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LISA CONSORTIUM

QUEST DMC

Cosmology and the AB transition in superfluid ^{3}He

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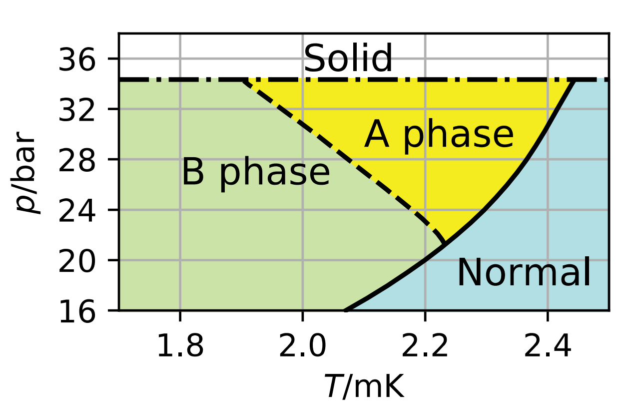
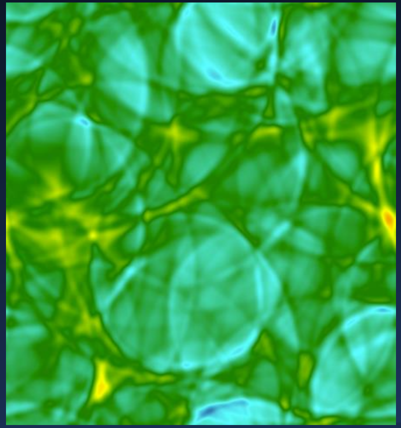
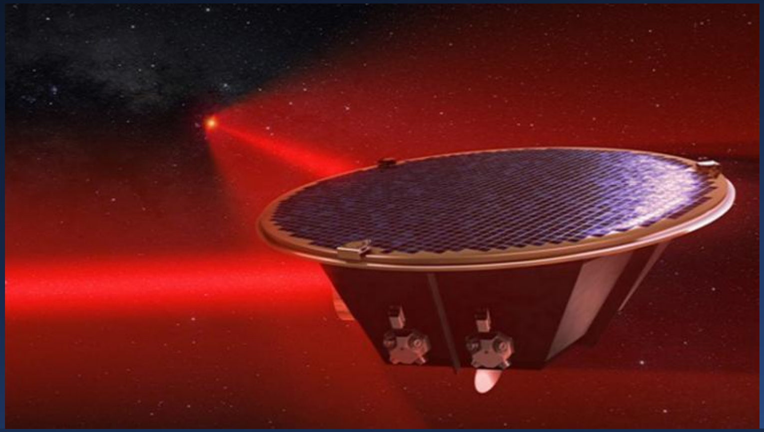
QUEST-DMC collaboration

Tunnelling in QFT

14. marraskuuta 2024



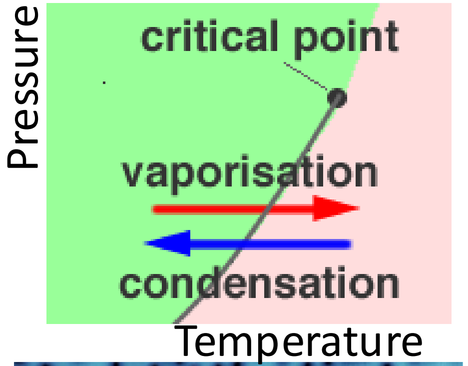


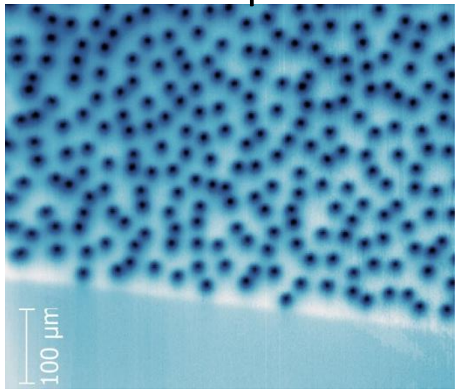


Phase transitions in the early Universe

$$\hbar = c = k_B = 1$$

- At very high temperatures and pressures, the state of matter in the Universe changes (1 eV ~ 10^4 K)
  - T_c ~ 100 MeV (0.1 ms) QCD
  - T_c ~ 100 GeV (10 ps) Electroweak
  - T_c >> 100 GeV new symmetries, interactions?
- Departures from equilibrium and homogeneity
  - First order phase transition: relativistic condensation or `fizz’
    Steinhardt (1982)
  - First or second order: formation of topological defects
    Kibble (1976)
  - First order: baryon asymmetry
    Sakharov (1967); Kuzmin, Rubakov, Shaposhnikov (1985)
- First order phase transitions can produce GWs
  - Stochastic gravitational wave background
    Witten (1984), Hogan (1986)

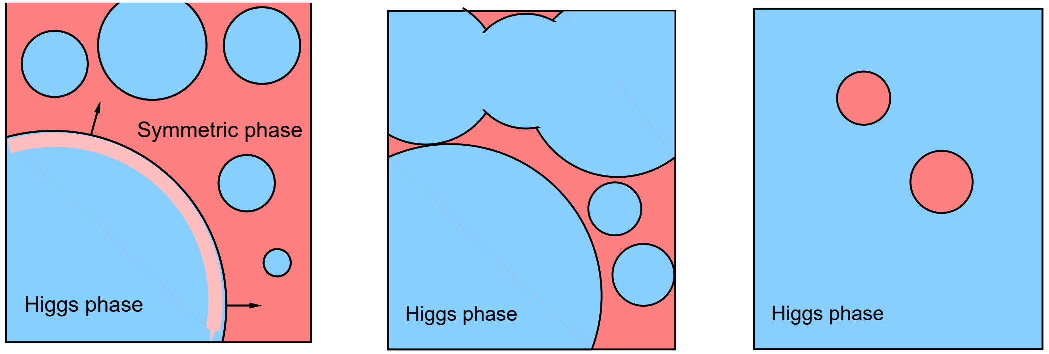




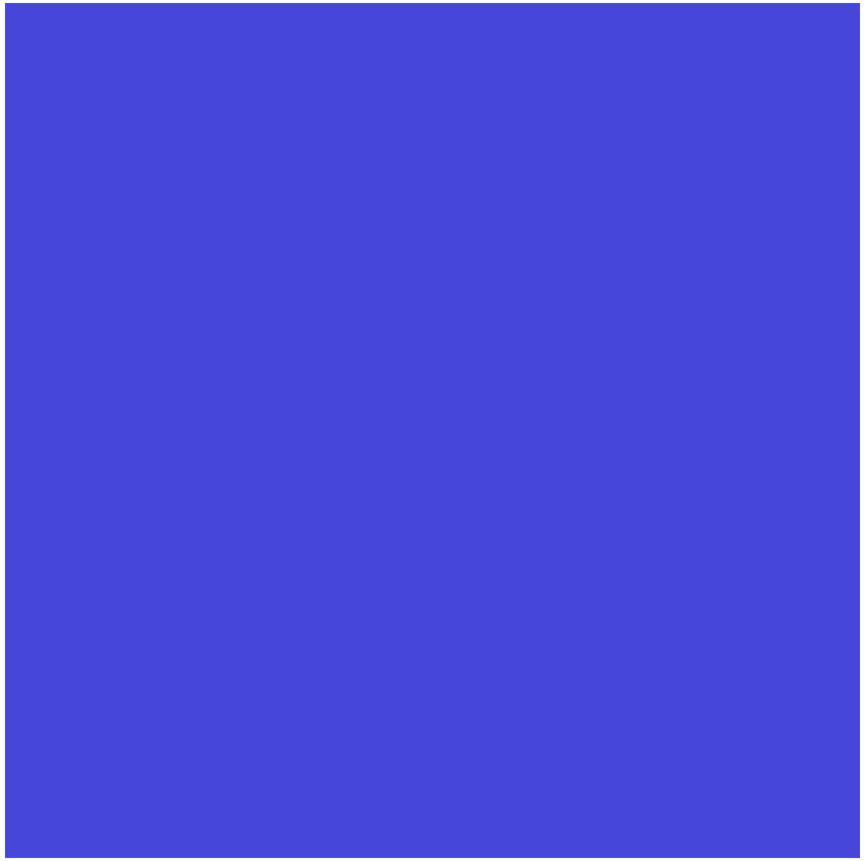
Abrikosov vortices

Little bangs in the Big Bang

- 1st order transition by nucleation of bubbles of low-T phase
  Cahn, Hilliard 1959, Langer 1969, Coleman 1974, Linde 1983
- Nucleation rate/volume $p(t)$ rapidly increases below T_c
- Expanding bubbles generate pressure waves in hot fluid
  Steinhardt (1982), Hogan (1983), Gyulassy et al (1984)
- Gravitational wave (GW) production
  Witten (1984), Hogan (1986)
- GW spectrum has information about phase transition
  MH, Huber, Rummukainen, Weir (2013,5,7); MH (2017); MH & Hijazi (2019); Cutting, MH, Weir (2019)



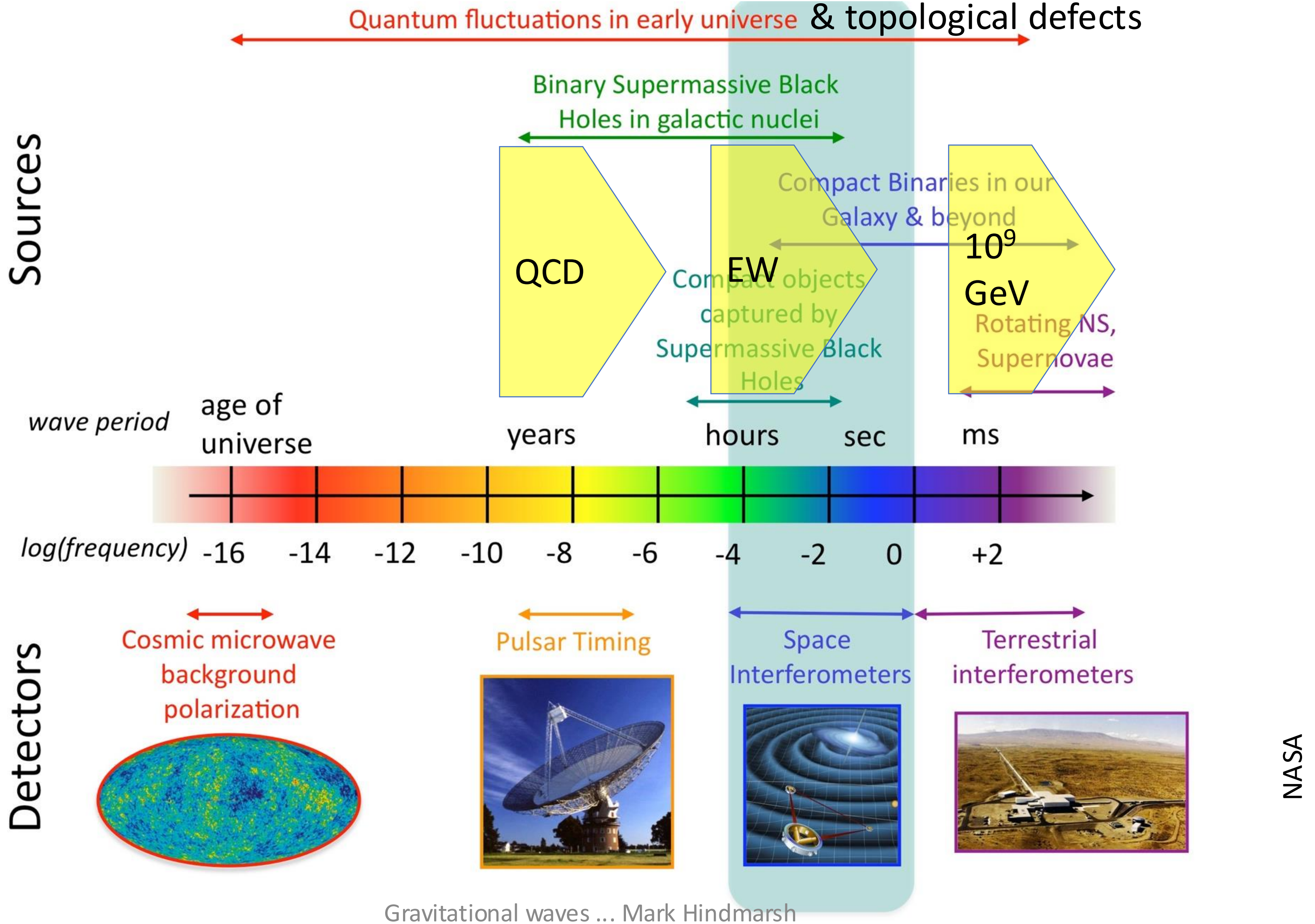
Fluid kinetic energy



David Weir

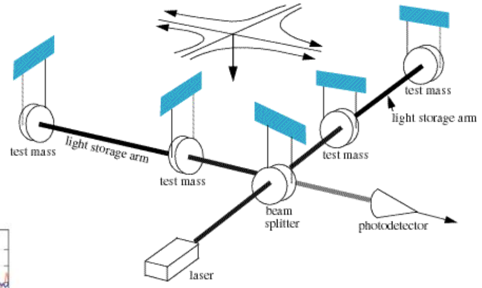
MH, Huber, Rummukainen, Weir (2013,5,7)
Cutting, MH, Weir (2018,9)

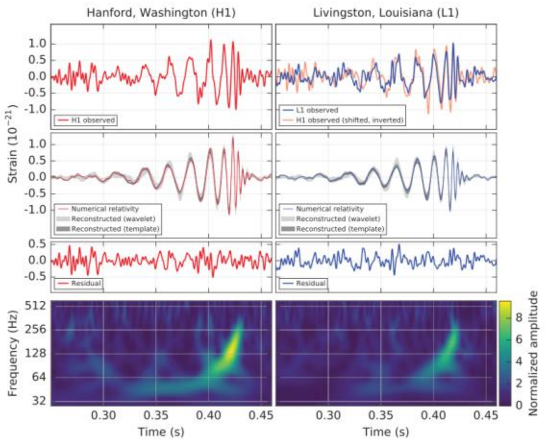
Gravitational wave spectrum



Observations of gravitational waves

- LIGO-Virgo-KAGRA
- GW150914

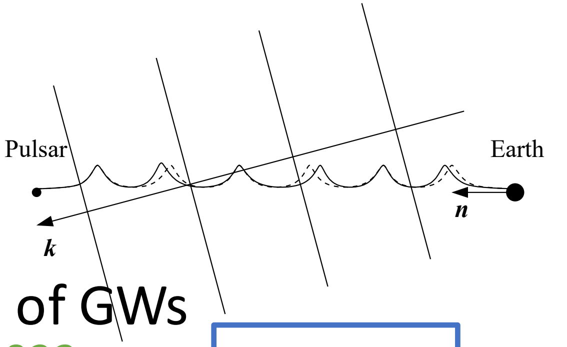




$f \sim 100$ Hz

- GWTC: 93 events up to O3 http://gwosc.org/
  - Mostly black hole binary mergers
  - 1 binary neutron star merger
  - 1 candidate BH-NS merger
- O4 under way (5/23 – 6/25)
  - 147 detection candidates http://gracedb.ligo.org/ (9.10.2024)

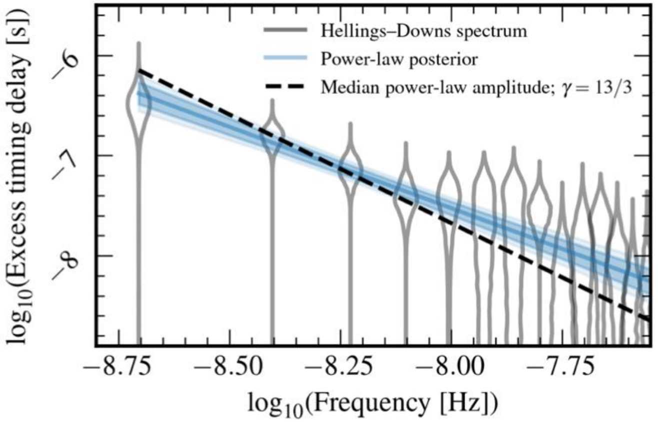
Pulsar timing arrays

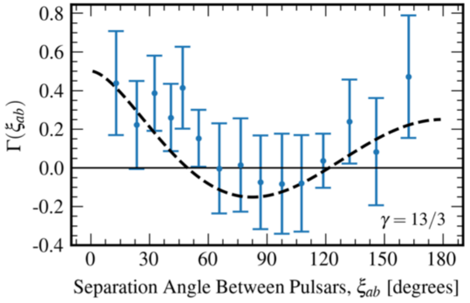


- NANOGrav:
  stochastic background of GWs

Afzal et al 2023

$f \sim$ nHz





- Support from EPTA and PPTA
- Leading candidate:
  merging supermassive black holes
- New physics? Agazie et al 2023
  e.g. phase transitions, cosmic strings, …

Current data on stochastic GW background

- LIGO O2 flat spectrum limit:
  Abbott et al (2019)

$$\Omega_{\rm gw,0}(f = 25{\rm Hz}) < 6.0 \times 10^{-8}$$

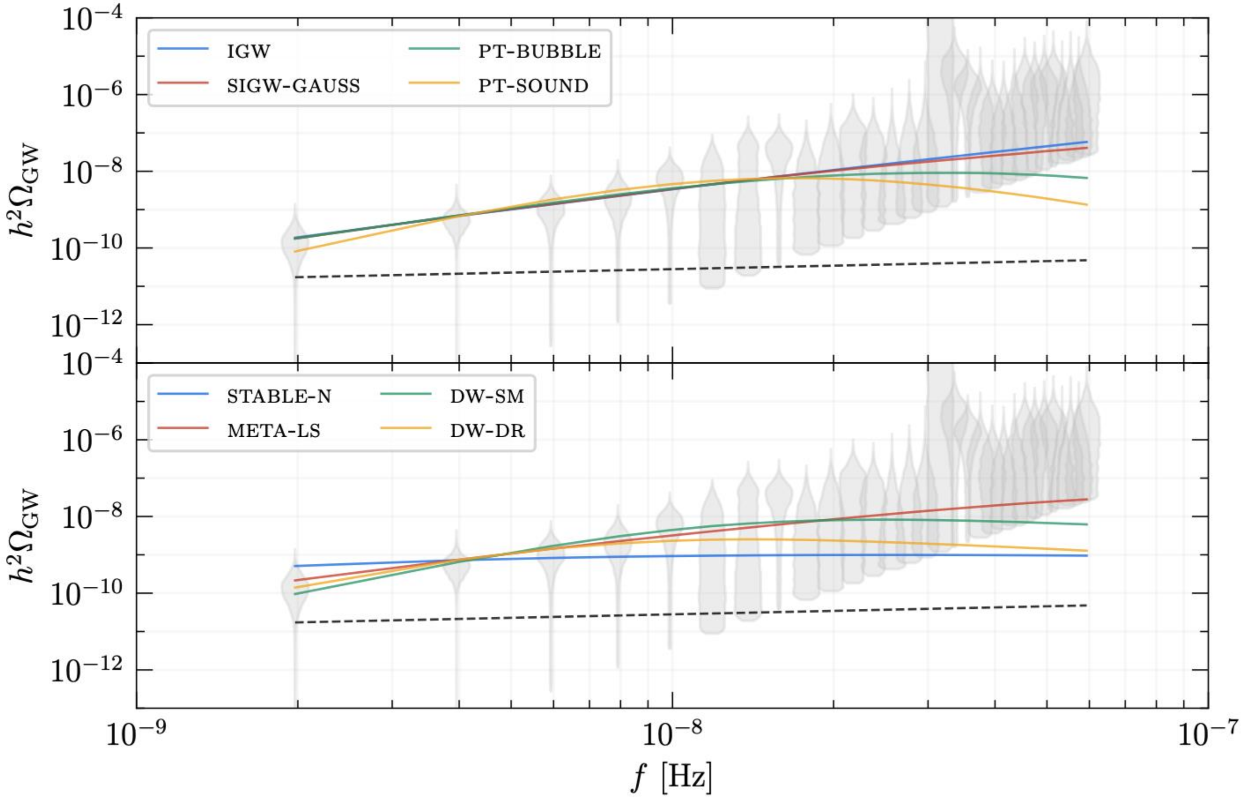
- "Gravitational wave power spectrum":

$$\Omega_{\rm gw} = \frac{1}{\rho_{\rm total}} \frac{d\rho_{\rm gw}(f)}{d\ln(f)}$$

- Density fraction in GWs in a log frequency interval around f

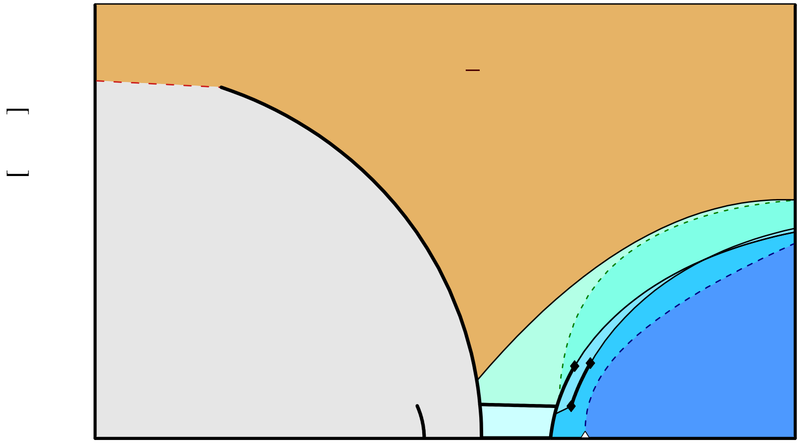
- NANOGrav 15 year: Agazie et al 2023
  - SMBH binaries + new physics?

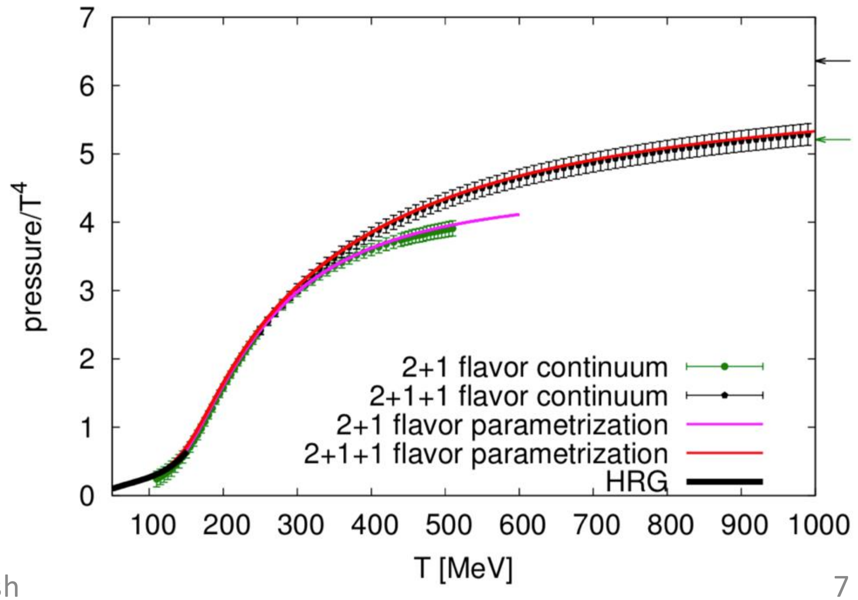
Afzal et al 2023



QCD transition: 100 MeV, 0.3 ms

- QCD: rich phase diagram in (T, μ)
- Universe: $n_B/n_\gamma \approx 6.1 \times 10^{-10}$
- Behaviour at low baryon chemical potential well-established by lattice QCD
- Transition from QGP to hadronic phase is a smooth cross-over Borsanyi et al (2016)
- No phase transition in standard cosmology
- No GWs

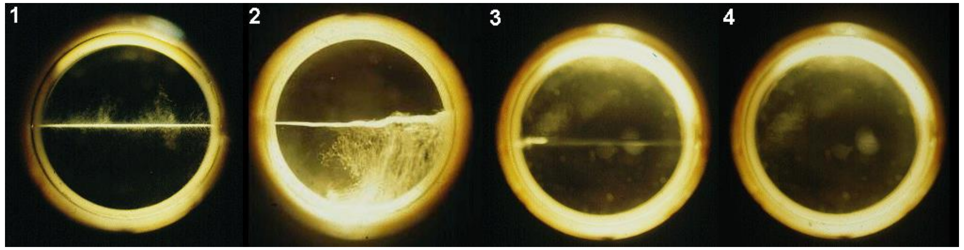


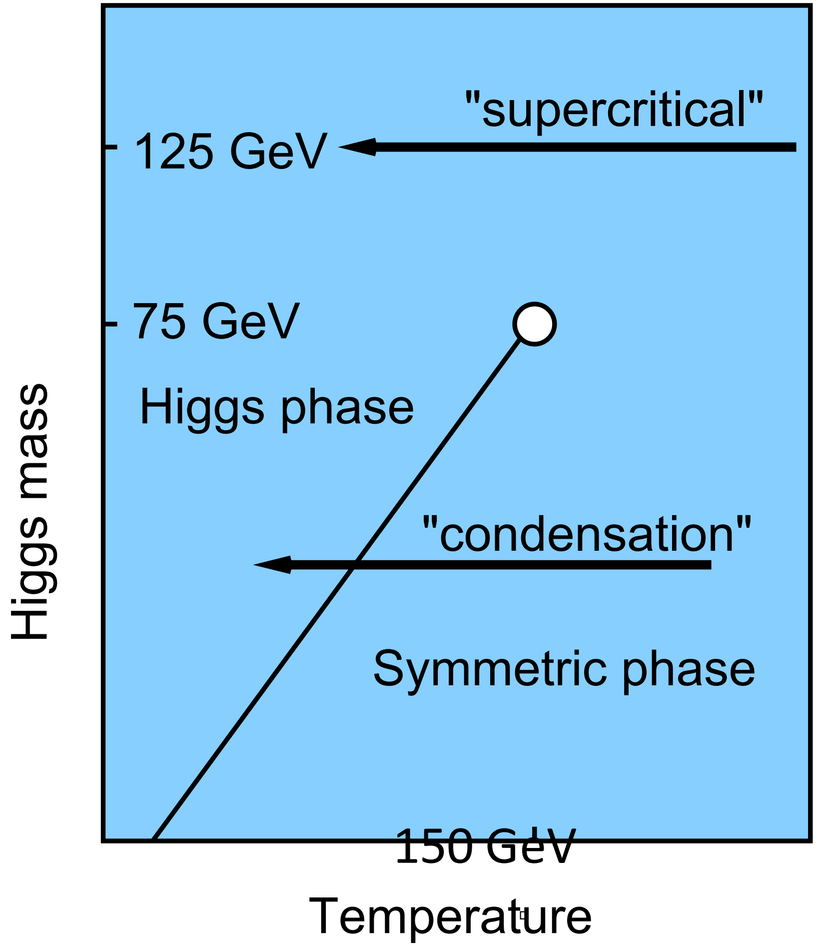


Borsanyi et al 2016

Electroweak transition: 100 GeV, 10 ps

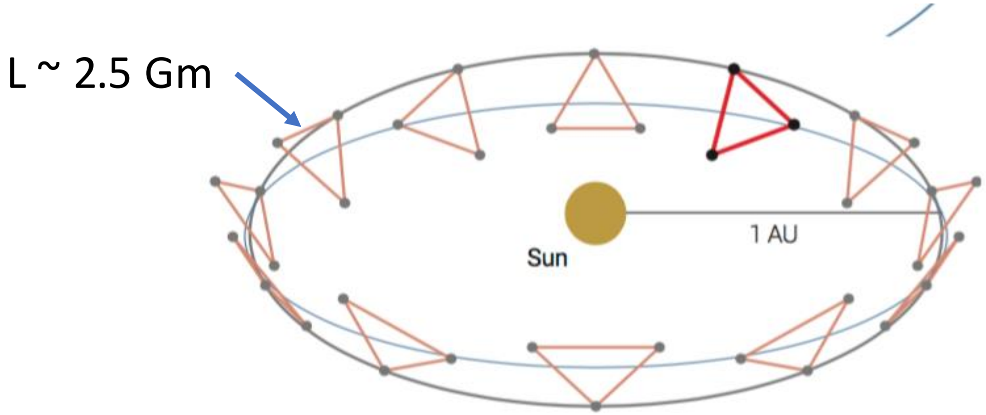
- In the Standard Model, transition is also a cross-over
  Kajantie, Laine, Rummukainen, Shaposhnikov (1995,6)
  - Susceptibility peak at $T \sim 160$ GeV
    D'Onofrio, Rummukainen (2015)
- Same universality class as liquid-vapour in water
  Rummukainen et al (1995)
  - SM transition like a supercritical fluid

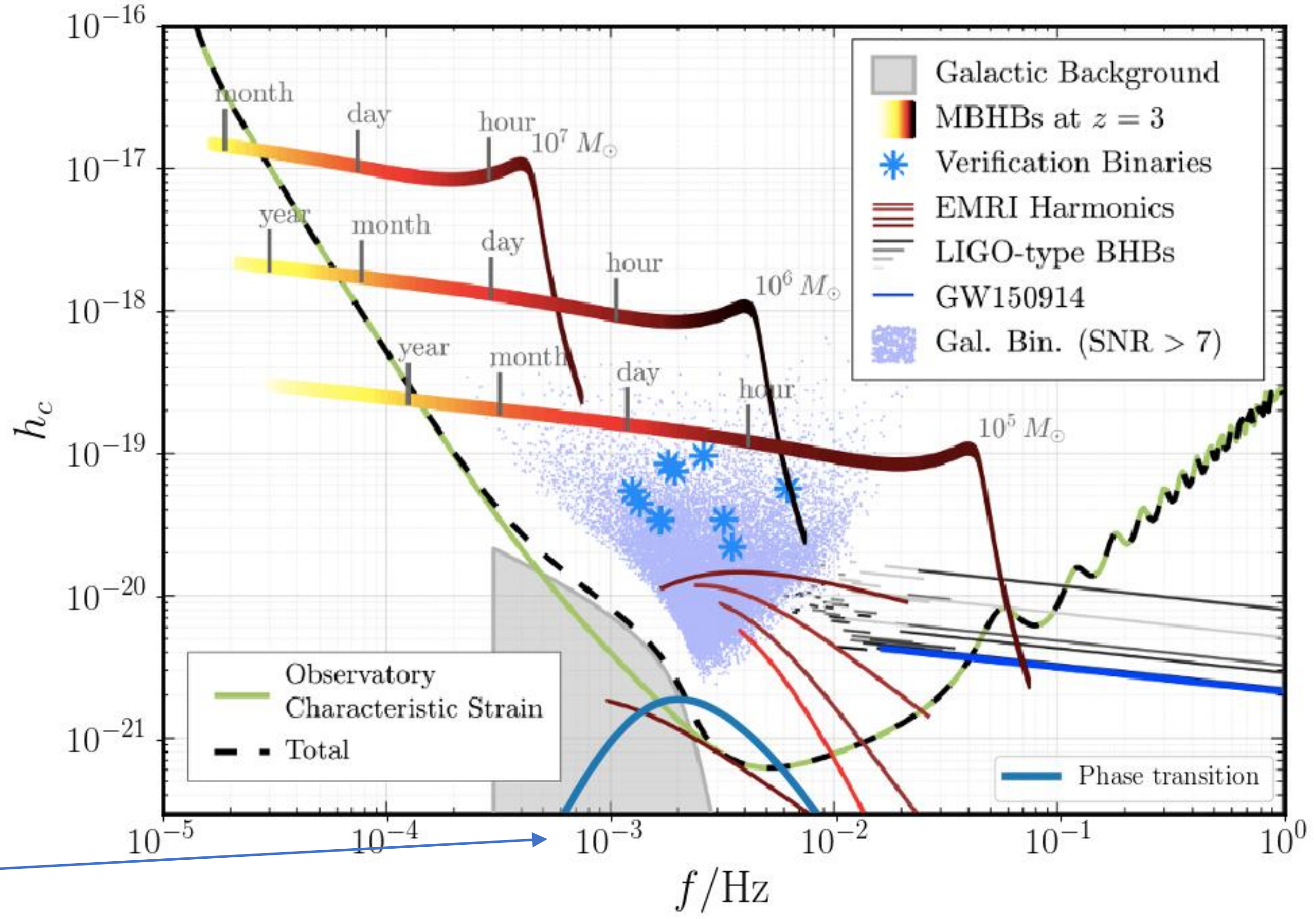




- SM stays close to equilibrium, no GWs
- But: New physics needed for baryon asymmetry, dark matter, …
- A search for GWs from phase transitions is a search for new physics

Laser Interferometer Space Antenna (~2035)





Characteristic strain $h_c \sim \Delta L/L$

- LISA Science objectives:
  - White dwarves
  - Black holes (galactic, stellar)
  - Galaxy mergers
  - Extreme gravity
  - Cosmological expansion
  - TeV-scale early Universe
  - ???
- Other missions: DECIGO, Taiji, Tianqin

GWs from 1st order phase transitions

Phase transition parameters :

Computable from theory

T_n = nucleation temperature

β = transition rate

α = strength parameter*

v_w = bubble wall speed

c_{s+}, c_{s-} = sound speeds

g_{eff} = effective d.o.f. in plasma

Simulations, Modelling

$H_n(T_n, g_{eff})$ (Hubble rate = cooling rate)

$K(v_w, \alpha, c_s)$ (kinetic energy fraction)

$R_*(\beta, v_w)$ (mean bubble separation)

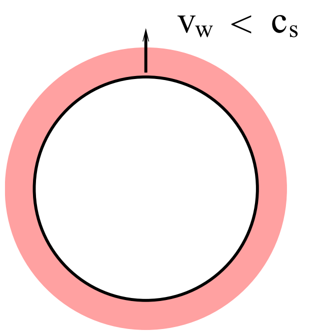
$t_v(R_*, K)$ (fluid motion lifetime)

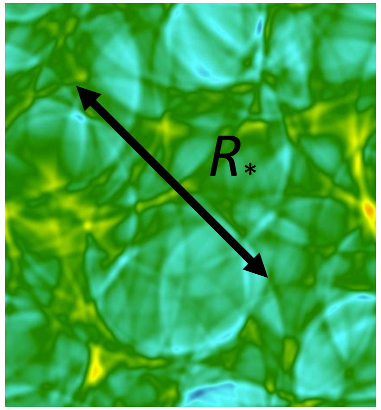
Spectral parameters :

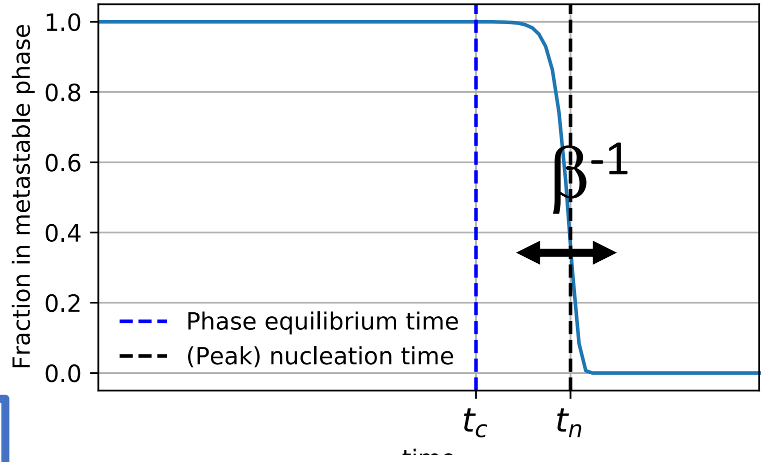
Ω_p = peak amplitude

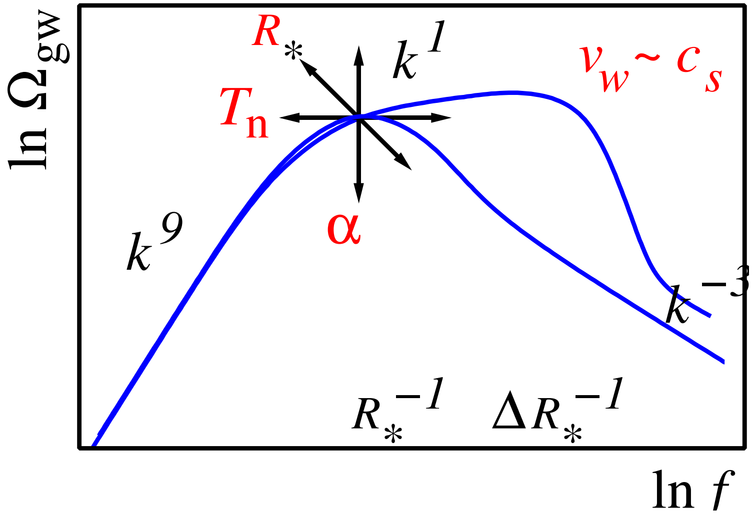
f_p = peak frequency

σ_i = shape parameters







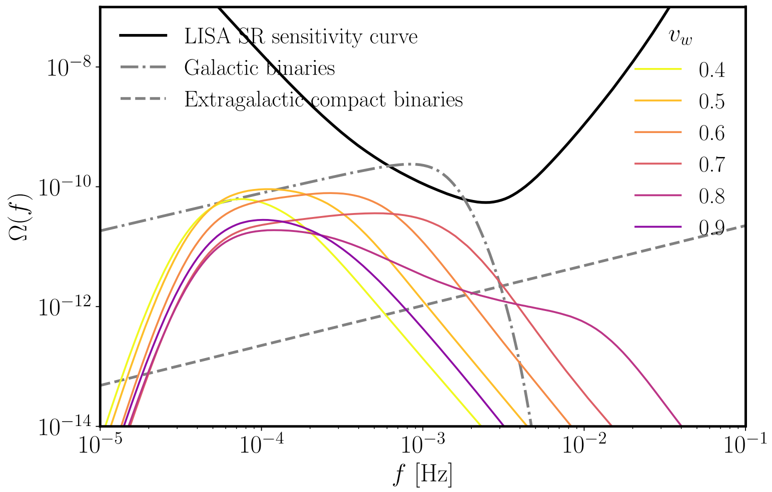


* α ~ (latent heat)/(total thermal energy)

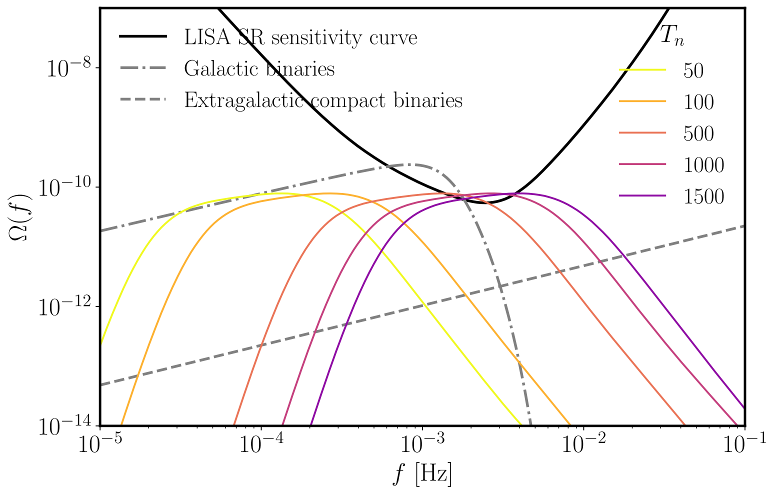
Review: MH, Lüben, Lumma, Pauly 2021

GW power spectra in the Sound Shell Model

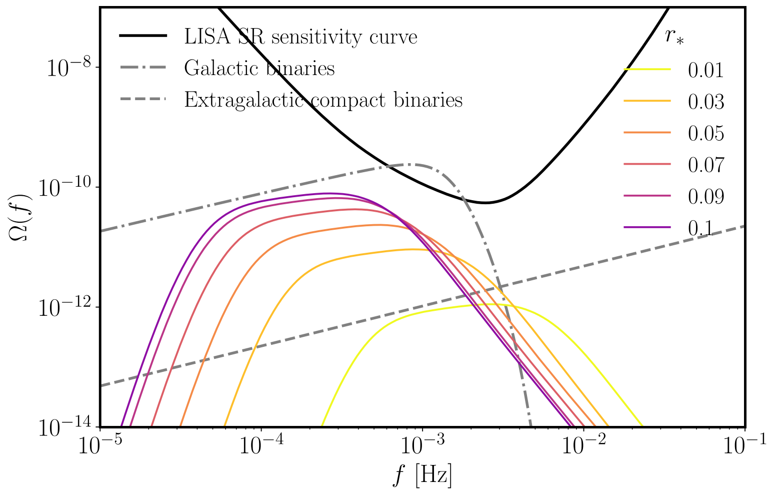
Gowling, MH (2021)



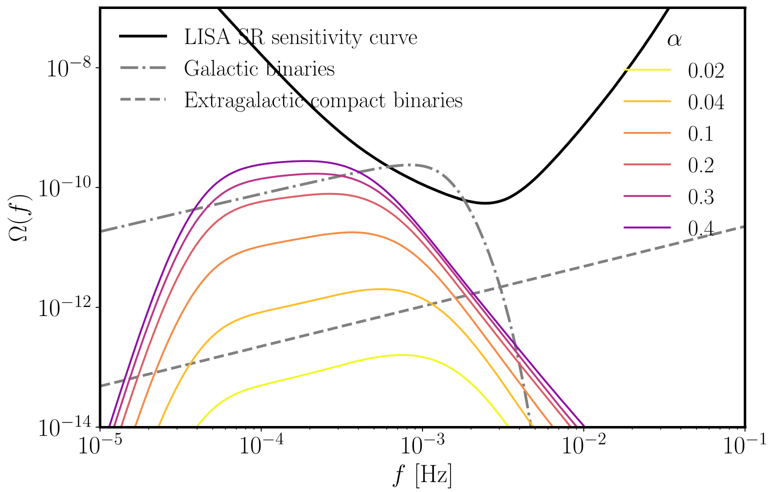
Vary wall speed



Vary transition temp

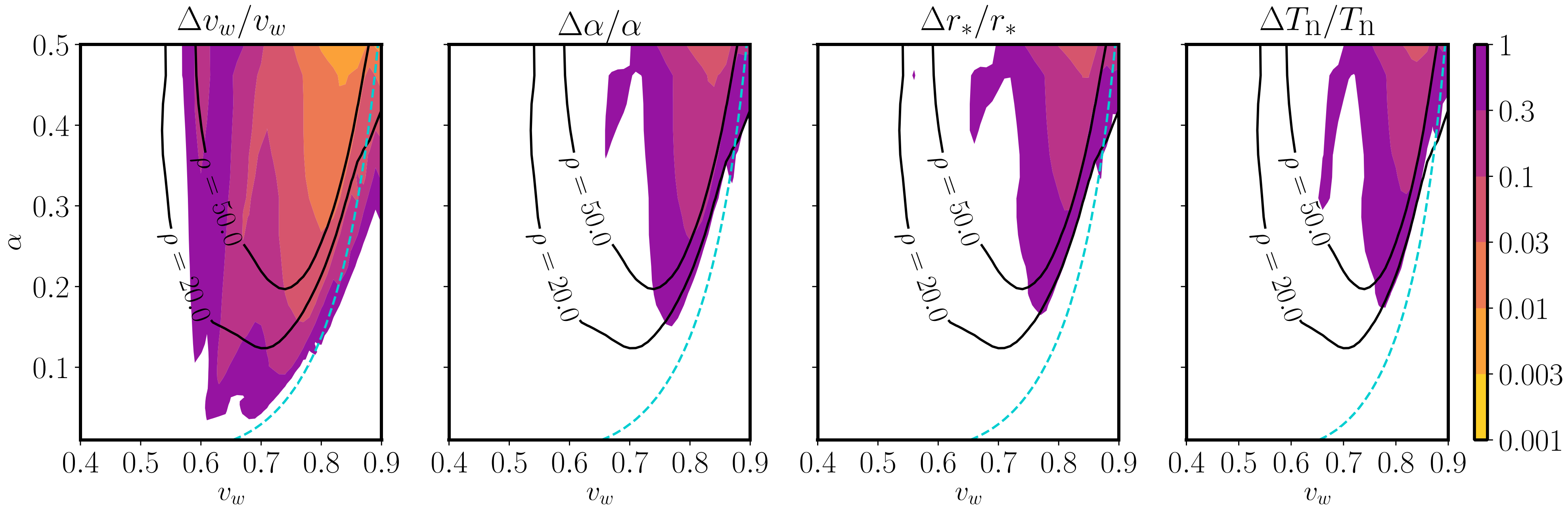


Vary bubble spacing



Vary transition strength

Observability of PT parameters at LISA

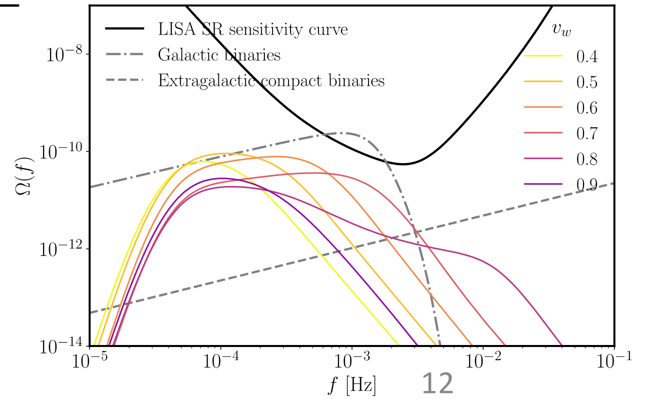


- T_n = 100 GeV
- $c_s = 1/\sqrt{3}$
- r_* = 0.1

Gowling, MH (2021)

- Covariance matrix ***C*** (t_obs = 4 years)
- Uncertainty from eigenvalues of ***C***
  - "noise" = GB foreground + "LISA science requirements" instrument noise
- **Wall speed** is best determined parameter
  - If signal is observed
  - shape dependence of power spectrum

$$C^{-1}_{ij} = 2T_\mathrm{obs} \int df \, \frac{\partial \ln \Omega_\mathrm{gw}}{\partial \theta_i} \frac{\partial \ln \Omega_\mathrm{gw}}{\partial \theta_j}$$



Can we test the theory of GW production?

- Lots of theory involved
- Equilibrium thermal quantum field theory
  - Free energy (T, ϕ)
  - Quantum tunnelling/thermal activation
  - Needed for T_n, β, α, c_s
- Non-equilibrium dynamics
  - Needed for v_w
- Relativistic hydrodynamics
  - Needed for fluid flow generation & decay

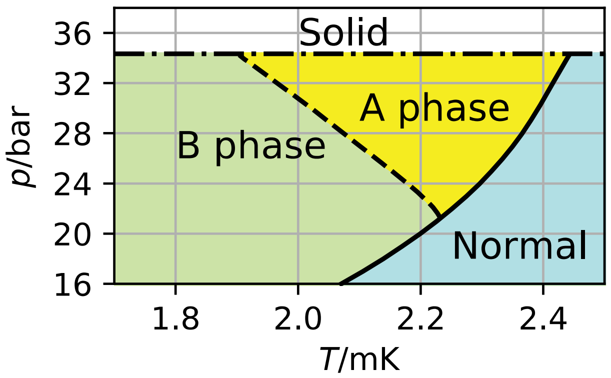
Laboratory system with a first order phase transition?

Cosmology & ^{3}He

"The Universe in a Helium Droplet"
Volovik (OUP, 2007)

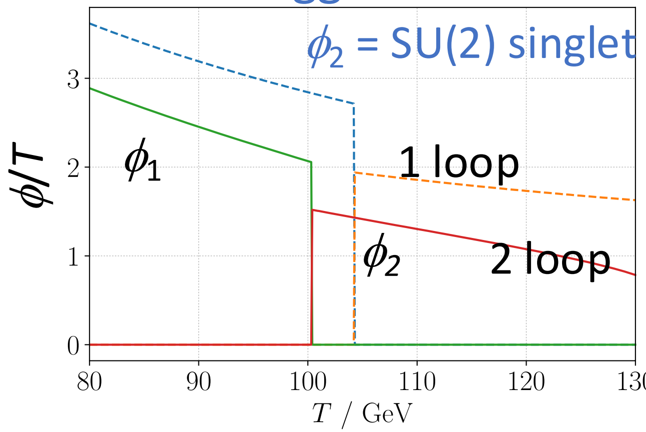
^{3}He

- Fermionic quasiparticles
- Triplet pairing interaction
- Order parameter A
  - 3x3 complex, spin x ang. mom.
- Ginzburg-Landau theory
  - Bulk free energy density $f_b(A)$
- Vortices and other topological defects Ruutu et al (1996)
- Multiple superfluid phases $T <$ O(mK)
- 1st order phase transition A/B



Standard Model (& beyond)

- Quarks & leptons
- --- (technicolour etc.)
- Fundamental (?) Higgs field(s) ϕ (ϕ_2, …)
  - Irreps of $SU(2)_W \times U(1)_Y$ (G_{GUT})
- High T effective action
  - Effective potential $V_T(\phi)$
- (cosmic strings and other defects)
- 1 (multiple) Higgs phase(s) $T <$ O(10^{16} K)
- (1st order PTs normal-Higgs & between Higgs phases)



Niemi, Schicho, Tenkanen 21

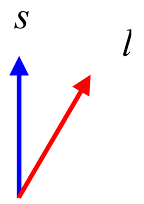
Bulk phases of superfluid ^{3}He

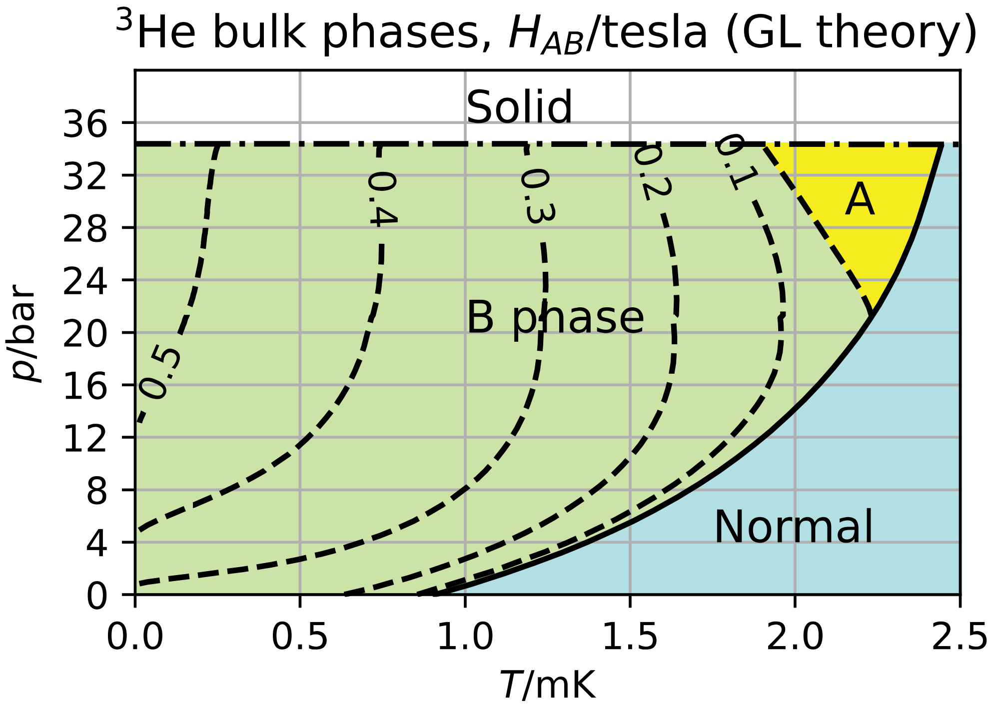
- Superfluid below T_c ~ 2 mK
  - T_c depends on pressure
- Symmetry group of free energy:
  - $SO(3)_S \times SO(3)_L \times U(1)_N$
- B phase: $SO(3)_J$ Balian, Werthamer (1964)

$$A_B = \frac{\Delta_B}{\sqrt{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$ isotropic

- A phase: $U(1)_{Sz} \times U(1)_{Lz+N}$ Anderson, Morel (1961) Anderson, Brinkman (1973)

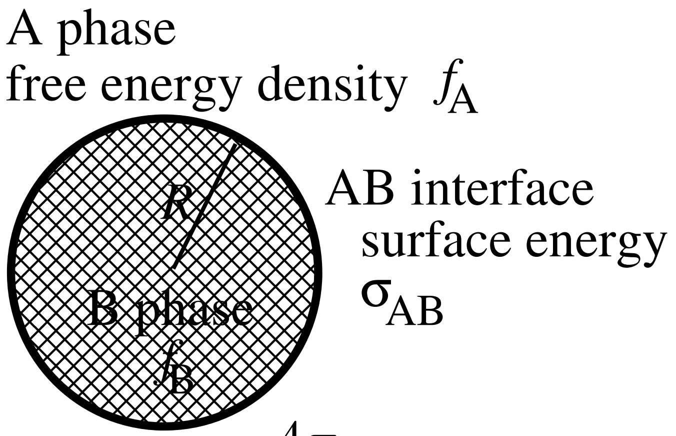
$$A_A = \frac{\Delta_A}{\sqrt{2}} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & i & 0 \end{pmatrix}$$ anisotropic





  - Stability: needs free energy beyond BCS theory
  - i.e. "strong coupling corrections" to free energy

^{3}He AB transition rate



- Homogeneous nucleation theory
  - nucleation rate per unit volume

Cahn, Hilliard 1958,
Langer 1969,
Coleman 1974, Linde 1983

$$p(T) \sim \omega \xi_{\rm GL}^{-3} \exp\left(-E_{\rm c}/T\right)$$

  - E_c (T) – energy of critical droplet/bubble
  - E_c/T ~ 10^4 - 10^6 in ^{3}He A/B GL theory
    - High density of states at Fermi surface N_F

$$E(R) = 4\pi R^2 \sigma_{\rm AB} - \frac{4\pi}{3} R^3 (f_A - f_B)$$

Critical bubble:

$$\left.\frac{dE}{dR}\right|_{R_c} = 0, \quad E_c = \frac{16\pi}{3} \frac{\sigma_{AB}^3}{|f_A - f_B|^2}$$

$$\sigma_{AB} \simeq \xi_{GL} |f_B|$$

- Prediction: supercooled A lasts a *long* time

Kaul, Kleinert 1980; Bailin, Love 1980

$$E_c = \frac{16\pi}{3} |f_B| \xi_{GL}^3 \frac{|f_B|^2}{(f_A - f_B)^2}$$

^{3}He AB transition rate puzzle

- But AB transition is observed

  Kleinberg et al 1974;
  Hakonen et al 1985; Fukuyama et al 1987; Swift, Buchanan 1987;
  Schiffer et al 1992; O'Keefe, Barker, Osheroff 1996
  Bartkowiak et al 2000; Zhelev et al 2017; Tian et al 2023

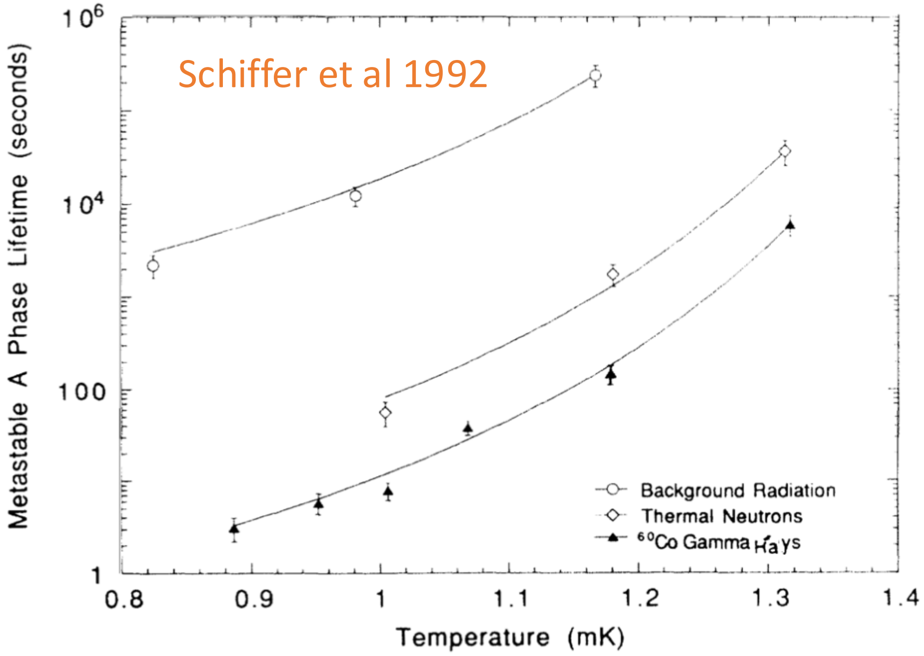
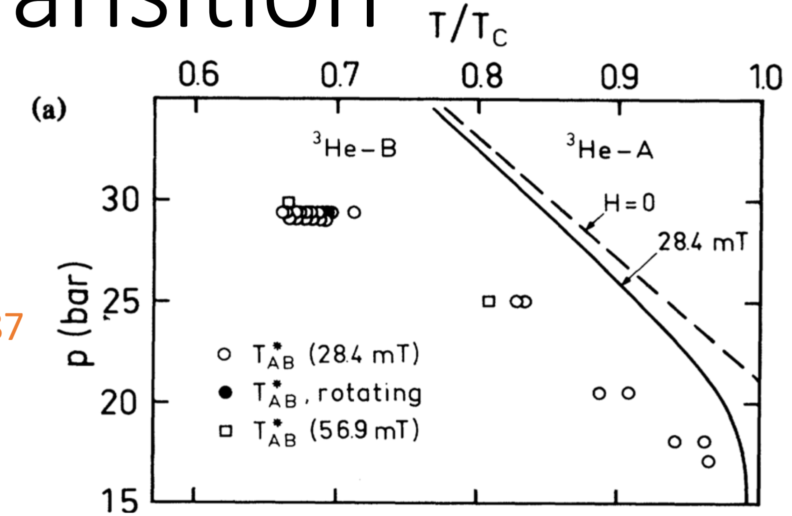


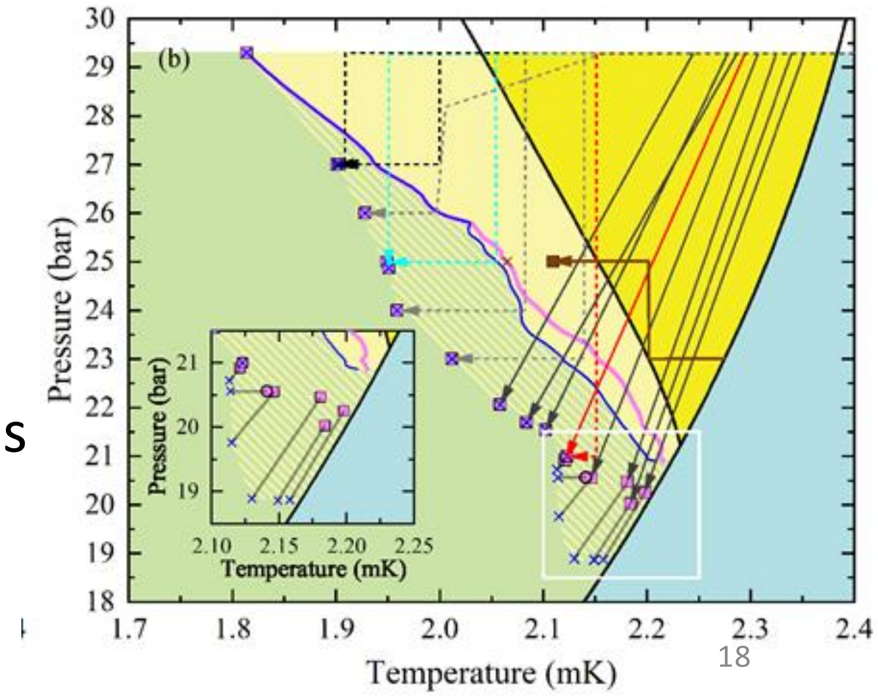
FIG. 3. Lifetime of the *A* phase in the presence of different radiation sources at $H = 28.2$ mT.

- How does transition happen?
  - Ionising radiation? ("baked Alaska", "cosmological") — Leggett 1984; Volovik, Kibble 1997; Bunkov, Timofeevskaya 1998
  - Boundary effects? (e.g. "lobster pots") — Leggett & Yip 1990
  - Textural singularities? (boojums)
  - Heterogeneous nucleation? — Balibar, Mizusaki, Sasaki 2000
  - Non-topological solitons, resonant tunnelling? — Hong 1988 — Tye, Wohns 2011

Boundary effects in AB transition

- Nucleation with rough surfaces
  - Spontaneous, AB "catastrophe line" Hakonen et al 1983 Fukuyama et al 1987
  - Can be induced by mechanical shock
    O'Keefe, Barker, Osheroff 1996; Bartkowiak et al 2000
- Path-dependent nucleation, rough surfaces
  - "Catastrophe line" depends on P_{max} in A phase Tian et al 2022
  - Model: B-phase "seeds" in O(μm) cavities
    - Heat exchanger?
  - $T > T_{AB}$: Higher P forces A phase into smaller spaces
  - $T < T_{AB}$: Smaller seeds need larger ΔP to expand





^{3}He A-B transition by energetic particles?

- Cosmic ray muons at sea level ~ 1 /cm^2/min
- Radioactivity
- Energy injection: local heating to normal phase, cooling to B phase
- Baked Alaska: expanding hot shell (ballistic quasiparticles) Leggett 1985
- Cosmological scenario: shrinking hot blob (quasiparticle diffusion)

Volovik 1996; Volovik, Kibble 1997; Bunkov Timofeevskaya 1998

Cosmological | Baked Alaska

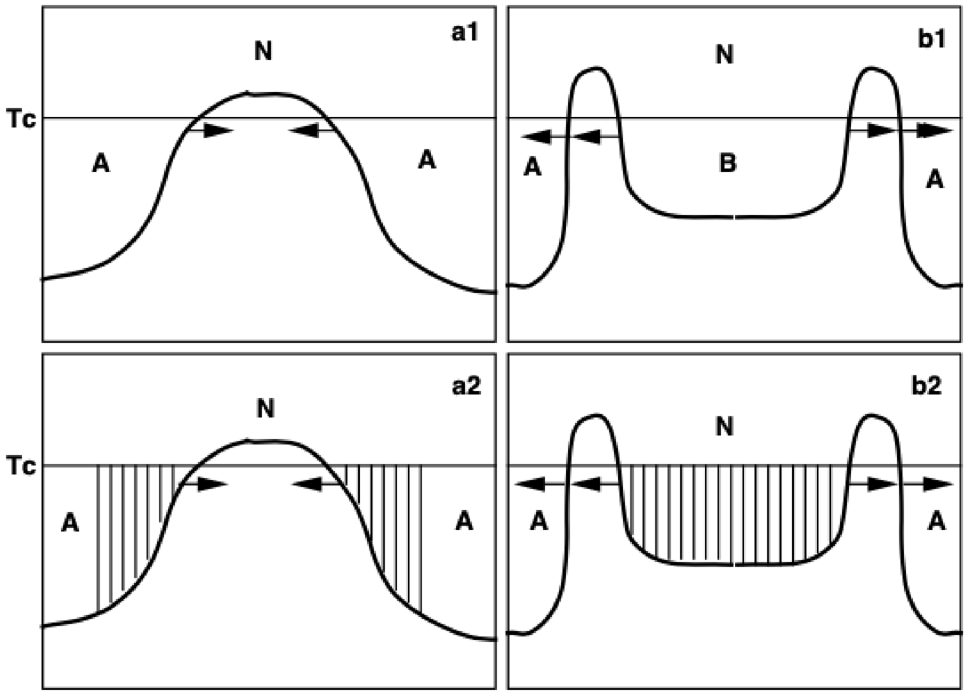


FIG. 1. The different scenario of transitions after a local heating. Vertical lines indicate a region of independently nucleated superfluid seeds. Bunkov Timofeevskaya 1999

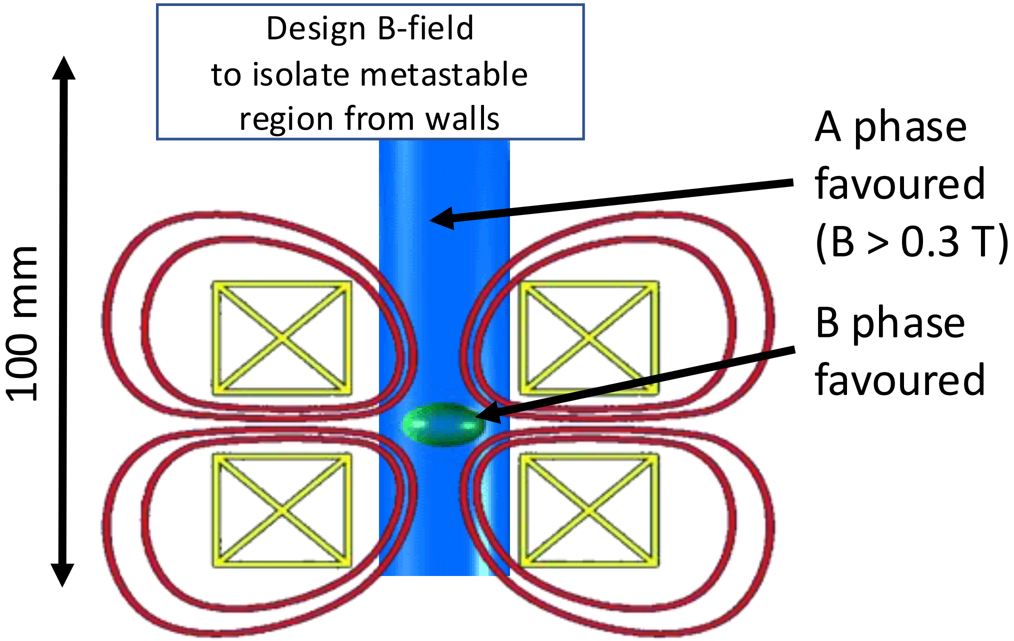
QUEST-DMC: solving the nucleation puzzle

Quantum-Enhanced Superfluid Technologies for Dark Matter and Cosmology

PTs with magnetic fields

University of Lancaster

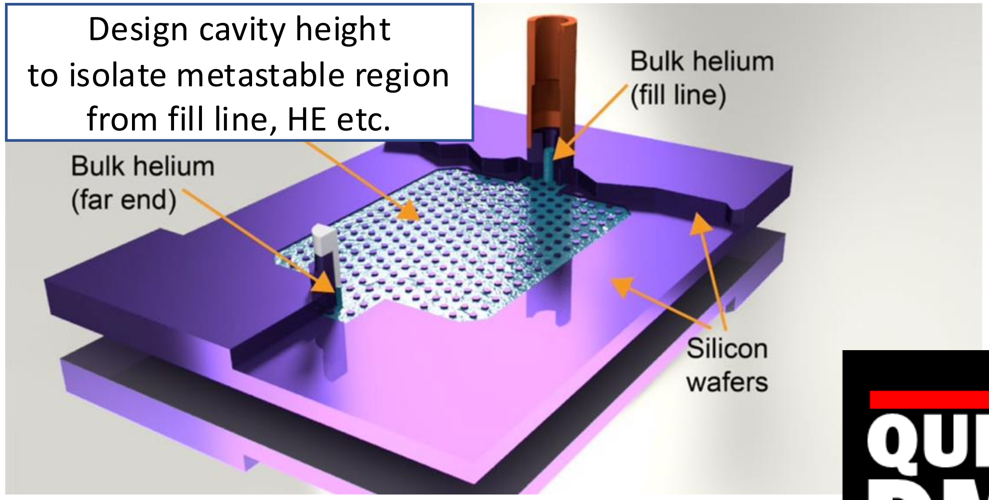
- Eliminate boundary effects
- Transition in shaped magnetic fields



PTs under confinement

Royal Holloway, University of London

- Control boundary effects with atomically smooth walls
- Confinement controls phases

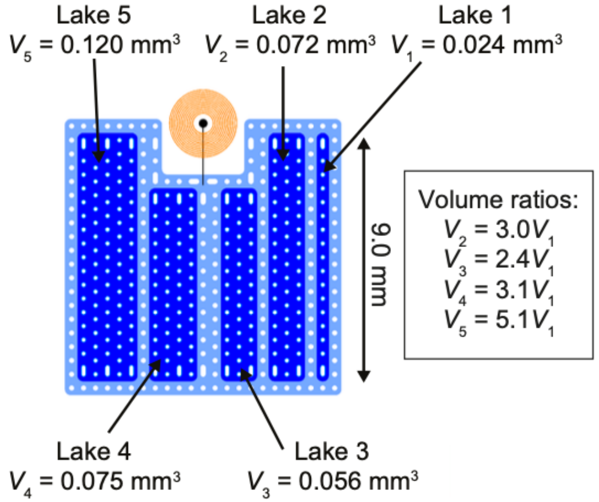


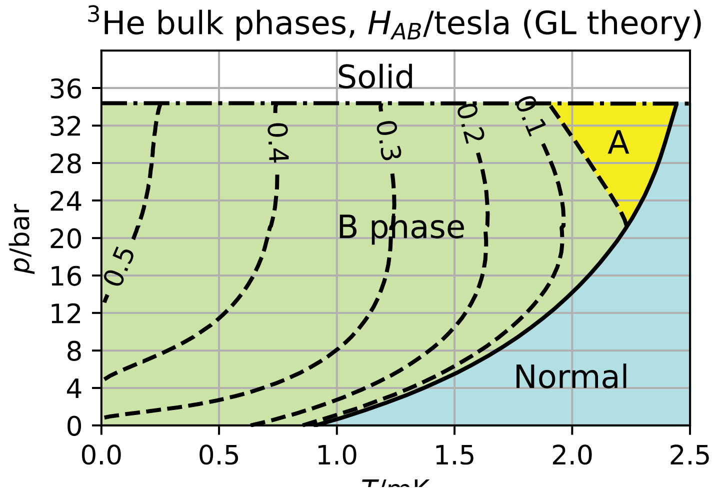
QUEST DMC

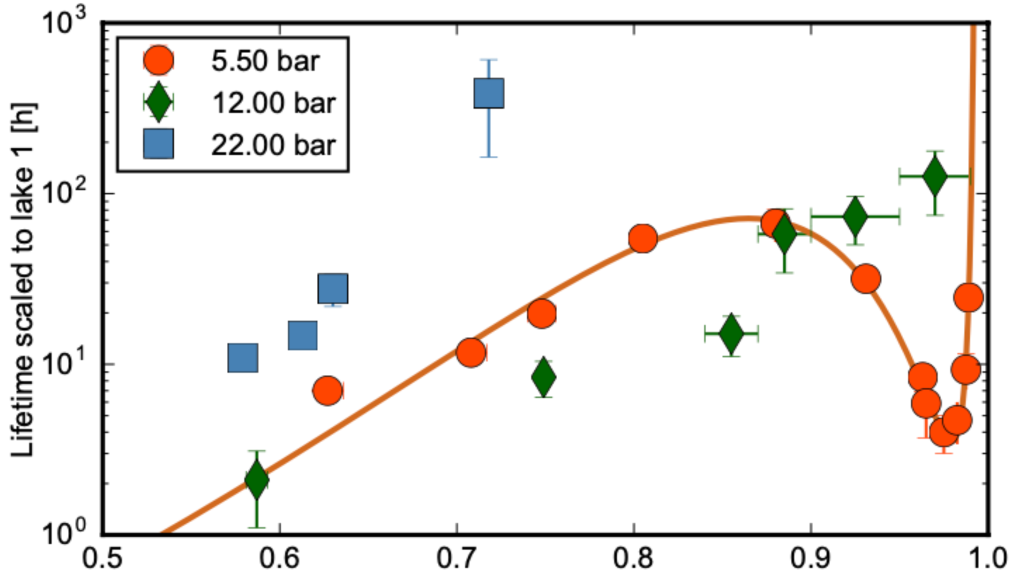
First results from thin slab experiment

Heikkinen talk QFS 2024

- Detect phase with NMR (B = 30 mT)
- Measure lifetimes of A phase in "lakes" at metastability
- Scale to same volume
- Fit 5.5 bar: $\tau = \tau_0 \exp(-E_c(T,p)/E_0)$
  - Fluctuation energy: $E_0 \sim 10^{-4}$ eV
  - $E_0 >> T$ (!?)
  - 12 bar???
- Attempt rate: $\tau_0 \sim 100$ Hz
  - Flux of e^- from ^{40}K?







Numerical simulations of ^{3}He order parameter

MH, J Sauls, K Zhang et al JLTP 2024

- Time-dependent Ginzburg-Landau model (effective theory near T_c)

$$\tau_0 \ddot{A}_{\alpha i} + \gamma \dot{A}_{\alpha i} - K_1 \partial^2 A_{\alpha i} - (K_2 + K_3)\partial_i \partial_j A_{\alpha j} + \alpha A_{\alpha i} + 2\left[\beta_1 A^*_{\alpha i} \mathrm{Tr} A A^T + \beta_2 A_{\alpha i} \mathrm{Tr} A A^\dagger + \beta_3 (A A^T A^*)_{\alpha i} + \beta_4 (A A^\dagger A)_{\alpha i} + \beta_5 (A^* A^T A)_{\alpha i}\right] = \zeta_{\alpha i}(\mathbf{r}, t) .$$

- $A_{\alpha i}$ – superfluid ^{3}He order parameter (3 x 3 complex)
- $\alpha(T)$, $\beta_a(T,P)$ – material parameters (β_a strong coupling corrected)
- γ - (Re) damping rate (quasiparticles), (Im) particle-hole asymmetry
- ζ – Langevin noise $\langle \zeta_{\alpha i}(\mathbf{r}, t) \zeta_{\beta j}(\mathbf{r}', t') \rangle = 2\gamma\, k_B T\, \delta_{\alpha\beta} \delta_{ij} \delta(\mathbf{r} - \mathbf{r}') \delta(t - t') .$
- Boundary conditions used so far:
  - periodic (for bulk simulations)
  - maximal pair breaking ($A_{\alpha i}$(boundary) = 0)
- Aim: understanding the dynamics of phase transition

QUEST DMC

Numerical simulations of ^{3}He order parameter

MH, A. Lopez-Eiguren, K. Rummukainen, K Zhang

- Discretisation
  - Space: Nearest-neighbour finite differences
  - Time: leapfrog
- Parallelism: HILA (https://cft-hy.github.io/HILA/)
  - C++ lattice field theory HPC programming framework
  - “Write once – run anywhere” (including GPU)

```
Field<Complex<double>> f;
Field<double> g = 0;

onsites(ALL) f[X].gaussian_random();

foralldir(d){
  g[ALL] += abs(f[X+d] - 2*f[X] + f[X-d]);
}
```

- Resources: CSC Finland

CSC ICT Solutions for Brilliant Minds

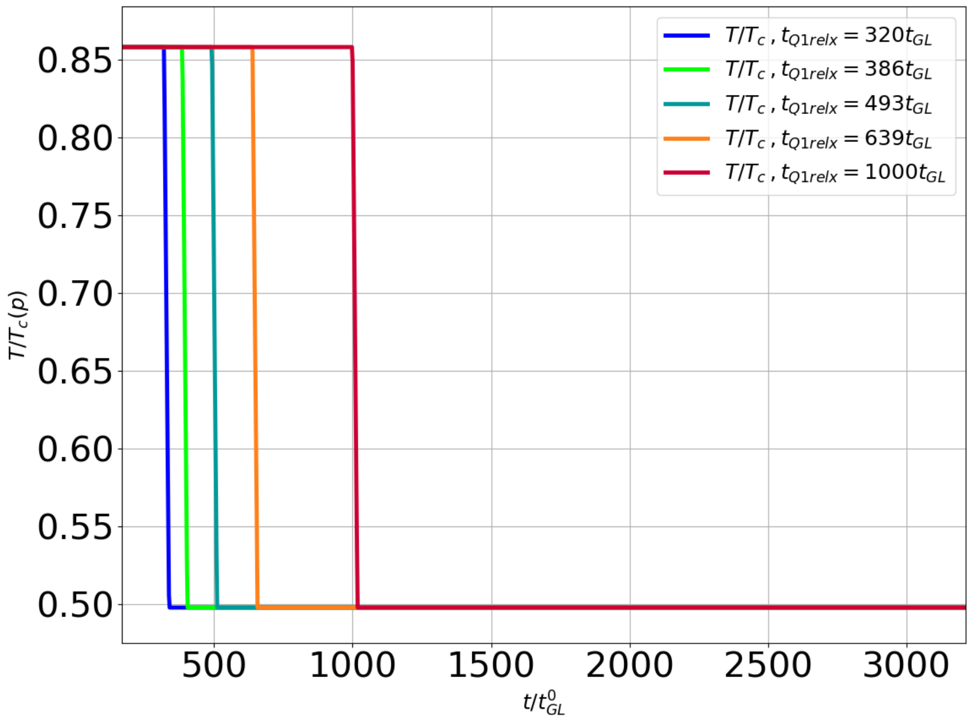
  - Mahti (1404 x 128 CPUs)
  - Lumi (2560 x 8 AMD 64GB GPU)
    Europe No 1, World No 3

QUEST DMC

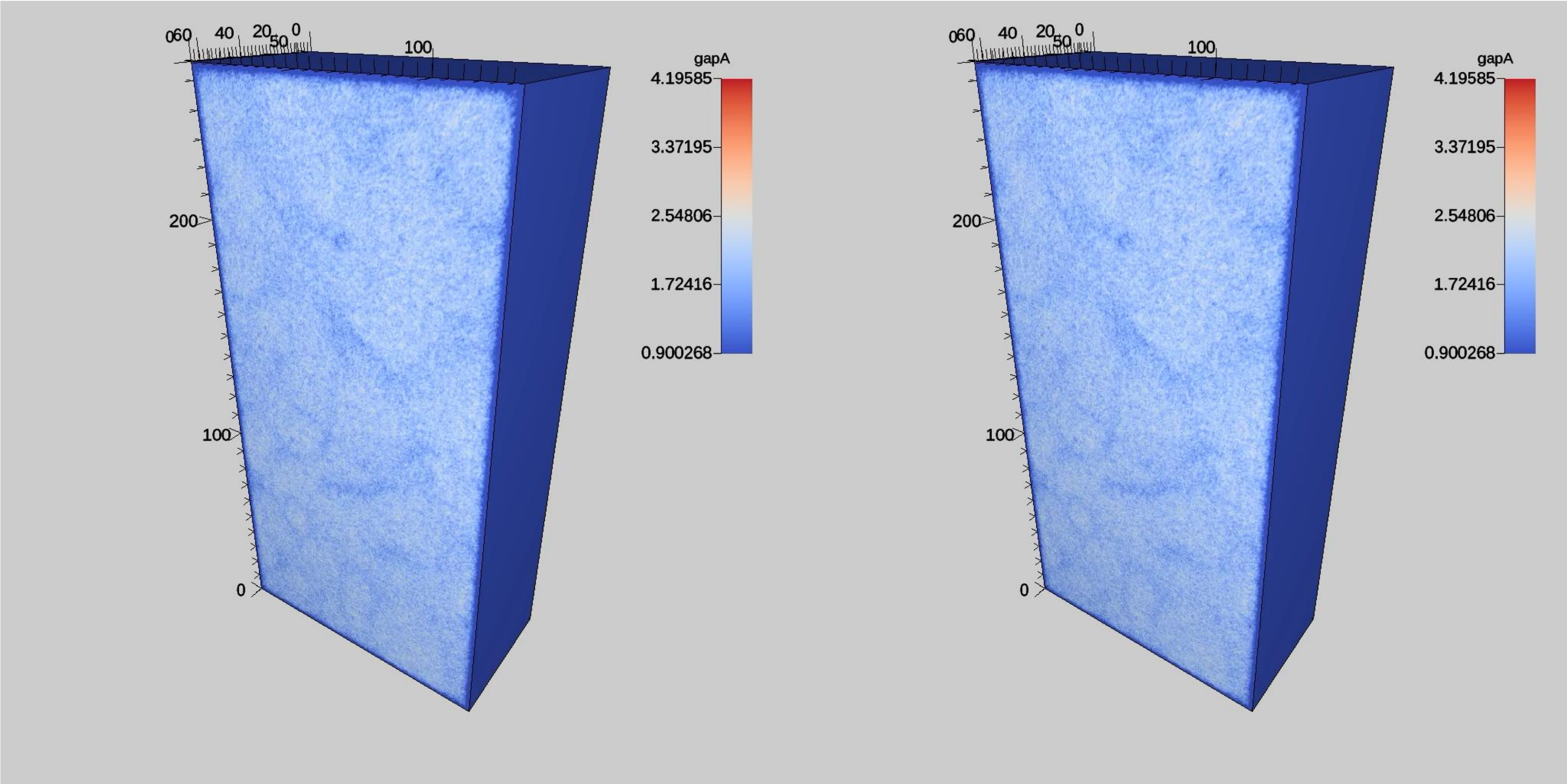
"Cosmological" simulations of ^{3}He (preliminary)

MH, A. Lopez-Eiguren, K. Rummukainen, K Zhang, QUEST-DMC

- Model rapid cooling after energy injection
- P = 26.0 bar
  - T_c = 2.33 mK, T_{AB} = 2.06 mK = 0.88 T_c
  - ξ_{GL} = 12 nm
  - $t_{GL} = \xi_{GL} / v_F$ = 0.3 ns
- Protocol:
  - Quench from normal to A phase (0.86 Tc)
  - Equilibrate in A phase
  - Quench into B phase (0.5 Tc)
  - $\Delta_A = 3.81\ T_c$, $\Delta_B = 3.99\ T_c$
  - Quench time $\tau_Q = 50\ t_{GL}$
- Dissipation $\gamma = 0.05/t_{GL}$
- Lattice spacing $\Delta x = 0.5\xi_{GL}$
- Lattice volume 512 x 256^2 Δx^3 = 3.1 x 1.5 μm^3
- Total simulation time 3000 t_{GL} = 1000 ns



“Cosmological” simulations of ^{3}He (preliminary)

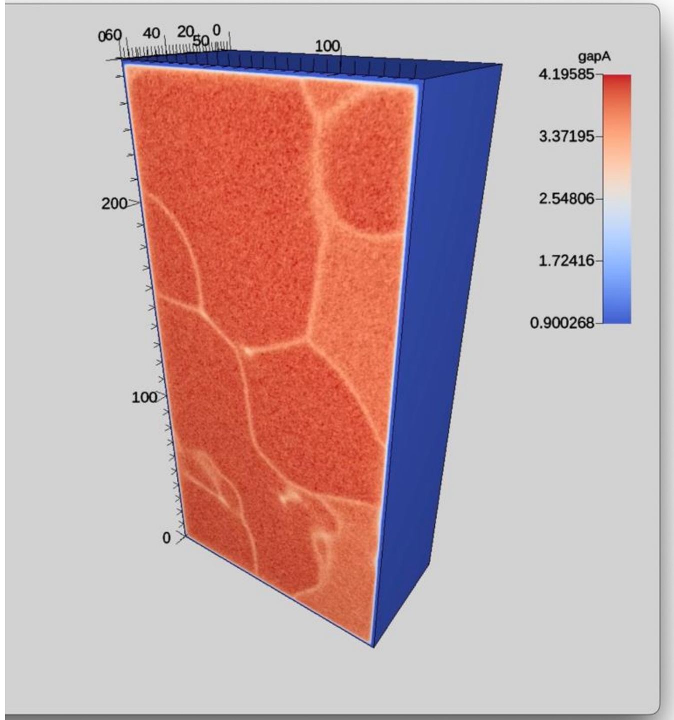


Quenches through T_{AB}
Starting in A phase

Left:
Late quench
Stays in A phase

Right:
Early quench
B phase appears

"Cosmological" simulations of ^{3}He



Conclusions:

- Rapid quenches leave "seeds" for B phase
- Topological defects from Kibble-Zurek mechanism (?)

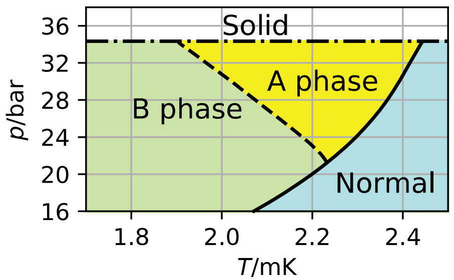
Further work needed:

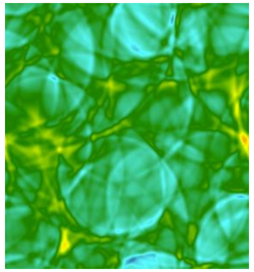
- What are the "seeds"?
- How likely? Rate?
- More physics:
  - better modelling of energy injection by particles ("hot blob") [synergy with QUEST-DMC dark matter search]
  - improve on Ginzburg-Landau free energy for low *T*
  - TDGL kinetic terms: improve in A phase

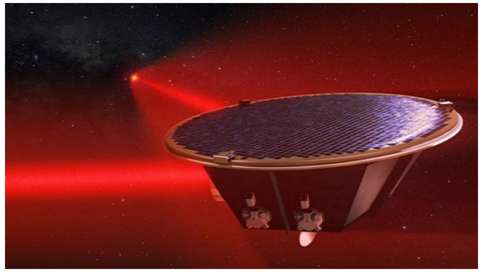
QUEST DMC

Summary

- Future space-based gravitational wave detectors (e.g. LISA) will probe the early Universe at $T \sim 100$ GeV and $t \sim 10$ ps.
- Theories extending the Standard Model to account for dark matter and the baryon asymmetry often have 1st order phase transitions, leading to GW production.
- Calculations of the GW power spectrum depend on homogeneous nucleation theory – but ^{3}He A-B doesn't obey the theory
- Better understanding of superfluid ^{3}He A/B transition is critical for reliable predictions for early Universe GW production for LISA.
- QUEST-DMC is addressing the nucleation puzzle with confinement, magnetic fields, & HPC tools from cosmology







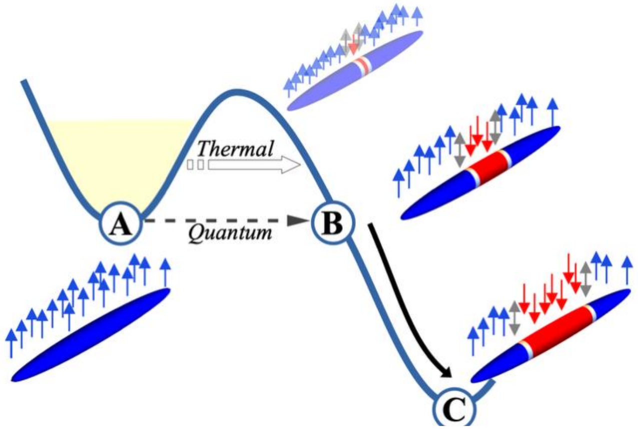
Back-up slides

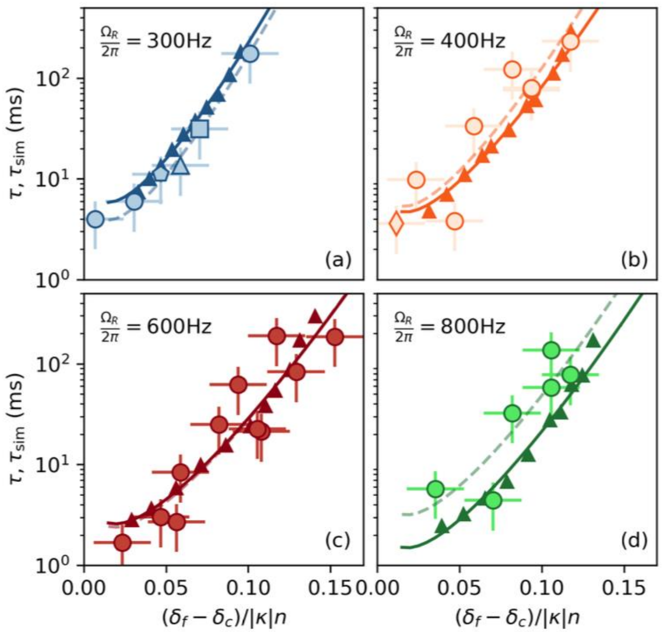
Thermal 1st order transition in BEC

- Is nucleation theory correct?
  - Support from BEC   Zenesini et al 2023
- ^{23}Na (10^6 atoms) trapped and cooled below T_{BEC}
  - 1D BEC, atoms in two states: ”up”, “down”,
  - Ferromagnetic superfluid
  - magnetic field: states separated by energy $\hbar\omega$
  - Control energy (magnetization Z)
  - µwave amplitude Ω_{R}, tune B field (δ_{f})

$$E(Z,\phi) \propto -\frac{\delta_f}{2} Z + \frac{\kappa n}{2} Z^2 - \Omega_{\mathrm{R}} \sqrt{1 - Z^2} \cos\phi$$

- Good agreement with theory

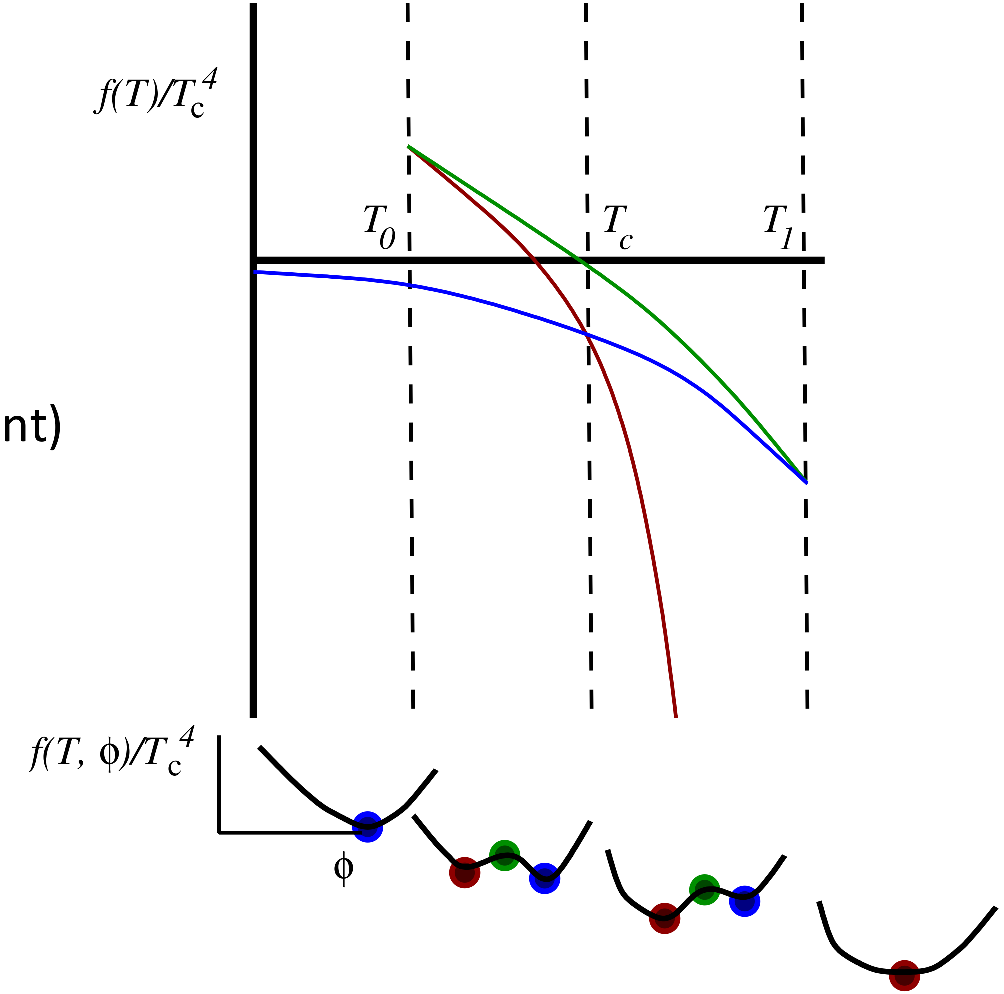




First order phase transitions

e.g. Binder 1987

- Phase: a local minimum of the free energy density f
- Behaviour of free energy around 1st order PT:
  - $T > T_2$: one equilibrium phase
  - $T_0 < T < T_1$: two equilibrium phases, one unstable
  - $T = T_c$: equal free energy, critical temperature
  - $T_0 < T < T_c$: high temperature phase is metastable
  - $T = T_0$: high temperature phase is unstable (spinodal point)
- Metastable phase can persist to $T = 0$
  - Example: superfluid ^{3}He, A phase
- Keep track of phase with order parameter ϕ
- In equilibrium: $$\partial_\phi f(T, \phi_{\text{eq}}) = 0$$
- Equilibrium free energy: $$f(T) \equiv f(T, \phi_{\text{eq}})$$



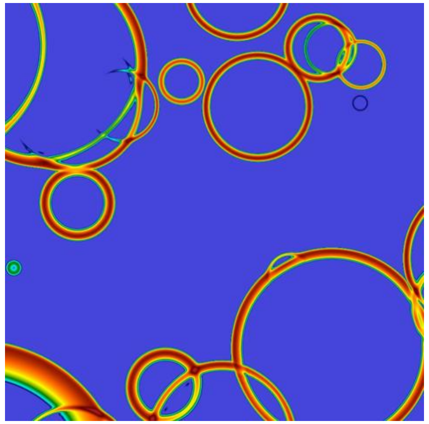
GWs from phase transitions: Sound shell model

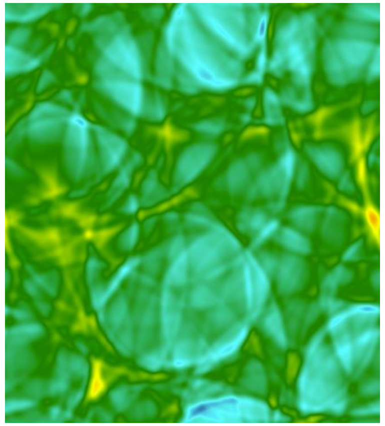
- Gaussian velocity field assumed linear
- Weighted addition of self-similar sound "shells" surrounding bubbles $\mathbf{v}_{\mathbf{q}}(t_i)$

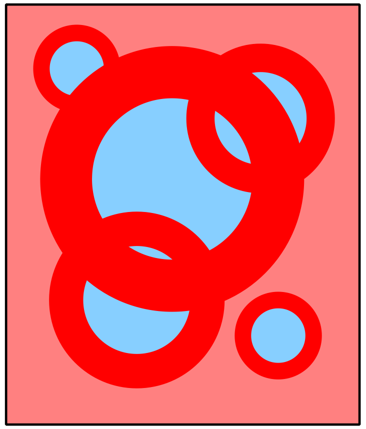
  MH 2017, MH, Hijazi 2019

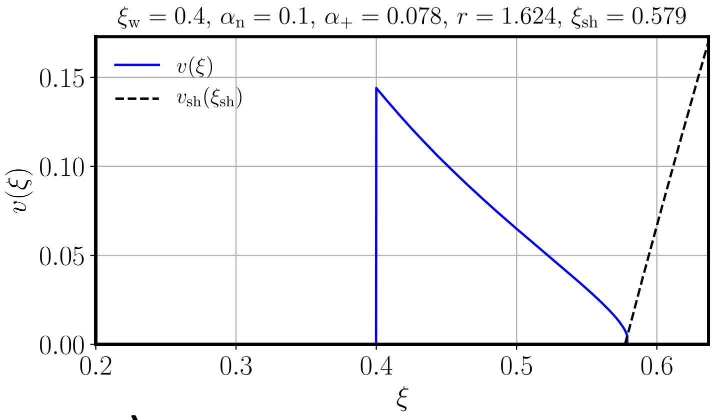
- Two length scales:
  - Bubble spacing R_*
  - Shell width $\Delta R_* \sim R_* |v_w - c_s|/c_s$
- Double broken power law
  - $P_{gw} \sim k^9, k^b, k^{-3}, b \sim 1$
- Amplitude proportional to:
  - Bubble spacing R_*
  - Shear stress lifetime
  - (Fluid kinetic energy)^2
- Similar: bulk flow model (real space)

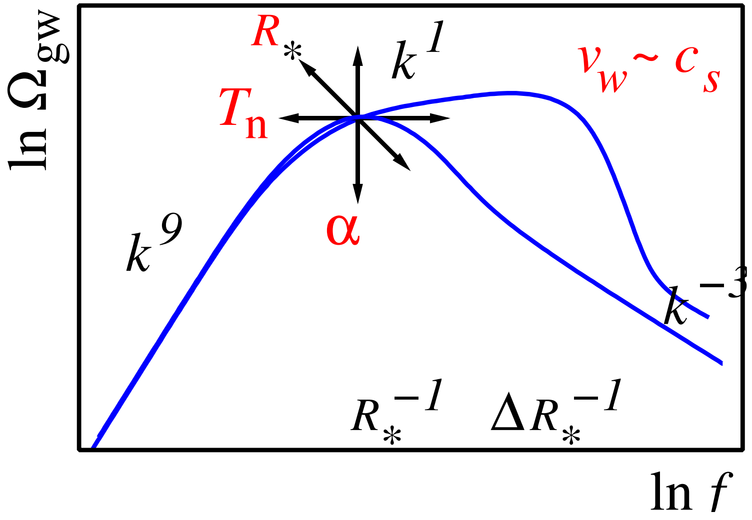
  Jinno, Konstandin, Rubira 2020











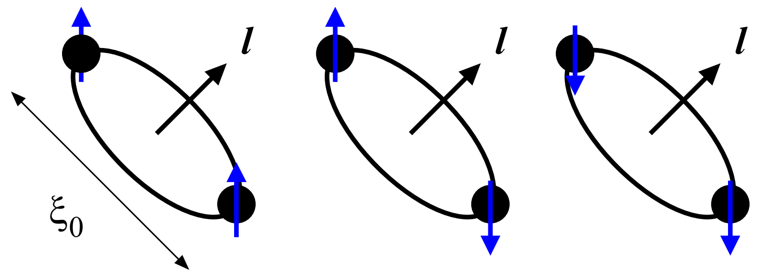
^{3}He overview

- T < 3.19 K, Fermi liquid of spin-1/2 particles
  - Fermi energy E_F = 4.25 x 10^{-4} eV (T_F = 4.94 K)
  - Effective mass $m_* = (3\text{-}6) m_{He3}$
- Attractive interaction in L = 1 channel
  - Cooper pairs L = 1, S = 1
  - Pair correlation length ξ_0 ~ 20 – 80 nm
  - Quasiparticle mass gap $\Delta_{\boldsymbol{k}ab}$ at low T
- Superfluid below T_c ~ 2 mK
- Order parameter A, complex 3x3

$$\Delta_{\mathbf{k}} = (i\sigma_2 \sigma^\alpha) A_{\alpha i} \hat{k}^i$$

- Effective theory for A: Ginzburg-Landau
  - Expand free energy in powers of A/T
  - Free energy for uniform states U(A)

$$V_{\mathbf{k}\mathbf{k}'} \simeq \begin{cases} 3V_1 P_1(\mathbf{k}\cdot\mathbf{k}') & \mathbf{k}, \mathbf{k}' \text{ near Fermi surface} \\ 0 & \text{elsewhere} \end{cases}$$



$$\Delta_{\mathbf{k}ab} = \int \frac{d^3k'}{(2\pi)^3} V_{\mathbf{k}\mathbf{k}'} \frac{\tanh(\beta E_{\mathbf{k}'a}/2)}{2E_{\mathbf{k}'a}} \Delta_{\mathbf{k}'ab}$$

spin indices a, b $\quad E^2_{\mathbf{k}a} = \left(\frac{\hbar^2 k^2}{2m_*} - \mu\right)^2 + (\Delta_{\mathrm{k}} \Delta^\dagger_{\mathrm{k}})_a$

$$U(A) = \alpha'_c (T - T_c)\mathrm{tr}(AA^\dagger) + \beta_1 |\mathrm{tr}(AA^T)|^2 + \beta_2 \mathrm{tr}(AA^\dagger)^2 + \beta_3 \mathrm{tr}(AA^T A^* A^\dagger) + \beta_4 \mathrm{tr}(AA^\dagger AA^\dagger) + \beta_5 \mathrm{tr}(AA^\dagger A^* A^T).$$

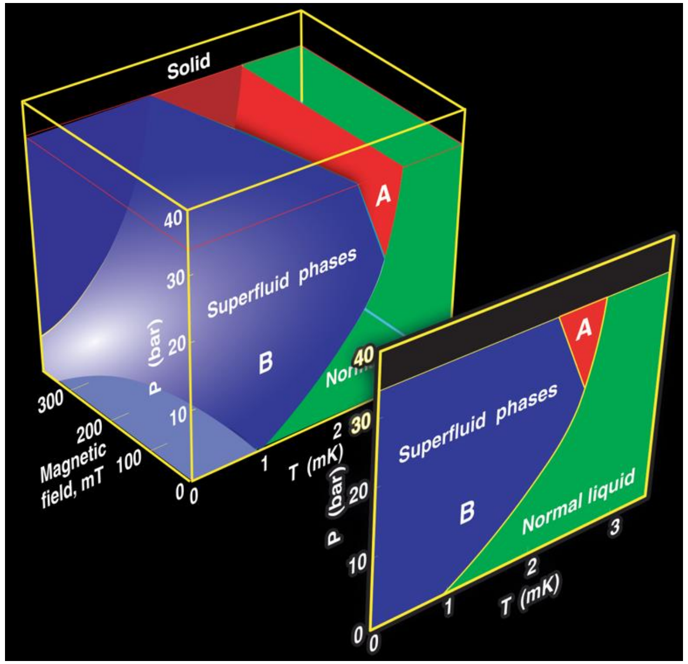
Eliminating boundaries with magnetic fields

- Magnetic field favours A phase
  - A has spins aligned
  - No B-phase for H > 0.3 T
- Changing H also induces transition
- Undermagnetisation of A phase

$$f_H = g_H H_\alpha (AA^\dagger)_{\alpha\beta} H_\beta$$

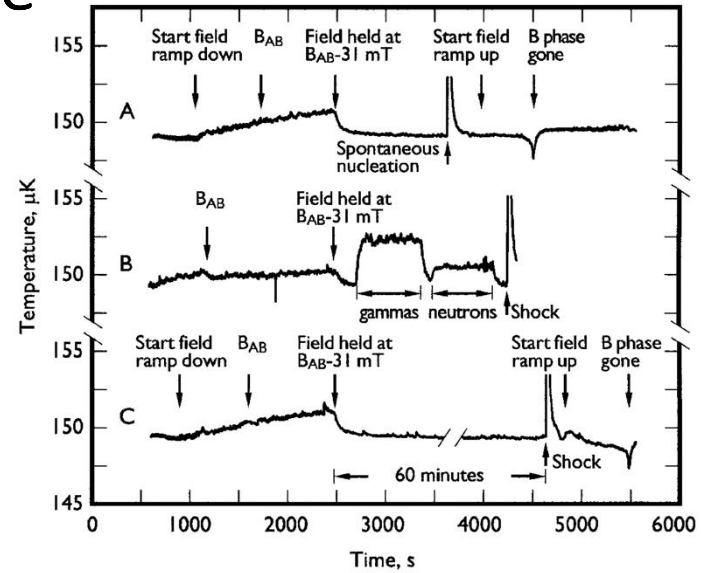
$$(AA^\dagger)^B \propto 1,$$

$$(AA^\dagger)^A \propto \mathrm{diag}(0,0,1)$$



Bartkowiak et al 2000

- Lifetime should still be long
- Spontaneous nucleation
- Also mechanical shock
- (Sinter particle found in cell)

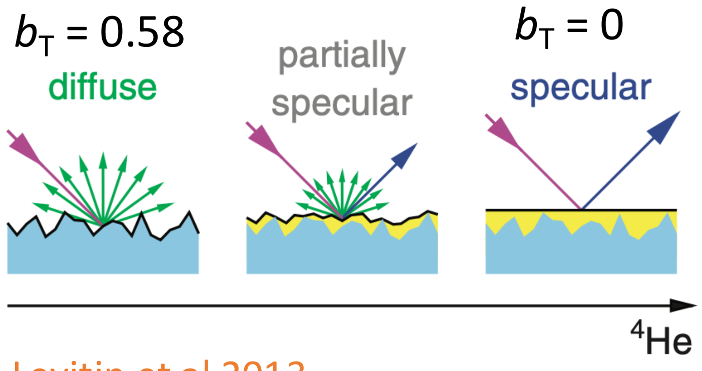


Controlling phase with confinement

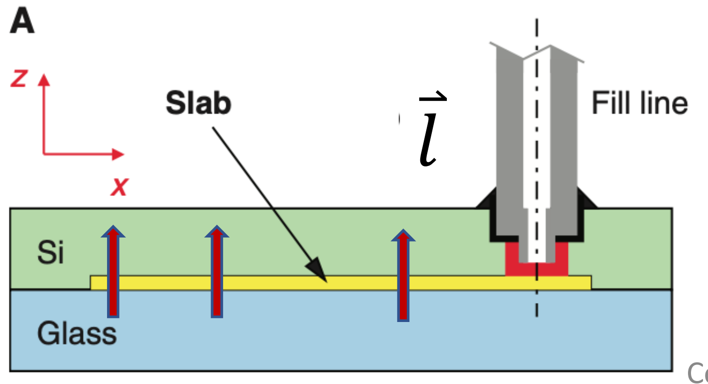
- Boundaries break Cooper pairs & suppress order parameter Ambegaokar, de Gennes, Rainer 1973

$$A_{\alpha i} n_i = 0, \quad (b_T + \mathbf{n}\cdot\nabla) A^{\parallel}_{\alpha i} = 0$$

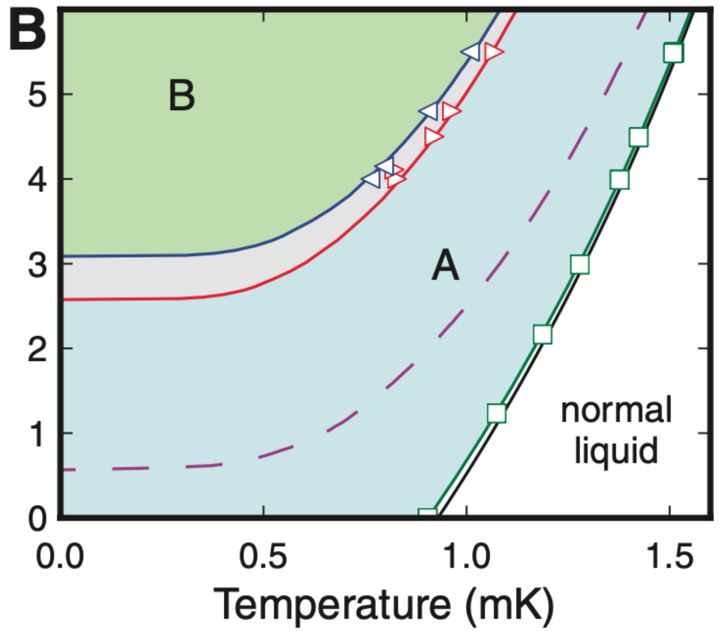
  - Control b_T with ^{4}He coating
- Angular momentum of Cooper pairs normal to boundaries
- Thin slab favours A phase



Levitin et al 2013



Slab with 'specular' walls

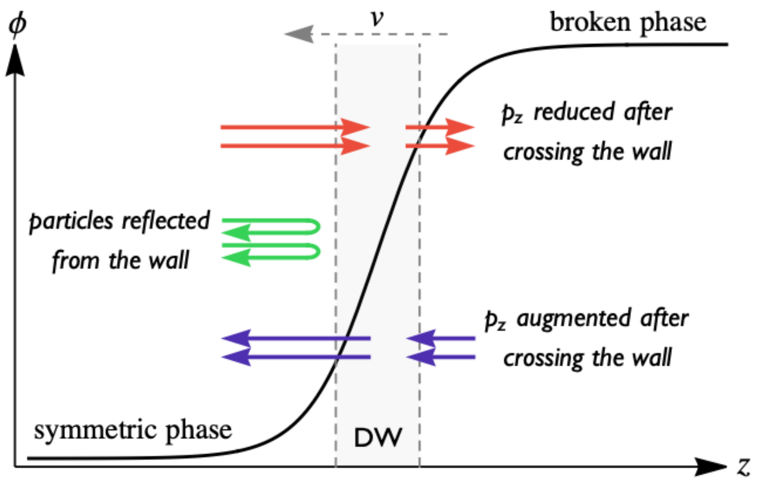


Phase boundary speed

- GW signal sensitive to phase boundary speed v_w
  - Difficult non-equilibrium problem

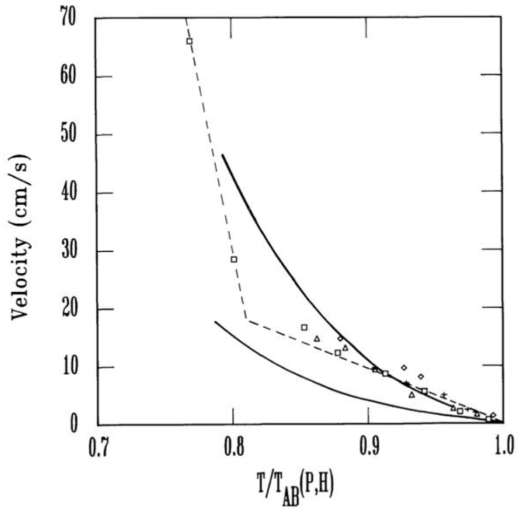
    McLerran, Turok, Liu 1994; Moore, Prokopec 1994; Huber, Sopena 2013; Laurent, Cline 2022; De Curtis et al 2022

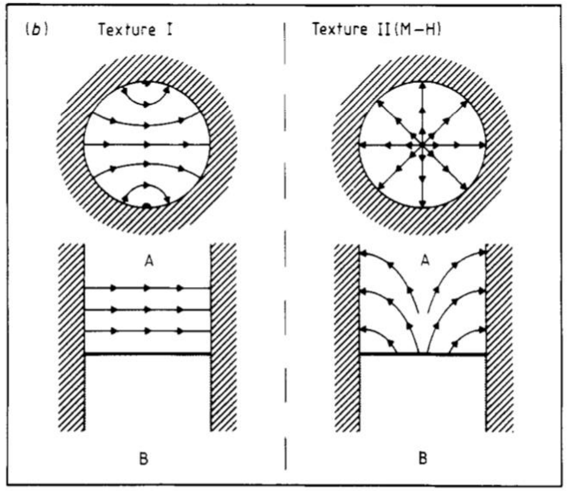
- AB Phase boundary speed measured Swift, Buchanan 1



- Quasiparticle scattering calculations

    Yip, Leggett 1986; Kopnin 1987; Palmieri 1990



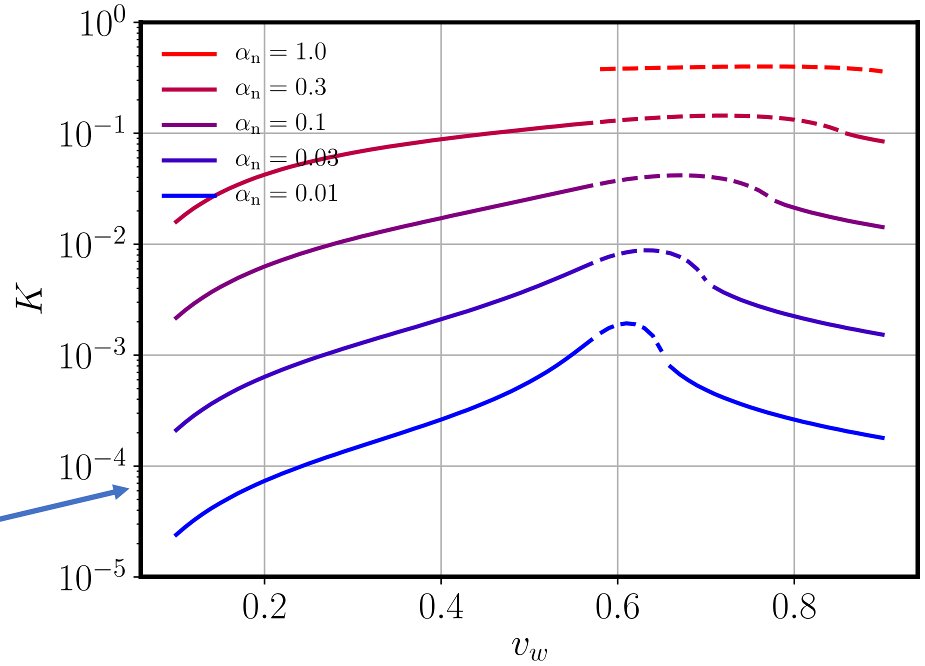


Salomaa 1988

- Dynamics complicated by S and L texture in A phase
  - E.g. interaction with boundary
  - Spin waves in front of wall?

Cosmological applications of AB transition

1. Transition rate
    - Theory predicts no AB transition in bulk ^{3}He
        - How long can supercooled A phase be maintained?
    - Rate with boundaries/magnetic fields?
    - If rate observable, compare to probability flux through inhomogeneous "critical configuration"
2. Phase boundary propagation
    - Fluid kinetic energy & hence GWs have strong dependence on wall speed
    - Same microscopic theory (quasiparticles + OP)
3. Generation of ^{3}He collective modes
    - Efficiency of hydrodynamic sound wave production
    - Analogue GW (massless J = 2) production?



Mode	Symmetry	Mass	Name
$D^{(+)}_{0,m}$	$J=0, c=+1$	2Δ	Amplitude
$D^{(-)}_{0,m}$	$J=0, c=-1$	0	Phase Mode
$D^{(+)}_{1,m}$	$J=1, c=+1$	0	NG Spin-Orbit Modes
$D^{(-)}_{1,m}$	$J=1, c=-1$	2Δ	AH Spin-Orbit Modes
$D^{(+)}_{2,m}$	$J=2, c=+1$	$\sqrt{\frac{8}{5}}\Delta$	2^+ AH Modes
$D^{(-)}_{2,m}$	$J=2, c=-1$	$\sqrt{\frac{12}{5}}\Delta$	2^- AH Modes

B phase bosonic modes

Sauls, Mizushima 16

Ginzburg-Landau and AB phase boundary

Schopohl (1987); Thuneberg (1991)

- Ginzburg-Landau for inhomogeneous states:
  - Gradient energy $f_{\rm g}(A) = K_1 \partial_k A_{\alpha j} \partial_k A^*_{\alpha j} + K_2 \partial_j A_{\alpha j} \partial_k A^*_{\alpha k} + K_3 \partial_k A_{\alpha j} \partial_j A^*_{\alpha k}.$
  - In GL theory B phase $K_a = K$
- Solve GL equation at T_{AB}, AB boundary conditions

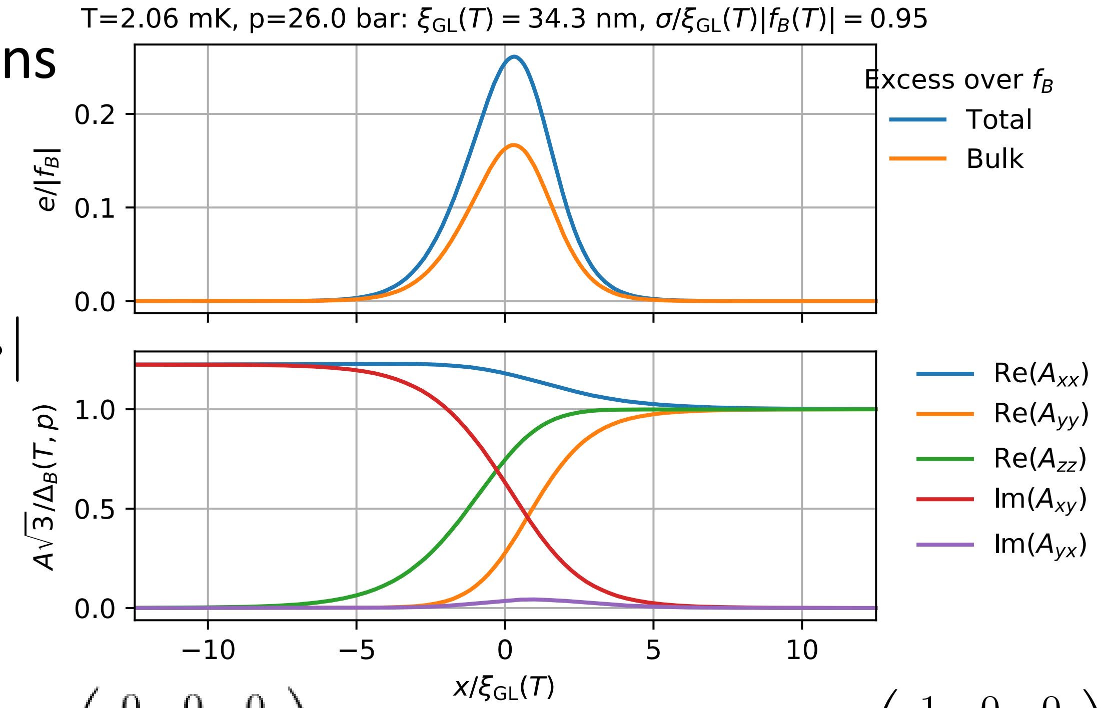
$$-K \left(2\hat{x}_i \hat{x}_j + \delta_{ij}\right) \partial_x^2 A_{\alpha j} + \frac{\partial U(A)}{\partial A^*_{\alpha i}} = 0$$

- Surface energy ($p_{\rm pcp} < p < p_{\rm melt}$) $\sigma_{AB} \simeq \xi_{\rm GL} |f_B|$
  - $\xi_{\rm GL}$: order parameter coherence length

$$\xi_{\rm GL}(T) = \sqrt{\frac{7\zeta(3)}{20}}\, \xi_0 \left(1 - T/T_c\right)^{-1/2}$$

  - $f_{\rm B}$: B-phase condensation energy density

$$f_B \sim -f_0 (1 - T/T_c)^2, \quad f_0 \sim \text{J m}^{-3}$$



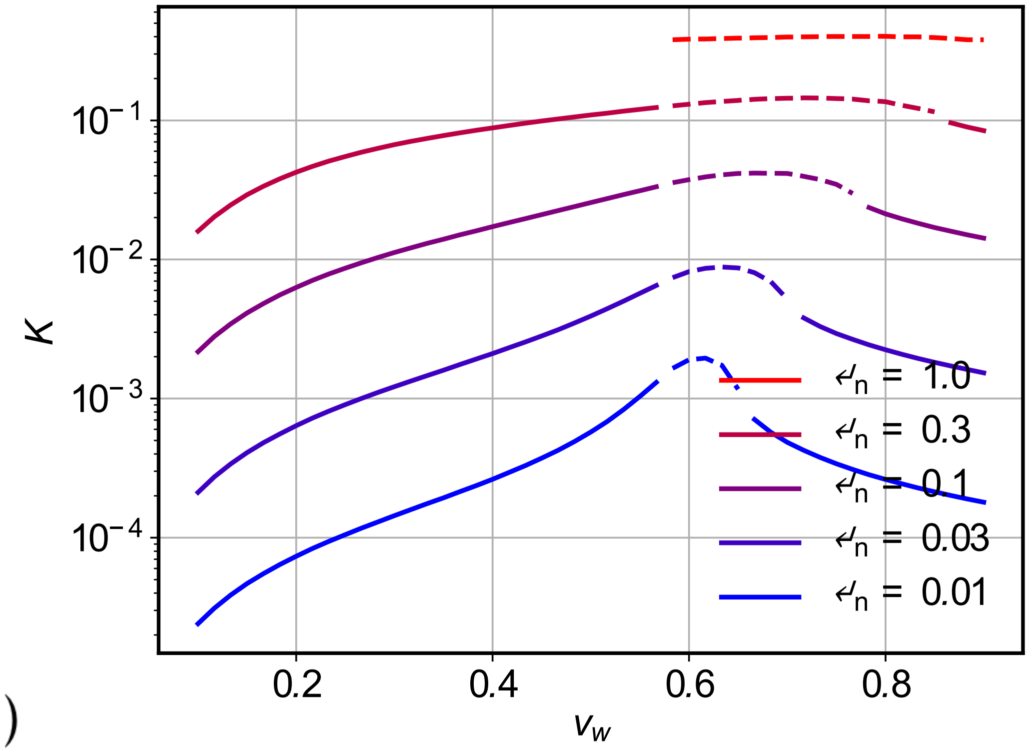
$$A_A = \frac{\Delta_A}{\sqrt{2}} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & i & 0 \end{pmatrix} \qquad A_B = \frac{\Delta_B}{\sqrt{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Estimating GW power in the SSM

$$\Omega_{\rm gw} = \rho_{\rm gw}/\rho_{\rm c} \sim (H_{\rm n}\tau_{\rm v})(H_{\rm n}\tau_{\rm c})K^2$$

- GW energy fraction:
  - H_n Hubble rate at nucleation
  - τ_v duration of stresses
  - τ_c coherence time
- Coherence time:
  - $\tau_c \sim R_*$ (bubble spacing)
- Shear stress lifetime (shocks):
  - $\tau_v = H_n^{-1}/(1 + U_f/r_*)$
  - U_f (~ RMS velocity) ~ $\sqrt{K}$
  - or $K = (4/3)U_f^2$

Estimate K (kinetic energy fraction) from self-similar hydro solution



$$\Omega_{\rm gw} \simeq \tilde{\Omega}_{\rm gw} \frac{r_*}{1+\sqrt{K}/r_*} K^2$$

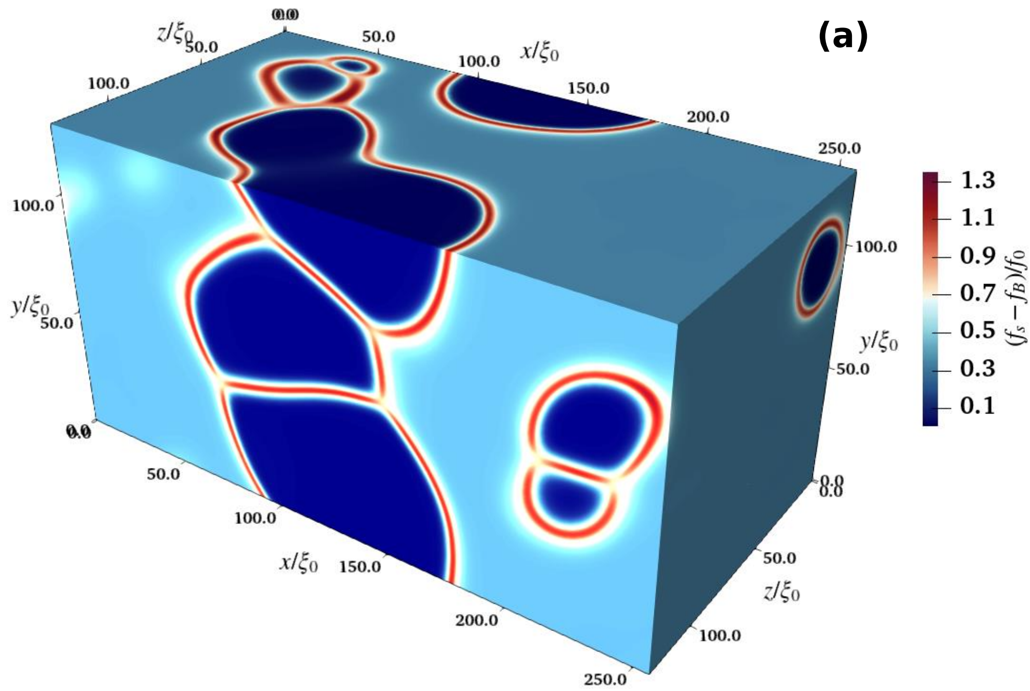
$$\tilde{\Omega}_{\rm gw} = {\rm O}(10^{-2})$$

“Cosmological” simulations of ^{3}He

- Model energy injection e.g. by cosmic ray (1st try)
- Initial condition:
  - *A*(x) Gaussian random at each x

$$\langle A\rangle = \frac{\Delta_A(T,P)}{\sqrt{2}}\begin{pmatrix}1 & i & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{pmatrix} \quad \sigma(\mathrm{Re}\,A_{\alpha i}) = \sigma(\mathrm{Im}\,A_{\alpha i}) = \sigma_0$$

- (T, P) = (1.23 mK, 25.0 bar)
- $\Delta_A = 3.7\ T_c$, $\Delta_B = 3.9\ T_c$
- Dissipation $\Gamma = 0.02/t_{GL}$ = 60 MHz
- Lattice spacing $\Delta x = \xi_{GL}$ = 12 nm, $\xi_{GL}(T)$ = 17 nm
- Lattice volume 512 x $256^2\ \Delta x^3$ = 6.2 x 3.0 μm^3
- Simulation time 600 t_{GL} = 200 ns
- $\sigma_0 = 1.9\ T_c$ -> energy injection Δf ~ 800 eV/μm^3



Free energy density above B phase $f_0 = N_F(\xi_{GL})^3/3$