CAN QUANTUM TUNNELLING INDUCE A COSMIC BOUNCE?

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1. **What is it about?**

   NEC violation in QFT (via tunnelling) and its consequences in cosmology.

$$\Lambda \to \Lambda_{eff}(a) = \Lambda - q\,\frac{e^{-\alpha^3}}{\alpha^{3/2}} \qquad \alpha^3 \propto a^3\, V_0\,.$$

2. **What is it *not* about?**
   - False vacuum decay.
   - Bubble nucleation.
   - DeSitter.

3. **Why is it important?**

   To *save the Universe* (with finite volume effects).

Can quantum tunneling induce a cosmological bounce? I will present a mechanism to induce a cosmological bounce that is purely generated by quantum fluctuations. I will also discuss the necessary conditions to have a bounce.

1. **Introduction and main results.**
   Framework, discussion and results.
   Assumptions, methods and limitations.
   Vacuum energy.
2. **The effective theory.**
   Partition function and saddle points.
   Effective action and vacuum energy.
3. **Towards a cosmic bounce.**
   Friedmann equations.
   Anisotropic universe.

Part I

Introduction, assumptions and main results

QFT in curved spaces studies the behaviour of quantum matter fields as *test fields* propagating in a specific background.

A second step to better understand the interaction between quantum fields and gravity is to study the *backreaction problem*, i.e., the effect of quantum fields on the background metric.

Semiclassical gravity.

We have to solve the semiclassical Einstein equations in a self-consistent way:

$$\kappa^{-1}\,\text{“}G_{\mu\nu}\text{”} = T_{\mu\nu}^{\text{class}} + \langle T_{\mu\nu}\rangle_{\text{ren}}$$

Singularity theorems. They show that singularities (in cosmology) are inevitable under very general circumstances.

However, they have in their assumptions a restriction on the energy-momentum tensor.

The **Null Energy Condition** (NEC)

$$T_{\mu\nu}\ell^{\mu}\ell^{\nu} \geq 0\,, \tag{1}$$

where ℓ^{μ} is a null vector. For a perfect fluid the null energy condition reads

$$\rho + p \geq 0\,. \tag{2}$$

- *Energy conditions can be violated by quantum fields.*

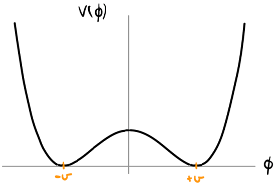
If the NEC is violated $\rightarrow$ can we avoid the "initial singularity"?

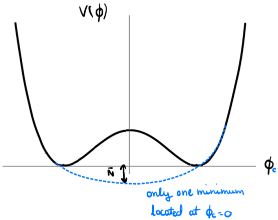
Starting point: the effective theory

We consider a real scalar field with a double-well bare potential with two degenerate minima in *finite volume* V_0 and in a *slowly varying* FLRW background metric.

$$U(\phi) = \frac{\lambda}{4!}(\phi^2 - v^2)^2 + \kappa^{-1}\Lambda\,, \qquad \mathrm{d}s^2 = -\mathrm{d}t^2 + a^2\mathrm{d}\mathbf{x}^2$$

In this context, quantum tunneling can happen between the two vacua. We compute the effective theory that captures this effect and study the properties of the resulting *vacuum state*.





The effective potential reads:

$$U_{eff}(\phi_c) = U_0 + M^2\phi_c^2 + \mathcal{O}(\phi_c^4)\,, \qquad \alpha^3 = a^3\,\frac{S_0}{\hbar} \quad \text{and} \quad S_0 \sim V_0$$

$$U_0 = \kappa^{-1}\Lambda\left(1 - r\,\frac{e^{-\alpha^3}}{\alpha^{3/2}}\right) \qquad r = \frac{\lambda\kappa v^4}{3\sqrt{3\pi}\Lambda}$$

From this result, we can compute the stress-energy tensor of the ground state: the Null Energy Condition (NEC) is violated

$$\frac{\kappa}{\Lambda}(\rho + p) = -r\,e^{-\alpha^3}\left(\alpha^{3/2} + \frac{1}{2}\alpha^{-3/2}\right) < 0\,.$$

We couple this “ground state fluid” to gravity via the Einstein equations.

If we start from a contracting phase $H < 0$, we can induce a cosmological bounce.

Assumptions, methods, and limitations

- **Finite Volume** $\to$ allows tunnelling [*3-torus, 3-sphere, box...*].
  - Fundamental volume cell V_0. Comoving volume $a^3\, V_0$.
  - We neglect spatial curvature K.
  - We assume that momentum quantisation can be ignored.
  - We assume that "periodic boundary conditions" do not play a role.
- **Adiabatic approximation.** Expansion rate $\frac{\dot{a}}{a} \ll$ tunneling rate (intantaneous effect).

  We set $a = cte$ for the computation of the effective theory.
  We couple the effective theory to gravity restoring $a = a(t)$.

- **Equilibrium QFT techniques:** Path integral approach in the $\langle in|out\rangle$ formalism. **Wick rotation**. Euclidean/imaginary time is needed to study this effect.

  Naively $t \to \tau = it\,;\ \sqrt{-g} \to \sqrt{g_E}\,;\ iS \to -S_E \ldots$

Part II

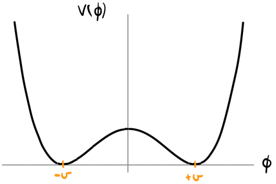
The effective theory

We start from the classical theory:

required for renormalization.

$$V(\phi) = \frac{\lambda}{4!}(\phi^2 - v^2)^2 + \bar{\Lambda} + j\phi$$

essential to obtain the effective action.



How do we arrive to $U_{eff}(\phi_c)$?

- We should find the partition function $Z[j]$ describing the system.
  - Semiclassical approximation: *several saddle points* ϕ_n are relevant for our analysis. We sum over all relevant configurations.

$$Z_E[j] = \int \mathcal{D}[\phi]\, e^{-S[\phi]/\hbar} \simeq \sum_n Z_n\,; \qquad Z_n = F_n\, e^{-S[\phi_n]/\hbar}\,.$$

  - Saddle point expansion (one-loop expansion)

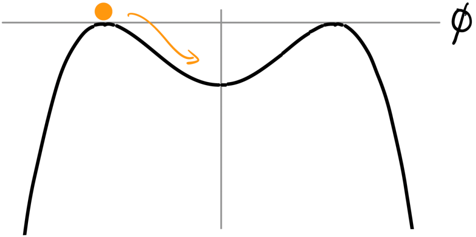
$$\phi = \phi_s + \delta\phi\,; \quad \phi_s \equiv \text{classical solution} \quad \delta\phi \equiv \text{quantum fluctuation}$$

Relevant saddle points. We focus on homogeneous configurations $\phi_s = \phi_s(t)$.

$$\phi'' + \omega^2\phi - \frac{\omega^2}{v^2}\phi^3 = 0$$

- Static saddle points.
- Instantons [all possible ways to connect the static solutions].

In imaginary time $U(\phi) \to -U(\phi)$. "Upside down potential".



Let's focus on $j = 0$ (the vacuum).

- **Static Saddle points.**

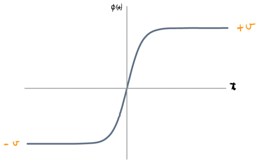
$$\phi_{1,2} = \pm v \qquad S_{1,2} = a^3 \, V_0 \, T \, \bar{\Lambda} \, .$$

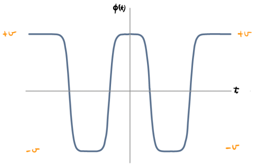
- **Instantons.** The basic solution is

$$\phi_i = \pm v \tanh(\omega(t - t_1)) \, , \qquad S = a^3 \, S_0$$

We can also have multiple jumps

$$\phi^{(p)} \simeq v^p \tanh(\omega(t - t_1)) \tanh(\omega(t - t_2)) \cdots \tanh(\omega(t - t_p)) \, , \qquad S = p a^3 \, S_0$$





Remember: $S_0 \propto V_0$

Relevant saddle points

- We have to sum over all the classical configurations.

$$Z_E = [v] + [-v] + [K] + [\bar{K}] + [K\bar{K}] + [\bar{K}K] + [K\bar{K}K] + \cdots$$

where

$$[v] = [-v] = e^{-a^3 V_0 T\bar{\Lambda}}, \qquad [K] = \int_{T/2}^{T/2} dt_1 [I][v] = T[I][v]$$

and with

$$[I] = \sqrt{\frac{a^3 S_0}{2\pi}} e^{-a^3 S_0} \left(\frac{\det' S_E''[\phi_i]}{\det S_E''[v]} \right)^{-\frac{1}{2}} = \sqrt{12}\,\omega \sqrt{\frac{a^3 S_0}{2\pi}} e^{-a^3 S_0} .$$

- For N-jumps we find

$$\left[K_1 \overline{K}_2 \ldots K_{N-1} \overline{K}_N\right] \approx \frac{T^N}{N!} [I]^N [v] ,$$

- **The partition function can be resummed!**

$$Z_E = 2[v] e^{-T[I]}$$

From $Z_E[0]$, we can find the effective action at the ground state (i.e., the zero-point energy)

$$\Gamma[0] = -\ln Z_E[0] \simeq \int \mathrm{d}^4x\sqrt{g}\,\bar{\Lambda}\left(1 - r\frac{e^{-\alpha^3}}{\alpha^{3/2}}\right)$$

If we want to obtain the full effective action, need to include j.

- From $Z_E[j]$ we obtain the *effective theory* in terms of an effective field ϕ_c

$$\phi_c[j] = \frac{1}{\sqrt{g}}\frac{\hbar}{Z_E[j]}\frac{\delta Z_E[j]}{\delta j} \qquad \longrightarrow \qquad j[\phi_c]$$

- The effective action $\Gamma[\phi_c]$ is defined through a Legendre transformation as a functional of ϕ_c.

$$\Gamma[\phi_c] = -\hbar\ln(Z_E[j[\phi_c]]) - \int \mathrm{d}^4x\sqrt{g}\,\phi_c\,j[\phi_c]$$

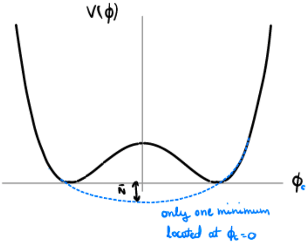
In practice, it is very difficult to obtain $\phi_c[j]$ and invert it. However, for small j we find

$$\phi_c = -M^{-2}\, j + \mathcal{O}(j^3)\,; \qquad \phi_c \text{ is linear in } j!$$

For $j = 0$ we find $\phi_c = 0$. **Symmetry restoration.**

Around the minimum, the effective action reads:

$$\Gamma[\phi_c] = \Gamma[0] + \frac{1}{2}\int \mathrm{d}^4x\, \sqrt{g}\, M^2 \phi_c^2 + \mathcal{O}(\phi_c^4)\,.$$



The vacuum energy (density) is obtained for $\phi_c = 0$, and, when coupled to gravity gives rise to the effects I showed you in the first part of the talk.

The main result (from the QFT side) is that [for now on we assume $a = 1$]

$$E_0 = -\lim_{T\to\infty} \frac{1}{T} \ln Z_E[0] = E_{\text{stat}} - [I] \tag{3}$$

where

$$E_{\text{stat}} = -\lim_{T\to\infty} \frac{1}{T} \log[v] = V_0 \bar{\Lambda}\,, \qquad [I] = \sqrt{12}\omega \sqrt{\frac{S_0}{2\pi}} e^{-S_0}\,, \qquad S_0 \propto V_0\,. \tag{4}$$

How would this result change if we ...

- Do not ignore the Casimir effect (in finite volume, momentum should be quantised)?
- Do consider the full one-loop corrections when computing $[I]$?

- Considering momentum quantisation implies

$$V_0\bar{\Lambda} \to V_0\bar{\Lambda} + E_{\text{casimir}} \,, \tag{5}$$

where

$$E_{\text{casimir}} = \frac{1}{2}\sum_{\mathbf{n}} \sqrt{\mathbf{k}_n^2 + m^2} - \frac{V_0}{2}\int_{\mathbb{R}^3} \frac{d^3\mathbf{k}}{(2\pi)^3}\sqrt{\mathbf{k}^2 + m^2} \tag{6}$$

This resut is strongly dependent on the geometry/topology. For a 3-torus (periodic boundary conditions) $E_{\text{casimir}} < 0$ and bigger than $[I]$.

- Considering the full one-loop corrections implies computing the fluctuation determinant

$$[I] = \sqrt{\frac{S_0}{2\pi}}\, e^{-S_0} \left(\frac{\det'\left(-\partial_\tau^2 - \nabla^2 + \frac{m^2}{2}\left(\frac{3\phi_K^2}{v^2} - 1\right)\right)}{\det(-\partial_\tau^2 - \nabla^2 + m^2)} \right)^{-\frac{1}{2}} \,, \tag{7}$$

Work in progress with W. Ai, J. Alexandre, M. Carosi and B. Garbrecht...

Part III

Towards a cosmic bounce

In more detail

We can derive the properties of the quantum vacuum from the minimum of the effective action, i.e., the minimum of the potential

$$U_{eff}(\phi_c = 0) = \kappa^{-1}\Lambda\left(1 - r\frac{e^{-\alpha^3}}{\alpha^{3/2}}\right) ,$$

with

$$\alpha^3 = a^3\frac{S_0}{\hbar} \quad \text{and} \qquad r = \frac{\lambda\kappa v^4}{3\sqrt{3\pi}\Lambda} .$$

In particular, we can obtain the *energy density and the pressure*

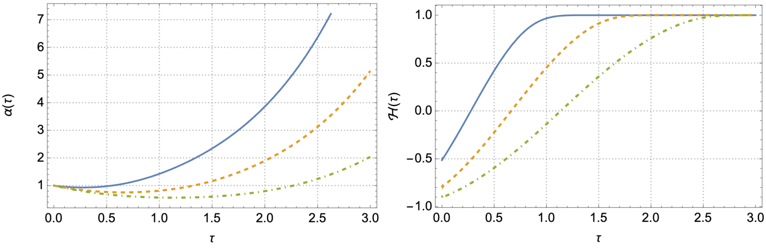
$$\frac{\kappa\rho}{\Lambda} = \tilde{\rho} = +1 - r\frac{e^{-\alpha^3}}{\alpha^{3/2}}$$
$$\frac{\kappa p}{\Lambda} = \tilde{p} = -1 - re^{-\alpha^3}\left(\alpha^{3/2} - \frac{\alpha^{-3/2}}{2}\right) .$$

We couple this ground state fluid to the Einstein equations and study its consequences.

Friedmann equations

$$H^2 = \frac{\kappa}{3}\rho \quad \text{(constraint)}; \qquad \frac{\ddot{a}}{a} = -\frac{\kappa}{6}(\rho + 3p) \quad \text{(dynamics)};$$

We solved the Friedmann equations for different values of r, and focusing on $H(t_0) < 0$ (initial contracting phase).



Note: If a bounce occurs, then $H = 0$ and $H' > 0$. This implies $\rho = 0$ and $\rho + 3p < 0$.

Question: How robust is this solution?

We can add a *second component* to test it. We focus on the case of anisotropy, since it is the component that dominates the energy budget the quickest during a collapse. Bianchi-I metric:

$$\mathrm{d}s^2 = -\mathrm{d}t^2 + a_1^2\,\mathrm{d}x^2 + a_2^2\,\mathrm{d}y^2 + a_3^2\,\mathrm{d}z^2$$

We work with averaged quantities: $a^3 = a_1 a_2 a_3$

$$H = \frac{1}{3}\left(H_1 + H_2 + H_3\right), \quad \sigma^2 = \frac{1}{18}\left[(H_1 - H_2)^2 + (H_2 - H_3)^2 + (H_1 - H_3)^2\right]$$

Note: ρ and p remain the same, with the change $\alpha_{iso} \to \alpha = \sqrt{\alpha_1\alpha_2\alpha_3}$.

Friedmann equations

$$H^2 = \frac{\kappa}{3}\rho + \sigma^2\,; \qquad \dot{H} + H^2 = -\frac{\kappa}{6}(\rho + 3p) - 2\sigma^2$$

It can be shown that $\sigma^2 \sim a^{-6}$. The anisotropy plays a role of a homogeneous fluid with equation of state $w = 1$ (i.e., $\rho_a = p_a = 3\kappa^{-1}\sigma^2$).

A critical solution $(\rho_c, p_c, a_c, \sigma_c^2)$ can be found by imposing the condition

$$\boxed{H = H' = 0}$$

$$\tilde{p}_c = \tilde{\rho}_c = -\tilde{\sigma}_c^2$$
$$\downarrow$$
$$4\alpha_c^{3/2} + re^{-\alpha_c^3}\left(-3 + 2\alpha_c^3\right) = 0$$

This solution gives *the minimum value* α *can have* for the bounce to succeed.

Two different regimes:

$$\alpha_c \to (3/2)^{1/3} \quad \text{for} \;\; r \gg 1$$
$$\alpha_c \sim (3r/4)^{2/3} \quad \text{for} \;\; r \ll 1$$

The anisotropy can be expressed as

$$\tilde{\sigma}_c^2 = \frac{1 + 2\alpha_c^3}{3 - 2\alpha_c^3}$$

and gives us the maximum anisotropy.

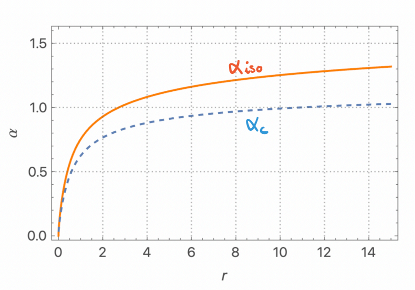
The critical point is unstable: a value of α that is slightly larger than α_c leads to a bounce and a value which is slightly smaller leads to a collapse.

One can also infer a *maximum value* for the rescaled scale factor α_{iso} at the bounce, which happens when the anisotropy is negligible and the quantum field dominates.

$$\tilde{\rho} = 0$$
$$\downarrow$$
$$\alpha_{iso}^{3/2} = r\, e^{-\alpha_{iso}^3}$$

Which leads to the two regimes

$$\begin{aligned} \alpha_{iso} &\sim (\ln r)^{1/3} \quad \text{for} \quad r \gg 1 \\ \alpha_{iso} &\sim r^{2/3} \quad \text{for} \quad r \ll 1\,, \end{aligned}$$



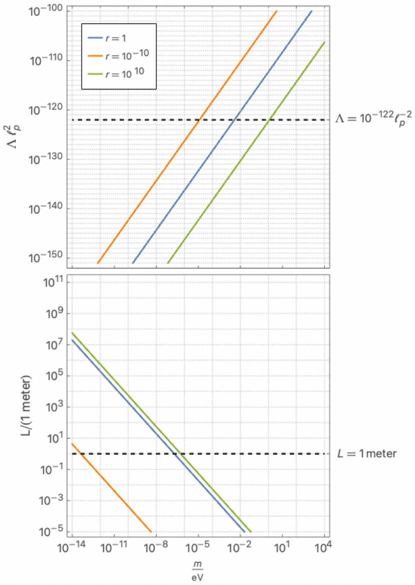
The critical solutions allow us to estimate a size of the universe at the bounce. Assuming $\lambda \sim 1$:

$$\frac{\alpha_c(r)}{m} \lesssim L_b \lesssim \frac{\alpha_{iso}(r)}{m}$$

Example: for $r = 1$, if the bounce occurred when $L \sim 1$ m, the scalar field would need a mass of $\sim 10^{-7}$ eV and a vacuum energy Λ of order $10^{-140}\ \ell_p^{-2}$.

Remember:

$$r \sim \frac{\kappa m^4}{\Lambda}$$



- We computed the effective theory allowing tunneling between two degenerate vacua.
- **Effective theory** $\rightarrow$ one true vacuum at $\phi_c = 0$, with a vacuum energy which is not proportional to the comoving volume $\rightarrow$ NEC violation.
- This feature can induce a cosmic bounce.
- We extended our result to include **anisotropies**: a bounce is still possible if the anisotropy is significant but *not dominant*.
- The improved result for [I] (considering 1+3 quantum fluctuations) shows that the approximations we took are justified (specially for small λ).
- The Casimir effect can be as important (or even more important) than [I], but it strongly depends on the geometry.

THANKS FOR YOUR ATTENTION