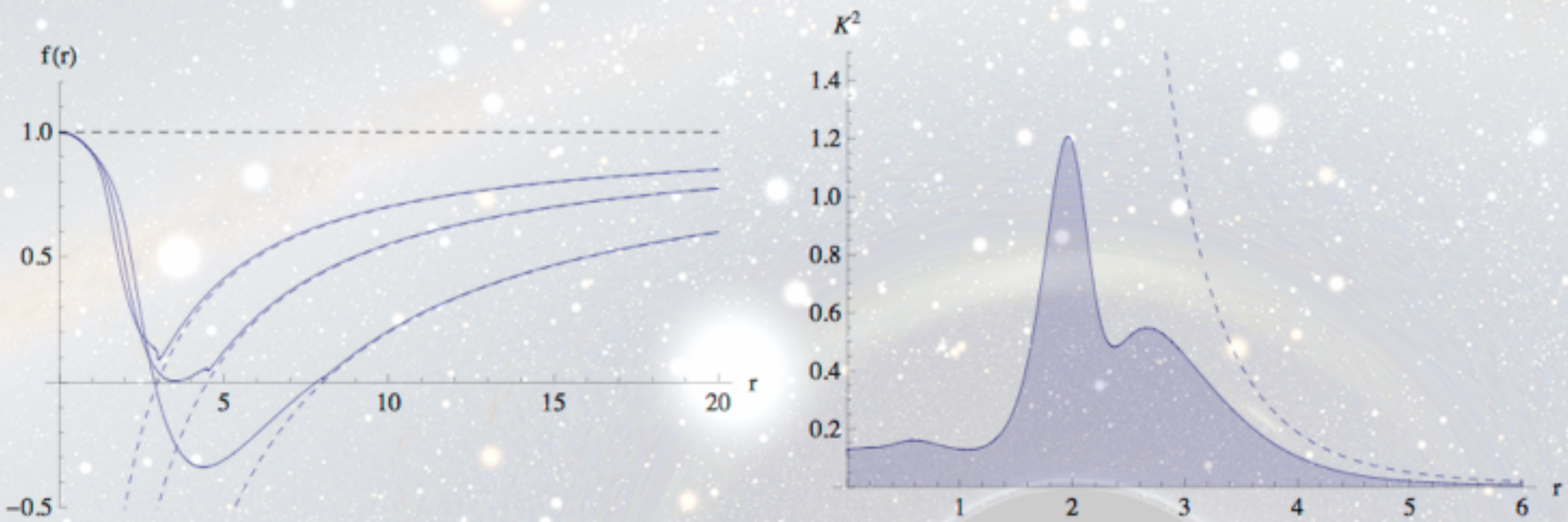


Figure 2: The left diagram illustrates the horizon structure for the RG improved Schwarzschild black holes with $m = 1.5$ (top curve), $m = m_{\rm crit} \approx 2.25$ (middle curve) and $m = 4$ (bottom curve), while the Kretschmann scalar curvature $K^2 = R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}$ for the case $m = 4 > m_{\rm crit}$ is shown in the right diagram. All quantities are measured in Planck units. The classical result is visualized by the dashed curves for comparison.

k is momentum cutoff, **not** a scale

Bonanno and Reuter,
"RG improved black hole space-times"
Phys. Rev. D**62** (2000) 043008

$$G(k) \mapsto G(1/r)$$

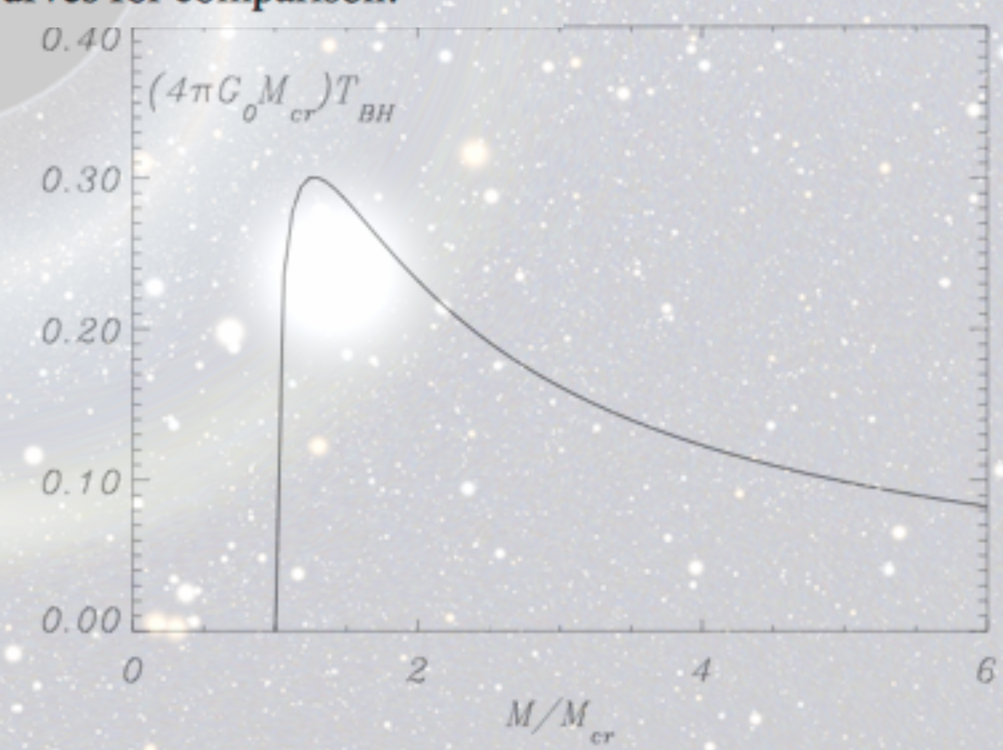
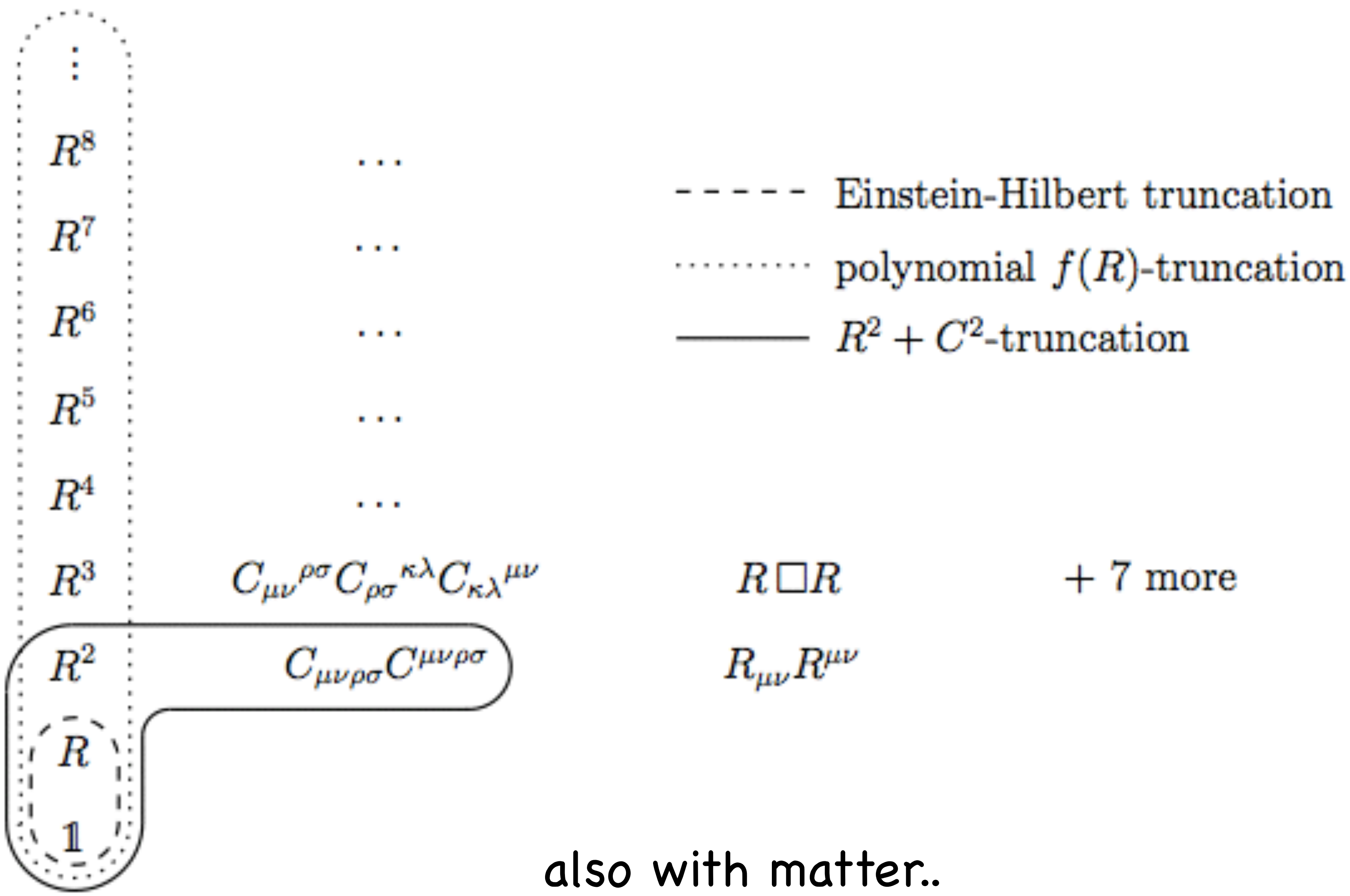


FIG. 5. The Hawking temperature of the quantum black hole (multiplied by $4\pi G_0 M_{\rm cr}$) as a function of $M/M_{\rm cr}$. The maximum temperature is reached for $\widetilde{M}_{\rm cr} \approx 1.27 M_{\rm cr}$.

Less severe truncations



also with matter..

Figure 3: Overview of the various truncations employed in the systematic exploration of the theory space of QEG. The lines indicate the interaction monomials contained in the various truncation ansätze for $\bar{\Gamma}_k[g]$, eq. (4.4). All truncations have confirmed the existence of a non-trivial UV fixed point of the gravitational RG flow.

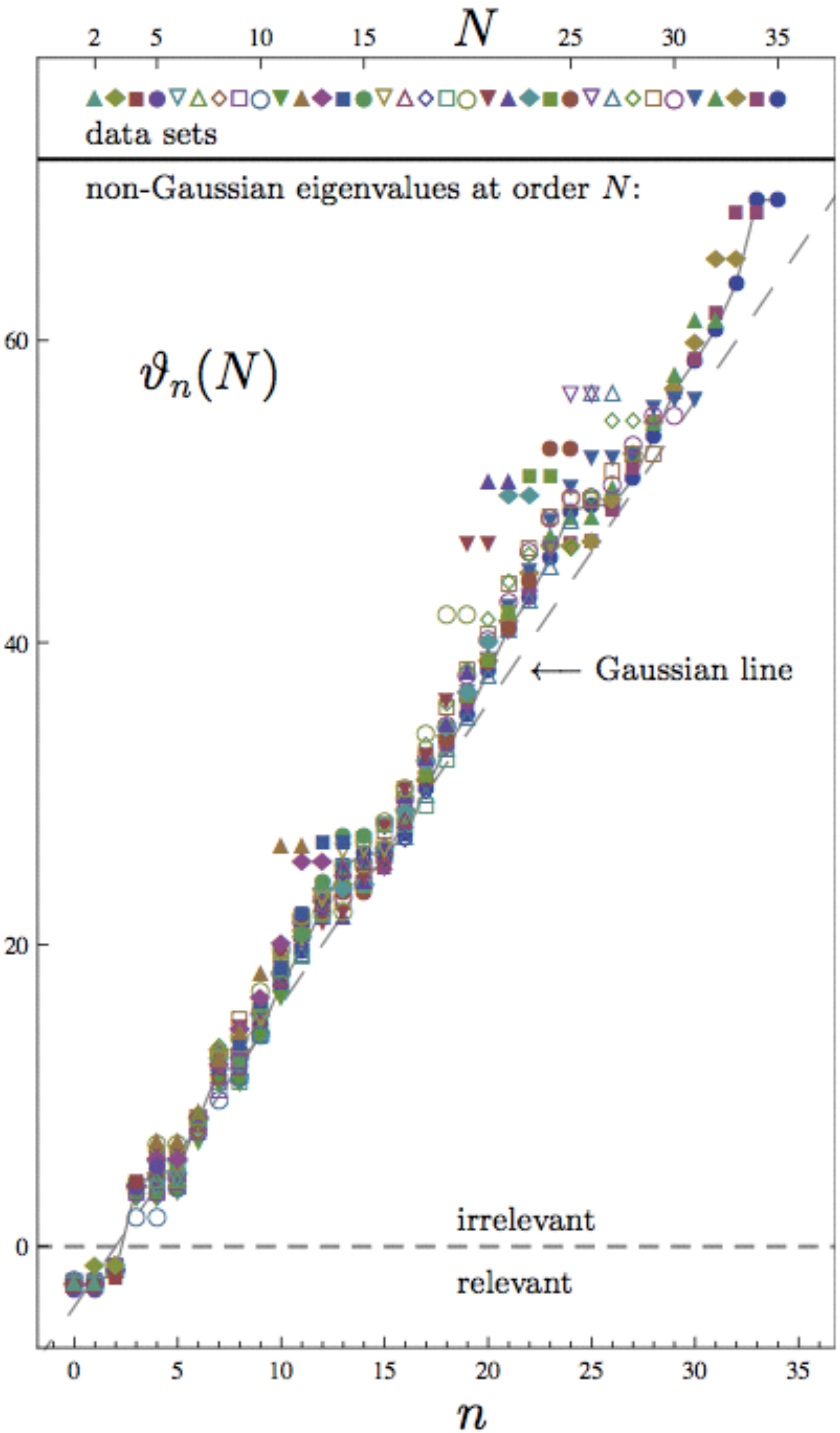
M. Reuter & F. Saueressig, New J. Phys. **14** (2012) 055022; M. Reuter, F. Saueressig, O. Lauscher, D. Benedetti, P. F. Machado, A. Codello, R. Percacci, C. Rahmede, M. Nierdermaier, K. Groh, S. Rechenberger, O. Zanusso ...

Figure 1: The complete sets of eigenvalues at the ultraviolet fixed point (8) for all N, sorted by magnitude. The results at the highest order ($N = 35$) are linked by a line to guide the eye. The long dashed line indicates Gaussian scaling. The inset (upper panel) relates the data sets at approximation order N with the symbols used in the lower panel.

K. Falls, D.F. Litim, K. Nikolakopoulos & C. Rahmede, arXiv:1301.4191

Go beyond polynomial truncations to explore

$$\bar{R} \sim O(1)$$



Split into background + fluctuation:

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu}$$

$$Z \sim \int \mathcal{D}h \; e^{-S[\bar{g}+h]}$$

Intuitively results should be background independent

Split into background + fluctuation:

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu}$$

IR cutoff: $\mathcal{R} \sim \mathcal{R}(-\bar{\nabla}^2/k^2)$

Impose Landau gauge: D. Litim & J. Pawlowski, Phys. Lett. **B435** (1998) 181

$$\bar{\nabla}^\mu h_{\mu\nu} = \frac{1}{4}\bar{\nabla}_\nu h$$

Ghosts get IR cutoff too, so BRS invariance recovered only in the limit k→0 (if we're careful).

TT decomposition (ghosts similarly):

$$h_{\mu\nu} = h^T_{\mu\nu} + \bar{\nabla}_\mu \xi_\nu + \bar{\nabla}_\nu \xi_\mu + \bar{\nabla}_\mu \bar{\nabla}_\nu \sigma + \frac{1}{4}\bar{g}_{\mu\nu}\bar{h}$$

Jacobian ⇒ auxiliary fields & they get IR cutoff too.

M. Reuter, Phys. Rev. **D57** (1998) 971

Conformal factor problem?

$$h_{\mu\nu} = h^T_{\mu\nu} + \bar\nabla_\mu \xi_\nu + \bar\nabla_\nu \xi_\mu + \bar\nabla_\mu \bar\nabla_\nu \sigma + \frac{1}{4}\bar g_{\mu\nu} \bar h$$

G. W. Gibbons, S. W. Hawking and M. J. Perry,
``Path Integrals and the Indefiniteness of the Gravitational Action,''
Nucl. Phys. B **138** (1978) 141

$$\frac{\partial}{\partial k}\Gamma[\varphi] = \frac{1}{2}\mathrm{tr}\left[\mathcal{R} + \frac{\delta^2\Gamma}{\delta\varphi\delta\varphi}\right]^{-1}\frac{\partial}{\partial k}\mathcal{R}\,.$$

M. Reuter, Phys. Rev. **D57** (1998) 971

IR cutoffs (basic idea):

$$\frac{\partial}{\partial k}\Gamma[\varphi] = \frac{1}{2}\,\mathrm{tr}\,\left[\mathcal{R} + \frac{\delta^2\Gamma}{\delta\varphi\delta\varphi}\right]^{-1}\frac{\partial}{\partial k}\mathcal{R}\,.$$

$$\mathcal{R} \sim \mathcal{R}_L = k^2 r(-\bar{\nabla}^2/k^2) = (k^2+\bar{\nabla}^2)\theta(k^2+\bar{\nabla}^2)$$

D.F. Litim, Phys. Rev. **D64** (2001) 105007

Effectively in inverse 2-pt: $-\bar{\nabla}^2 + \mathcal{R}_L \equiv k^2$

Adaptive IR cutoffs:

On constant scalar curvature $\bar{R}$ background...

$$\frac{\delta^2\Gamma}{\delta\sigma\delta\sigma} \sim \left(-\bar{\nabla}^2 - \frac{\bar{R}}{3}\right)^2 (-\bar{\nabla}^2)$$

$$\mathcal{R} \sim \left(-\bar{\nabla}^2 + \mathcal{R}_L - \frac{\bar{R}}{3}\right)^2 (-\bar{\nabla}^2 + \mathcal{R}_L) - \frac{\delta^2\Gamma}{\delta\sigma\delta\sigma}$$

so effectively: $$\frac{\delta^2\Gamma}{\delta\sigma\delta\sigma} + \mathcal{R} \equiv \left(k^2 - \frac{\bar{R}}{3}\right)^2 k^2$$

$$h_{\mu\nu} = h^T_{\mu\nu} + \bar{\nabla}_\mu \xi_\nu + \bar{\nabla}_\nu \xi_\mu + \bar{\nabla}_\mu \bar{\nabla}_\nu \sigma + \frac{1}{4}\bar{g}_{\mu\nu}\bar{h}$$

Performing the space-time trace

$$\frac{\partial}{\partial k}\Gamma[\varphi] = \frac{1}{2}\int_x \left\{ \mathrm{tr}\, \left[\mathcal{R} + \frac{\delta^2\Gamma}{\delta\varphi\delta\varphi}\right]^{-1} \frac{\partial}{\partial k}\mathcal{R}\right\}(x,x) \, .$$

$$\frac{\partial}{\partial k}\Gamma[\varphi] = \frac{1}{2}\sum_{\lambda_i<k} \left\langle i \left| \left[\mathcal{R} + \frac{\delta^2\Gamma}{\delta\varphi\delta\varphi}\right]^{-1} \frac{\partial}{\partial k}\mathcal{R} \right| i \right\rangle$$

- heat kernel expansion
- short distance (small R̄) expansion
- background independence
- background universality
- But **asymptotic** so terms are neglected
- E.g. on 4-sphere really "staircase"

$$-\bar{\nabla}^2 \left| i \right\rangle = \lambda_i^2 \left| i \right\rangle$$

D. Benedetti and F. Caravelli, JHEP 1206 (2012) 017.
J. A. Dietz & TRM, JHEP 1301 (2013) 108.

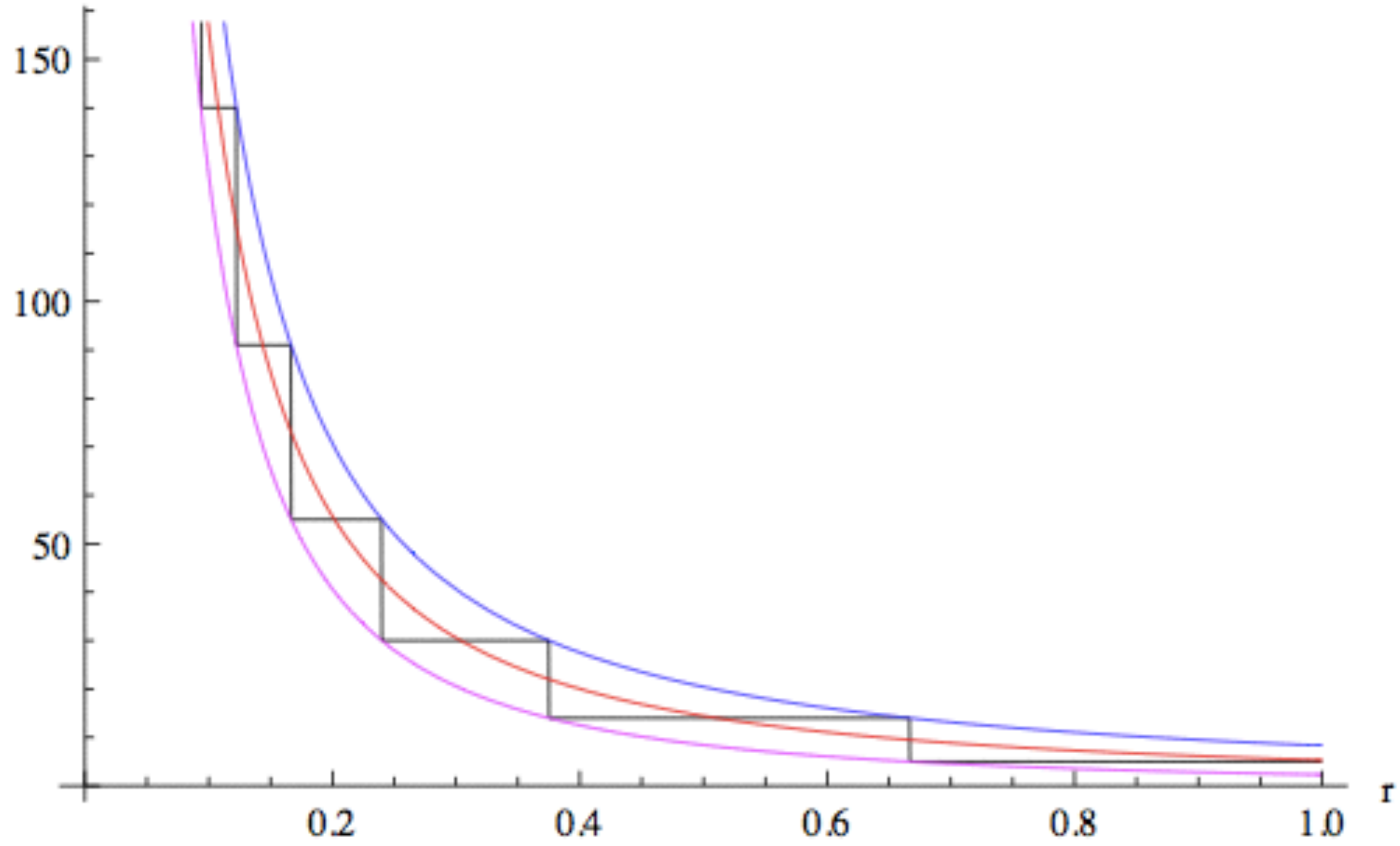


Figure 2. Smooth approximations of the staircase function $\sum_n \theta\left(1 - \frac{r}{6}n^2\right) n^4$ (black line). The curves obtained from the replacement (5.11), truncating the sum at N_r and $N_r - 1$ are given by the blue (top) and magenta (bottom) curve, respectively. The average of the two approximations resulting from eq. (5.12) gives rise to the red (middle) line.

M. Demmel, F. Saueressig & O. Zanusso, arXiv:1401.5459

Other typical approximations:

- k dependence of ghosts & auxiliaries neglected
- Mixed $h_{\alpha\beta}$ and $g_{\mu\nu}$ terms neglected (single field approximation)

Exceptions:

A. Eichhorn, H. Gies & M.M. Schere, Phys. Rev. **D80** (2009) 104003; K. Groh & F. Saueressig, J. Phys. **A43** (2010) 365403; A. Eichhorn & H. Gies, Phys. Rev. **D81** (2010) 104010;
E. Manrique, M. Reuter, Ann. Phys. 325 (2010) 785;
E. Manrique, M. Reuter & F. Saueressig, Ann. Phys. 326 (2011) 440 & 463;
A. Codello, G. D'Odorico & C. Pagani, Phys. Rev. D**89** (2014) 8, 081701
P. Dona, A. Eichhorn & R. Percacci, Phys. Rev. D**89** (2014) 8, 084035 arXiv:1410.4411
D. Becker & M. Reuter, Ann. Phys. **350** (2014) 225
J.A. Dietz & TRM, JHEP **1504** (2015) 118

Beyond polynomial truncations

E.g.

- Project on four-sphere background $(R \geq 0)$
- Effective action: $\Gamma = \int d^4x \, \sqrt{g} \, f(R,t)$
- Get non-linear PDE flow equation for $f(R,t)$

Fixed points: $f(R,t) \mapsto f(R)$

$$
\begin{aligned}
&768\pi^2\left(2f - Rf'\right) = \\
&\left[5R^2\theta\left(1-\tfrac{R}{3}\right) - \left(12 + 4\,R - \tfrac{61}{90}\,R^2\right)\right]\left[1-\tfrac{R}{3}\right]^{-1} + \Sigma \\
&+\left[10\,R^2\,\theta(1-\tfrac{R}{4}) - R^2\,\theta(1+\tfrac{R}{4}) - \left(36 + 6\,R - \tfrac{67}{60}\,R^2\right)\right]\left[1-\tfrac{R}{4}\right]^{-1} \\
&+\left[(2f' - 2Rf'')\left(10 - 5R - \tfrac{271}{36}R^2 + \tfrac{7249}{4536}R^3\right) + f'\left(60 - 20R - \tfrac{271}{18}R^2\right)\right]\left[f + f'(1-\tfrac{R}{3})\right]^{-1} \\
&+\tfrac{5R^2}{2}\left[(2f' - 2Rf'')\left\{r(-\tfrac{R}{3}) + 2r(-\tfrac{R}{6})\right\} + 2f'\theta(1+\tfrac{R}{3}) + 4f'\theta(1+\tfrac{R}{6})\right]\left[f + f'(1-\tfrac{R}{3})\right]^{-1} \\
&+\Big[(2f' - 2Rf'')f'\left(6 + 3R + \tfrac{29}{60}R^2 + \tfrac{37}{1512}\,R^3\right) \\
&\quad -2Rf'''\left(27 - \tfrac{91}{20}R^2 - \tfrac{29}{30}\,R^3 - \tfrac{181}{3360}\,R^4\right) \\
&+f''\left(216 - \tfrac{91}{5}R^2 - \tfrac{29}{15}R^3\right) + f'\left(36 + 12R + \tfrac{29}{30}R^2\right)\Big]\left[2f + 3f'(1-\tfrac{2}{3}R) + 9f''(1-\tfrac{R}{3})^2\right]^{-1},
\end{aligned}
$$

$$r(z) = (1-z)\theta(1-z) \qquad \Sigma = 10R^2\theta(1-R/3)$$

P. F. Machado & F. Saueressig, Phys. Rev. D 77 (2008) 124045

A. Codello, R. Percacci & C. Rahmede, Annals Phys. 324 (2009) 414

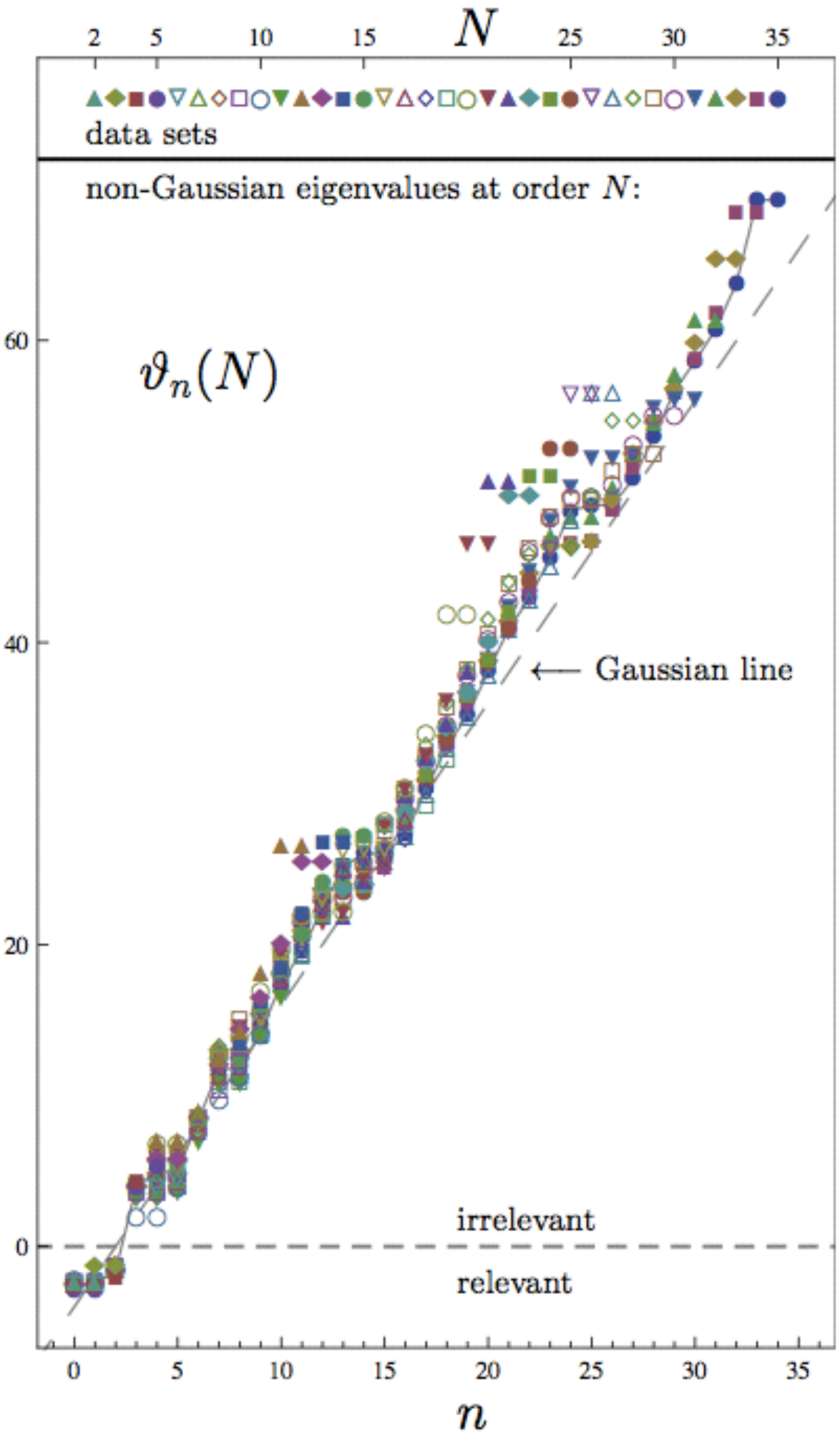
Figure 1: The complete sets of eigenvalues at the ultraviolet fixed point (8) for all N, sorted by magnitude. The results at the highest order ($N = 35$) are linked by a line to guide the eye. The long dashed line indicates Gaussian scaling. The inset (upper panel) relates the data sets at approximation order N with the symbols used in the lower panel.

K. Falls, D.F. Litim, K. Nikolakopoulos & C. Rahmede, arXiv:1301.4191

Polynomial truncations of these equations don't see

$$\bar{R} \sim O(1)$$

effects



Fixed singularities and parameter space

Suppose we have normal form: $$f'''(R) = \frac{F(f, f', f'', R)}{R}$$

with fixed singularity at R=0.

Substitute: $$f(R) = a_0 + a_1 R + \frac{1}{2}\, a_2 R^2 + \cdots$$

=> $$\text{regular in } R \; = \frac{u(a_0, a_1, a_2)}{R} + \text{regular in } R$$

$u(a_0, a_1, a_2)$ is non-trivial constraint on parameters a_0, a_1, a_2

Fixed points: $f(R,t) \mapsto f(R)$

$$768\pi^2\left(2f - Rf'\right) =$$

$$\left[5R^2\theta\left(1-\tfrac{R}{3}\right) - \left(12+4\,R-\tfrac{61}{90}\,R^2\right)\right]\left[1-\tfrac{R}{3}\right]^{-1} + \Sigma$$

$$+\left[10\,R^2\,\theta(1-\tfrac{R}{4}) - R^2\,\theta(1+\tfrac{R}{4}) - \left(36+6\,R-\tfrac{67}{60}\,R^2\right)\right]\left[1-\tfrac{R}{4}\right]^{-1}$$

$$+\left[(2f'-2Rf'')\left(10-5R-\tfrac{271}{36}R^2+\tfrac{7249}{4536}R^3\right)+f'\left(60-20R-\tfrac{271}{18}R^2\right)\right]\left[f+f'(1-\tfrac{R}{3})\right]^{-1}$$

$$+\tfrac{5R^2}{2}\left[(2f'-2Rf'')\left\{r(-\tfrac{R}{3})+2r(-\tfrac{R}{6})\right\}+2f'\theta(1+\tfrac{R}{3})+4f'\theta(1+\tfrac{R}{6})\right]\left[f+f'(1-\tfrac{R}{3})\right]^{-1}$$

$$+\Big[(2f'-2Rf'')f'\left(6+3R+\tfrac{29}{60}R^2+\tfrac{37}{1512}\,R^3\right)$$

$$-2Rf'''\left(27-\tfrac{91}{20}R^2-\tfrac{29}{30}\,R^3-\tfrac{181}{3360}\,R^4\right)$$

$$+f''\left(216-\tfrac{91}{5}R^2-\tfrac{29}{15}R^3\right)+f'\left(36+12R+\tfrac{29}{30}R^2\right)\Big]\left[2f+3f'(1-\tfrac{2}{3}R)+9f''(1-\tfrac{R}{3})^2\right]^{-1},$$

$R = 0,\ 2.0065$

$$r(z) = (1-z)\theta(1-z) \qquad \Sigma = 10R^2\theta(1-R/3)$$

Parameter counting => no global solutions

J. A. Dietz & TRM, JHEP 1301 (2013) 108.

D. Benedetti and F. Caravelli, JHEP 1206 (2012) 017; D. Benedetti, New J. Phys. **14** (2012) 015005

$$768\pi^2\left(2f - Rf'\right) = \frac{40\left(Rf'' - 4f'\right)}{(R-2)f' - 2f} - 48 - 5R^2$$

$$+\frac{R\left(R^4 - 54R^2 - 54\right)f''' - \left(R^3 + 18R^2 + 12\right)\left(Rf'' - f'\right) + 18\left(R^2 + 2\right)\left(f' + 6f''\right)}{9f'' - (R-3)f' + 2f}.$$

$R = 0,\ 7.4150$

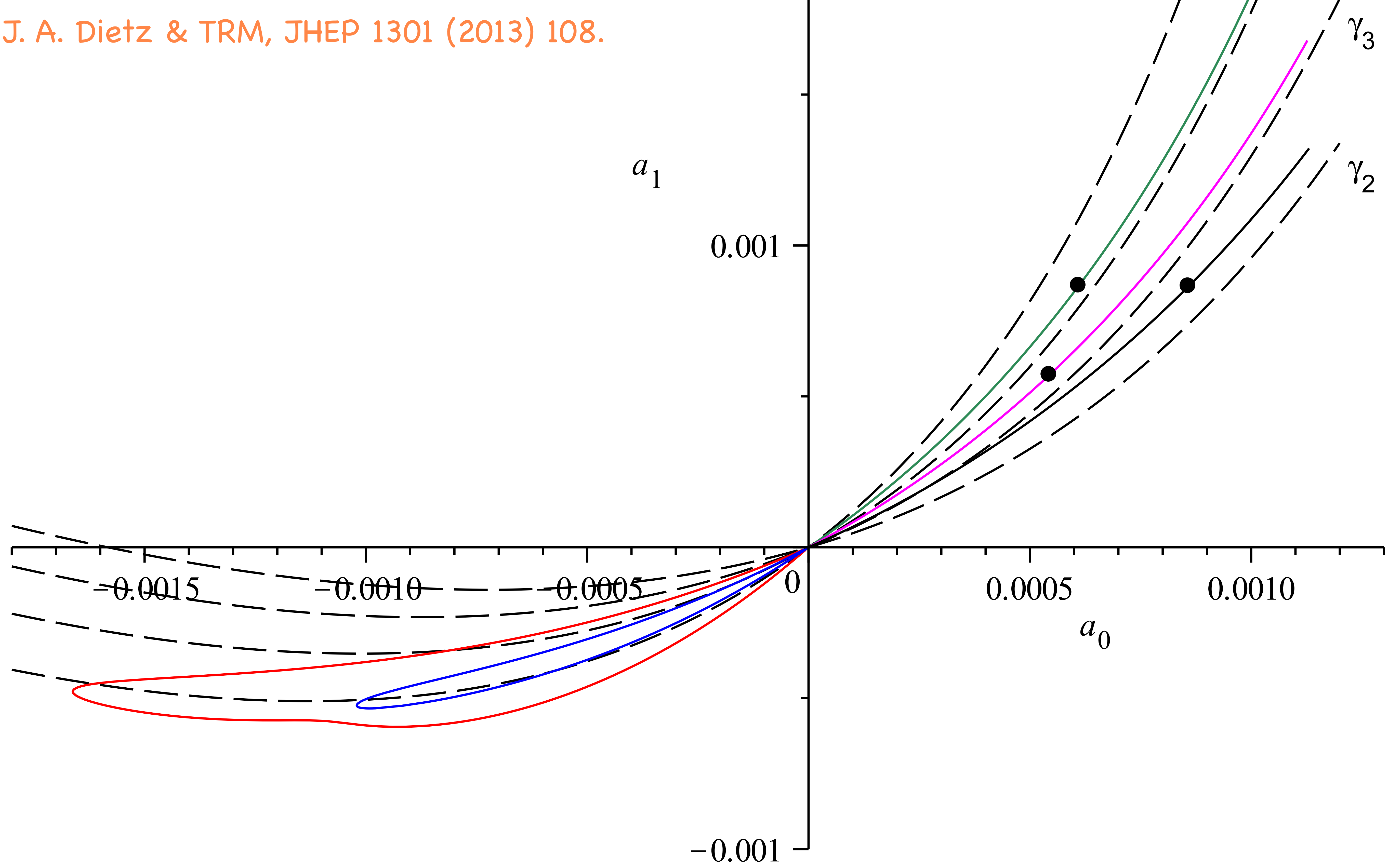
$$f(R) = A\,R^2 + R\left\{\frac{3}{2}A + B\cos\ln R^2 + C\sin\ln R^2\right\} + O(1)$$

$$\frac{121}{20}A^2 > B^2 + C^2$$

Parameter counting => lines of fixed points

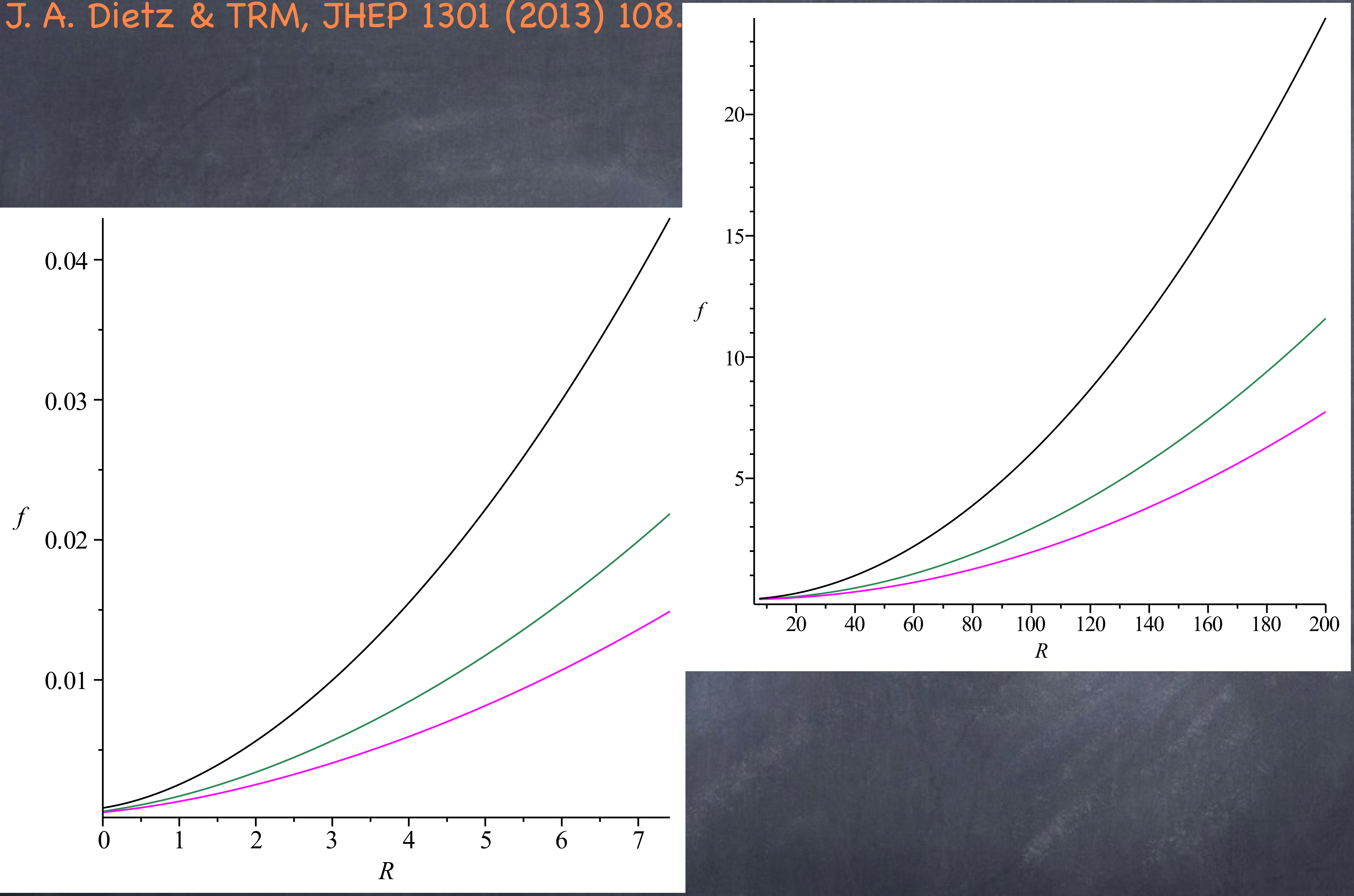
J. A. Dietz & TRM, JHEP 1301 (2013) 108.

J. A. Dietz & TRM, JHEP 1301 (2013) 108.



$$f(R) = a_0 + a_1 R + \frac{1}{2}\, a_2(a_0, a_1)\, R^2 + \cdots$$

J. A. Dietz & TRM, JHEP 1301 (2013) 108.



Continuous eigenspectrum!

J. A. Dietz & T.R. Morris, JHEP 1301 (2013) 108.

M. Demmel, F. Saueressig & O. Zanusso, arXiv:1401.5459

Extend to $-\infty<R<\infty$

D. Benedetti, Europhys. Lett. **102** (2013) 20007;

T.R. Morris, **B495** (1997) 477.

$$768\pi^2\left(2f-Rf'\right)=\frac{40\left(Rf''-4f'\right)}{(R-2)f'-2f}-48-5R^2$$

$$+\frac{R\left(R^4-54R^2-54\right)f'''-\left(R^3+18R^2+12\right)\left(Rf''-f'\right)+18\left(R^2+2\right)\left(f'+6f''\right)}{9f''-(R-3)f'+2f}.$$

$$R=0,\pm 7.4150$$

$$f(R)=A\,R^2+R\left\{\frac{3}{2}A+B\cos\ln R^2+C\sin\ln R^2\right\}+O(1)$$

No global solutions!

$$\frac{121}{20}A^2>B^2+C^2$$

Parameter counting => discrete set of fixed points

Quantised Eigenoperator spectrum

Break-down of f(R) approximation

FP action $\Gamma = \int d^4x\,\sqrt{g}\,f(R)$

J.A. Dietz & T.R. Morris, JHEP **07** (2013) 064

Eigenoperator $\int d^4x\,\sqrt{g}\,v(R)$

Wegner, J. Phys. C7 (1974) 2098.

$$g_{\mu\nu}(x) \mapsto g_{\mu\nu}(x) + \varepsilon F_{\mu\nu}[g](x)$$

Eigenoperator redundant if of the form:

$$\int d^dx\,\sqrt{g}\,F_{\mu\nu}\left\{\frac{1}{2}g^{\mu\nu}f - R^{\mu\nu}f' + \nabla^\mu\nabla^\nu f' - g^{\mu\nu}\Box f'\right\}.$$

$$F_{\mu\nu} = \zeta(R)\,g_{\mu\nu} \quad \Longrightarrow \quad v(R) = \zeta(R)\,\{2f(R) - Rf'(R)\}.$$

Does not vanish for any R≥0 $\Rightarrow$ entire space is redundant!

Split into background + fluctuation:

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu}$$

$$Z \sim \int \mathcal{D}h \; e^{-S[\bar{g}+h]}$$

Intuitively results should be background independent

Scalar field theory @ LPA

I.H. Bridle, J. Dietz & TRM, JHEP **03** (2014) 093

$$\Gamma[\phi] = \int d^dx \left\{ \frac{1}{2} (\partial_\mu \phi)^2 + V(\phi) \right\}$$

$$\phi = \bar{\varphi} + \varphi$$

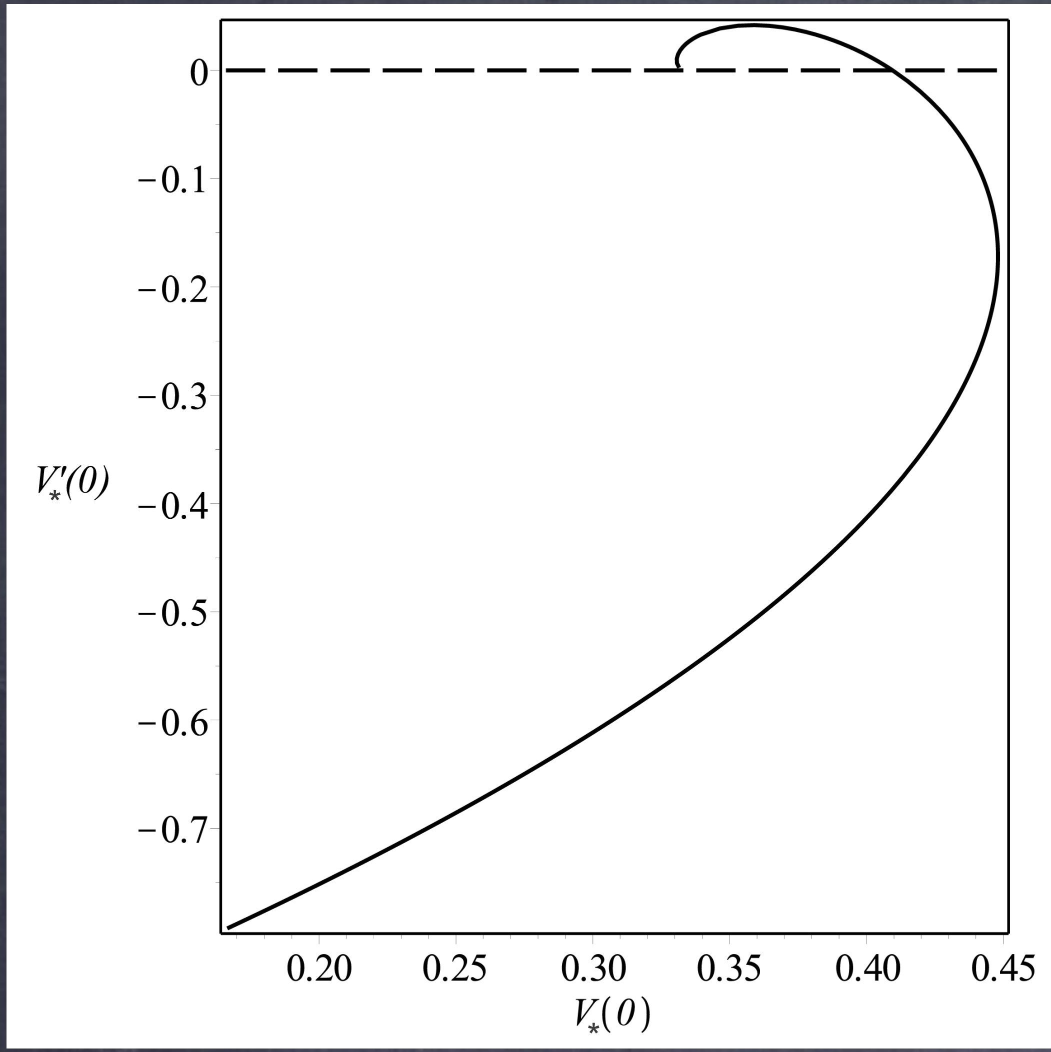
$$\mathcal{R}\big(-\partial^2, \bar{\varphi}(x)\big) = \big(k^2 + \partial^2 - h(\bar{\varphi})\big)\, \theta\big(k^2 + \partial^2 - h(\bar{\varphi})\big)$$

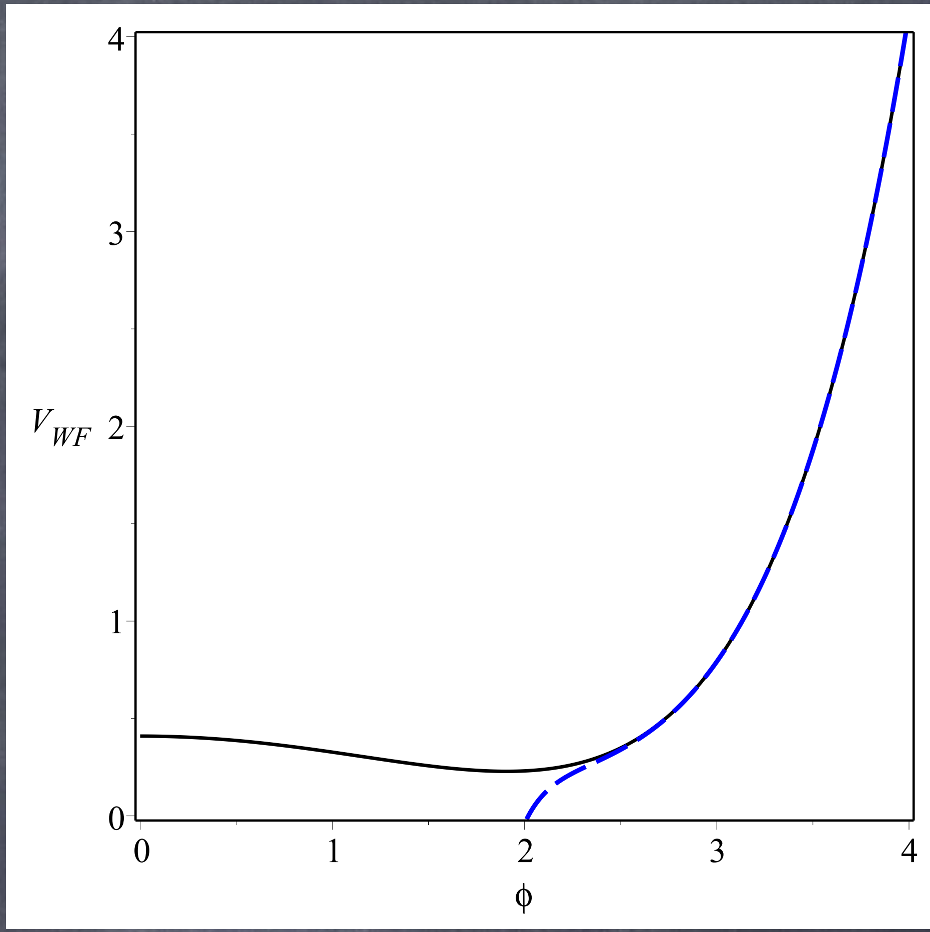
Single field approximation:

$$\partial_t V - \frac{1}{2}(d-2)\phi V' + dV = \frac{(1-h)^{d/2}}{1-h+V''} \left(1 - h - \frac{1}{2}\partial_t h + \frac{1}{4}(d-2)\phi h'\right) \theta(1-h)$$

$\Longrightarrow$ pathologies

h=0 (d=3):

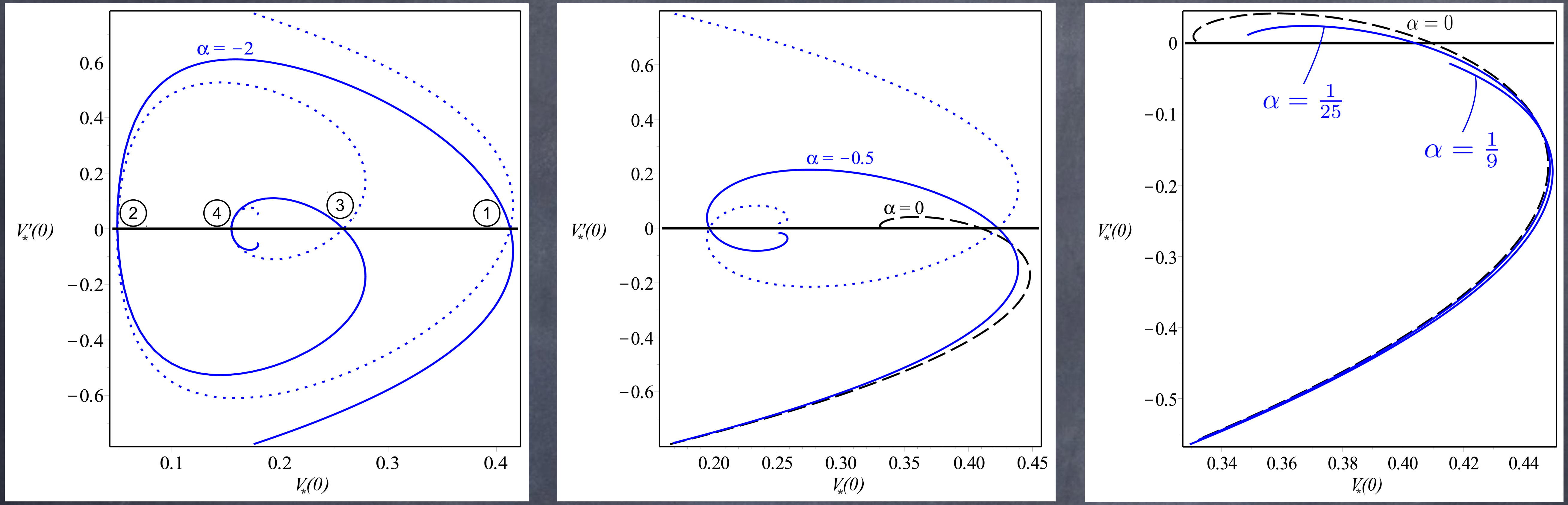




$$\partial_t V + dV - \frac{1}{2}(d-2)\varphi\partial_\varphi V = \frac{1}{1+\partial_\varphi^2 V}$$

Fixed points: $\{V_*(0),\ \pm V_*'(0)\}$

$h=\alpha\bar{\varphi}^2$ (d=3):



Fixed points: $\{V_*(0),\ \pm V_*'(0)\}$

Keep both fields & impose Ward Identity:

$$\phi = \varphi + \bar{\varphi} \qquad \bar{\varphi} \mapsto \bar{\varphi} + \varepsilon(x) \quad \text{and} \quad \varphi \mapsto \varphi - \varepsilon(x)$$

$$\frac{\delta \Gamma}{\delta \bar{\varphi}_a} - \frac{\delta \Gamma}{\delta \varphi_a} = \frac{1}{2} \mathrm{Tr} \left[\left(\mathcal{R} + \frac{\delta^2 \Gamma}{\delta \varphi \delta \varphi} \right)^{-1} \frac{\delta \mathcal{R}}{\delta \bar{\varphi}_a} \right].$$

Reuter, Wetterich, Litim, Pawlowski. E. Manrique, M. Reuter, Ann. Phys. 325 (2010) 785; E. Manrique, M. Reuter, F. Saueressig, Ann. Phys. 326 (2011) 440 & 463; D. Becker & M. Reuter, arXiv:1404.4367

$$\frac{\partial}{\partial k} \Gamma[\varphi, \bar{\varphi}] = \frac{1}{2} \mathrm{tr} \left[\left(\mathcal{R} + \frac{\delta^2 \Gamma}{\delta \varphi \delta \varphi} \right)^{-1} \frac{\partial}{\partial k} \mathcal{R} \right].$$

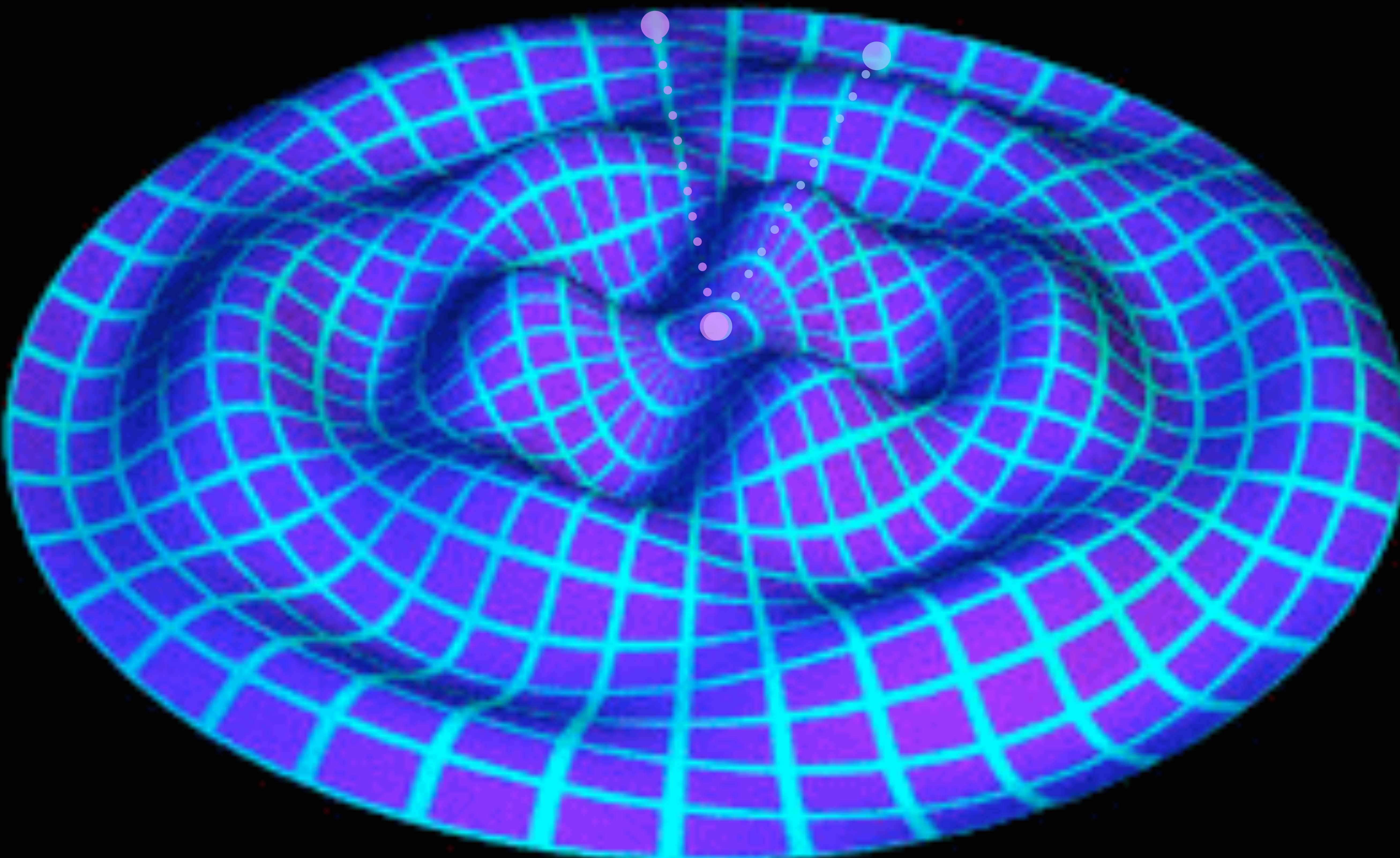
I.H. Bridle, J. Dietz & TRM, JHEP **03** (2014) 093

$$V = (1-h)^{d/2} \hat{V}, \qquad \varphi = (1-h)^{\frac{d-2}{4}} \hat{\varphi} - \bar{\varphi}, \qquad t = \hat{t} - \ln \sqrt{1-h}$$

$$\partial_{\hat{t}} \hat{V} + d\hat{V} - \frac{1}{2}(d-2)\hat{\varphi}\partial_{\hat{\varphi}} \hat{V} = \frac{1}{1 + \partial^2_{\hat{\varphi}} \hat{V}}$$

⇒implements background independence!

What does scale 1/k mean?



Conformal field theory? How to separate scales?

What does scale invariant mean? What is large/small scale?

What is short wavelength? What is long wavelength?

How to form a continuum limit?

Use background field method:

$$\tilde{g}_{\mu\nu} = \bar{g}_{\mu\nu} + \tilde{h}_{\mu\nu}$$

cutoff:

$$R_k \sim r\left(-\frac{\bar{\nabla}^2}{k^2}\right)$$

- Now the scale k is defined through $\bar{g}_{\mu\nu}$ so the notion depends on the choice of background
- The RG itself depends on both $g_{\mu\nu}$ **and** $\bar{g}_{\mu\nu}$ so also is inherently background dependent, and necessarily bi-metric

- Is there a background independent notion of scale k?

Conformally reduced gravity

Bimetric truncations

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu}$$

$$\phi = \varphi + \chi$$

$$g_{\mu\nu} = f(\phi)\,\delta_{\mu\nu} \qquad \text{and} \qquad \bar{g}_{\mu\nu} = f(\chi)\,\delta_{\mu\nu}$$

- Remnant diffeomorphism invariance ...

$$x^\mu \mapsto x^\mu/\lambda\,, \qquad f(\chi) \mapsto \lambda^2 f(\chi)$$

$$\Gamma_k[\varphi,\chi] = \int d^dx\sqrt{\bar{g}}\left(-\frac{1}{2}K(\varphi,\chi)\,\bar{g}^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi + V(\varphi,\chi)\right)$$

J.A. Dietz & TRM, JHEP **1504** (2015) 118

msWIs, flow equations

$$\partial_t V(\varphi,\chi) = f(\chi)^{-\frac{d}{2}} \int_0^\infty dp\, p^{d-1} \frac{\dot R(p^2/f)}{\partial_\varphi^2 V - K p^2/f + R(p^2/f)}$$

$$\partial_\chi V - \partial_\varphi V + \frac{d}{2}\partial_\chi \ln f\, V = f(\chi)^{-\frac{d}{2}} \int_0^\infty dp\, p^{d-1} \frac{\partial_\chi R + \frac{d}{2}\partial_\chi \ln f\, R}{\partial_\varphi^2 V - K p^2/f + R(p^2/f)}$$

$$\hat k = k^{\frac{1}{1+d_f/\alpha}}, \quad \tilde\chi = k^{\frac{1}{1+d_f}}\, f(\chi)^{\frac{1}{\alpha+d_f}}$$

$$f^{-1}\partial_t K(\varphi,\chi) = 2 f^{-\frac{d}{2}} \int_0^\infty dp\, p^{d-1}\, P \dot R$$

$$f^{-1}\left\{\partial_\chi K - \partial_\varphi K + \frac{d-2}{2}\partial_\chi \ln f\, K\right\} = 2 f^{-\frac{d}{2}} \int_0^\infty dp\, p^{d-1}\, P \left[\partial_\chi R + \frac{d}{2}\partial_\chi \ln f\, R\right]$$

$$\tilde V = \hat k^d \hat V_k(\hat\phi), \quad \tilde K = \hat k^{d-2-\eta}\hat K_k(\hat\phi), \quad \phi = \hat k^{\frac{\eta}{2}}\hat\phi, \quad p = \hat k \hat p$$

$$P = -\frac{1}{2}\frac{\partial_\varphi^2 K}{f} Q_0^2 + \frac{\partial_\varphi K}{f}\left(2\partial_\varphi^3 V - \frac{2d+1}{d}\frac{\partial_\varphi K}{f}p^2\right) Q_0^3$$
$$- \left[\left\{\frac{4+d}{d}\frac{\partial_\varphi K}{f}p^2 - \partial_\varphi^3 V\right\}\left(\partial_{p^2}R - \frac{K}{f}\right) + \frac{2}{d}p^2 \partial_{p^2}^2 R \left(\frac{\partial_\varphi K}{f}p^2 - \partial_\varphi^3 V\right)\right]\left(\partial_\varphi^3 V - \frac{\partial_\varphi K}{f}p^2\right) Q_0^4$$
$$- \frac{4}{d}p^2\left(\partial_{p^2}R - \frac{K}{f}\right)^2\left(\partial_\varphi^3 V - \frac{\partial_\varphi K}{f}p^2\right)^2 Q_0^5$$

$$Q_0 = \frac{1}{\partial_\varphi^2 V - K p^2/f + R}$$

Background Independent flow equations

$$\partial_{\hat{t}}\hat{V} + d\hat{V} - \frac{\eta}{2}\hat{\phi}\hat{V}' = -(d-\eta+2n)\int_0^\infty d\hat{p}\;\hat{p}^{d-1}\,\hat{Q}_0\,r(\hat{p}^2)$$

- No χ : $\hat{V} \equiv \hat{V}_{\hat{k}}(\hat{\phi})\,, \quad \hat{K} \equiv \hat{K}_{\hat{k}}(\hat{\phi})\,!$

$$\partial_{\hat{t}}\hat{K} + (d-2-\eta)\hat{K} - \frac{\eta}{2}\hat{\phi}\hat{K}' = -2(d-\eta+2n)\int_0^\infty d\hat{p}\;\hat{p}^{d-1}\,\hat{P}\,r(\hat{p}^2)$$

- No f !

$\hat{t} = \ln(\hat{k}/\mu)$

$$\begin{aligned}\hat{P} = &-\frac{1}{2}\hat{K}''\hat{Q}_0^2 + \hat{K}'\left(2\hat{V}''' - \frac{2d+1}{d}\hat{K}'\hat{p}^2\right)\hat{Q}_0^3 \\ &+\left[\left\{\frac{4+d}{d}\hat{K}'\hat{p}^2 - \hat{V}'''\right\}\left(r'(\hat{p}^2)+\hat{K}\right) + \frac{2}{d}\hat{p}^2 r''(\hat{p}^2)\left(\hat{K}'\hat{p}^2 - \hat{V}'''\right)\right]\left(\hat{V}''' - \hat{K}'\hat{p}^2\right)\hat{Q}_0^4 \\ &-\frac{4}{d}\hat{p}^2\left(r'(\hat{p}^2)+\hat{K}\right)^2\left(\hat{V}''' - \hat{K}'\hat{p}^2\right)^2\hat{Q}_0^5\end{aligned}$$

$$\hat{Q}_0 = \frac{1}{\hat{V}'' - \hat{K}\hat{p}^2 - r(\hat{p}^2)}$$

Bimetric truncations:

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu}$$

Matter matters: keeping separately the wavefn renormalisation of $h_{\mu\nu}$ and all other fields in an EH truncation

P. Dona, A. Eichhorn & R. Percacci, Phys. Rev. D**89** (2014) 8, 084035 arXiv:1410.4411

TABLE IV: Fixed-point values, critical exponents and anomalous graviton dimension for specific matter content.

model	N_S	N_D	N_V	$\tilde{G}_*$	$\tilde{\Lambda}_*$	θ_1	θ_2	η_h
no matter	0	0	0	0.77	0.01	3.30	1.95	0.27
SM	4	45/2	12	1.76	-2.40	3.96	1.64	2.98
SM +dm scalar	5	45/2	12	1.87	-2.50	3.96	1.63	3.15
SM+ 3 ν's	4	24	12	2.15	-3.20	3.97	1.65	3.71
SM+3ν's + axion+dm	6	24	12	2.50	-3.62	3.96	1.63	4.28
MSSM	49	61/2	12	-	-	-	-	-
SU(5) GUT	124	24	24	-	-	-	-	-
SO(10) GUT	97	24	45	-	-	-	-	-

Conclusions

- Significant for black holes, cosmology & BSM
- Increasing sophistication... general importance for QG
- Going beyond polynomial truncations to explore $\bar{R} \sim O(1)$
- Incorporating background independence
- New effects become visible in this regime & much more sensitive to issues with approximations.
- Unitarity? conformal factor problem? Background independence? break-down of Wilsonian RG...?
- Huge body of supporting evidence for asymptotic safety in various truncations/approximations, but there are still many dangers...!