Black holes, geons and metric-affine gravity

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Motivations

- Motivations
- Gravitation and Geometry
- Beyond GR: metric-affine gravity
- Conclusions
- The End

- Black Holes (BH's) are promising windows on quantum gravity:
  - BHs were born as a dramatic prediction of GR.
  - Their properties also set the limits of the theory.
  - BHs pose fundamental questions for **Physics**.
  - Exploring them may bring us new insights on high-energy physics.

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  - Their properties also set the limits of the theory.
  - BHs pose fundamental questions for **Physics**.
  - Exploring them may bring us new insights on high-energy physics.
- In this talk we will discuss:
  - Gravitation and geometry: a matter of metrics or something else?
  - Black hole structure in non-Riemannian spaces.
  - Physical characterization of BH singularities:

    geodesic incompleteness - Vs - curvature pathologies

  - Two examples of how BH singularities can be removed in metric-affine space-times.

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Gravitation and Geometry
- Gravity as a geometric phenomenon
- Topology change
- Lessons from the lab
- Effective geometry of crystals

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Gravitation and Geometry

Gravity as a geometric phenomenon

- The Einstein Equivalence Principle tells us that gravitation is a curved space-time phenomenon.
    - It tells us that matter fields couple to a metric.
    - It tells nothing about the form of the gravity Lagrangian.

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- Obviously, whether the space-time geometry is Euclidean, Riemannian, or something else, is a question that must be answered by experiments.
  - Experiments provide information about a certain range of energies and length scales.
  - The kind of effective geometries that arise close to the scales typically attributed to quantum gravity could be very different from traditional **Riemannian structures** in which GR is framed.

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- This question is as fundamental as the number of space-time dimensions or the existence of supersymmetry, for instance.

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- This question is as fundamental as the number of space-time dimensions or the existence of supersymmetry, for instance.
- The phenomenology of gravitation in non-Riemannian geometries has been poorly investigated.
  - Is there any good reason to explore non-Riemannian geometries???

Topology change and space-time foam

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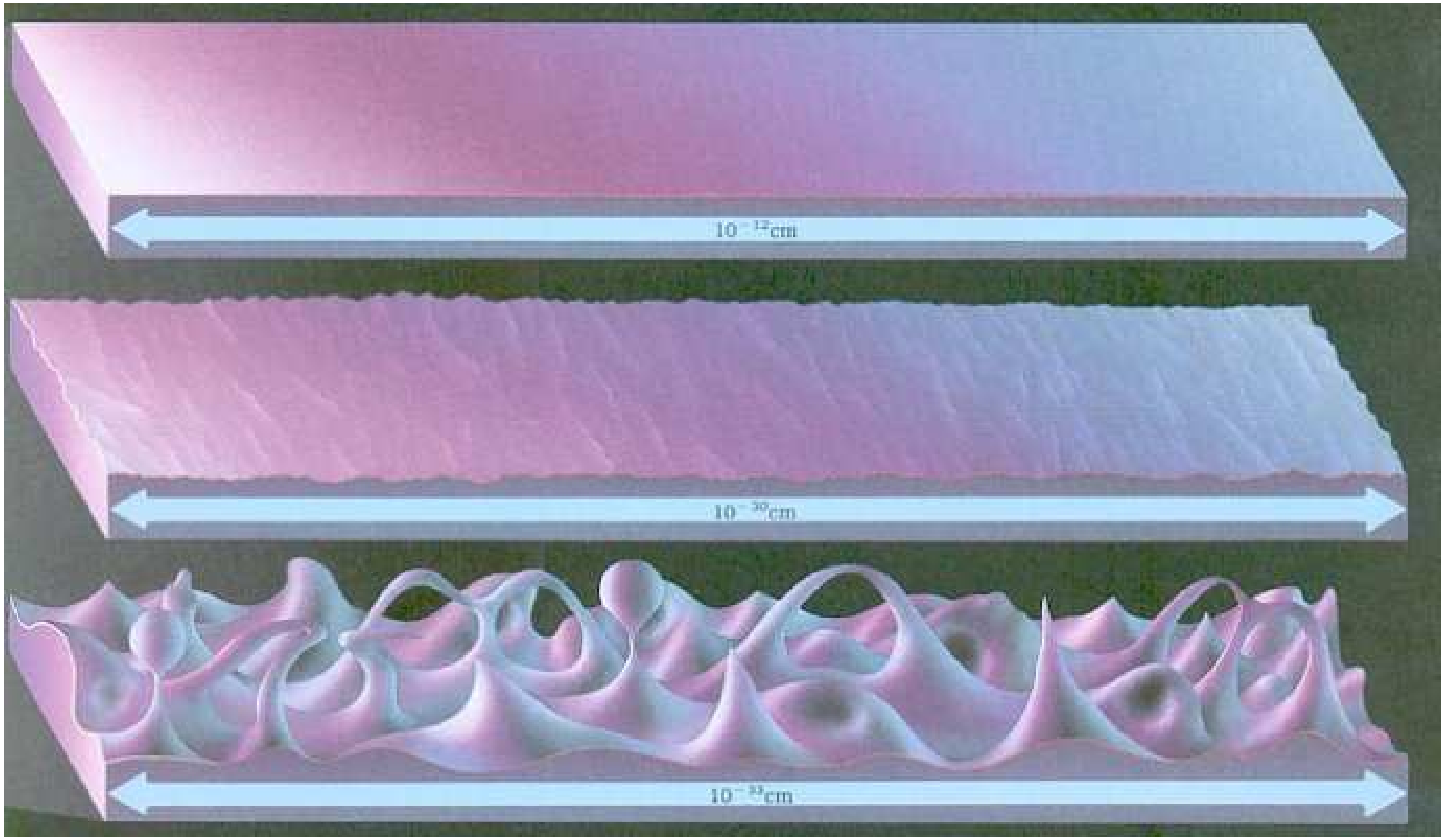
- If topology change could occur dynamically ($l_P \sim 10^{-35}$ m , $t_P \sim 10^{-44}$ s):
  - The smoothness of Minkowski space disappears at Planckian scales.
  - Quantum fluctuations would lead to creation/annihilation of wormholes.
  - Fluxes through wormholes appear as pairs of elementary particles.



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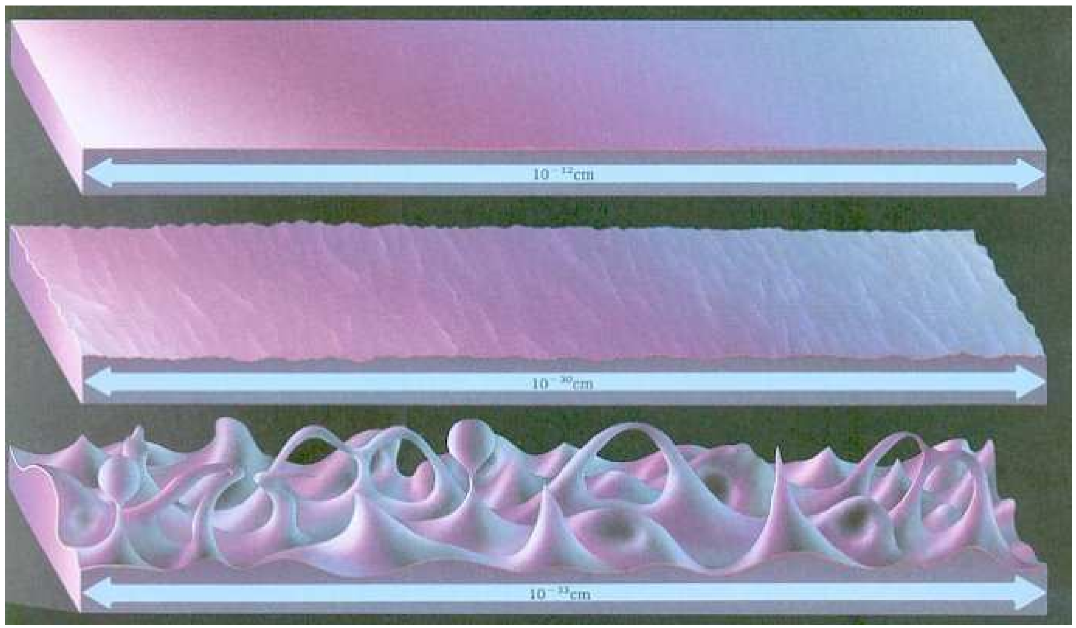

- A microstructure with holes and other topological defects raises questions:
  - What kind of framework should we use to describe this scenario?
  - What properties should those effective geometries have?
  - What are their relevant degrees of freedom?

Lessons from crystalline structures

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- A microstructure with topological defects and a macroscopic continuum limit:
  - Is what the idea of space-time foam suggests.
  - Is what we find in ordered structures such as Bravais crystals, graphene, ...

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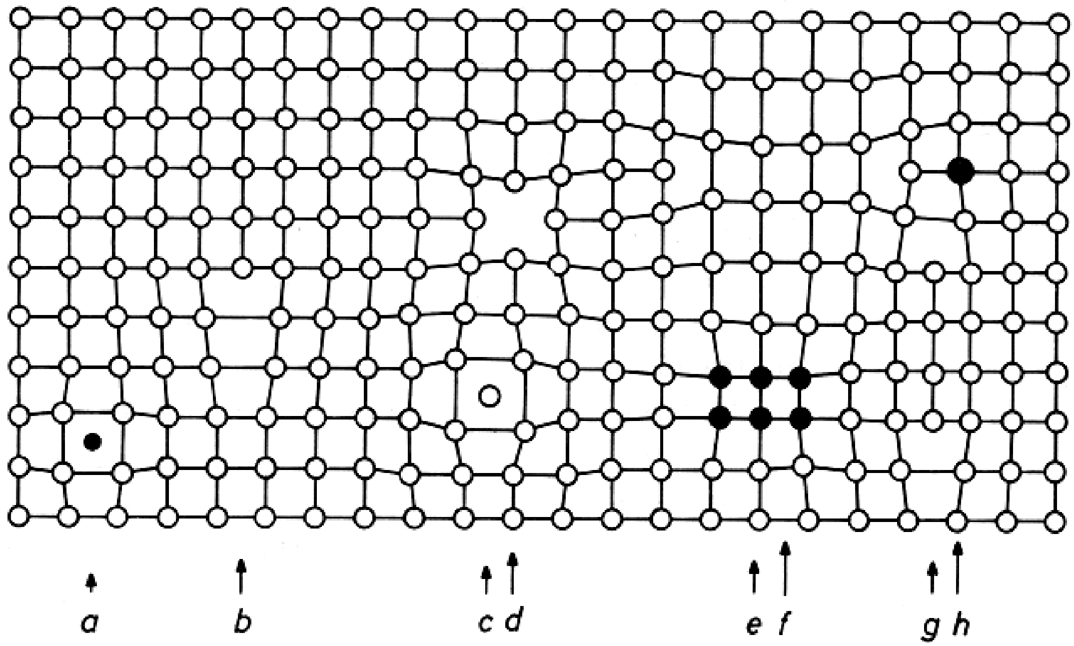
- Crystalline structures may have different kinds of defects:



a) Interstitial impurity atom, b) Edge dislocation, c) Self interstitial atom
d) Vacancy, e) Precipitate of impurity atoms, f) Vacancy type dislocation loop,
g) Interstitial type dislocation loop, h) Substitutional impurity atom

  - In real crystals, the density of defects is generally non-zero.
  - There are interactions between different kinds of defects.

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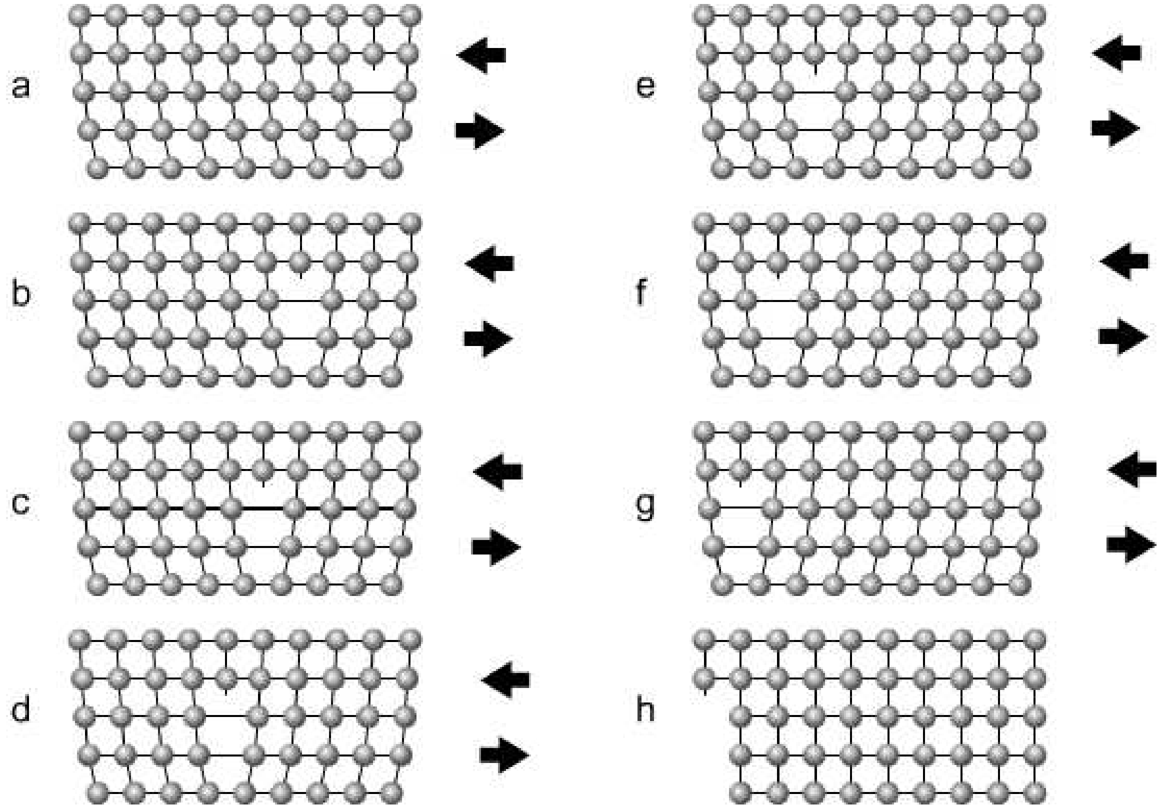

  - Defects have dynamics.
  - Upon the action of forces or heat, defects can move and interact.

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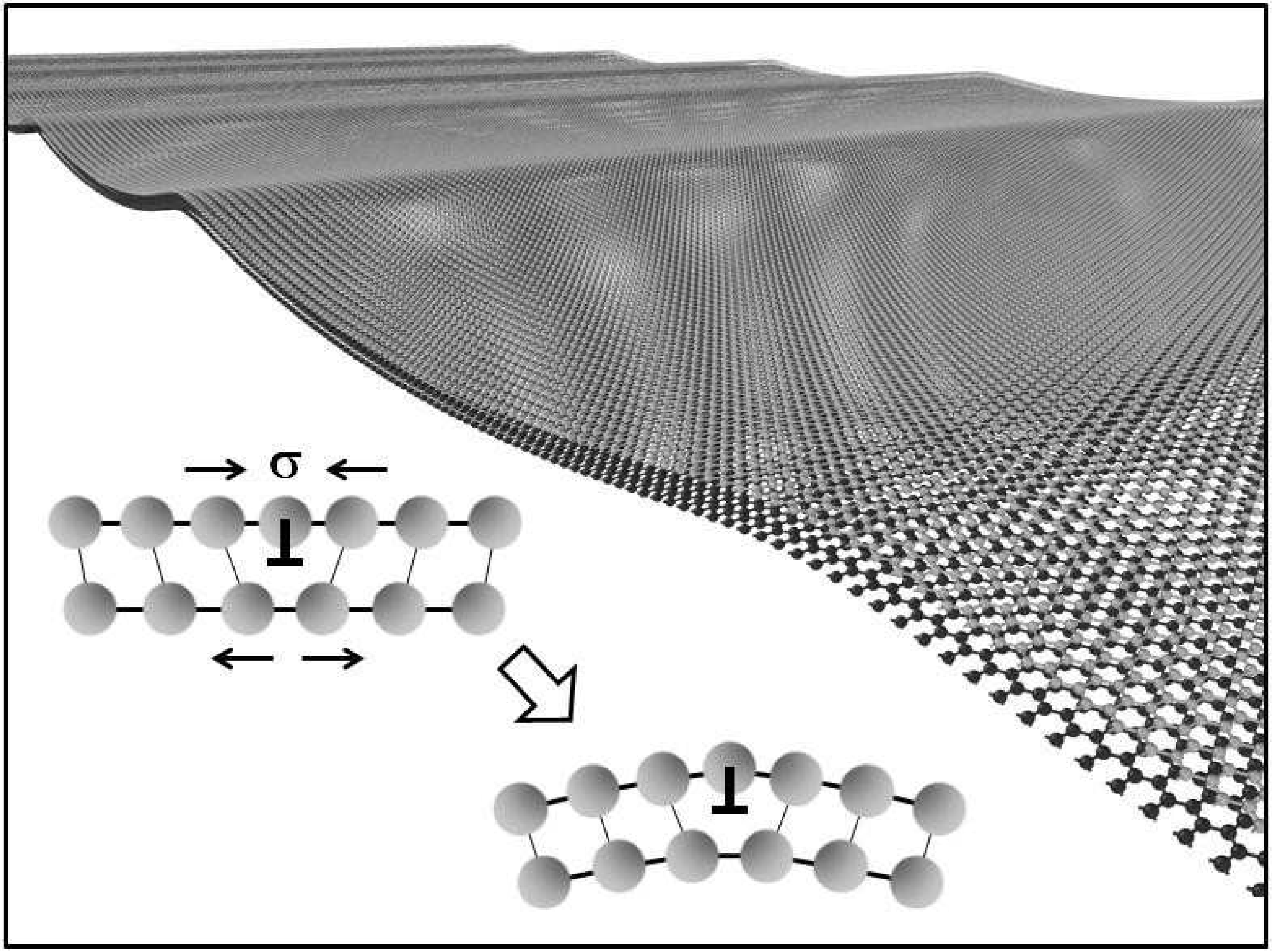
Effective geometry of crystals

- Microscopic structures may yield a continuum effective geometry:



  - Wave propagation in bilayer graphene.

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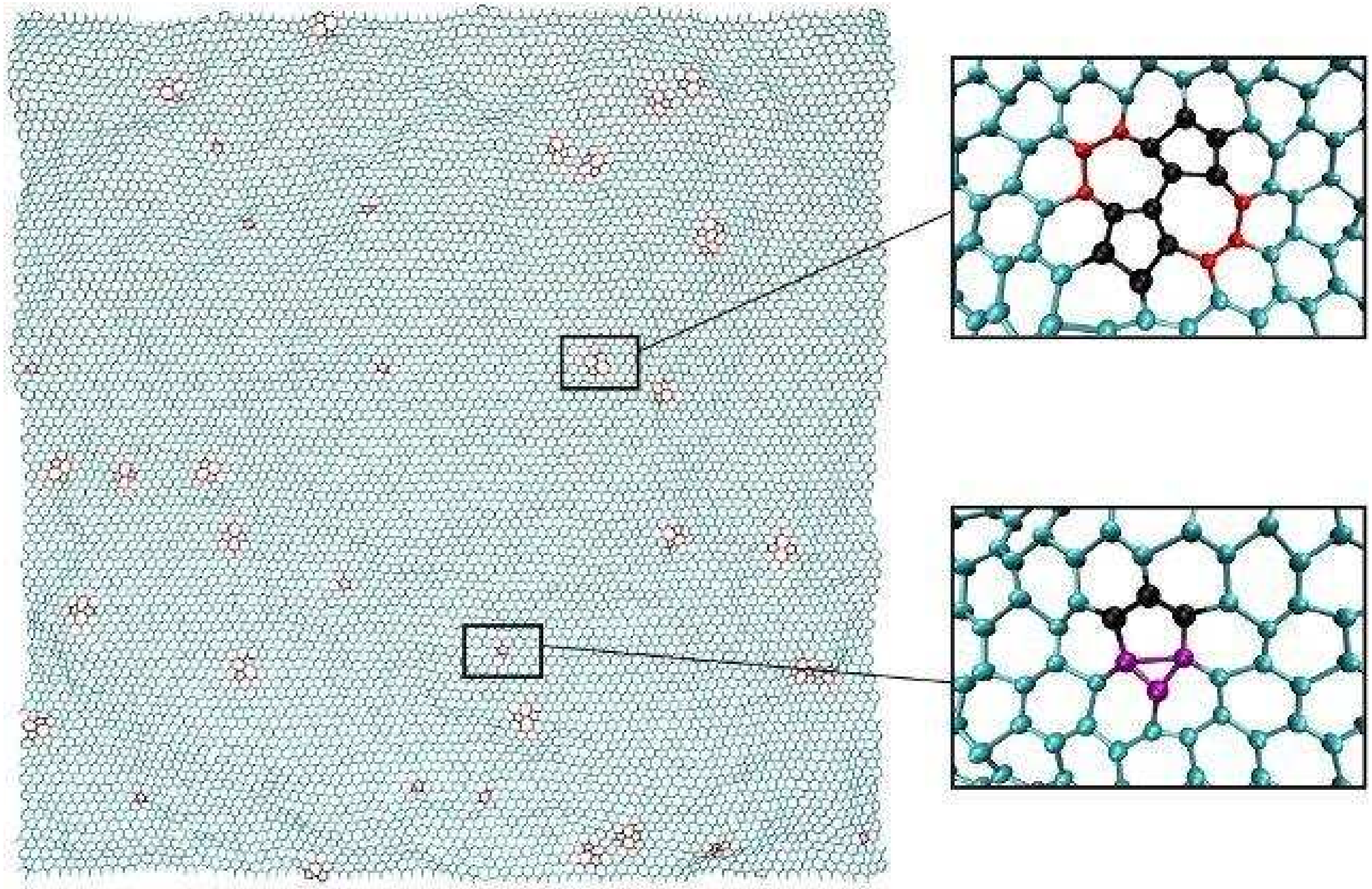

  - Microscope image of a graphene layer with defects.

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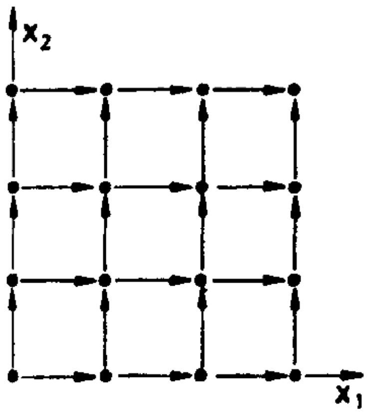
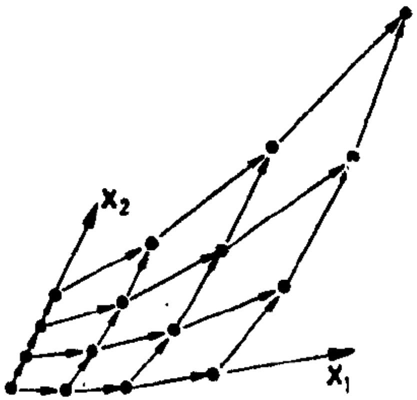
Effective geometry of crystals

- Microscopic structures may yield a continuum effective geometry:
- The continuum limit of these structures is most naturally described in terms of differential geometry:
  - At each point we find 2 or 3 lattice vectors defining the microstructure.
  - Moving along those vectors we jump from atom to atom.
  - Distances can be measured by step counting along lattice vectors.
  - $ds^2 = g_{ij}dx^i dx^j$, with $g_{ij} = \delta_{ij}$ and $\Gamma^a_{bc} = 0$ in suitable coordinates.



Ideal crystal



Perfect (deformed) crystal

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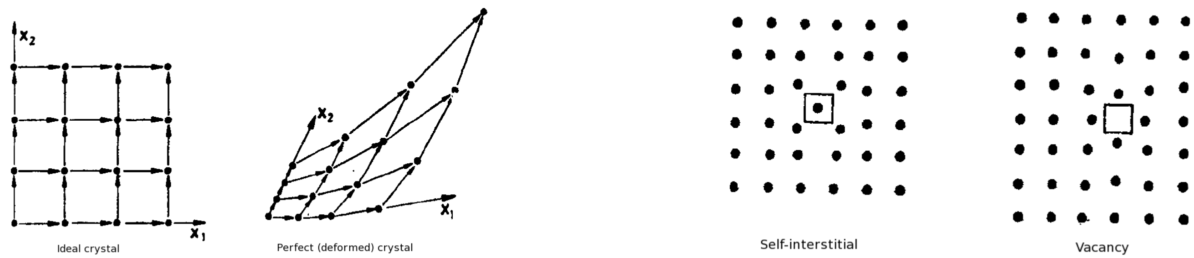

- However, the step-counting procedure breaks down with point defects:
  - An auxiliary idealized metric structure is necessary: $g^{Phys}_{\mu\nu} = D_\mu{}^\alpha h^{Aux}_{\alpha\nu}$
  - $D_\mu{}^\alpha$ depends on the density of defects. Non-metricity: $\nabla_\mu g_{\alpha\beta} \neq 0$

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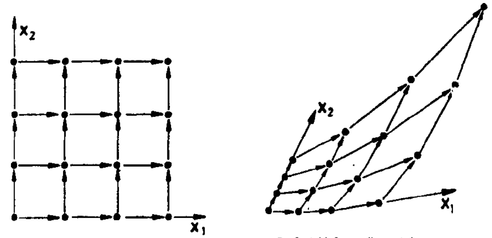

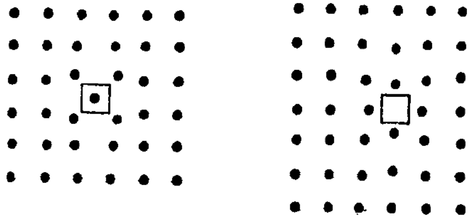

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- Dislocations are the microscopic realization of torsion: $T^\alpha_{\beta\gamma} = \Gamma^\alpha_{\beta\gamma} - \Gamma^\alpha_{\gamma\beta}$

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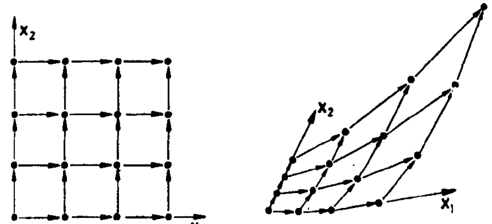

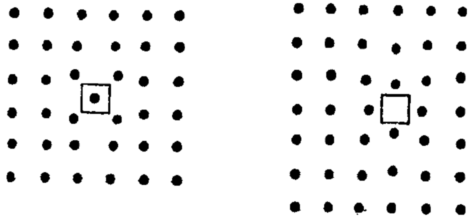

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- Dislocations are the microscopic realization of torsion: $T^\alpha_{\beta\gamma} = \Gamma^\alpha_{\beta\gamma} - \Gamma^\alpha_{\gamma\beta}$
- Independent $g_{\mu\nu}$ and $\Gamma^\alpha_{\beta\gamma}$ are necessary to account for microscopic defects.

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Gravitation and Geometry

Beyond GR: metric-affine gravity
- Need to go beyond GR
- Metric-affine gravity
- Three gravity models
- BHs with charge
- Wormhole structure
- Curvature
- Geodesics in Born-Infeld
- BH structure in f(R)
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  - At curvatures of order or above the Planck scale, $\sim 1/l_P^2$, the quantum degrees of freedom of the gravitational field should no longer be neglected.
  - Quantum gravity is expected to replace classical GR at these scales and cure the issues of space-time singularities.

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- Effective geometric descriptions can also be considered:
  - QFT in curved space-times and string theories suggest that GR should be supplemented with R^2 and $R_{\mu\nu}R^{\mu\nu}$ terms at higher energies.
  - Such extensions, however, are typically affected by ghost instabilities and higher-order equations $\Rightarrow$ **undesirable features**.

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  - Such extensions, however, are typically affected by ghost instabilities and higher-order equations $\Rightarrow$ **undesirable features**.
- Lessons from condensed matter physics:
  - The effective geometry of crystals is non-Riemannian. Rather, it requires a metric-affine description. Properties such as elasticity and plasticity are intimately related to Einstein's equations in 3D.
  - What impact could these geometries have on gravitation?
  - How do **black holes** look like in metric-affine quadratic gravity?

Metric-affine gravity

- In the metric-affine (or Palatini) formalism, one assumes that $g_{\mu\nu}$ and $\Gamma^{\alpha}_{\beta\gamma}$ are independent entities: $S = \int d^n x \sqrt{-g} L[g_{\mu\nu}, \Gamma^{\alpha}_{\beta\gamma}] + S_{matter}[g_{\mu\nu}, \psi_m]$

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- The field equations follow from variation of the action:
  - Palatini approach:

$$\delta S = \int d^n x \left[\sqrt{-g} \left(\frac{\delta L}{\delta g^{\mu\nu}} - \frac{L}{2} g_{\mu\nu} \right) \delta g^{\mu\nu} + \sqrt{-g} \frac{\delta L}{\delta \Gamma^{\alpha}_{\beta\gamma}} \delta \Gamma^{\alpha}_{\beta\gamma} \right] + \delta S_{matter}$$

$$\delta g^{\mu\nu} \Rightarrow \frac{\delta L}{\delta g^{\mu\nu}} - \frac{L}{2} g_{\mu\nu} = 8\pi G T_{\mu\nu}$$

$$\delta \Gamma^{\alpha}_{\beta\gamma} \Rightarrow \frac{\delta L}{\delta \Gamma^{\alpha}_{\beta\gamma}} = 0$$ (assuming no coupling of Γ to the matter)

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(assuming no coupling of Γ to the matter)

  - Metric approach:

The relation $\delta\Gamma^\alpha_{\beta\gamma}=\frac{g^{\alpha\rho}}{2}\left[\nabla_\beta\delta g_{\rho\gamma}+\nabla_\gamma\delta g_{\rho\beta}-\nabla_\rho\delta g_{\beta\gamma}\right]$ implies

$$\frac{\delta L}{\delta\Gamma^\alpha_{\beta\gamma}}\delta\Gamma^\alpha_{\beta\gamma}=\left\{g^{\alpha\mu}\frac{\delta L}{\delta\Gamma^\alpha_{\lambda\nu}}-\frac{g^{\alpha\lambda}}{2}\frac{\delta L}{\delta\Gamma^\alpha_{\mu\nu}}\right\}\nabla_\lambda\delta g_{\mu\nu}$$

and leads to

$$\delta g^{\mu\nu}\Rightarrow \left(\frac{\delta L}{\delta g^{\mu\nu}}-\frac{L}{2}g_{\mu\nu}\right)+\nabla_\lambda\left[g_{\gamma\nu}\frac{\delta L}{\delta\Gamma^\mu_{\lambda\gamma}}-g_{\beta\mu}g_{\gamma\nu}g^{\alpha\lambda}\frac{\delta L}{\delta\Gamma^\alpha_{\beta\gamma}}\right]=8\pi G T_{\mu\nu}$$

- Metric and **Palatini** variations generally lead to different field equations.

Three gravity models

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- In our discussion we will consider three different models à la Palatini.
  - A simple quadratic model: $f(R) = R - l_\varepsilon^2 R^2$
  - A not so simple quadratic model: $f(R,Q) = R + l_\varepsilon^2 \left(aR^2 + bR_{\mu\nu}R^{\mu\nu}\right)$
  - A Born-Infeld like model: $\frac{1}{\kappa^2\varepsilon}\left(\sqrt{-|g_{\mu\nu}+\varepsilon R_{\mu\nu}|} - \sqrt{-|g_{\mu\nu}|}\right)$
  - With $R_{\mu\nu} = R^{\rho}{}_{\mu\rho\nu}$, and $R^{\alpha}{}_{\beta\mu\nu} = \partial_\mu \Gamma^{\alpha}_{\nu\beta} - \partial_\nu \Gamma^{\alpha}_{\mu\beta} + \Gamma^{\alpha}_{\mu\lambda}\Gamma^{\lambda}_{\nu\beta} - \Gamma^{\alpha}_{\nu\lambda}\Gamma^{\lambda}_{\mu\beta}$

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- In these theories, the connection can always be formally solved in such a way that one obtains a set of modified second-order equations for the metric $g_{\mu\nu}$.

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- In these theories, the connection can always be formally solved in such a way that one obtains a set of modified second-order equations for the metric $g_{\mu\nu}$.
- For electrically charged black holes, the last two models yield exactly the same equations and solutions if $\epsilon = -2l_\epsilon^2$.

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Conclusions

The End

- In our discussion we will consider three different models à la Palatini.
  - A simple quadratic model: $f(R) = R - l_\varepsilon^2 R^2$
  - A not so simple quadratic model: $f(R,Q) = R + l_\varepsilon^2 \left(aR^2 + bR_{\mu\nu}R^{\mu\nu}\right)$
  - A Born-Infeld like model: $\frac{1}{\kappa^2\varepsilon}\left(\sqrt{-|g_{\mu\nu}+\varepsilon R_{\mu\nu}|} - \sqrt{-|g_{\mu\nu}|}\right)$
  - With $R_{\mu\nu} = R^\rho{}_{\mu\rho\nu}$, and $R^\alpha{}_{\beta\mu\nu} = \partial_\mu \Gamma^\alpha_{\nu\beta} - \partial_\nu \Gamma^\alpha_{\mu\beta} + \Gamma^\alpha_{\mu\lambda}\Gamma^\lambda_{\nu\beta} - \Gamma^\alpha_{\nu\lambda}\Gamma^\lambda_{\mu\beta}$
- In these theories, the connection can always be formally solved in such a way that one obtains a set of modified second-order equations for the metric $g_{\mu\nu}$.
- For electrically charged black holes, the last two models yield exactly the same equations and solutions if $\varepsilon = -2l_\varepsilon^2$.
- Exact solutions can be found for typical electric fields and also for non-linear theories of electrodynamics.

Three gravity models

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- Exact solutions can be found for typical electric fields and also for non-linear theories of electrodynamics.
- We will just focus on the solutions. Details on the equations can be found in the literature. See for instance: arXiv:1509.02430, 1508.03272, 1507.07763

BHs with charge in Born-Infeld gravity

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- Coupling our theory to a static, spherically symmetric electric field one finds:

$$ds^2 = -A(x)dt^2 + \frac{1}{A(x)\sigma_+^2}dx^2 + r^2(x)d\Omega^2$$ with:

  - $\sigma_\pm = 1 \pm \frac{r_c^4}{r^4}$, $A(x) = \frac{1}{\sigma_+}\left[1 - \frac{2M(r)}{r}\frac{1}{\sigma_-^{1/2}}\right]$ and $\left(\frac{dr}{dx}\right)^2 = \frac{\sigma_-}{\sigma_+^2}$,

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  - where $M(r) = M_0 + M_0\delta_1 G(z)$, $\frac{dG}{dz} = \frac{z^4+1}{z^4\sqrt{z^4-1}}$, and $z = \frac{r}{r_c}$.
  - $\delta_1 = \frac{1}{2r_S}\sqrt{\frac{r_q^3}{l_\varepsilon}}$, $r_S = 2M_0$, $r_c^2 = r_q l_\varepsilon$, $r_q^2 = \kappa^2 q^2/4\pi$.

- This problem admits an exact analytical solution.
- For not too small black holes (large mass and/or charge) and/or $r \gg r_c$:
  - GR solution recovered: $A(x) \approx 1 - \frac{r_S}{r} + \frac{r_q^2}{2r^2}$
  - with $\sigma_\pm \approx 1$ and $r(x)^2 \approx x^2$

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  - with $\sigma_\pm \approx 1$ and $r(x)^2 \approx x^2$
- Significant deviations arise as $r \to r_c$, near $x \to 0$.

Wormhole structure

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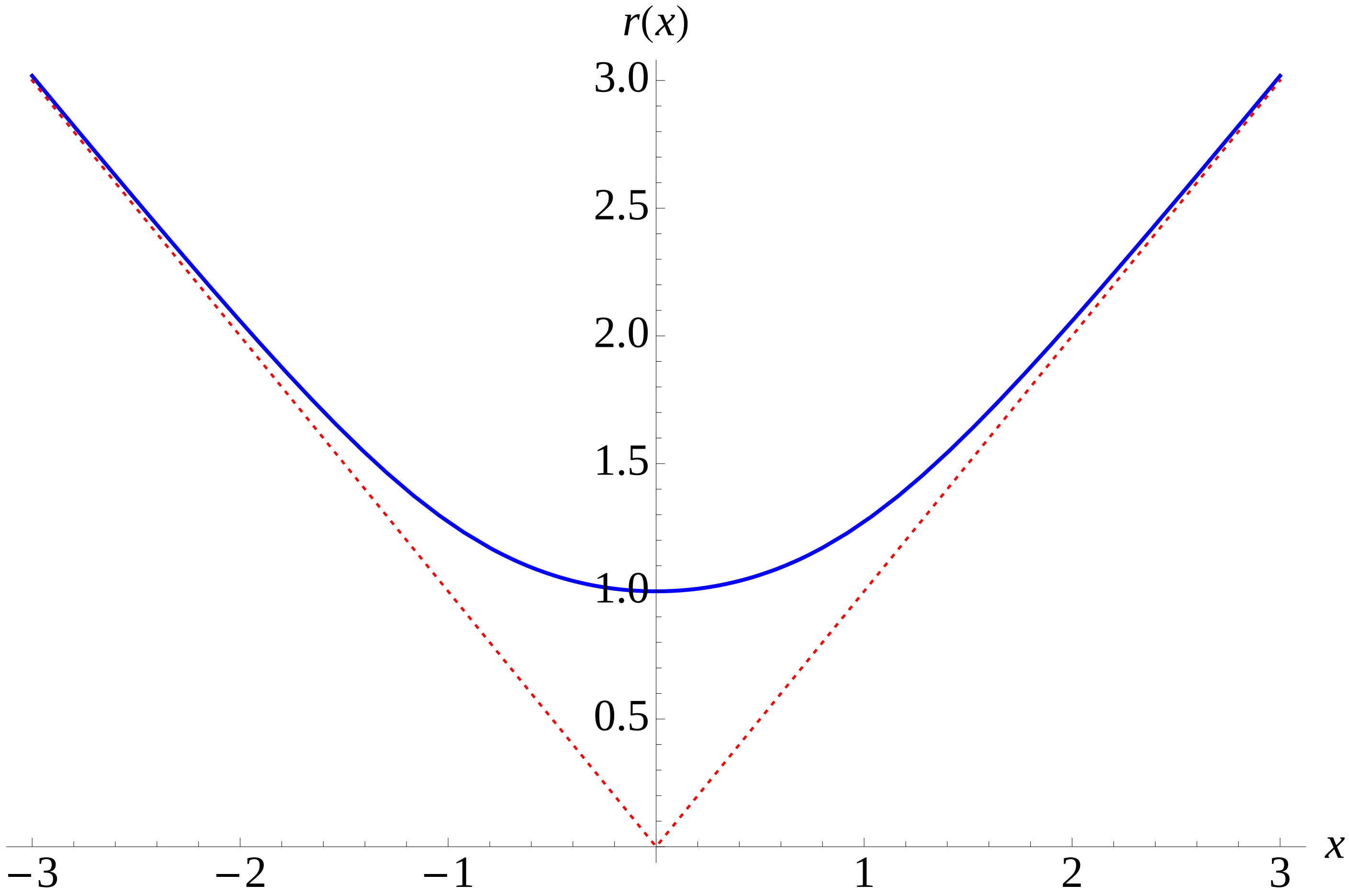
- From $\left(\frac{dr}{dx}\right)^2 = \frac{\sigma_-}{\sigma_+^2}$ we find $r^2(x) = \frac{x^2+\sqrt{x^4+4r_c^4}}{2}$, with a minimum at $x = 0$. This is reminiscent of a wormhole geometry.



Wormhole structure

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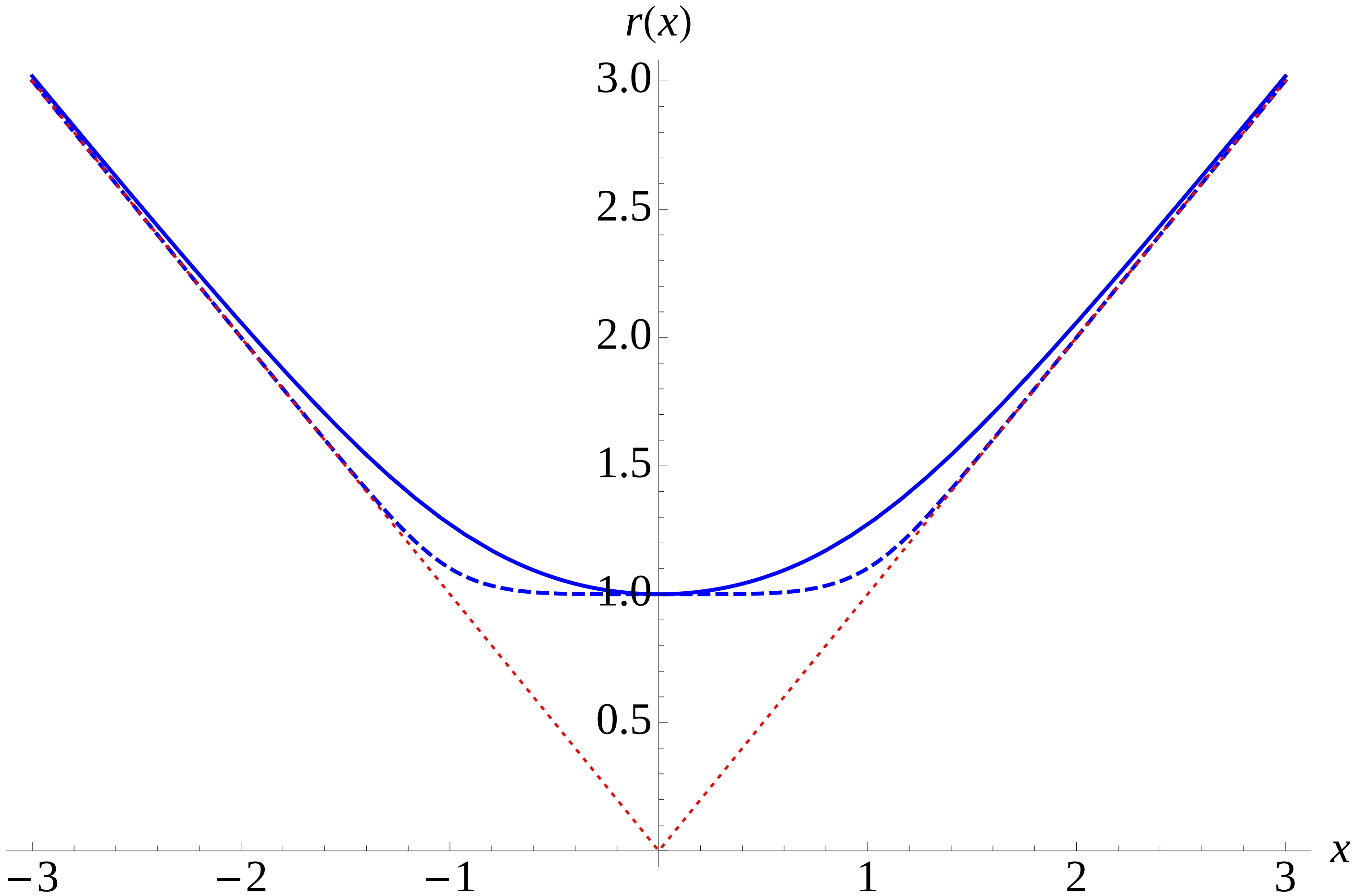
- From $\left(\frac{dr}{dx}\right)^2 = \frac{\sigma_-}{\sigma_+^2}$ we find $r^{D-2}(x) = \frac{|x|^{D-2} + \sqrt{|x|^{2(D-2)} + 4r_c^{2(D-2)}}}{2}$, with a minimum at $x = 0$. This is reminiscent of a wormhole geometry.



- $ds^2 = -A(x)dt^2 + \frac{1}{A(x)\sigma_+^2}dx^2 + r(x)^2 d\Omega^2$. $D = 4$ (solid), $D = 7$ (dashed).

Curvature

- In GR, the Reissner-Nordström solution is characterized by:

$$R_{GR} = 0\,, \qquad Q_{GR} \equiv R_{\mu\nu}R^{\mu\nu} = \frac{r_q^4}{r^8}\,, \qquad K_{GR} \equiv R^{\alpha}{}_{\beta\mu\nu}R_{\alpha}{}^{\beta\mu\nu} = \frac{12 r_S^2}{r^6} - \frac{24 r_S r_q^2}{r^7} + \frac{14 r_q^4}{r^8}$$

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- In our case, defining $r_c \equiv \sqrt{r_q l_P}$ and $r_q^2 = 2Gq^2$, when $r \gg r_c$:

$$R(g) \approx -\frac{48 r_c^8}{r^{10}} + O\left(\frac{r_c^9}{r^{11}}\right), \quad Q(g) \approx \frac{r_q^4}{r^8}\left(1 - \frac{16 l_P^2}{r^2} + \ldots\right), \quad K(g) \approx K_{GR} + \frac{144 r_S r_q^2 l_P^2}{r^9} + \ldots$$

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- But when $z \equiv r/r_c \to 1$: $\left[\,\delta_1 = \frac{1}{2r_S}\sqrt{\frac{r_q^3}{l_P}}\,,\ \delta_2 = \frac{\sqrt{r_q l_P}}{r_S},\ \text{and}\ \delta_c \approx 0.572\,\right]$

$$r_c^2 R(g) \approx \left(-4 + \frac{16\delta_c}{3\delta_2}\right) + O(z-1) + \ldots - \frac{1}{2\delta_2}\left(1 - \frac{\delta_c}{\delta_1}\right)\left[\frac{1}{(z-1)^{3/2}} - O\left(\frac{1}{\sqrt{z-1}}\right)\right]$$

$$r_c^4 Q(g) \approx \left(10 + \frac{86\delta_1^2}{9\delta_2^2} - \frac{52\delta_1}{3\delta_2}\right) + \ldots + \left(1 - \frac{\delta_c}{\delta_1}\right)\left[\frac{6\delta_2 - 5\delta_1}{3\delta_2^2 (z-1)^{3/2}} + \ldots\right] + \left(1 - \frac{\delta_c}{\delta_1}\right)^2\left[\frac{1}{8\delta_2^2 (z-1)^3} - \ldots\right.$$

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If $\delta_1=\delta_c$ then all curvature scalars are finite everywhere !!!

Geodesics in Born-Infeld

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The End

- The equation that governs the evolution of geodesics in this space-time is:

  $$\frac{1}{\sigma_+^2}\left(\frac{dx}{d\lambda}\right)^2 = E^2 - V_{eff} \quad , \text{ with } V_{eff} \equiv \left(\kappa + \frac{L^2}{r^2}\right)A(r) \, .$$

  - Where $\kappa = 0$ for null geodesics and $\kappa = 1$ for time-like geodesics.
  - L^2 and E^2 are the angular momentum and energy per unit mass.

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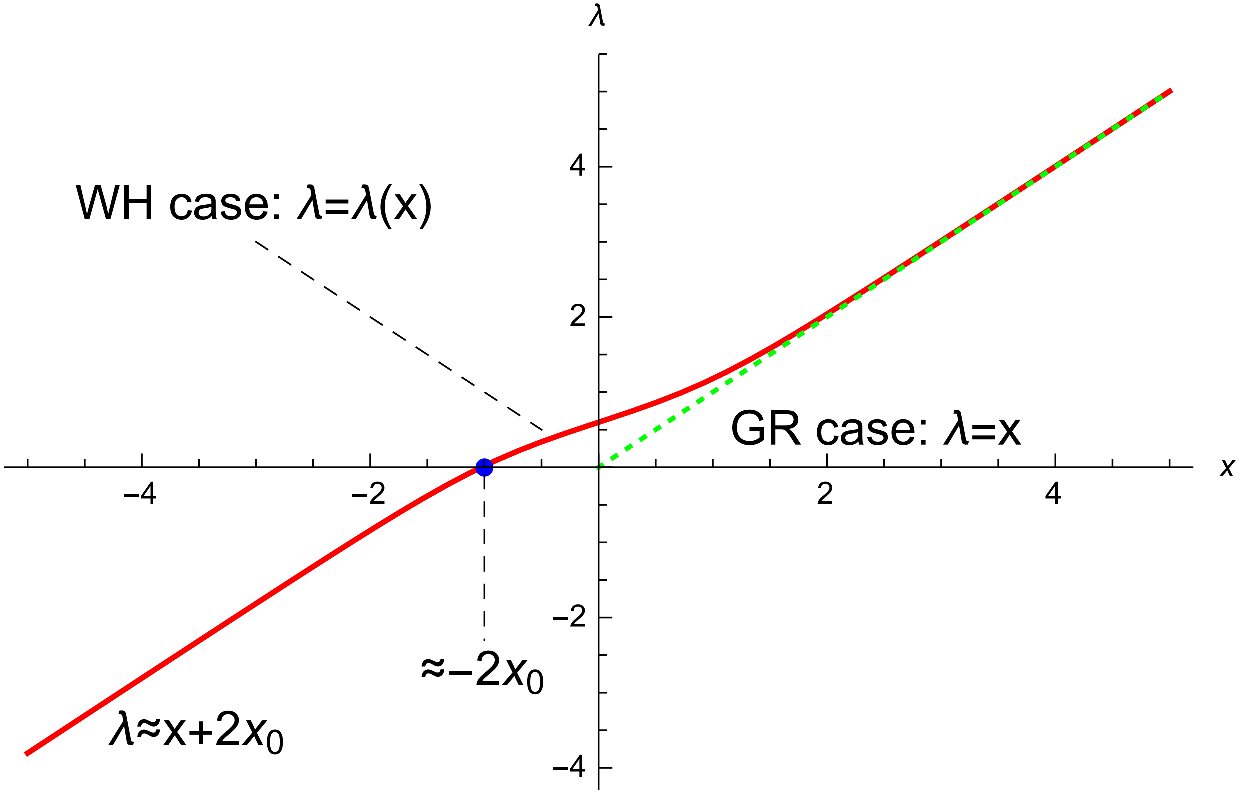

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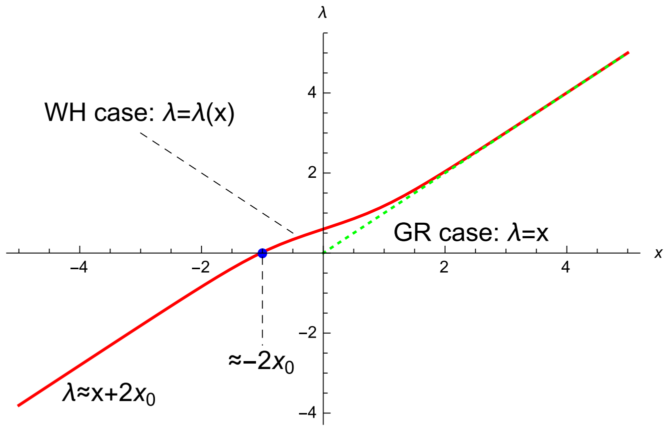

- Null and time-like geodesics with $L \neq 0$: $V_{eff} \approx -\left(\kappa + \frac{L^2}{r_c^2}\right)\frac{N_q(\delta_c - \delta_1) r_c}{2N_c \delta_c \delta_1 |x|}$.
  - WH case: $\lambda(x) \approx \pm \frac{x}{3}\left|\frac{x}{a}\right|^{\frac{1}{2}}$ - Vs - GR case: $\lambda(r) \approx \pm \frac{2}{3} r \left(\frac{r}{r_S}\right)^{\frac{1}{2}}$.
  - Main difference: $x \in]-\infty, +\infty[$ while $r \in [0, \infty[$. Complete - Vs- Incomplete.

BH structure in $f(R) = R - \lambda R^2$

- Consider a fluid like $T_\mu{}^\nu = \text{diag}[-\rho, -\rho, \alpha\rho, \alpha\rho]$, where $\rho(x) = \frac{C}{r(x)^{2+2\alpha}}$.

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  $ds^2 = \frac{1}{f_R}\left(-A(x)dt^2 + \frac{1}{A(x)}dx^2\right) + r^2(x)d\Omega^2$, with $A(x) = 1 - \frac{2M(x)}{x}$ and
  - $M(x) = M_0(1 + \delta_1 G(z))$, $\delta_1 = \frac{r_c^3}{8\lambda M_0}$, $z = \frac{r}{r_c}$ and $G_z = \frac{\left(1+\frac{\alpha}{z^{2+2\alpha}}\right)}{z^{2\alpha} f_R^{3/2}}\left[\frac{1}{1-\alpha} - \frac{1}{2z^{2+2\alpha}}\right]$

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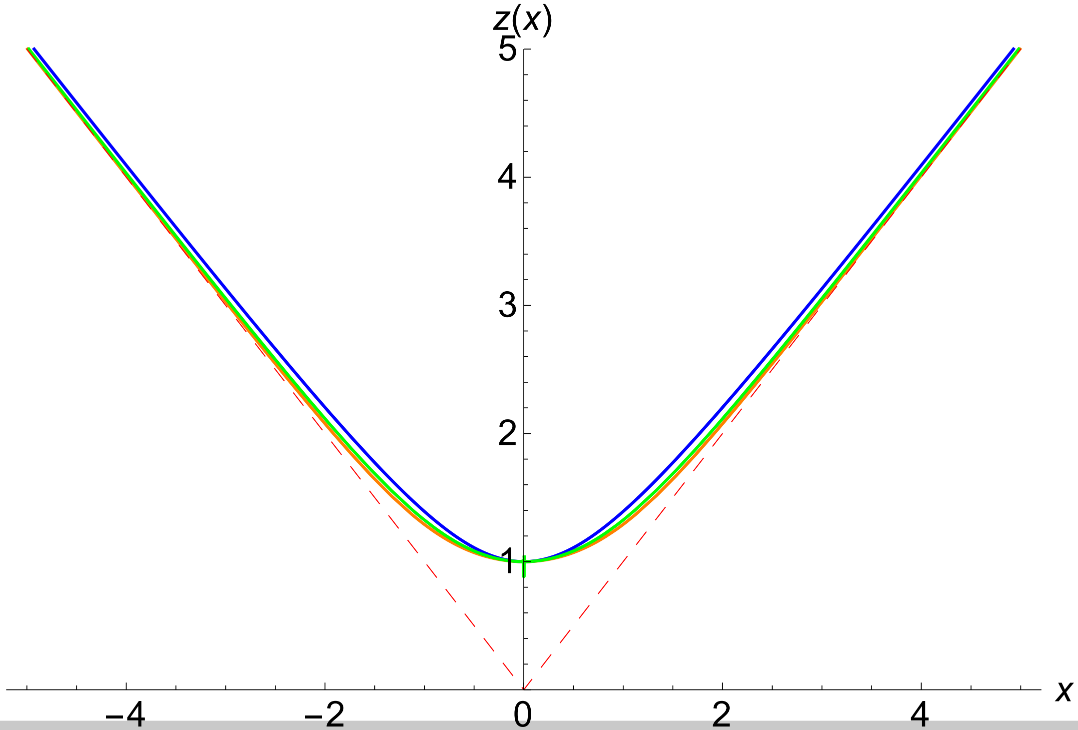
- These solutions have wormhole structure (here $\alpha = 1/10, 1/2, 4/5$):



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- The relevant equation for geodesics in $f(R)$ theories is

$$\frac{1}{f_R^2}\left(\frac{dx}{d\lambda}\right)^2 = E^2 + g_{tt}\left(k + \frac{L^2}{r^2(x)}\right) \quad ,$$

with $g_{tt} \approx -\frac{\delta_1}{8\delta_2(1-\alpha^2)}\frac{1}{(z-1)^2}$ as $z \to 1$.

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- For time-like geodesics, $\frac{dx}{d\lambda} = 0$ before reaching the wormhole at $x = 0$.
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- Need to go beyond GR
- Metric-affine gravity
- Three gravity models
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- Wormhole structure
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- BH structure in f(R)
- Geodesics in f(R)

Conclusions

The End

Geodesics in $f(R) = R - \lambda R^2$

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- The relevant equation for geodesics in $f(R)$ theories is

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$$\pm E\lambda(z) = -\frac{z}{\sqrt{1-z^{-2(\alpha+1)}}} + 2z\, {}_2F_1\left(\frac{1}{2}, -\frac{1}{2(\alpha+1)}; 1 - \frac{1}{2(\alpha+1)}; z^{-2(\alpha+1)}\right)$$

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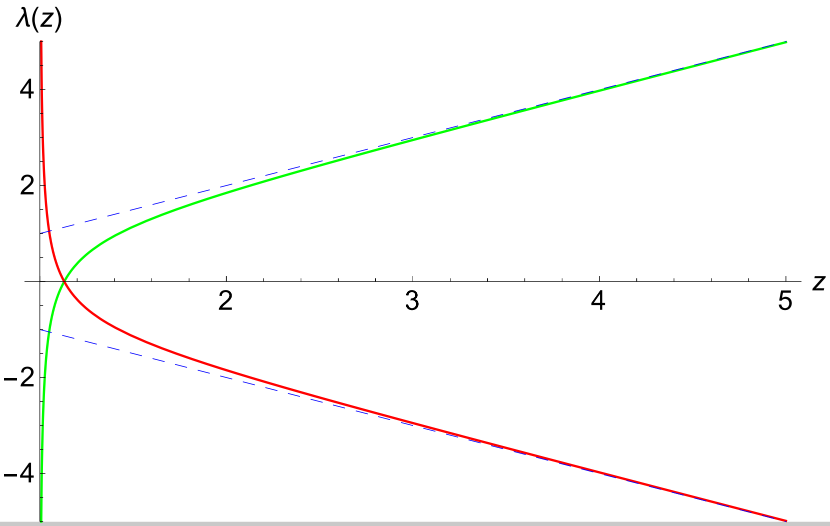

- As $z \to \infty$: $\pm E\lambda(z) \approx z \approx x$.
- As $z \to 1$:

$$\pm E\lambda(z) \approx -\frac{1}{\sqrt{2\alpha + 2\sqrt{z-1}}} \approx -\frac{1}{|\tilde{x}|} .$$

- Geodesically complete space.

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The **avoidance of singularities** can be achieved with simple models in 4D classical geometric scenarios.
What are the implications for quantum gravity?

Thank you !!!