Windows on Quantum Gravity
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Palatini-Born-Infeld Gravity and Bouncing Universe

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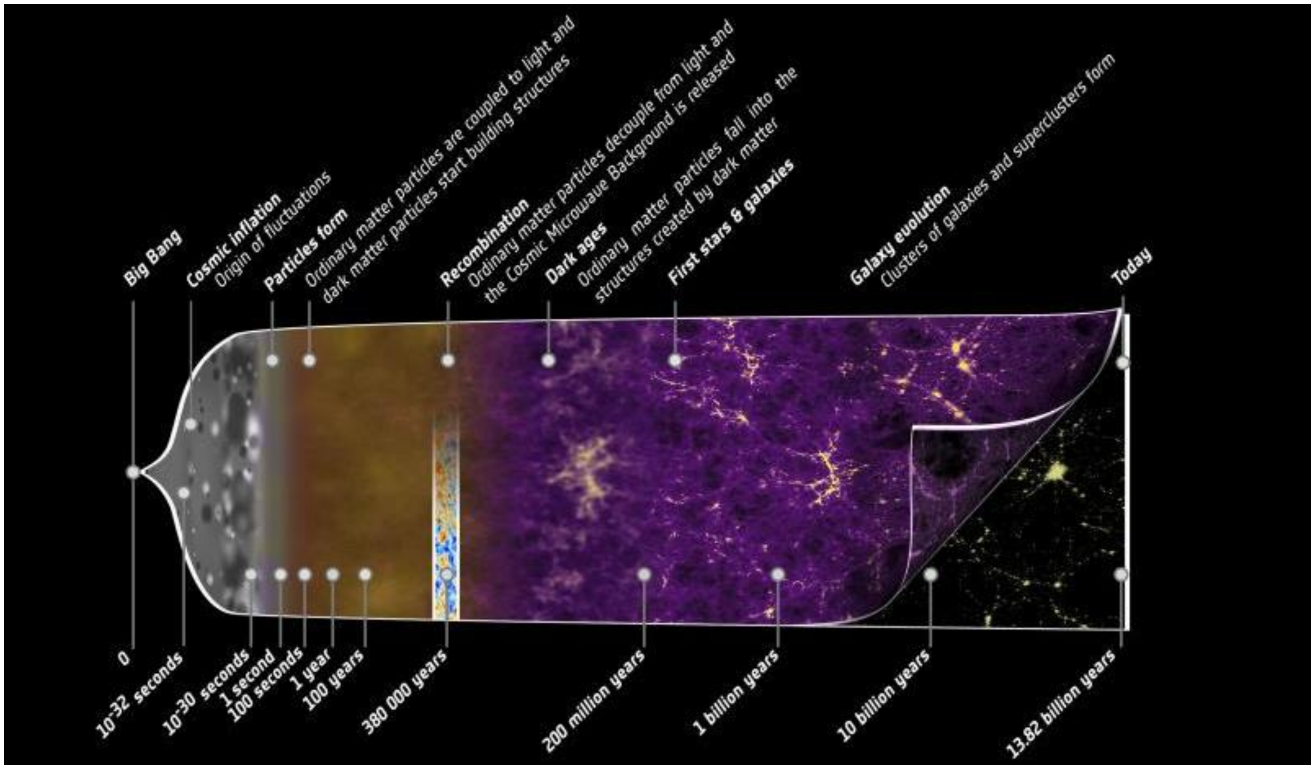
Alternative Theories to General Relativity

GR is simple but successful theory. Combining the SM based on QFT, our Universe is described well.



Planck (2013)

However, there are many reasons and motivations to consider alternative theories of gravity to GR.

Modification inspired by IR and UV Physics

Low energy scale

The observation implies the existence of Dark energy and Dark matter.

- Cosmological constant problem $\Lambda_{theo} \sim 10^{120}\Lambda_{obs}$
- Origin of Cold Dark Matter etc.

It may be possible **to explain these two "dark" components in terms of modified gravity**.

High energy scale

GR loses the predictability at the Planck scale where both GR and QFT are required simultaneously.

- Singularity and evaporation of black holes ← Today's topic
- Initial singularity (Big Bang scenario)

We can regard **modified gravity as effective field theory of quantum gravity**.

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Born-Infeld Gravity in Palatini Formalism

The Born-Infeld type theory was first proposed as a non-linear model of electromagnetics.

In the Born-Infeld model, a new scale is introduced.

$$\mathcal{L} = \frac{1}{\lambda}\left(1 - \sqrt{1 - \lambda(E^2 - B^2)}\right)$$

Born and Infeld (1934)

1. Gauge invariant
2. Lorentz invariant
3. To restore the Maxwell theory

where, λ is a parameter with dimension $(length)^2$.

Because the action includes the square root, there appear the upper limit in the strength given by the scale, which may have suggested that **there might not appear the divergence.**

The energy of point charge $U = \sqrt{\frac{Q^3}{4\pi\sqrt{\lambda}}} \int_0^\infty dx \left(\sqrt{1 + x^4} - x^2\right)$

Born-Infeld Gravity

The Born-Infeld type deformation was also considered in gravitational theory.

(Eddington-inspired) Born-Infeld Gravity

$$S = \frac{1}{\kappa^2 b} \int d^4x \left[\sqrt{|\det(g_{\mu\nu} + bR_{\mu\nu})|} - \sqrt{|\det(g_{\mu\nu})|} \right] + S_{\text{matter}}$$

where b is a parameter which introduce the new scale.

For $bR \ll 1$, one can expand the action.

$$S = \frac{1}{2\kappa^2} \int d^4x \sqrt{-\det(g)} \left[R + b\left(\frac{1}{4}R^2 - \frac{1}{2}R_{\mu\nu}R^{\mu\nu} \right) \cdots \right]$$

Leading term of the action is the Einstein-Hilbert action.

However, it includes **infinite number of higher curvature terms**, and they cause the **ghost propagating modes**.

Metric Formalism vs. Palatini Formalism

In the standard theory of gravity, metric and affine are not independent variables since we want to describe only the DoF corresponding to a massless spin-2 particle.

To relate these two fields, we impose the **metric postulate** (metric-compatibility).

$$\nabla_\lambda g_{\mu\nu} = 0 \to \partial_\lambda g_{\mu\nu} - \Gamma^\rho{}_{\lambda\mu} g_{\rho\nu} - \Gamma^\rho{}_{\lambda\nu} g_{\mu\rho} = 0$$

Still, the metric postulate leaves the torsion tensor.

To completely determine a connection by the metric, one impose the **torsion-free postulate**.

$$\Gamma^\lambda{}_{\mu\nu} = \Gamma^\lambda{}_{\nu\mu} \to T^\lambda{}_{\mu\nu} = 0$$

The torsionless, metric-compatible connection is called the **Levi-Civita connection (Christoffel symbols)** .

$$\Gamma^\lambda_{\mu\nu} = \frac{1}{2} g^{\lambda\rho} \left(\partial_\mu g_{\rho\nu} + \partial_\nu g_{\mu\rho} - \partial_\rho g_{\mu\nu} \right)$$

On the other hand, we can treat metric and connection as independent.

This is called **the Palatini formalism**.

(N.B. the connection is symmetric, and that the matter fields do not couple to the connection in this work)

However, **the Palatini formalism is equivalent to metric formalism in GR.**

$$S = \frac{1}{2\kappa^2}\int d^4x\sqrt{-g}\left[g^{\mu\nu}R_{\mu\nu}(\Gamma) - 2\Lambda\right]$$

→ Deviation by metric: $$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} - \Lambda g_{\mu\nu} = 0$$ Einstein's eq.

→ Deviation by connection: $$\nabla_\lambda\left(\sqrt{-g}g_{\mu\nu}\right) = 0$$ Metric-postulate

In modified gravity, the Palatini formalism is different from the metric formalism.

We consider the Born-Infeld gravity in the Palatini formalism.

$$S = \frac{1}{\kappa^2 b} \int d^4x \left[\sqrt{|\det(P)|} - \sqrt{|\det(g)|} \right] + S_{\rm matter}$$

where $P_{\mu\nu} = g_{\mu\nu} + bR_{\mu\nu}(\Gamma)$

Deviation by metric

$$\sqrt{|\det(P)|}\,(P^{-1})^{\mu\nu} - \sqrt{|\det(g)|}g^{\mu\nu} = -b\kappa^2\sqrt{|\det(g)|}\,T^{\mu\nu}$$

Deviation by connection

$$\nabla_\lambda \left(\sqrt{|\det(P)|}\,(P^{-1})^{\mu\nu} \right) = 0$$

Non-vanishing energy-momentum tensor is required.

Otherwise, $T_{\mu\nu} = 0$, the Born-Infeld gravity is equivalent to GR.

$$T_{\mu\nu} = 0 \quad \leftrightarrow \quad P_{\mu\nu} = g_{\mu\nu} \longrightarrow R_{\mu\nu} = 0, \quad \nabla_\lambda \left(\sqrt{-g} g_{\mu\nu} \right) = 0$$

Existence of matter fields modifies the gravitational theory.

And, there is no ghost mode around the Minkowski background.

Bouncing Universe

FRW Dust Universe

We consider the FRW space-time with flat spatial part filled with pressure-less dust.

$$ds^2 = -dt^2 + a^2(t) \sum_{i=1,2,3} \left(dx^i\right)^2 \qquad T^{\mu\nu} = \mathrm{diag}(\rho, 0, 0, 0)$$

Non-vanishing components of the connection by isometry

$$\Gamma^t{}_{tt} = A(t), \quad \Gamma^t{}_{ij} = a^2(t) B(t) \delta_{ij}, \quad \Gamma^i{}_{jt} = \Gamma^i{}_{tj} = C(t) \delta^i_j$$

When we solve the EOMs, we obtain following results,

$$A = C - H = -\frac{3}{4} b\kappa^2 H\rho \left(1 + b\kappa^2 \rho\right)^{-1}, \quad B = H + \frac{b\kappa^2}{4} H\rho,$$

$$C = H - \frac{3}{4} b\kappa^2 H\rho \left(1 + b\kappa^2 \rho\right)^{-1}, \quad b\kappa^2 \rho = \left\{1 + b\left(\dot{H} + 3H^2 + \frac{b\kappa^2}{2} \dot{H}\rho\right)\right\}^2 - 1$$

Conservation law :

$$\dot{\rho} + 3H\rho = 0\,, \;\; H = \frac{\dot{a}}{a} \quad \leftrightarrow \quad \rho = \rho_0 a^{-3}$$

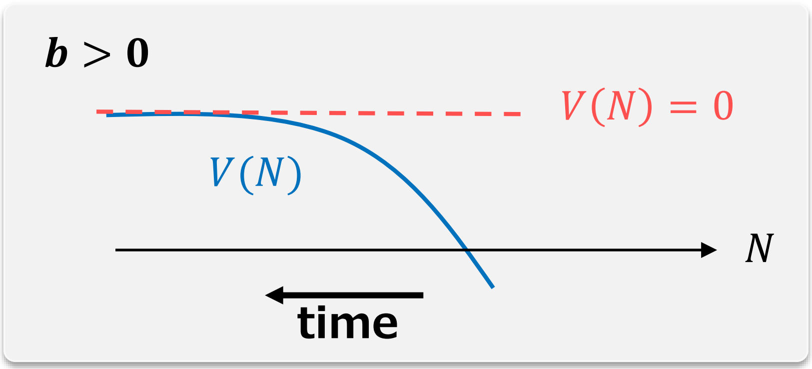
In terms of the **e-foldings number N**, we obtain

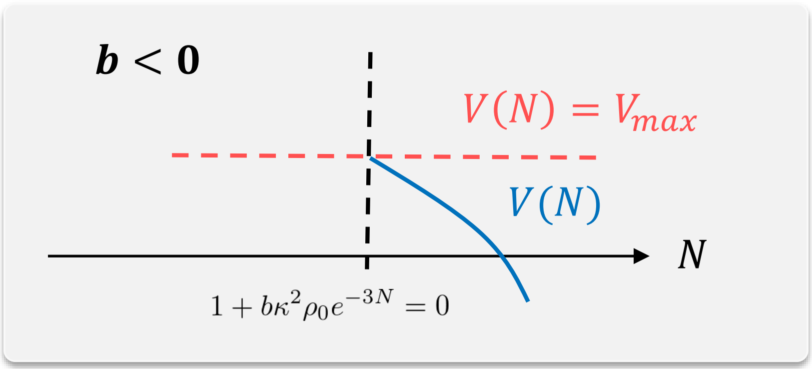
$$0 = \frac{d^2}{dt^2}\left(\frac{e^{3N}}{3} + \frac{b\kappa^2\rho_0}{4}N\right) + \frac{e^{3N}}{b}\left(1 - \sqrt{1 + b\kappa^2\rho_0 e^{-3N}}\right)\ , \ \rho = \rho_0 e^{-3N}$$

We can calculate the conserved quantity E which corresponds to a total energy in the classical mechanics.

$$E = \frac{1}{2}\left\{\frac{d}{dt}\left(\frac{e^{3N}}{3} + \frac{b\kappa^2\rho_0}{4}N\right)\right\}^2 + V(N),$$

$$V(N) = \frac{e^{6N}}{6b}\left(1 + \frac{1}{2}b\kappa^2\rho_0 e^{-3N}\right)\left(1 - \sqrt{1 + b\kappa^2\rho_0 e^{-3N}}\right) - \frac{b\kappa^2\rho_0}{12b}e^{3N}\sqrt{1 + b\kappa^2\rho_0 e^{-3N}}$$





For large N, we can estimate the conserved quantity E .

$$E = -\frac{(b\kappa^2\rho_0)^2}{16b} \quad \text{at} \quad N \to +\infty$$

- In case $b > 0$, the shrinking of the universe will always stop and turn to expand. → **Bouncing universe**
- In case $b < 0$, the shrinking universe will always reach the singularity.

Bounce happens at $N = N_b$

$$e^{3N_b} \sim \frac{3}{8} b\kappa^2\rho_0 \quad \text{when} \quad b\kappa^2\rho_0 \gg 1 \qquad e^{3N_b} \sim \frac{9}{64} b\kappa^2\rho_0 \quad \text{when} \quad b\kappa^2\rho_0 \ll 1$$

Modified FRW eq. $\frac{3}{\kappa^2}H^2 = \rho\left(1 - \frac{\rho}{\rho_l}\right) + \mathcal{O}\left(e^{-9N}\right), \quad \rho_l \equiv \frac{2}{b\kappa^2}$

Bounce of the Universe is characterized by b.

Black Hole Formation

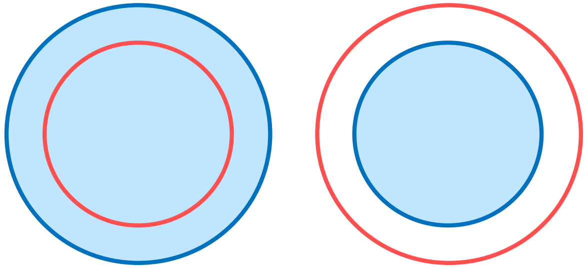
Speculations about Spherical Dust Collapse

Considering the spherical dust collapse, we expect the bouncing could occur as well as bouncing universe.

However, We cannot calculate the junction condition because **the effective repulsive force violates the homogeneity**.

We give a naïve speculations:

1. If the bounce occurs outside the Schwarzschild radius, BH does not form.
2. If the bounce occurs inside the Schwarzschild radius, **BH horizon forms but there would be a remnant inside BH without singularity**



If **BH horizon** is created outside **matter**, it appears to be BH from distant observer. But, the structure inside the BH horizon is different from usual BH.

Similar cases have been proposed.

In LQC, Friedmann eq. is **modified by QG effect**.

$$\frac{3}{\kappa^2}H^2 = \rho\left(1-\frac{\rho}{\rho_c}\right), \quad \rho_c \sim \rho_P = \frac{M_P}{l_P^3}$$

[Ashtekar et al. (2006)]

The analogy between on cosmological and BH singularity implies the possibility that **gravitational collapse of stars do not lead to singularity but to new phase in a life of stars**.

This phase of star is called "**Planck star**".

[Rovelli, Vidotto (2014)]

Mimicking the QG repulsive force, spherical space-time without singularity can be given as **regular black holes**

$$ds^2 = -F(r)dt^2 + \frac{1}{F(r)}dr^2 + r^2 d\Omega^2, \quad F(r) = 1 - \frac{2mr^2}{r^3 + 2\alpha^2 m}$$

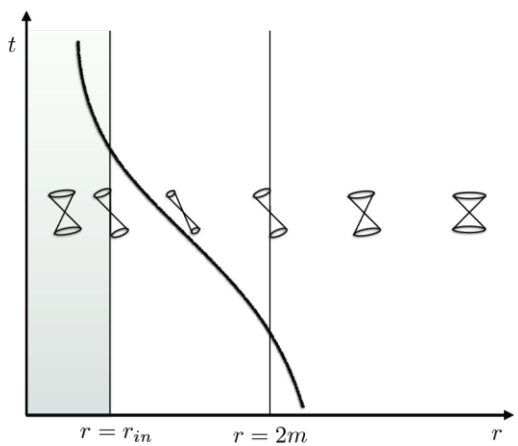
[Hayward (2005)]

When we expand the red-shift factor in $1/r$

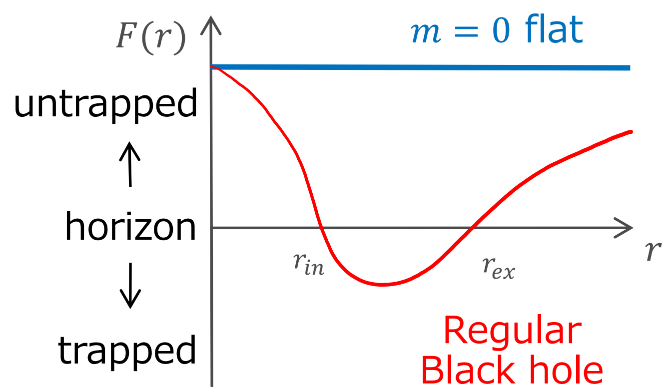
$$F(r) = 1 - \frac{2m}{r} + \frac{4\alpha^2 m^2}{r^4} + \mathcal{O}(r^{-5})$$

Repulsive force by QG effect

- Asymptotically flat with total mass m, $F(r) \sim 1 - \frac{2m}{r}$ as $r \to \infty$
- Flatness at the center r=0, $F(r) \sim 1 - \frac{r^2}{\alpha^2}$ as $r \to 0$ (**de Sitter core**)



[Rovelli, Vidotto (2014)]

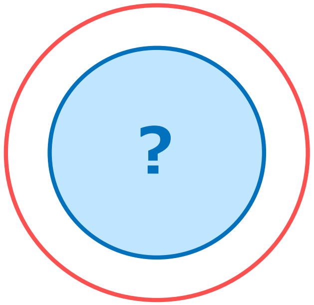


Non-singular Black Holes in Modified gravity

We can study the Planck star with the Born-Infeld gravity by using the similarity in bouncing, taking $b \to l_P^2$.
→ New regular black hole solutions?

In GR, we need non-linear electromagnetic source to explain the regular black hole models.
However, it may be possible that we obtain the **regular black hole in modified gravity** without non-linear source because of **the non-linearity of gravitational action.**

[Olmo and Garcia (2012)] etc.



We need to study
- Structure inside the BH horizon
- The EoS inside of central matter
- Behavior before/after the bounce

We go back to EOMs.

$$\sqrt{|\det(P)|}\,\left(P^{-1}\right)^{\mu\nu} - \sqrt{|\det(g)|}g^{\mu\nu} = -b\kappa^2\sqrt{|\det(g)|}\,T^{\mu\nu}$$

$$\nabla_\lambda\left(\sqrt{|\det(P)|}\,\left(P^{-1}\right)^{\mu\nu}\right) = 0$$

Second EoM implies that **connection is constructed by $P_{\mu\nu}$ (metric postulate for new "metric" $P_{\mu\nu}$)**

$$\Gamma^{\rho}{}_{\mu\nu}(P) = \frac{1}{2}\left(P^{-1}\right)^{\rho\lambda}\left(\partial_\nu P_{\mu\lambda} + \partial_\mu P_{\nu\lambda} - \partial_\lambda P_{\mu\nu}\right)$$

Thus, we can change the dynamical variables from $(g_{\mu\nu}, \Gamma^{\lambda}{}_{\mu\nu})$ to $(g_{\mu\nu}, P_{\mu\nu})$ and regard the Palatini formalism as "bi-metric" formalism.
Let us consider bi-metric type of theory which restores the Born-Infeld gravity in strong coupling limit (ignore the kinetic terms).

We find the **action which is classically equivalent**

$$S = \frac{1}{2\tilde{\kappa}^2}\int d^4x\sqrt{-g}\left\{R\left(g\right) - 2\Lambda_g\right\} + \frac{1}{2\kappa^2}\int d^4x\sqrt{-P}\left\{R\left(P\right) - 2\Lambda_P\right\}$$
$$+ \frac{1}{2\kappa^2 b}\int d^4x\sqrt{-P}\left(P^{-1}\right)^{\mu\nu} g_{\mu\nu} + S_{matter}\left(g_{\mu\nu}, \phi\right)$$

In strong coupling limit $\tilde{\kappa} \to \infty\,, \quad 2\Lambda_g = \frac{2\tilde{\kappa}^2}{\kappa^2 b}\,, \quad 2\Lambda_P = \frac{2}{b}$

By variating the action, we obtain EoMs for two metric

$$R^{\mu\nu}(g) - \frac{1}{2}R(g)g^{\mu\nu} + \Lambda_g g^{\mu\nu} = \frac{\tilde{\kappa}^2}{\kappa^2 b}\frac{\sqrt{-P}}{\sqrt{-g}}\left(P^{-1}\right)^{\mu\nu} + \tilde{\kappa}^2 T^{\mu\nu}$$

$$R_{\mu\nu}(P) - \frac{1}{2}R(P)P_{\mu\nu} + \Lambda_P P_{\mu\nu} = \frac{1}{b}\left[\frac{1}{2}\left(P^{-1}\right)^{\alpha\beta} g_{\alpha\beta}P_{\mu\nu} - g_{\mu\nu}\right]$$

N.B. Considering the localized matter distribution, we need to **patch BI gravity with GR** because BI gravity is equivalent to GR in vacuum (EM force propagates to infinity)

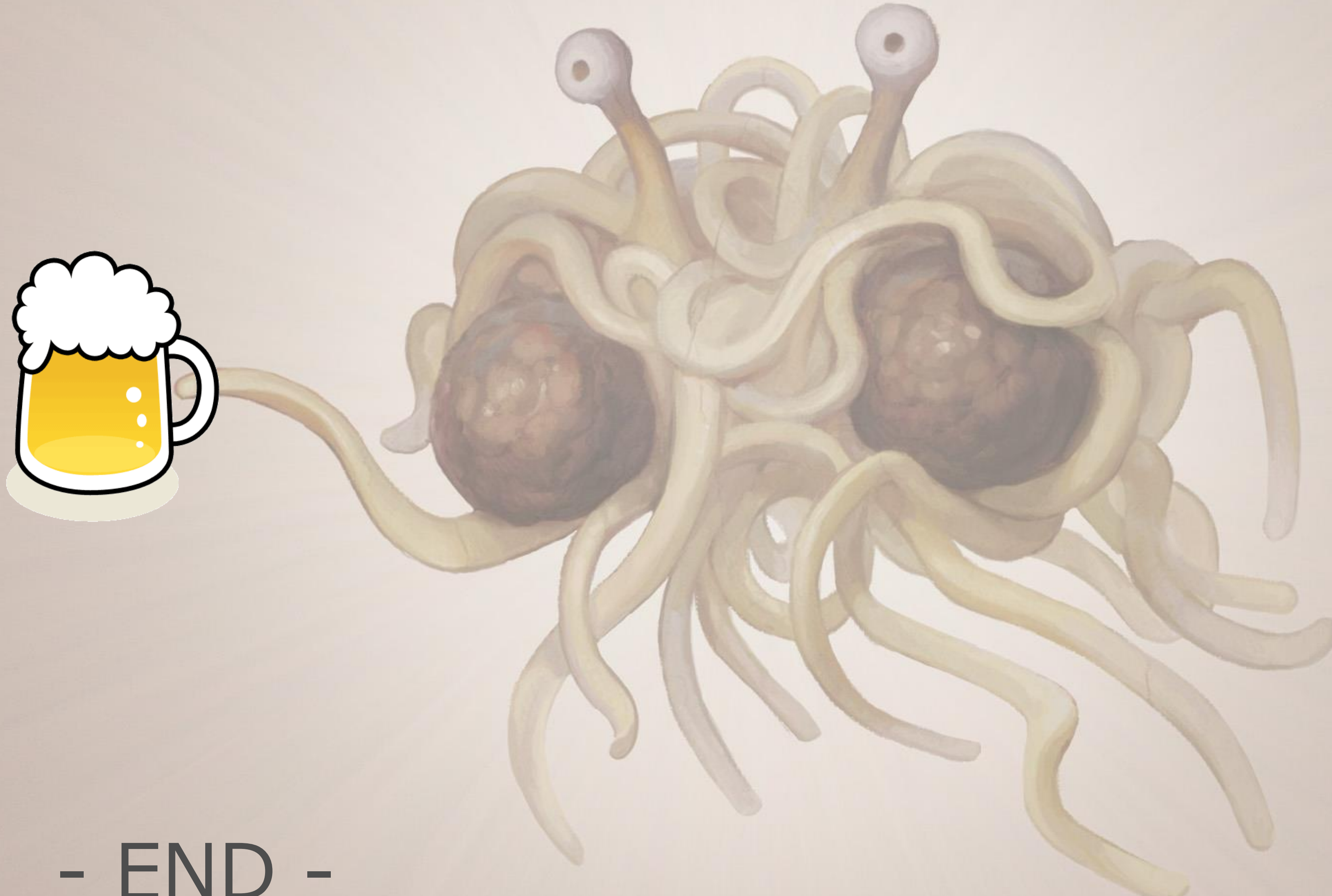
Summary and Discussion

Summary and Discussion

- We investigated the possibility to get rid of singularity in Born-Infeld gravity via Palatini formalism.
- $b > 0$, we found that the **Universe filled with dust will bounce**.
- $b < 0$, we cannot follow all time evolution because of the upper limit of density. (it may be unphysical?)
- We expect the **non-singular spherical solution** as well as bouncing universe.
- If we found a new regular black hole, it would be interesting to regard the Palatini-Born-Infeld gravity as the effective theory of QG and study the Planck star scenario.

Works in progress

We are trying to solve the field equation for non-vanishing pressure and study Mass-Radius relation. If we find the star whose radius is smaller than the Schwarzschild radius, it means non-singular black holes(?)



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Spherical and Static case

Relation btw. two metric

$$P_{\mu\nu} = g_{\mu\nu} + bR_{\mu\nu}(P) \equiv \Sigma_\mu{}^\alpha g_{\alpha\nu}\,, \quad \Sigma_\mu{}^\alpha = \delta_\mu{}^\alpha + bR_{\mu\nu}(P)g^\alpha$$

The EoM becomes

$$R_\mu{}^\nu(P) = \frac{1}{b\sqrt{\det(\Sigma)}}\left[\sqrt{\det(\Sigma)}\delta_\mu{}^\nu - \left(\delta_\mu{}^\nu - b\kappa^2 T_\mu{}^\nu\right)\right]$$

We assume the spherical and static case.

$$g_{\mu\nu}dx^\mu dx^\nu = -f(r)dt^2 + f^{-1}(r)dr^2 + r^2 d\Omega^2\,, \quad f(r) = 1 - \frac{2m(r)}{r}$$

$$T_\mu{}^\nu = \mathrm{diag}\left(-\rho, P, P, P\right)$$

On above assumption, $P_{\mu\nu}$ is given by

$$P_{\mu\nu}dx^\mu dx^\nu = -A^{-1/2}(r)B^{3/2}(r)f(r)dt^2 + A^{1/2}(r)B^{1/2}(r)f^{-1}(r)dr^2 + A^{1/2}(r)B^{1/2}(r)r^2 d\Omega^2$$

$$A(r) = 1 + b\kappa^2\rho\,, \quad B(r) = 1 - b\kappa^2 P$$

Modified TOV equation

From the energy conservation law, we obtain

$$\nabla_\nu T_\mu{}^\nu = 0 \to \frac{f'}{f} = \frac{2B'}{A-B}$$

Therefore, if the pressure P is zero, $f(r)$ is trivial (flat), and we need to consider the matter field with non-vanishing pressure.

This situation is very common and known as TOV equation. We need to solve the modified TOV equation and show that it is possible to form star in Palatini-Born-Infeld gravity

Works in progress

- To solve the TOV equations for certain EoS (e.g. polytrope)
- To assume general ansatz for $g_{\mu\nu}$
- To consider the non-static case for studying the bounce

etc.