Hot big bang in a Cold Gas

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January 2024

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I Carusotto, G Lamporesi, A Recati, G Ferrari arXiv:2305.05225

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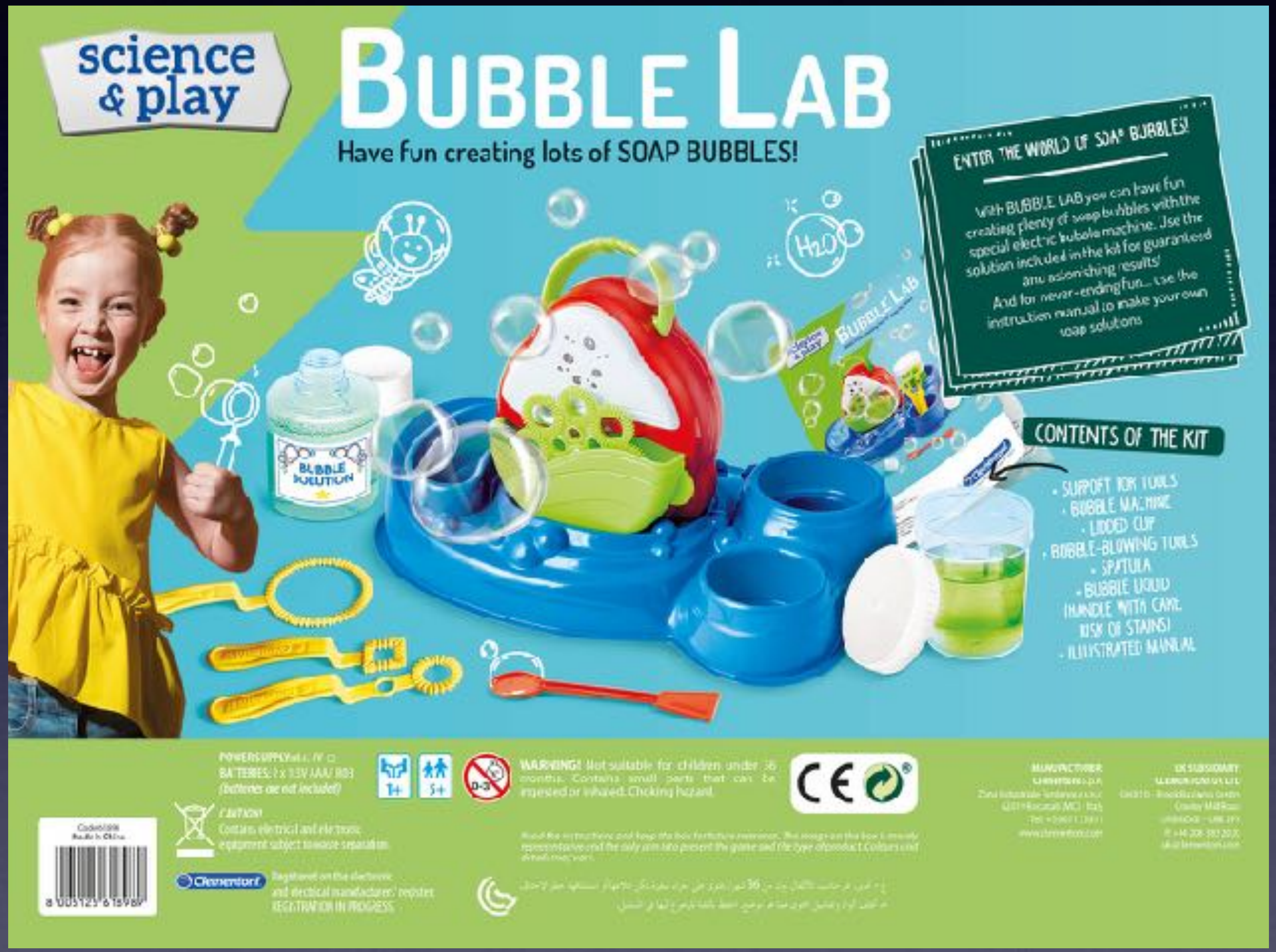
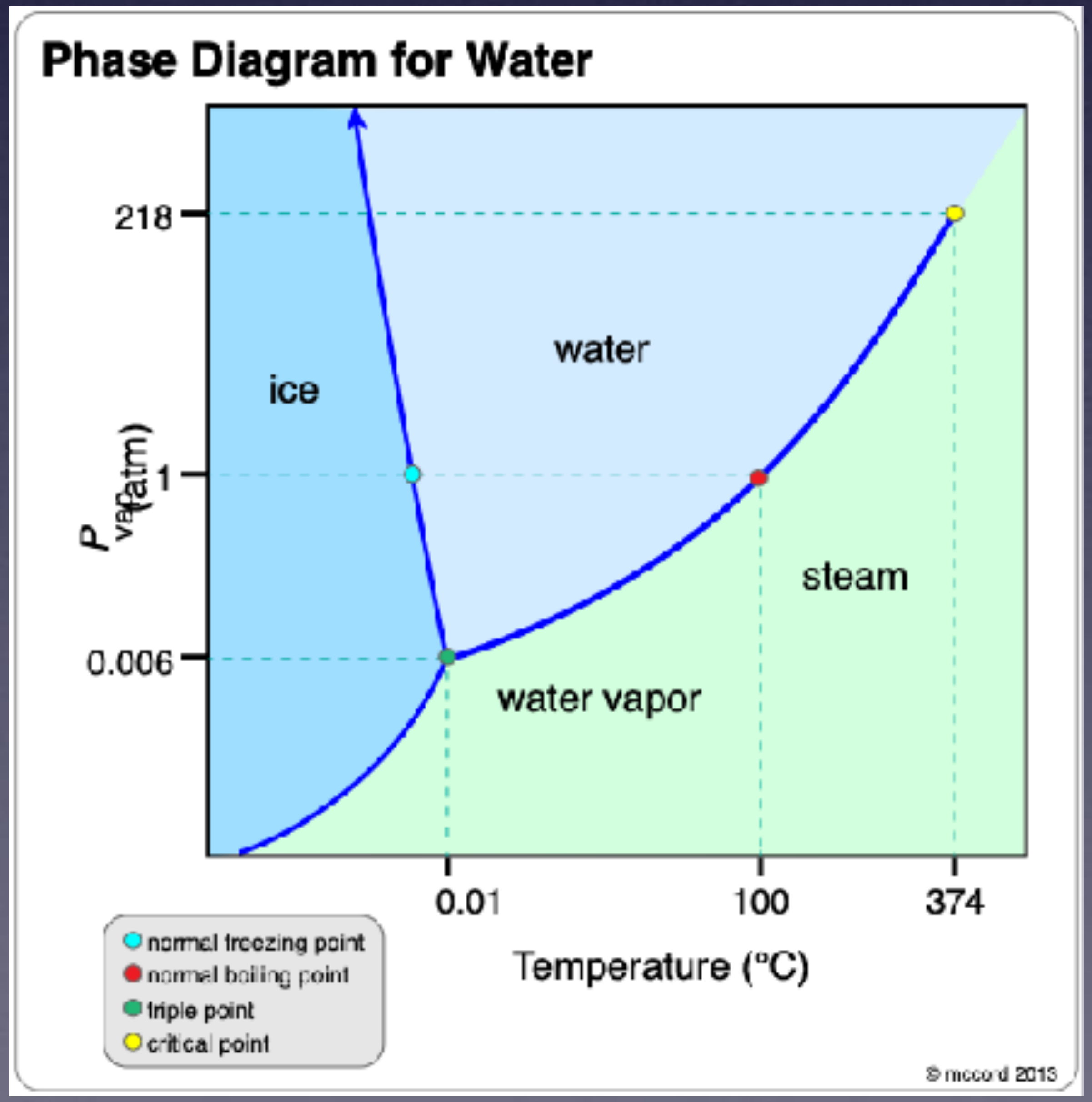
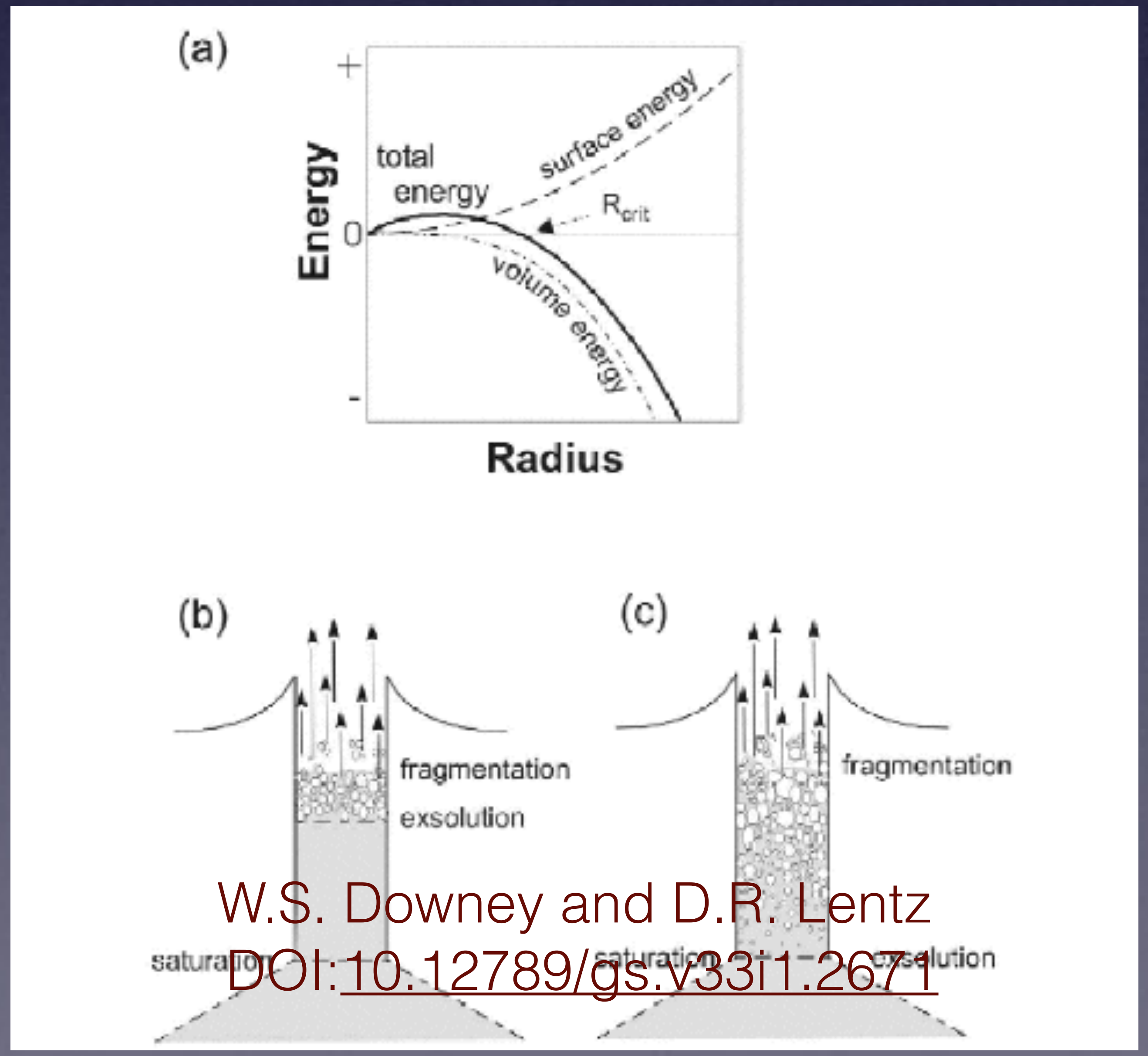


Bubble nucleation

Cooling into a metastable state → Escape by bubble (or crystal) formation





W.S. Downey and D.R. Lentz
DOI:10.12789/gs.v33i1.2671

Bubbles in QFT

bubble nucleation in quantum field theory

application: early universe cosmology

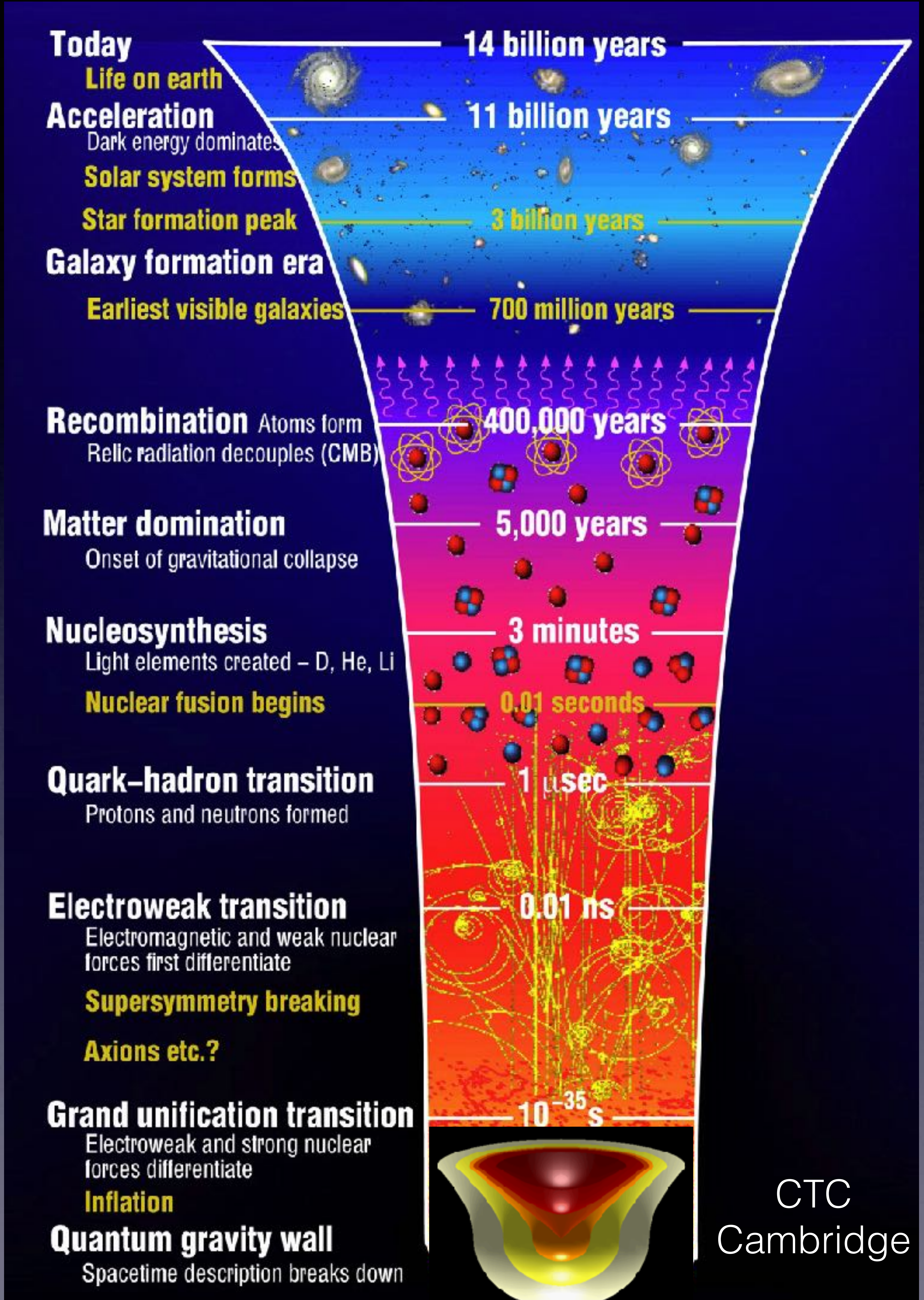
gravitational waves

baryogenesis

black holes

metaverse

universe from nothing

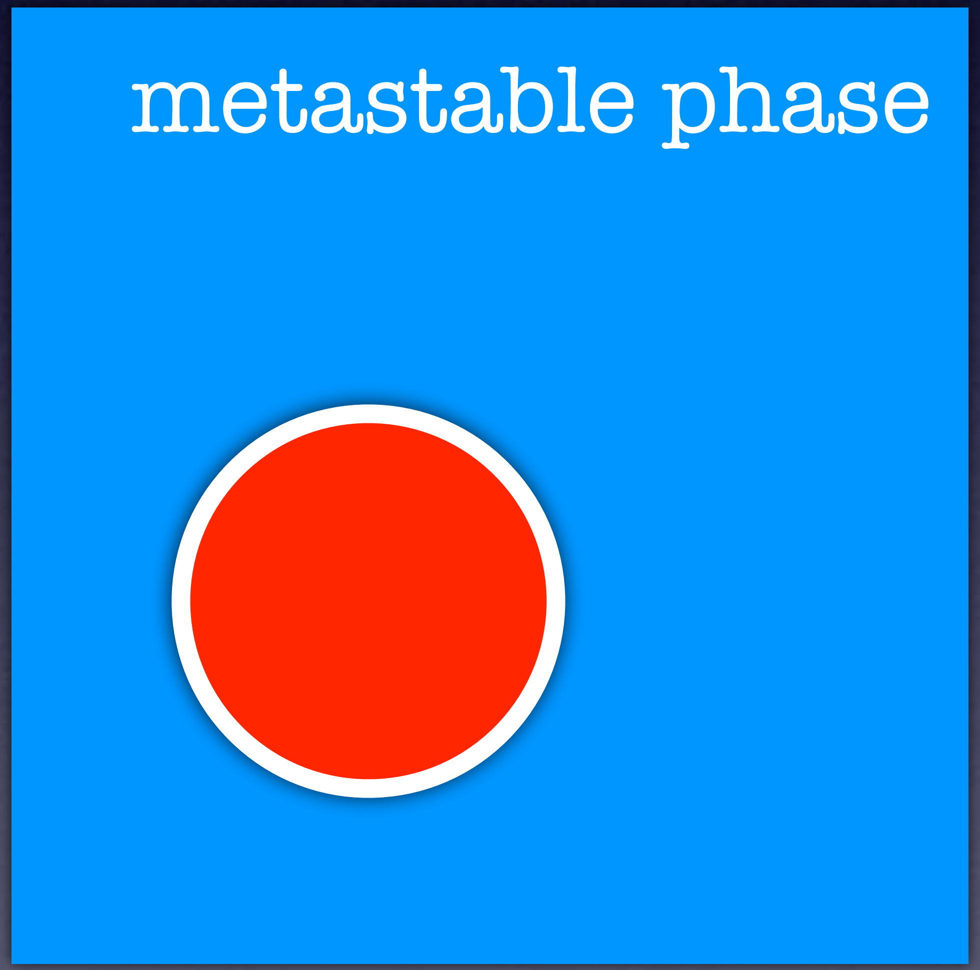


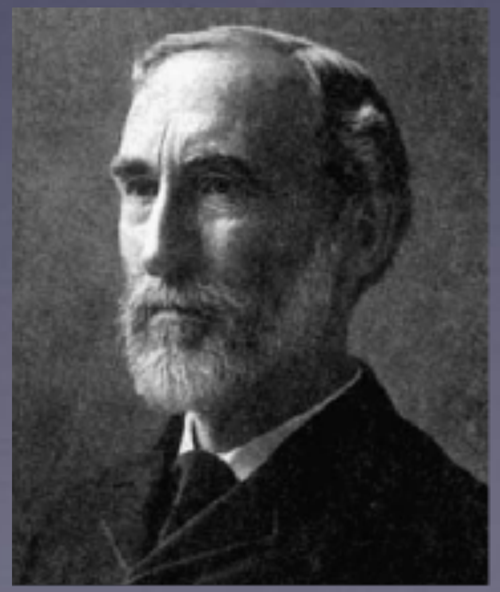
Critical Nucleation Theory

Critical bubbles in a thermal metastable state: Josiah Gibbs 1878!

Probability of a bubble fluctuation $\propto e^{\Delta S/k_B}$

Entropy changes by $\Delta S = -\dfrac{\Delta E}{T}$

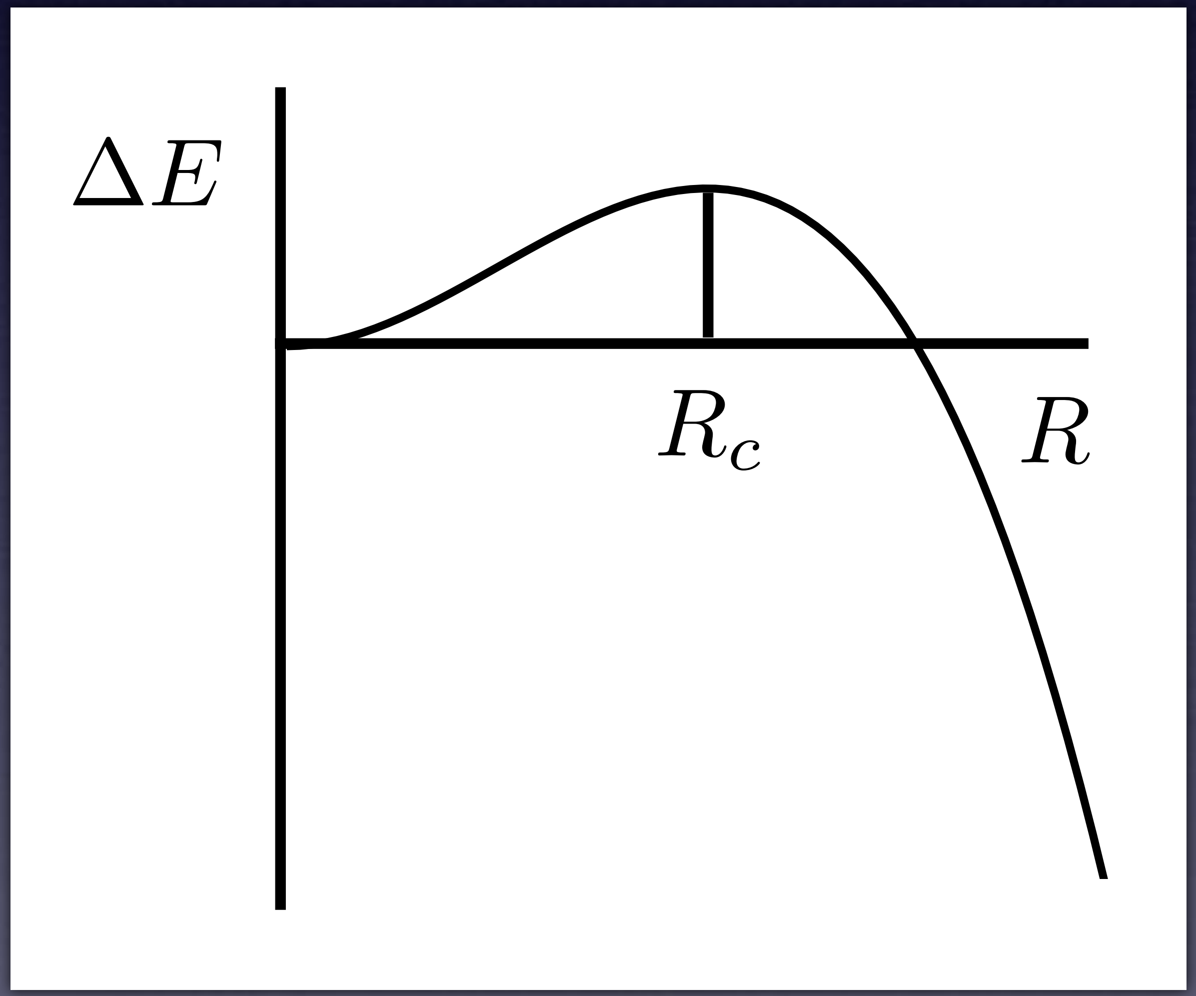




J. W Gibbs, Equilibrium of heterogeneous substances, 1878, p416

Critical Nucleation Theory

$$\Delta E = 4\pi R^2 \sigma - \frac{4}{3}\pi R^3 \epsilon$$



σ surface tension

ϵ energy density

Critical case $R_c = \frac{2\sigma}{\epsilon}$

$\Delta S/k_B$

Nucleation rate $$\Gamma = \frac{N}{R_c^3}\frac{k_B T}{h}\left(\frac{\Delta E_c}{k_B T}\right)^{1/2} e^{-\Delta E_c/k_B T}$$

Lifshitz and Pitaevski, Statistical Physics

Frenkel, Kinetic Theory of Liquids 1947

False vacuum decay

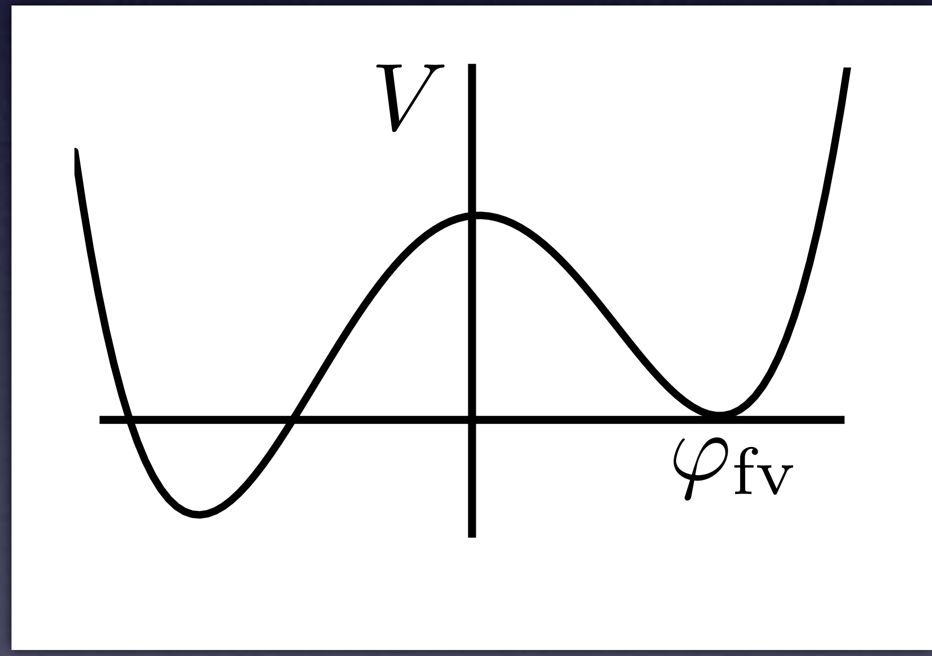
In quantum field theory, replace the bubble by a ‘bounce’ instanton, a solution to the field equations with **imaginary** time τ.

$$S_E = \int \left\{ \frac{1}{2} \nabla\varphi^2 + V(\varphi) \right\}$$

local potential minimum

$\varphi_{\rm fv}$

$$\beta = \hbar / k_B T$$



false vacuum decay

$$\varphi = \varphi(\sqrt{\mathbf{x}^2 + \tau^2})$$

false vacuum decay
at finite temperature

$$\varphi = \varphi(x)$$

Coleman, Phys Rev D15 2929 1977

False vacuum decay rate

product of the eigenvalues

excluded zero modes (translations)

$$\Gamma = \left| \frac{\det' S''_E[\varphi_b]}{\det S''_E[\varphi_{fv}]} \right|^{-1/2} \left(\frac{S_E[\varphi_b]}{2\pi\hbar} \right)^{n/2} e^{-S_E[\varphi_b]/\hbar}$$

exists?
singular?

redistribute some of the vacuum polarisation?

renormalisation?

inertial frame?

correlations?

does some other process dominate vacuum decay?

big difference between QFT and QM of single particles

Real time approaches

Stochastic nucleation: nucleate bubbles at random and evolve (semi-)classically. Assumes bubble nucleation rate is known.

Hindmarsh,Weir,Gould

Stochastic sources: add a random source term and dissipation to the classical equations.

Suited to relatively high temperature simulations

Alford,Feldman,Gleiser,1993 Billam,Brown,Moss, 2020

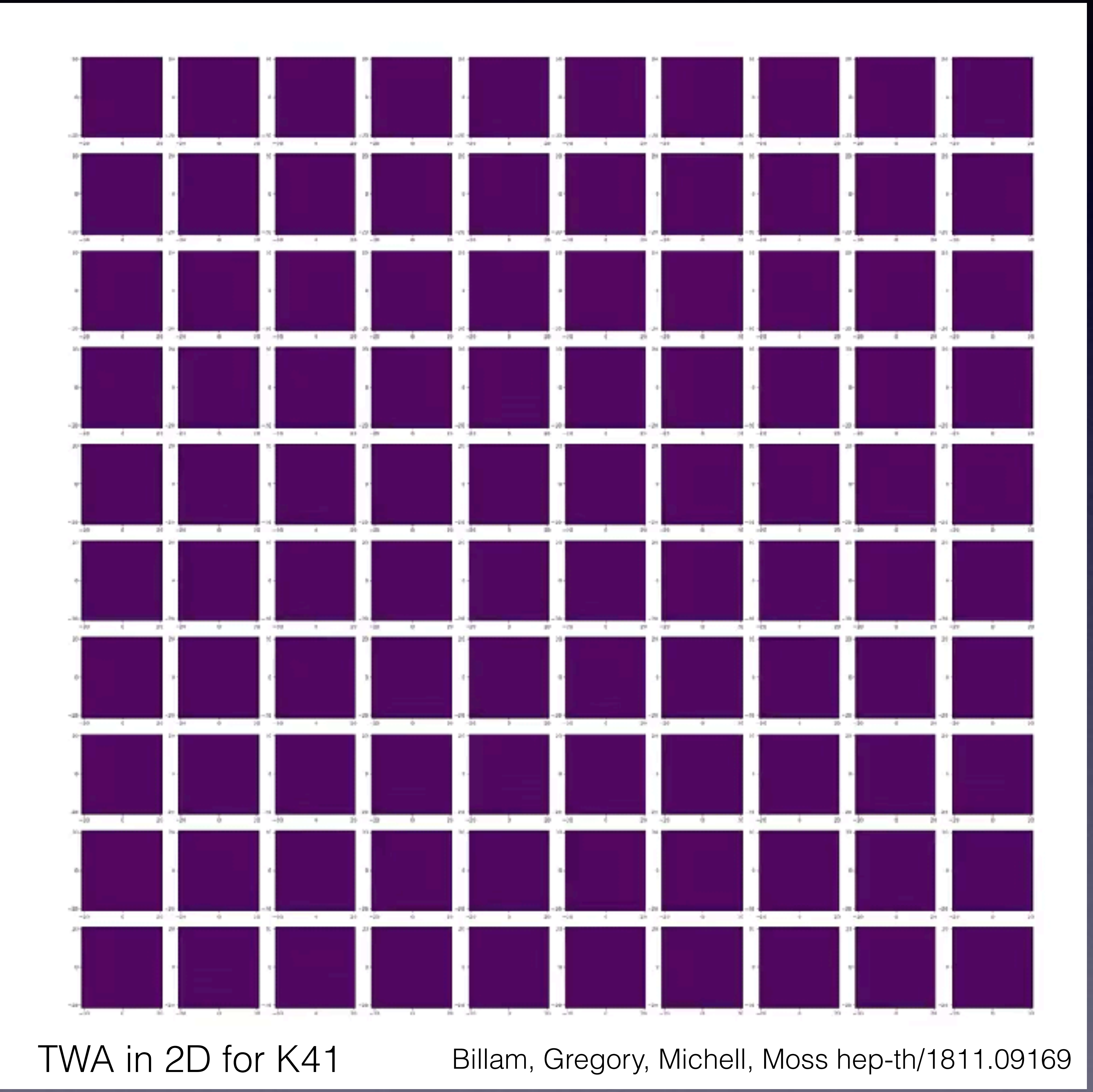
Truncated Wigner: draw initial conditions at random from a Wigner distribution and evolve classically.

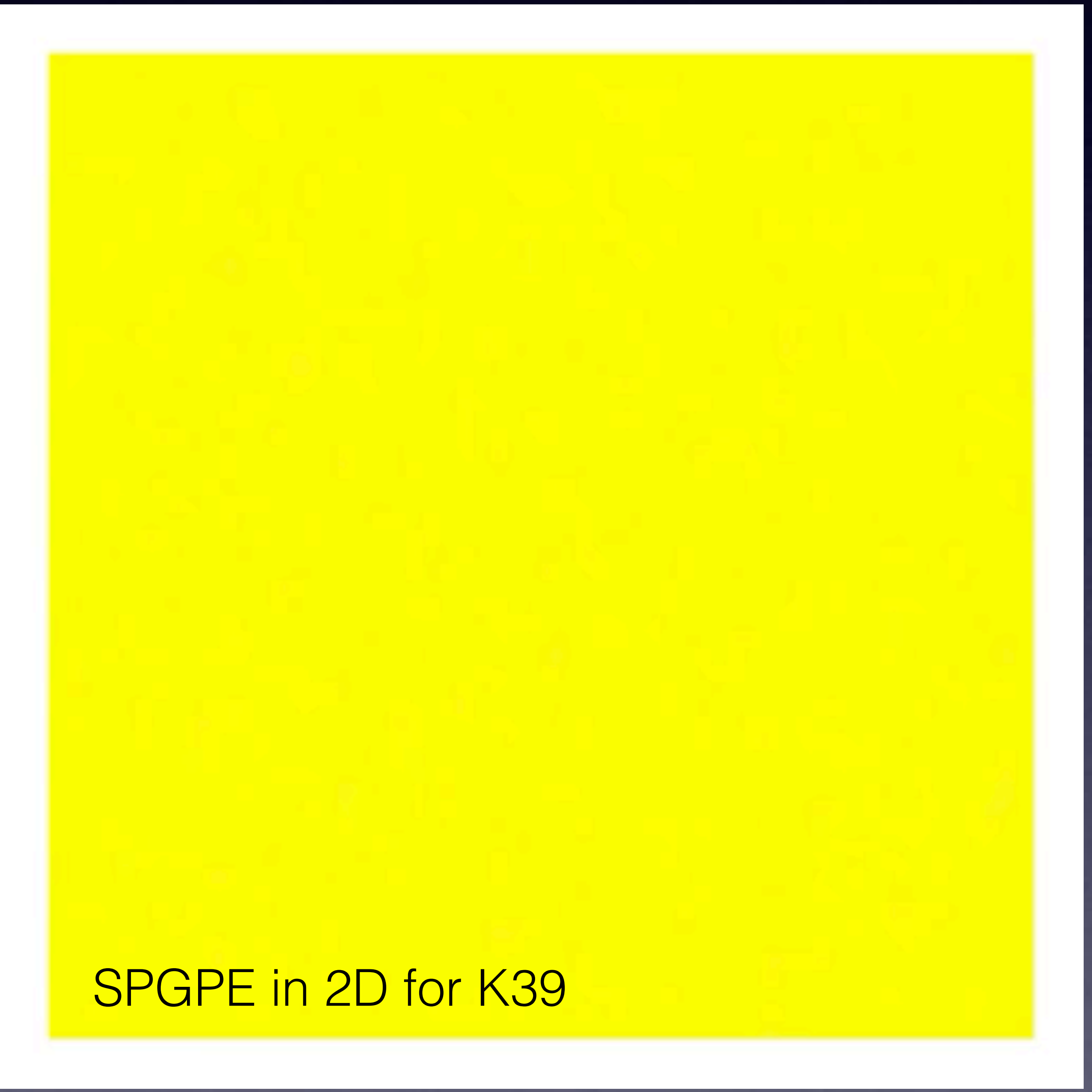
Possibly exact if we use the quantum effective field equations.

Billam,Gregory,Michel,Moss, 2019 Braden,Johnson,Peiris,Pontzen,Weinfurtner, 2019

Real time approaches: Bose Atom Condensates

Plots show phases of the mean fields





No agreement with instantons so far

Good agreement with instantons

FVD in the laboratory

Ferromagnetic bubbles in a sodium BEC

Two Zeeman levels $|\uparrow\rangle = |1,-1\rangle \quad |\downarrow\rangle = |2,-2\rangle$

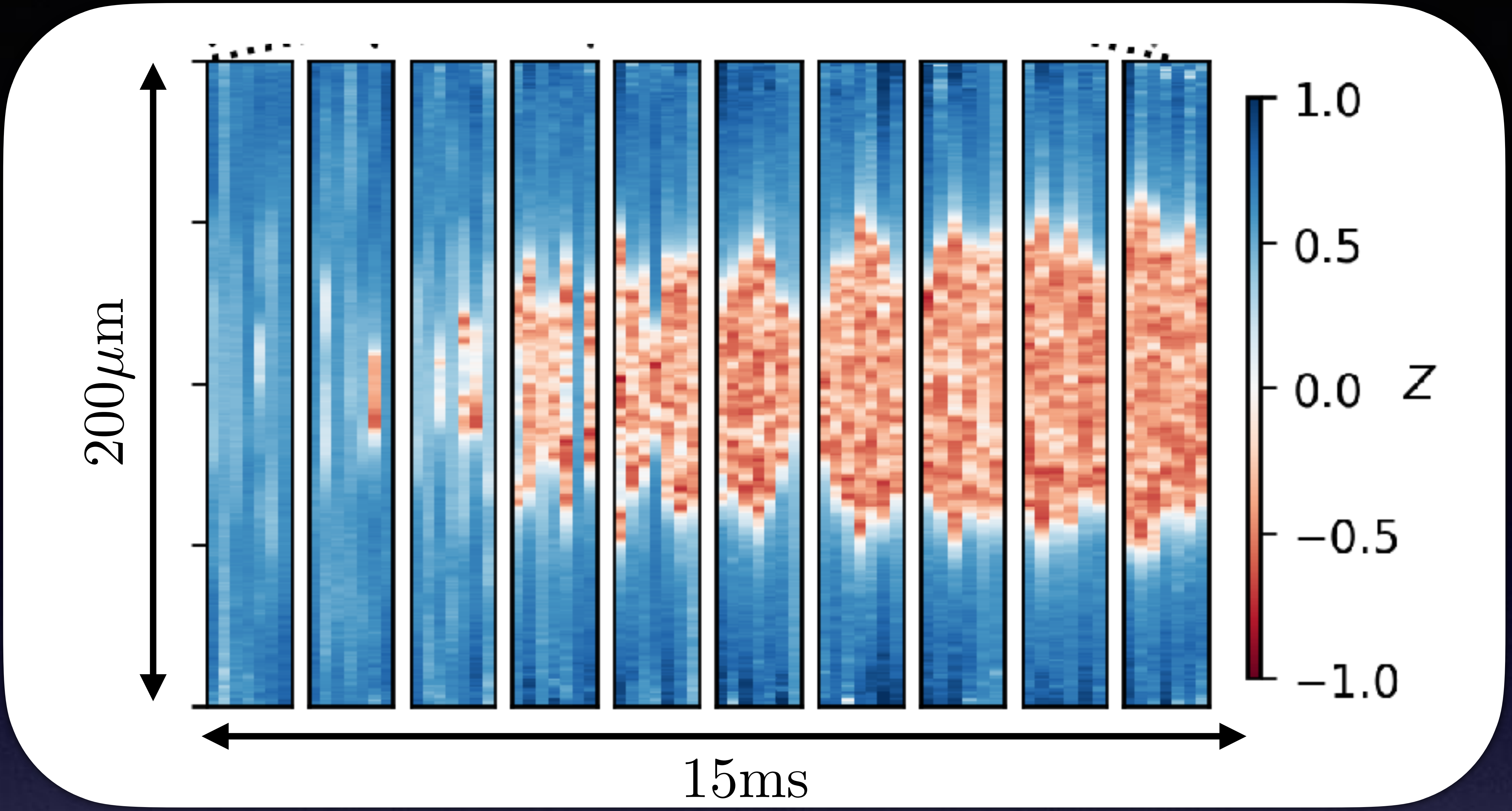
RF mixing $\Omega_R \quad \delta_{\text{eff}}$ (Rabi+detuning)

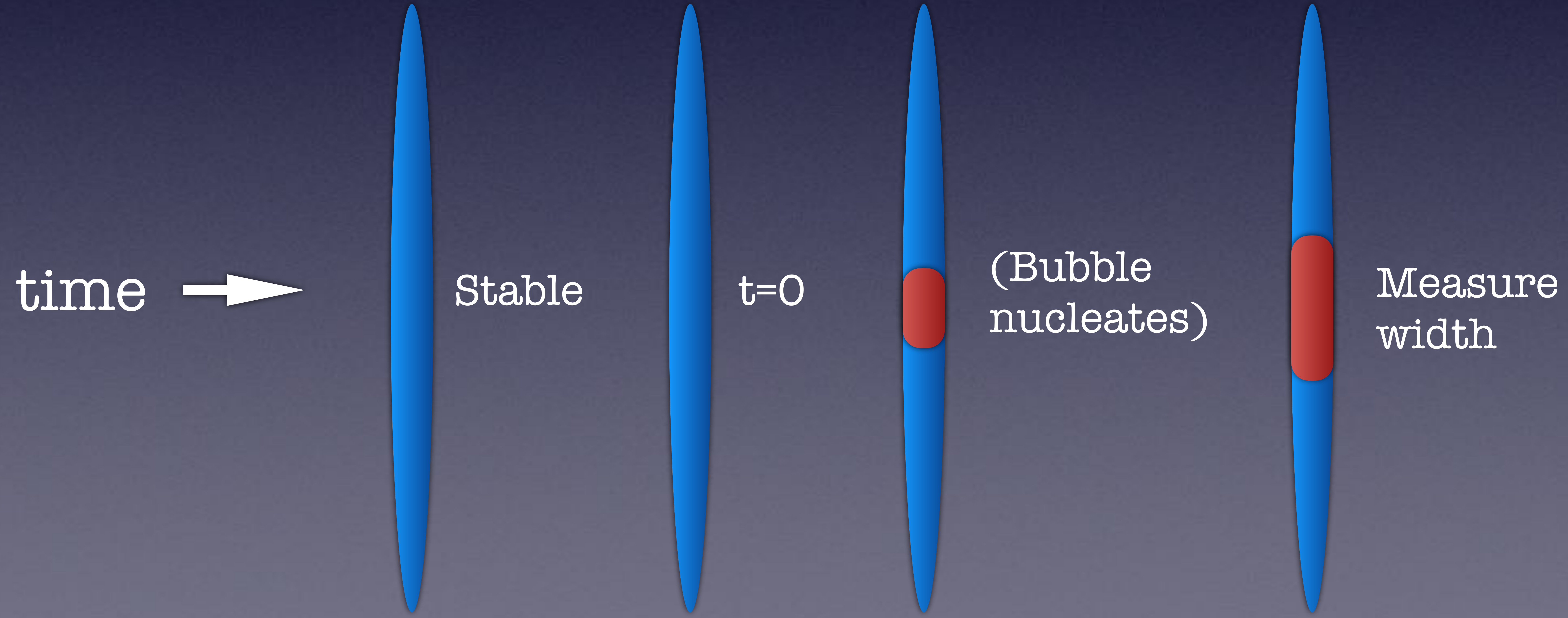
Collisions $a_{\uparrow\uparrow} \quad a_{\uparrow\downarrow} \quad a_{\downarrow\downarrow}$

Ferromagnetic ground states exist when $\kappa \propto a_{\uparrow\uparrow} + a_{\downarrow\downarrow} - 2a_{\uparrow\downarrow} < 0$

A Zenesini, A Berti, R Cominotti, C Rogora, IG Moss, TP Billam, I Carusotto, G Lamporesi, A Recati, G Ferrari arXiv:2305.05225

FVD Observations

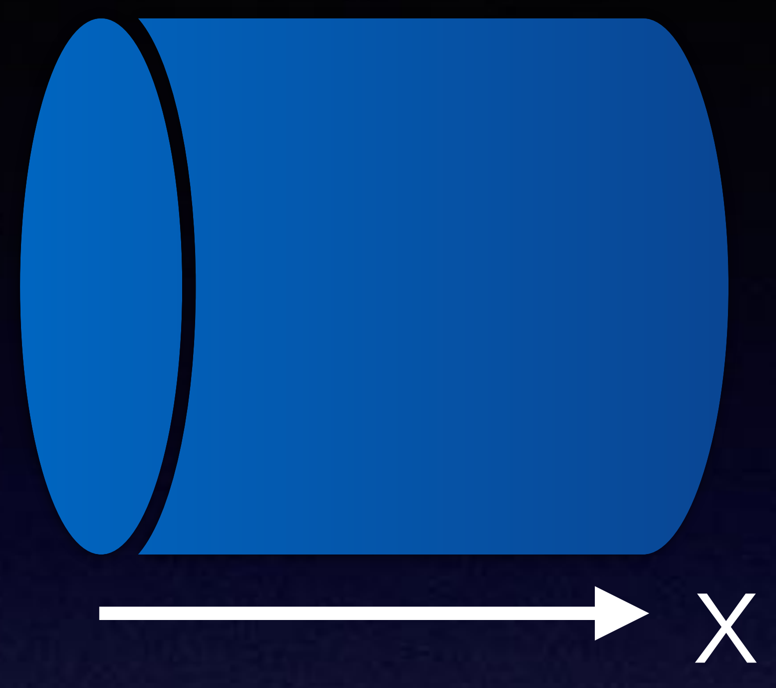




1D Model

Density $n(x)$

Magnetisation $Z = \dfrac{n_\uparrow - n_\downarrow}{n_\uparrow + n_\downarrow}$

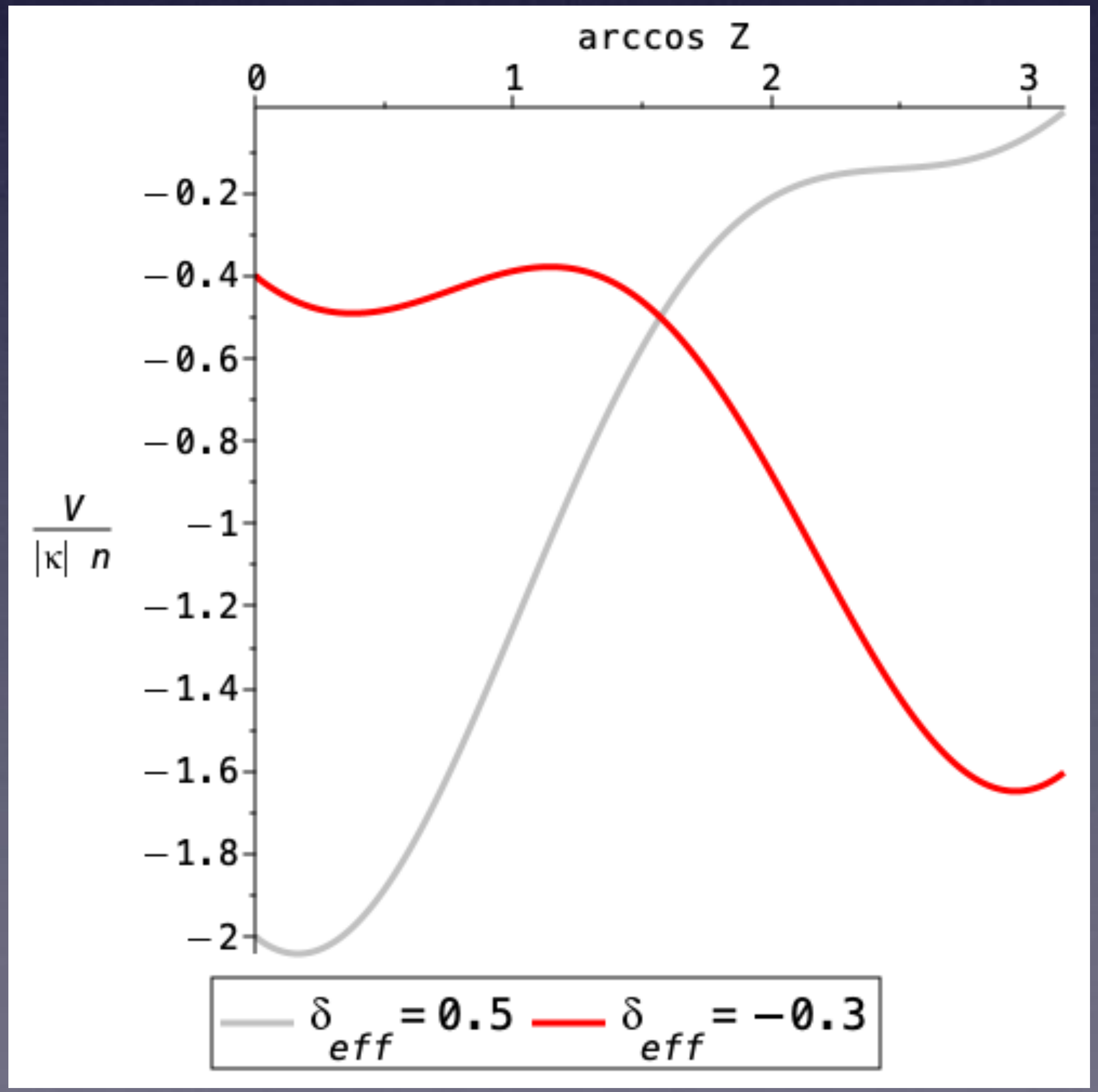


Potential

$$V = -|\kappa| n Z^2 - 2\Omega_R \sqrt{1 - Z^2} - 2\delta_{\text{eff}} Z$$

Depends on x
(1200Hz at centre)

Choose



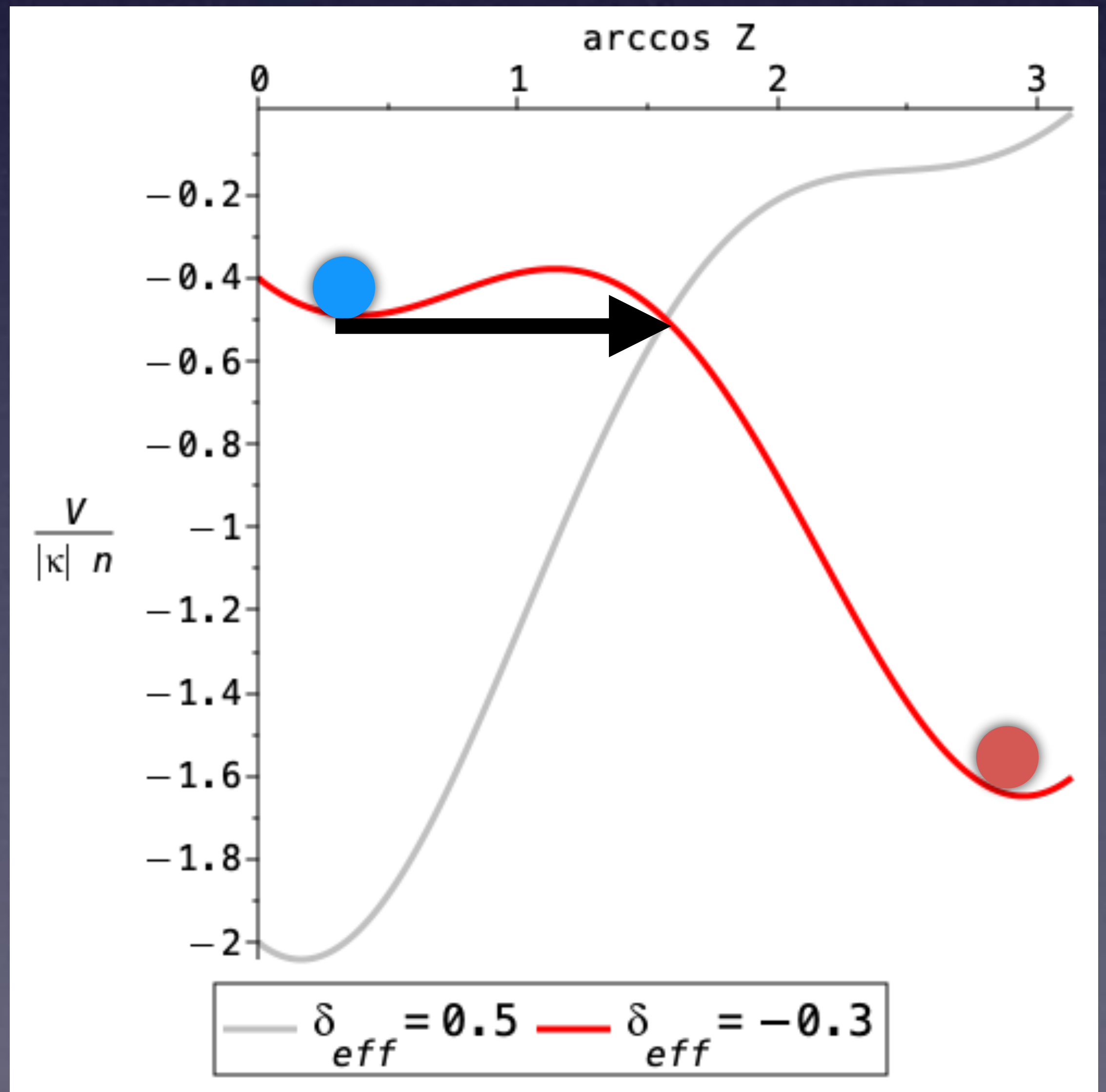
False vacuum decay rate

Thermal rate* $\Gamma = A \left(\frac{E_c}{k_B T} \right)^{1/2} e^{-E_c/k_B T}$

$$E_c = \frac{\hbar n}{4} \int \left\{ \frac{n}{2m} \frac{(\nabla Z_c)^2}{1 - Z_c^2} + V(Z_c) \right\} dx$$

Small barrier limit

$$\frac{E_c}{k_B T} \approx 1.77 \frac{n}{T} \left(\frac{\delta_{\text{eff}} - \delta_{\text{crit}}}{|\kappa| n} \right)^{5/4} \left(\frac{\Omega_R}{|\kappa| n} \right)^{1/6} \left(\frac{|\delta_{\text{crit}}|}{|\kappa| n} \right)^{-1/4}$$



*M Hindmarsh, M Ruben, J Lumma 2021

Stochastic GPE simulation

In healing length units

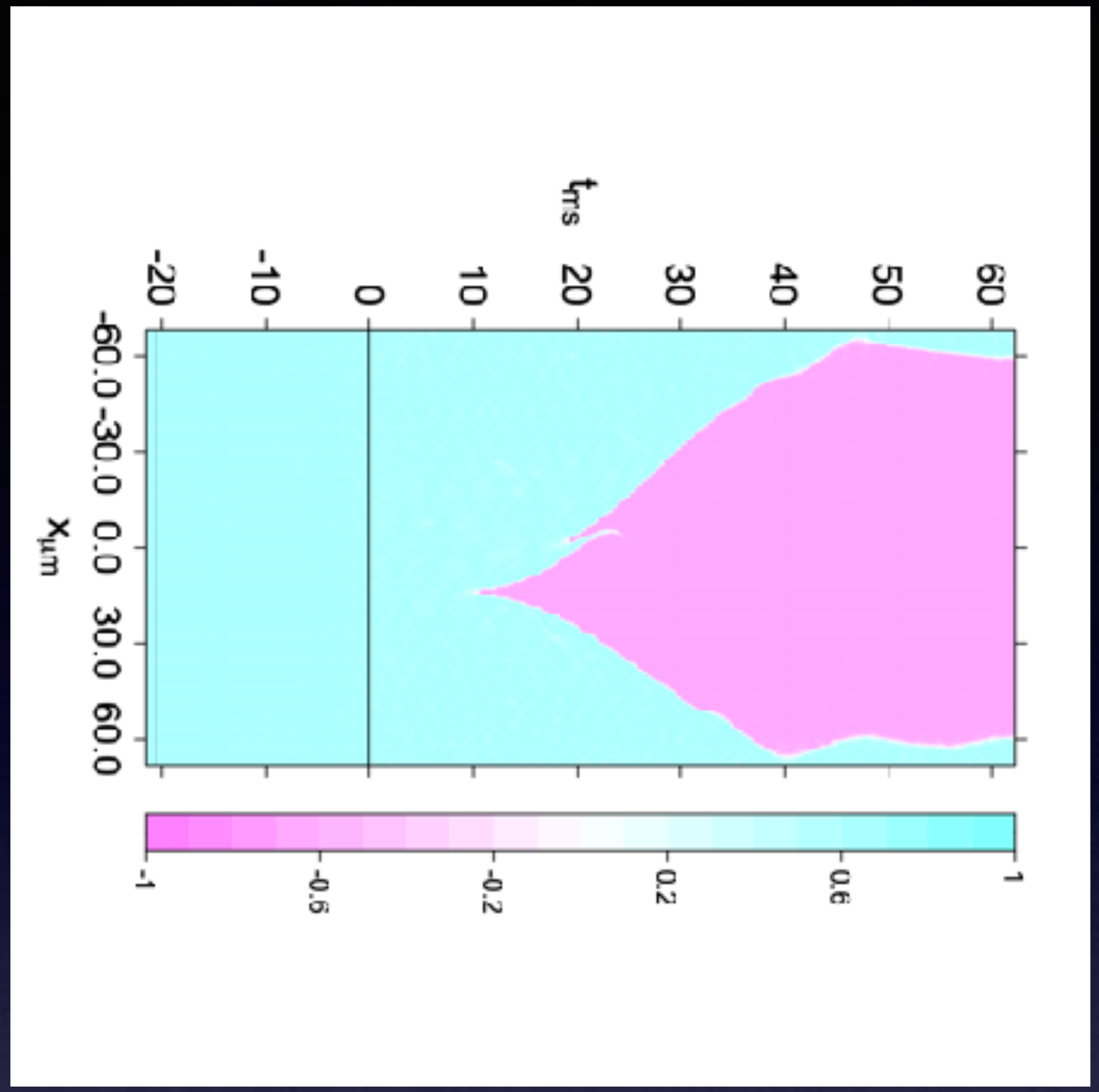
$$i\frac{\partial \psi_m}{\partial t} = \mathcal{P}\left\{(1-i\gamma)\left[-\frac{1}{2}\nabla^2\psi_m + \frac{\partial V}{\partial \psi_m^\dagger}\right] + \eta_m\right\}$$

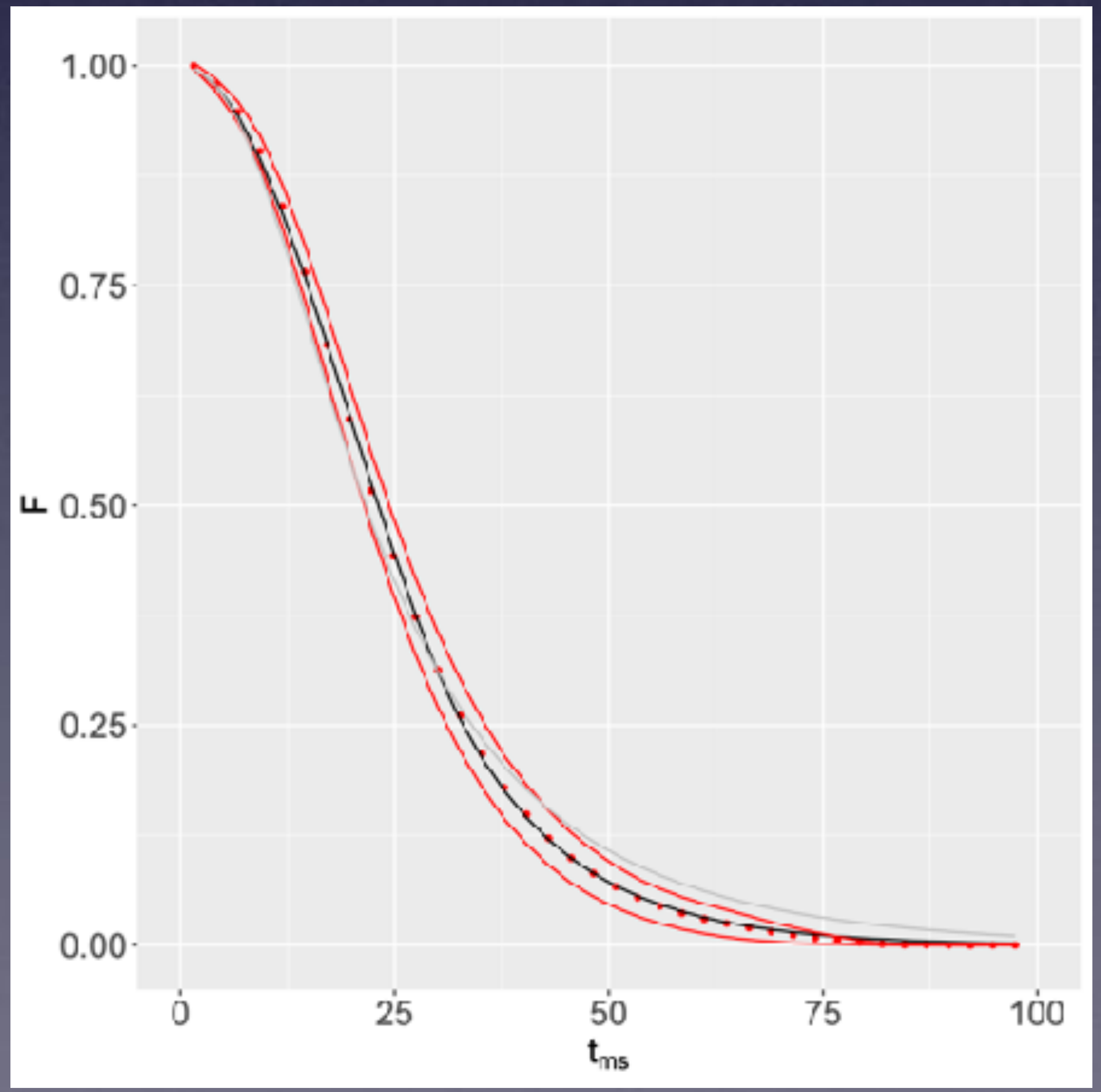
Thermal noise

$$\left\langle \eta_m(x,t)\eta_{m'}^\dagger(x',t')\right\rangle = \frac{2\gamma T}{n}\,\delta(x-x')\delta(t-t')\delta_{mm'}$$

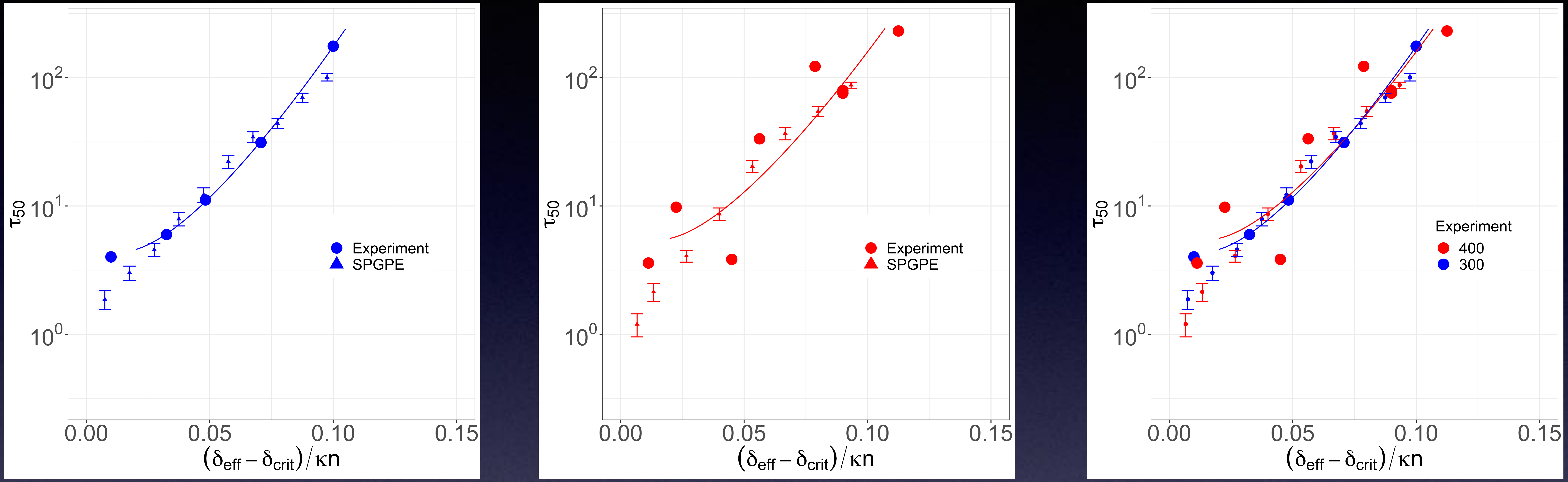
Measure mean nucleation time τ

or vacuum fraction $F(t) = \text{fraction in FV}$





Theory matches experiment



$\Omega_R/2\pi = 300\text{Hz}$ $\Omega_R/2\pi = 400\text{Hz}$

Instanton (line) is a two parameter fit (pre-factor, temperature)

SPGPE data has one parameter (temperature)

$$\frac{E_c}{k_B T} \approx 1.77\frac{n}{T}\left(\frac{\delta_{\text{eff}} - \delta_{\text{crit}}}{|\kappa|n}\right)^{5/4}\left(\frac{\Omega_R}{|\kappa|n}\right)^{1/6}\left(\frac{|\delta_{\text{crit}}|}{|\kappa|n}\right)^{-1/4}$$

Summary of the experiment

We see a metastable field state that decays by bubble nucleation.

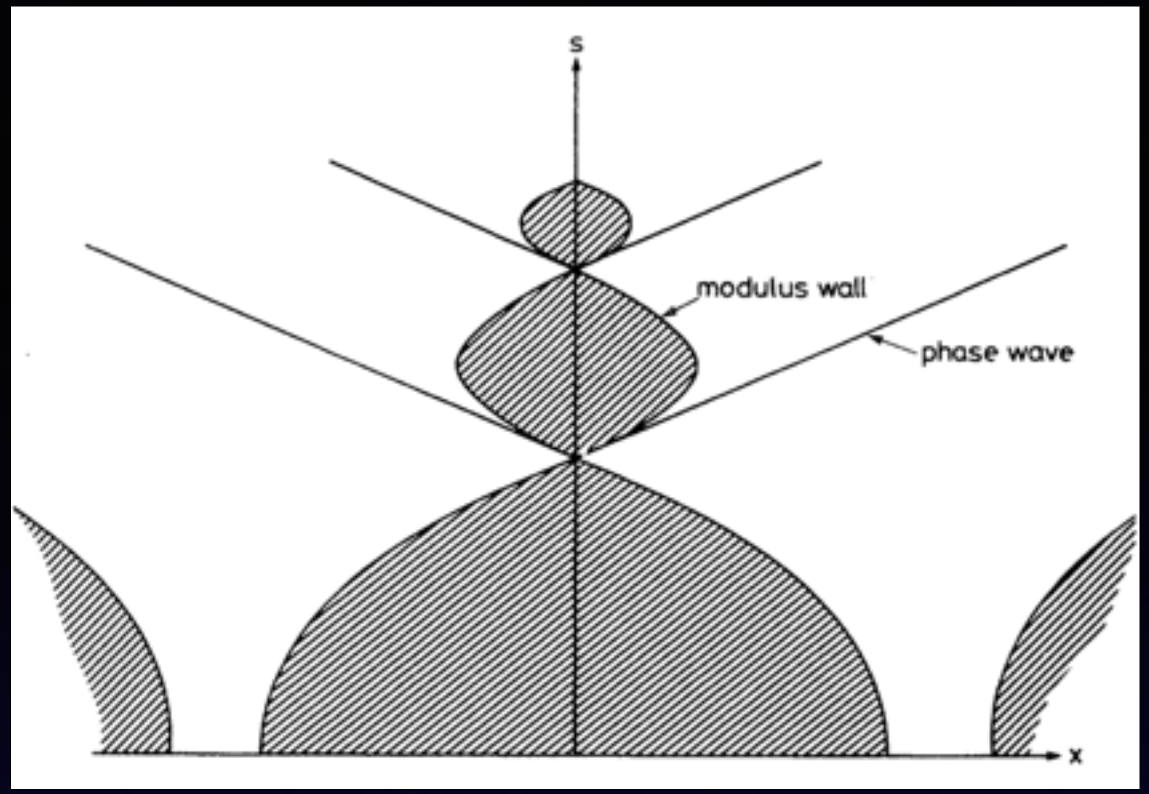
Nucleation timescales show an exponential dependence on the parameters.

Good agreement between SPGPE simulations, the thermal instanton decay rate and the experiment* *.

*The value of $\delta_{\rm crit}$ in the SPGPE is shifted (by fluctuations?).

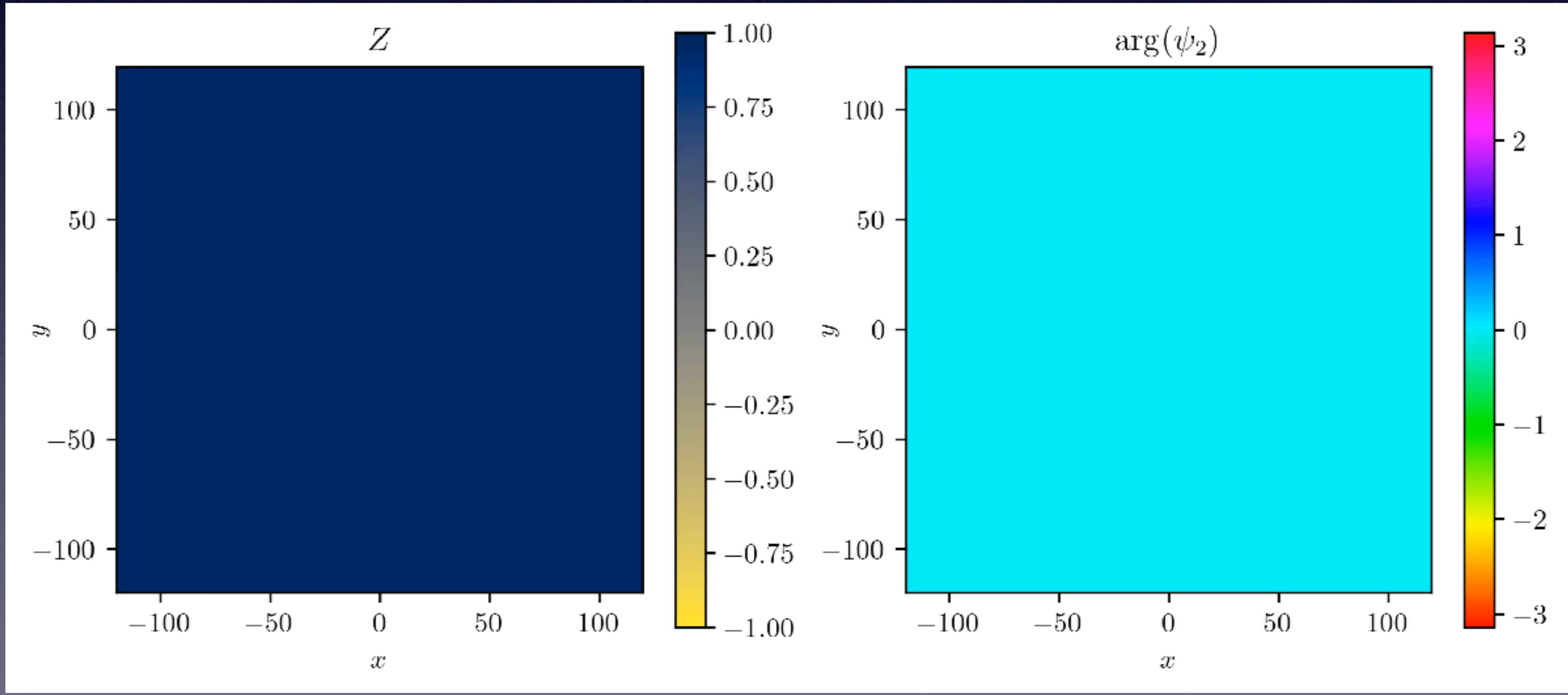
* *Value of n/T smaller than TW simulations (<50%).

Bubble collisions, wave phenomena, ...



BEC analogue of monopole production in the early universe!

Simulations



Hawking, Moss, Stewart 1982

From fantasy to reality?

We have seen the first experimental demonstration of false vacuum decay at finite temperature in a BEC.

False vacuum decay is a non-perturbative quantum field theory phenomenon that may be important in understanding the origin of the universe.

We are on the verge of seeing a whole range of vacuum decay phenomena in the lab: zero temperature decay, bubble collisions, defect formation, seeded decay, wave production....

End

Relativistic?

$$S = \frac{n}{4}\int \left\{ \frac{1}{2}\frac{\dot{Z}^2}{\sqrt{1-Z^2}} - \frac{1}{2}\frac{(\nabla Z)^2}{1-Z^2} - V(Z) \right\} dxdt$$ + higher derivatives

1. During nucleation

2. Bubble growth   Use ansatz: $f = \tanh \frac{\gamma\mu}{2}(x - R(t))$

$$L = \frac{1}{2}\gamma\left(\dot{R}^2 - 1 - \gamma^{-2}\right) + \epsilon R$$

$$L = -\sqrt{1-\dot{R}^2} + \epsilon R$$

But: correction terms $O(\gamma^{-3})$

Dispersion Relation

In healing length units

$$\omega^2 = \left(\epsilon_k + \tilde{\Omega}\right)\left(\epsilon_k + \sqrt{6}\tilde{\Omega}^{1/2}(\delta_{\text{eff}} - \delta_{\text{crit}})^{1/2}\right)$$

where $\epsilon_k = k^2/2 \qquad \tilde{\Omega} = \Omega/(1 - Z^2)^{1/2}$

$$\omega^2 \approx \frac{\tilde{\Omega}}{2}\left(k^2 + 2\sqrt{6}\tilde{\Omega}^{1/2}(\delta_{\text{eff}} - \delta_{\text{crit}})^{1/2}\right)$$