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Gauge invariance and gauge zero modes of bubble nucleation rates

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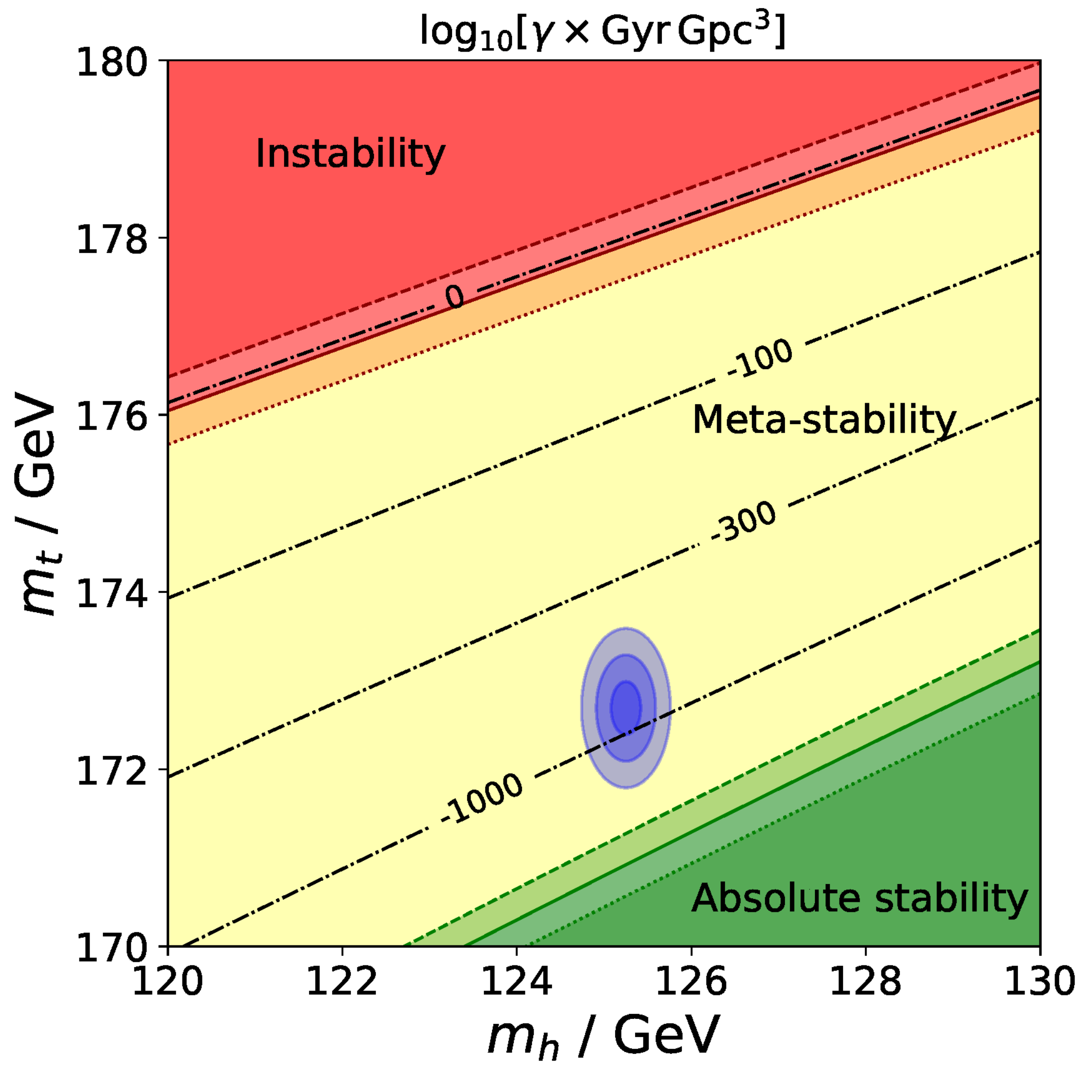


[S. Chigusa, T. Moroiu, YS, '17 (Updated 2023)]

Theory: JHEP 11(2020)006; S. Chigusa, T. Moroi, YS
JHEP 11(2017)074; M. Endo, T. Moroi, M. M. Nojiri, YS
PLB 771(2017)281; M. Endo, T. Moroi, M. M. Nojiri, YS

Applications: JHEP 03(2020)011; S. Oda, YS, D-S Takahashi
PRD 97(2018)11; S. Chigusa, T. Moroi, YS
PRL 119(2017)21; S. Chigusa, T. Moroi, YS
JHEP 11(2023)027; S. Chigusa, T. Moroi, YS
PLB 831(2022)137163; S. Chigusa, T. Moroi, YS

Tunneling in QFT 2024, 14 Mar 2024

Introduction

Bubble nucleation rate

[S. R. Coleman, ’77; G. G. Callan, S. R. Coleman, ’77]

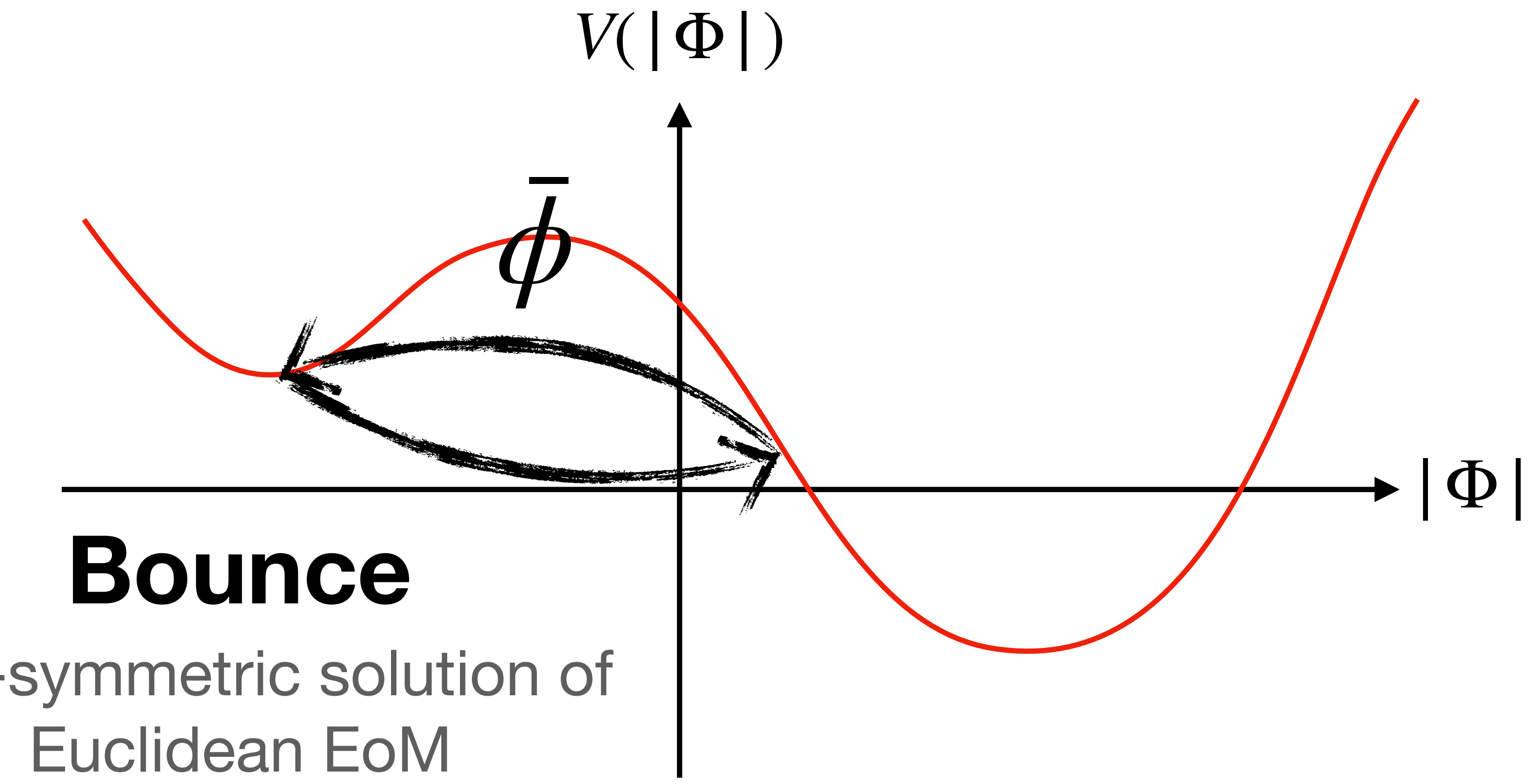
Rate of bubble nucleation in unit volume

$$\gamma = \mathscr{A} e^{-\mathscr{B}}$$

(Euclidean) Bounce action

$$\mathscr{B} = S_{\mathrm{B}} - S_{\mathrm{FV}}$$

FV potential



Bounce

O(4)-symmetric solution of Euclidean EoM

Jacobian for the translational and the gauge zero modes

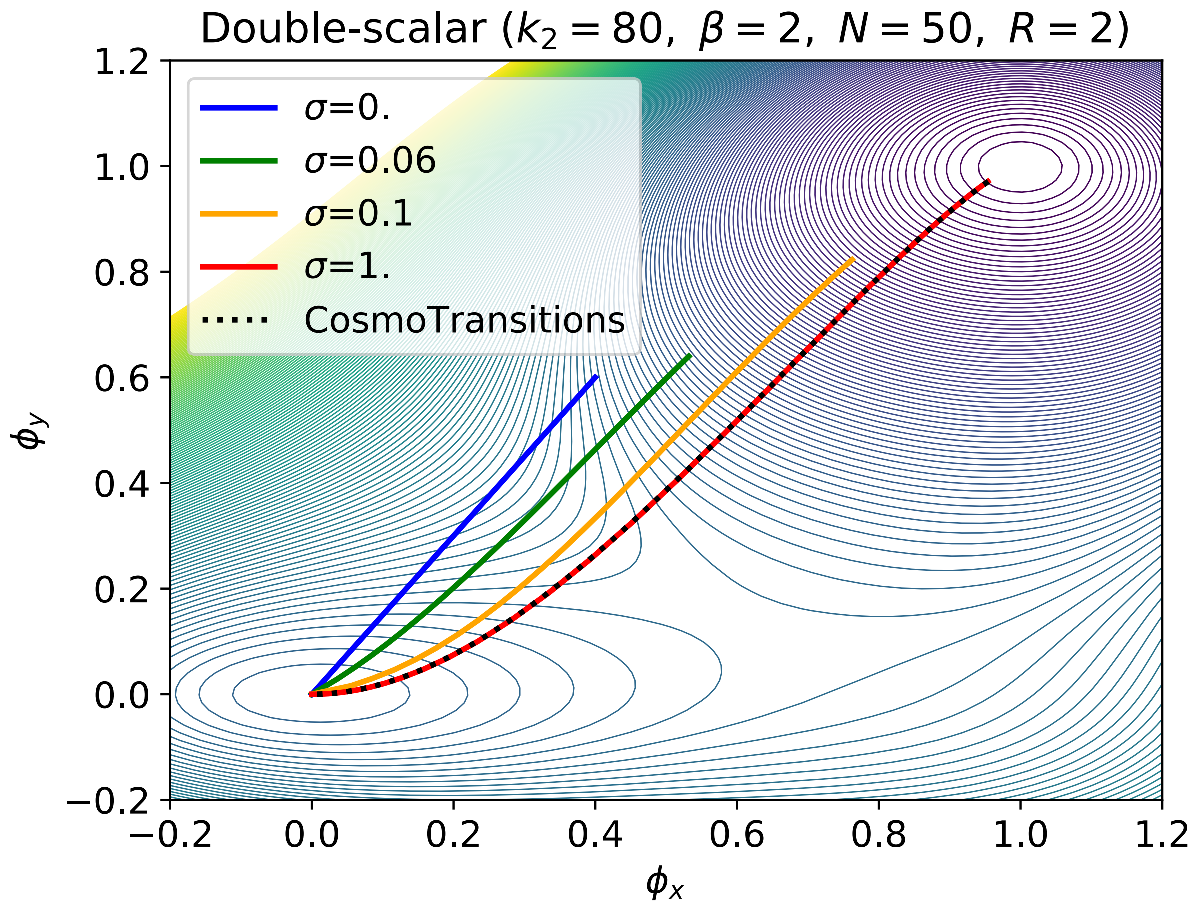
Subtraction of zero modes

$$J_T = \frac{\mathscr{B}^2}{4\pi^2}$$

$$\mathscr{A} = V_G J_G J_T \left| \frac{\mathrm{SDet}' S''_{\mathrm{B}}}{\mathrm{SDet}\, S''_{\mathrm{FV}}} \right|^{-1/2}$$

Super-determinants of all the fields coupled to the bounce

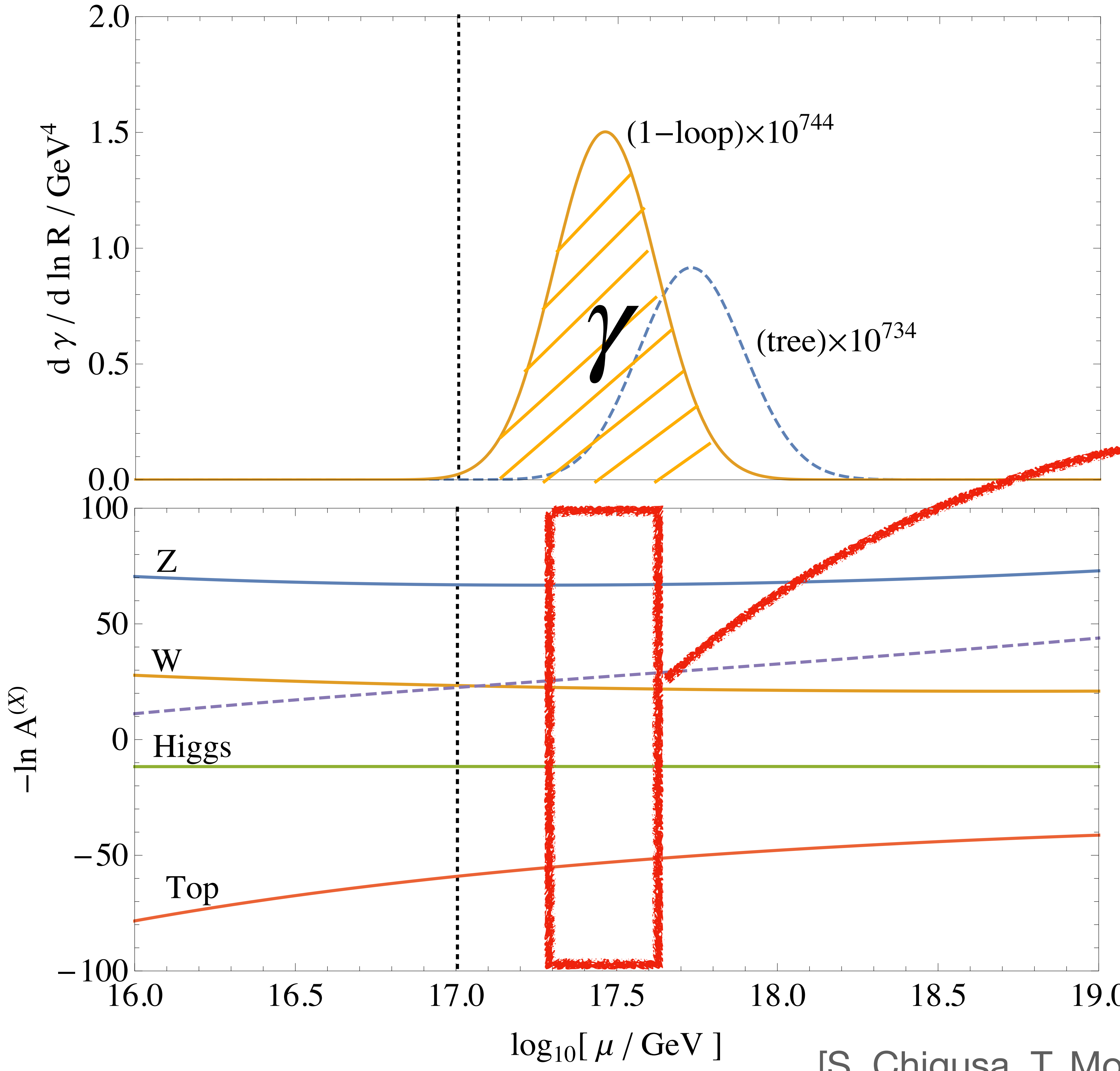
Volume of the group broken by the bounce



Bounce from gradient flow of [S. Chigusa, T. Moroi, YS, ’20]

Importance of the prefactor

Standard Model



[S. Chigusa, T. Moroi, YS, '18]

$$\mathscr{A} = A^{\mathrm{Top}} A^{\mathrm{Higgs}} A^{W} A^{Z} \neq m^4$$

Typical scale

$$A^{\mathrm{Top}} \sim \exp(50)$$

$$A^{\mathrm{Higgs}} \sim \exp(10)\ \mathrm{GeV}^4$$

$$A^{W} \sim \exp(-25)$$

$$A^{Z} \sim \exp(-70)$$

Today's topic

$$\mathscr{A} e^{-\mathscr{B}} \lesssim H_0^4 = \exp(-386)\ \mathrm{GeV}^4$$

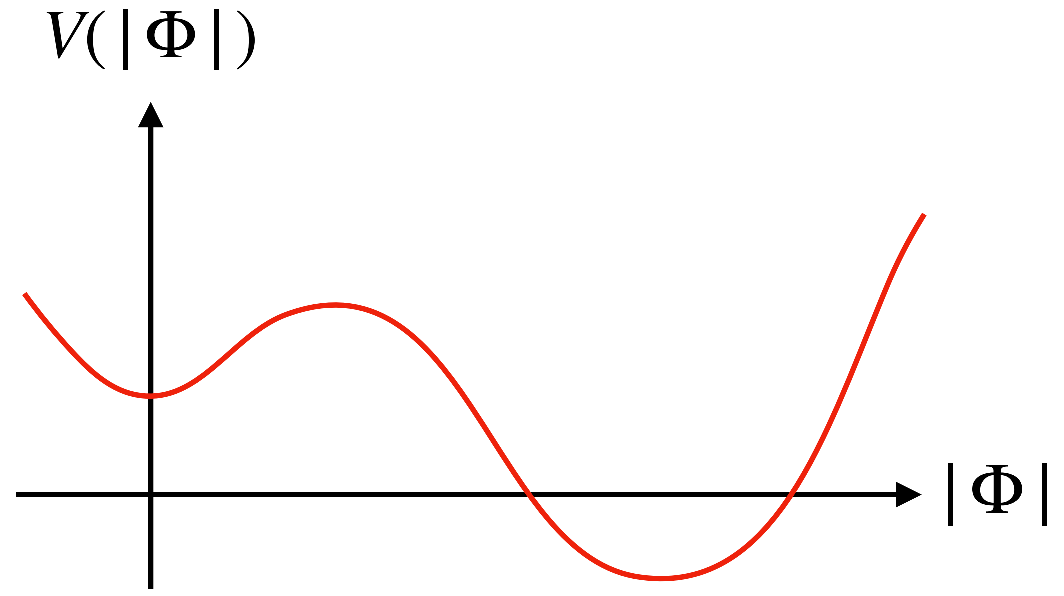
Contents

Gauge independence

Setup

One complex scalar + U(1) gauge boson in 4D

$$\mathscr{L} = \frac{1}{4} F_{\mu\nu} F_{\mu\nu} + |D\Phi|^2 + V(|\Phi|) + \mathscr{L}_{\rm GF} + \mathscr{L}_{\rm ghost}$$



Background gauge

~R_ξ gauge in a broken phase~

$$\mathscr{L}_{\rm GF} = \frac{1}{2\xi} \left[\partial_\mu A_\mu - 2\xi g (\Re\Phi)(\Im\Phi) \right]^2$$

$$\mathscr{L}_{\rm ghost} = \bar{c} \left[-\partial^2 + 2\xi g^2 |\Phi|^2 \right] c$$

Fermi gauge

~R_ξ gauge in a symmetric phase~

$$\mathscr{L}_{\rm GF} = \frac{1}{2\xi} \left[\partial_\mu A_\mu \right]^2$$

$$\mathscr{L}_{\rm ghost} = \bar{c} \left[-\partial^2 \right] c$$

Partial wave expansions

Lorentz scalar

$$h(x) = h_{l,m_A,m_B}(r) Y_{l,m_A,m_B}(\Omega)$$

$$a(x) = a_{l,m_A,m_B}(r) Y_{l,m_A,m_B}(\Omega)$$

$$c(x) = c_{l,m_A,m_B}(r) Y_{l,m_A,m_B}(\Omega)$$

Lorentz vector

$$A_\mu(x) = a^S_{l,m_A,m_B}(r) \frac{x_\mu}{r} Y_{l,m_A,m_B}(\Omega)$$

$$+ a^L_{l,m_A,m_B}(r) \frac{r}{L} \partial_\mu Y_{l,m_A,m_B}(\Omega)$$

$$+ a^{T1}_{l,m_A,m_B}(r) i \epsilon_{\mu\nu\rho\sigma} V^{(1)}_\nu L_{\rho\sigma} Y_{l,m_A,m_B}(\Omega)$$

$$+ a^{T2}_{l,m_A,m_B}(r) i \epsilon_{\mu\nu\rho\sigma} V^{(2)}_\nu L_{\rho\sigma} Y_{l,m_A,m_B}(\Omega)$$

They mix with each other

$$\Phi = \frac{\bar{\phi} + h + ia}{\sqrt{2}}$$

$\bar{\phi}$: **Bounce**

$$L_{\mu\nu} = \frac{i}{\sqrt{2}}(x_\mu \partial_\nu - x_\nu \partial_\mu)$$

$V^{(i)}_\mu$: independent vectors

Functional determinant

Background gauge

$$\Delta_l = \partial_r^2 + \frac{3}{r}\partial_r - \frac{L^2}{r^2} \qquad L = \sqrt{l(l+2)}$$

$[S, L, a]$

$l = 0$

BG vs Fermi

$$\mathcal{M}_0^{(Sa)} = \begin{pmatrix} \frac{1}{\xi}\left(-\Delta_0 + \frac{3}{r^2} + \xi g^2\bar{\phi}^2\right) & 2g\bar{\phi}' \\ 2g\bar{\phi}' & -\Delta_0 + \frac{(\Delta_0\bar{\phi})}{\bar{\phi}} + \xi g^2\bar{\phi}^2 \end{pmatrix}$$

$l > 0$

Diagonalisable

Zero at $r = 0$ and $r = \infty$

$$\mathcal{M}_l^{(SLa)} = \begin{pmatrix} -\Delta_l + \frac{3}{r^2} + g^2\bar{\phi}^2 & -\frac{2L}{r^2} & \boxed{2g\bar{\phi}'} \\ -\frac{2L}{r^2} & -\Delta_l - \frac{1}{r^2} + g^2\bar{\phi}^2 & 0 \\ \boxed{2g\bar{\phi}'} & 0 & -\Delta_l + \frac{(\Delta_0\bar{\phi})}{\bar{\phi}} + \xi g^2\bar{\phi}^2 \end{pmatrix}$$

$$+\left(1 - \frac{1}{\xi}\right)\begin{pmatrix} \partial_r^2 + \frac{3}{r}\partial_r - \frac{3}{r^2} & -L\left(\frac{1}{r}\partial_r - \frac{1}{r^2}\right) & 0 \\ L\left(\frac{1}{r}\partial_r + \frac{3}{r^2}\right) & -\frac{L^2}{r^2} & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$[c, \bar{c}]$

$l \geq 0$

$$\mathcal{M}_l^{(c\bar{c})} = -\Delta_l \boxed{+ \xi g^2\bar{\phi}^2}$$

$\times 2$

cancel accidentally when $\xi = 1$

$[T1, T2]$

$l > 0$

$$\mathcal{M}_l^{(T)} = -\Delta_l + g^2\bar{\phi}^2$$

$\times 2$

BG gauge with $\xi = 1$ is often used for numerical calculations

Functional determinant

Fermi gauge

$$\Delta_l = \partial_r^2 + \frac{3}{r}\partial_r - \frac{L^2}{r^2} \qquad L = \sqrt{l(l+2)}$$

$[S, L, a]$

$l = 0$

BG vs Fermi

$$\mathscr{M}_0^{(Sa)} = \begin{pmatrix} \frac{1}{\xi}\left(-\Delta_0 + \frac{3}{r^2} + \xi g^2 \bar{\phi}^2\right) & g\bar{\phi}' - g\bar{\phi}\partial_r \\ 2g\bar{\phi}' + g\bar{\phi}\partial_r + \frac{3}{r}g\bar{\phi} & -\Delta_0 + \frac{(\Delta_0\bar{\phi})}{\bar{\phi}} \end{pmatrix}$$

$l > 0$

$$\mathscr{M}_l^{(SLa)} = \begin{pmatrix} -\Delta_l + \frac{3}{r^2} + g^2\bar{\phi}^2 & -\frac{2L}{r^2} & g\bar{\phi}' - g\bar{\phi}\partial_r \\ -\frac{2L}{r^2} & -\Delta_l - \frac{1}{r^2} + g^2\bar{\phi}^2 & -\frac{L}{r}g\bar{\phi} \\ 2g\bar{\phi}' + g\bar{\phi}\partial_r + \frac{3}{r}g\bar{\phi} & -\frac{L}{r}g\bar{\phi} & -\Delta_l + \frac{(\Delta_0\bar{\phi})}{\bar{\phi}} \end{pmatrix}$$

$$+\left(1 - \frac{1}{\xi}\right)\begin{pmatrix} \partial_r^2 + \frac{3}{r}\partial_r - \frac{3}{r^2} & -L\left(\frac{1}{r}\partial_r - \frac{1}{r^2}\right) & 0 \\ L\left(\frac{1}{r}\partial_r + \frac{3}{r^2}\right) & -\frac{L^2}{r^2} & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$[c, \bar{c}]$

$l \geq 0$

$$\mathscr{M}_l^{(c\bar{c})} = -\Delta_l \boxed{}$$

$\times 2$

$[T1, T2]$

$l > 0$

$$\mathscr{M}_l^{(T)} = -\Delta_l + g^2\bar{\phi}^2$$

$\times 2$

Gauge (in)dependence

$$\mathscr{A} = J_T \left[\prod_{l=0}^{\infty} \left| \frac{\det' \mathscr{M}_l^{(h)}}{\det \widehat{\mathscr{M}}_l^{(h)}} \right|^{-(l+1)^2/2} \right]$$

$$\times \left[\prod_{l=1}^{\infty} \left(\frac{\det \mathscr{M}_l^{(T)}}{\det \widehat{\mathscr{M}}_l^{(T)}} \right)^{-(l+1)^2} \right]$$

Manifestly gauge independent

$$\times \left[\prod_{l=0}^{\infty} \left(\frac{\det \mathscr{M}_l^{(c\bar{c})}}{\det \widehat{\mathscr{M}}_l^{(c\bar{c})}} \right)^{(l+1)^2} \right]$$

=1 for Fermi gauge

$$\times V_G J_G \left(\frac{\det' \mathscr{M}^{(Sa)_0}}{\det \widehat{\mathscr{M}}_0^{(Sa)}} \right)^{-1/2} \left[\prod_{l=1}^{\infty} \left(\frac{\det \mathscr{M}_l^{(SLa)}}{\det \widehat{\mathscr{M}}_l^{(SLa)}} \right)^{-(l+1)^2/2} \right]$$

Gauge dependence should be canceled

(BG)

$$\mathscr{M}_l^{(SLa)} = \begin{pmatrix} -\Delta_l + \frac{3}{r^2} + g^2\bar{\phi}^2 & -\frac{2L}{r^2} & 2g\bar{\phi}' \\ -\frac{2L}{r^2} & -\Delta_l - \frac{1}{r^2} + g^2\bar{\phi}^2 & 0 \\ 2g\bar{\phi}' & 0 & -\Delta_l + \frac{(\Delta_0\bar{\phi})}{\bar{\phi}} + \xi g^2\bar{\phi}^2 \end{pmatrix}$$

$$+ \left(1 - \frac{1}{\xi}\right) \begin{pmatrix} \partial_r^2 + \frac{3}{r}\partial_r - \frac{3}{r^2} & -L\left(\frac{1}{r}\partial_r - \frac{1}{r^2}\right) & 0 \\ L\left(\frac{1}{r}\partial_r + \frac{3}{r^2}\right) & -\frac{L^2}{r^2} & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

Related works

Our work is along this line

Nielsen-Fukuda-Kugo identity

[N. K. Nielsen, ’75; R. Fukuda, T. Kugo, ’76]

$$\left(\xi\frac{\partial}{\partial\xi} + C(v,\xi)\frac{\partial}{\partial v}\right) V_{\mathrm{eff}}(v,\xi) = 0$$

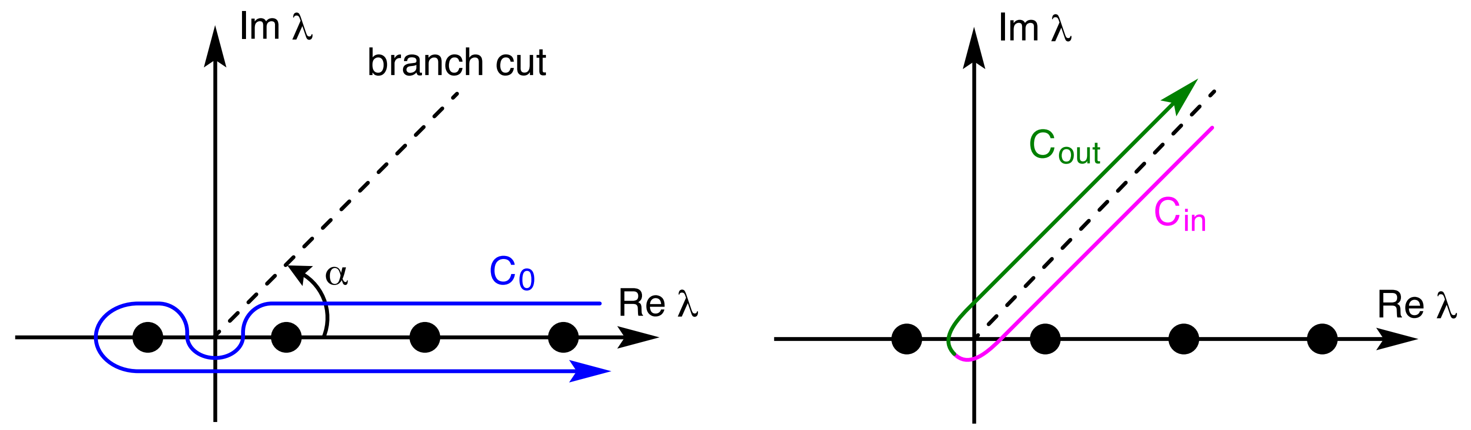
Should be applicable also to a saddle point

Indirect proof

The treatment of gauge zero modes is not clear

From Gelfand-Yagrom theorem

[J. Baacke, K. Heitmann, ’99]



Found one FP determinant in the gauge determinant

Rather a hand waving discussion for the other FP determinant

Not a complete proof (no asymptotic analysis)

No discussion on gauge zero modes

Gelfand-Yaglom for gauge sector

Proof for the gauge determinant is available in [M. Endo, T. Moroi, M. M. Nojiri, YS, '17]

$$\frac{\det \mathscr{M}_l^{(SLa)}}{\det \widehat{\mathscr{M}}_l^{(SLa)}} = \left[\lim_{r\to\infty} \frac{\det\begin{pmatrix}\psi_1(r) & \psi_2(r) & \psi_3(r)\end{pmatrix}}{\det\begin{pmatrix}\hat{\psi}_1(r) & \hat{\psi}_2(r) & \hat{\psi}_3(r)\end{pmatrix}}\right] \left[\lim_{r\to 0} \frac{\det\begin{pmatrix}\psi_1(r) & \psi_2(r) & \psi_3(r)\end{pmatrix}}{\det\begin{pmatrix}\hat{\psi}_1(r) & \hat{\psi}_2(r) & \hat{\psi}_3(r)\end{pmatrix}}\right]^{-1}$$

$$\mathscr{M}_l^{(SLa)}\psi_i(r) = 0 \qquad \widehat{\mathscr{M}}_l^{(SLa)}\hat{\psi}_i(r) = 0$$

Three independent solutions that are regular at r=0

Semi-analytic decomposition

Case1: gauge symmetry broken at the false vacuum

BG gauge [M. Endo, T. Moroi, M. M. Nojiri, YS, '17]

$$\frac{\det \mathscr{M}_l^{(c\bar{c})}}{\det \widehat{\mathscr{M}}_l^{(c\bar{c})}} = \frac{f_l^{(c\bar{c})}(\infty)}{\hat{f}_l^{(c\bar{c})}(\infty)}$$

$$\frac{\det \mathscr{M}_0^{(Sa)}}{\det \widehat{\mathscr{M}}_0^{(Sa)}} = \frac{\bar{\phi}(\infty)}{\bar{\phi}(0)} \left(\frac{f_0^{(c\bar{c})}(\infty)}{\hat{f}_0^{(c\bar{c})}(\infty)} \right)^2$$

$$\frac{\det \mathscr{M}_l^{(SLa)}}{\det \widehat{\mathscr{M}}_l^{(SLa)}} = \frac{\bar{\phi}(0)}{\bar{\phi}(\infty)} \frac{f_l^{(\eta)}(\infty)}{\hat{f}_l^{(\eta)}(\infty)} \left(\frac{f_l^{(c\bar{c})}(\infty)}{\hat{f}_l^{(c\bar{c})}(\infty)} \right)^2$$

Fermi gauge [M. Endo, T. Moroi, M. M. Nojiri, YS, '17]

$$\frac{\det \mathscr{M}_l^{(c\bar{c})}}{\det \widehat{\mathscr{M}}_l^{(c\bar{c})}} = 1$$

$$\frac{\det \mathscr{M}_0^{(Sa)}}{\det \widehat{\mathscr{M}}_0^{(Sa)}} = \frac{\bar{\phi}(\infty)}{\bar{\phi}(0)}$$

$$\frac{\det \mathscr{M}_l^{(SLa)}}{\det \widehat{\mathscr{M}}_l^{(SLa)}} = \frac{\bar{\phi}(0)}{\bar{\phi}(\infty)} \frac{f_l^{(\eta)}(\infty)}{\hat{f}_l^{(\eta)}(\infty)}$$

FP modes

$$\left(\Delta_l - \xi g^2 \bar{\phi}^2\right) f_l^{(c\bar{c})} = 0$$

Physical modes

$$\left(\Delta_l - g^2 \bar{\phi}^2\right) f_l^{(\eta)} - \frac{2\bar{\phi}'}{r^2 \bar{\phi}} \partial_r \left(r^2 f_l^{(\eta)} \right) = 0$$

Gauge zero mode

Gauge zero mode

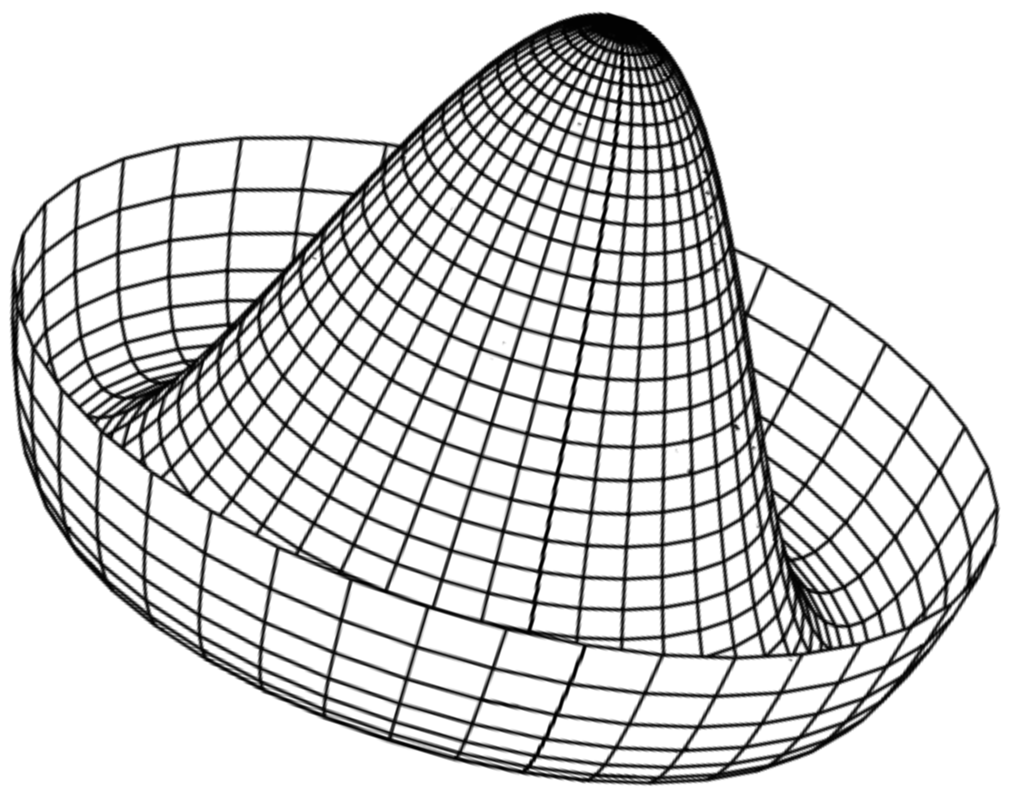
Case2: gauge symmetry restored at the false vacuum

[A. Kusenko, K. M. Lee, E. J. Weinberg, ’97]

BG gauge

$$\begin{pmatrix} \frac{1}{\xi}\left(-\Delta_0+\frac{3}{r^2}+\xi g^2\bar{\phi}^2\right) & 2g\bar{\phi}' \\ 2g\bar{\phi}' & -\Delta_0+\frac{(\Delta_0\bar{\phi})}{\bar{\phi}}+\xi g^2\bar{\phi}^2 \end{pmatrix}\begin{pmatrix} \partial_r f^{(c\bar{c})} \\ g\bar{\phi} f^{(c\bar{c})} \end{pmatrix}=0$$

Global symmetry: $\Phi \to ?,\ A_\mu \to ?$



Scalar potential is lifted

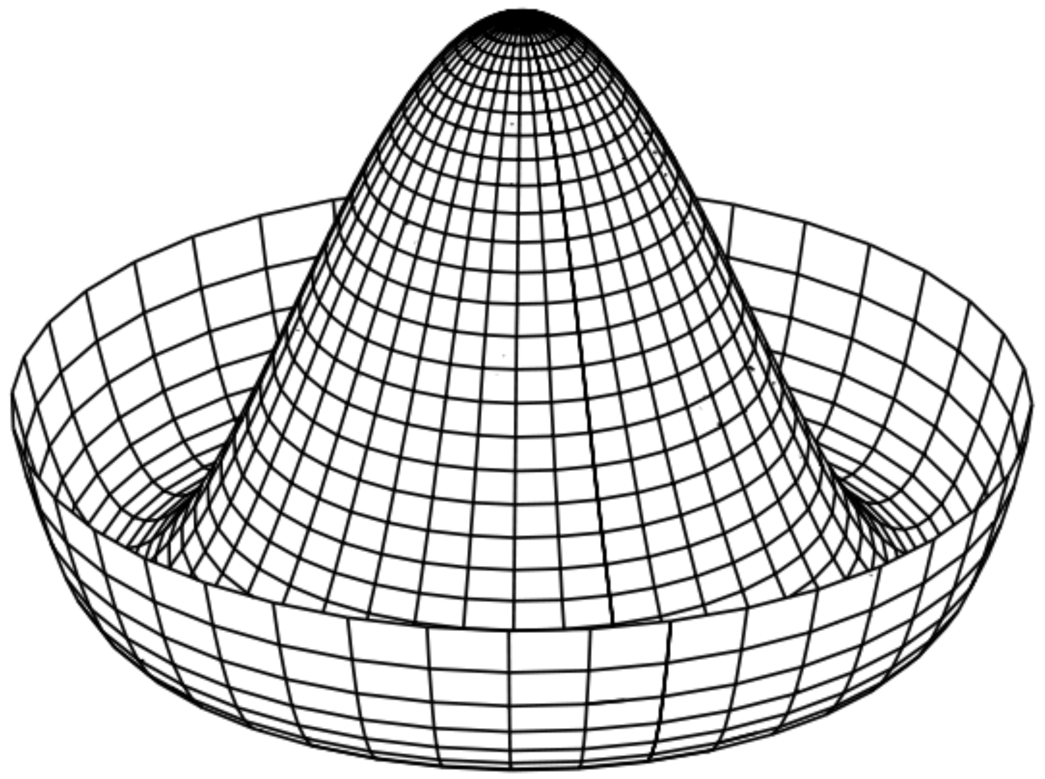
$$\boxed{\begin{aligned} J_G &= ? \\ V_G &= ? \end{aligned}}$$

Determine these to reproduce the Fermi gauge result

Fermi gauge

$$\begin{pmatrix} \frac{1}{\xi}\left(-\Delta_0+\frac{3}{r^2}+\xi g^2\bar{\phi}^2\right) & g\bar{\phi}'-g\bar{\phi}\partial_r \\ 2g\bar{\phi}'+g\bar{\phi}\partial_r+\frac{3}{r}g\bar{\phi} & -\Delta_0+\frac{(\Delta_0\bar{\phi})}{\bar{\phi}} \end{pmatrix}\begin{pmatrix} 0 \\ g\bar{\phi} \end{pmatrix}=0$$

Global symmetry: $\Phi \to e^{i\theta}\Phi,\ A_\mu \to A_\mu$



$$J_G=\sqrt{\pi\int \mathrm{d}r r^3\bar{\phi}^2}$$

$$V_G=2\pi$$

Semi-analytic decomposition

Case2: gauge symmetry restored at the false vacuum

BG gauge [M. Endo, T. Moroi, M. M. Nojiri, YS, '17]

$$\frac{\det \mathscr{M}_l^{(c\bar{c})}}{\det \widehat{\mathscr{M}}_l^{(c\bar{c})}} = \frac{f_l^{(c\bar{c})}(\infty)}{\hat{f}_l^{(c\bar{c})}(\infty)}$$

$$\frac{\det' \mathscr{M}_0^{(Sa)}}{\det \widehat{\mathscr{M}}_0^{(Sa)}} = \frac{1}{2\pi m_h \bar{\phi}(0)} \lim_{r\to\infty} \frac{1}{r^3 \bar{\phi}(r) f_0^{(h)}(r)} \frac{\pi r^3 \partial_r f_0^{(c\bar{c})}(r)}{\xi g^2 f_0^{(c\bar{c})}(r)} \left(\frac{f_0^{(c\bar{c})}(\infty)}{\hat{f}_0^{(c\bar{c})}(\infty)} \right)^2$$

$$\frac{\det \mathscr{M}_l^{(SLa)}}{\det \widehat{\mathscr{M}}_l^{(SLa)}} = \frac{l}{2} \frac{\bar{\phi}(0)}{m_h} \lim_{r\to\infty} \frac{f_l^{(\eta)}(r)}{r \hat{f}_l^{(h)}(r) \bar{\phi}(r)} \left(\frac{f_l^{(c\bar{c})}(\infty)}{\hat{f}_l^{(c\bar{c})}(\infty)} \right)^2$$

Fermi gauge [M. Endo, T. Moroi, M. M. Nojiri, YS, '17]

$$\frac{\det \mathscr{M}_l^{(c\bar{c})}}{\det \widehat{\mathscr{M}}_l^{(c\bar{c})}} = 1$$

$$\frac{\det' \mathscr{M}_0^{(Sa)}}{\det \widehat{\mathscr{M}}_0^{(Sa)}} = \frac{1}{2\pi m_h \bar{\phi}(0)} \lim_{r\to\infty} \frac{1}{r^3 \bar{\phi}(r) f_0^{(h)}(r)} J_G^2$$

$$\frac{\det \mathscr{M}_l^{(SLa)}}{\det \widehat{\mathscr{M}}_l^{(SLa)}} = \frac{l}{2} \frac{\bar{\phi}(0)}{m_h} \lim_{r\to\infty} \frac{f_l^{(\eta)}(r)}{r \hat{f}_l^{(h)}(r) \bar{\phi}(r)}$$

FP modes

$$\left(\Delta_l - \xi g^2 \bar{\phi}^2\right) f_l^{(c\bar{c})} = 0$$

Physical modes

$$\left(\Delta_l - g^2 \bar{\phi}^2\right) f_l^{(\eta)} - \frac{2\bar{\phi}'}{r^2 \bar{\phi}} \partial_r \left(r^2 f_l^{(\eta)} \right) = 0 \qquad \left(\Delta_l - m_h^2\right) \hat{f}_l^{(h)} = 0$$

Jacobian for the BG gauge

$$\mathscr{A} = V_G J_G \left(\frac{\det' \mathscr{M}^{(Sa)_0}}{\det \widehat{\mathscr{M}}_0^{(Sa)}} \right)^{-1/2} \left(\frac{\det \mathscr{M}_0^{(c\bar{c})}}{\det \widehat{\mathscr{M}}_0^{(c\bar{c})}} \right) \times \cdots$$

BG gauge

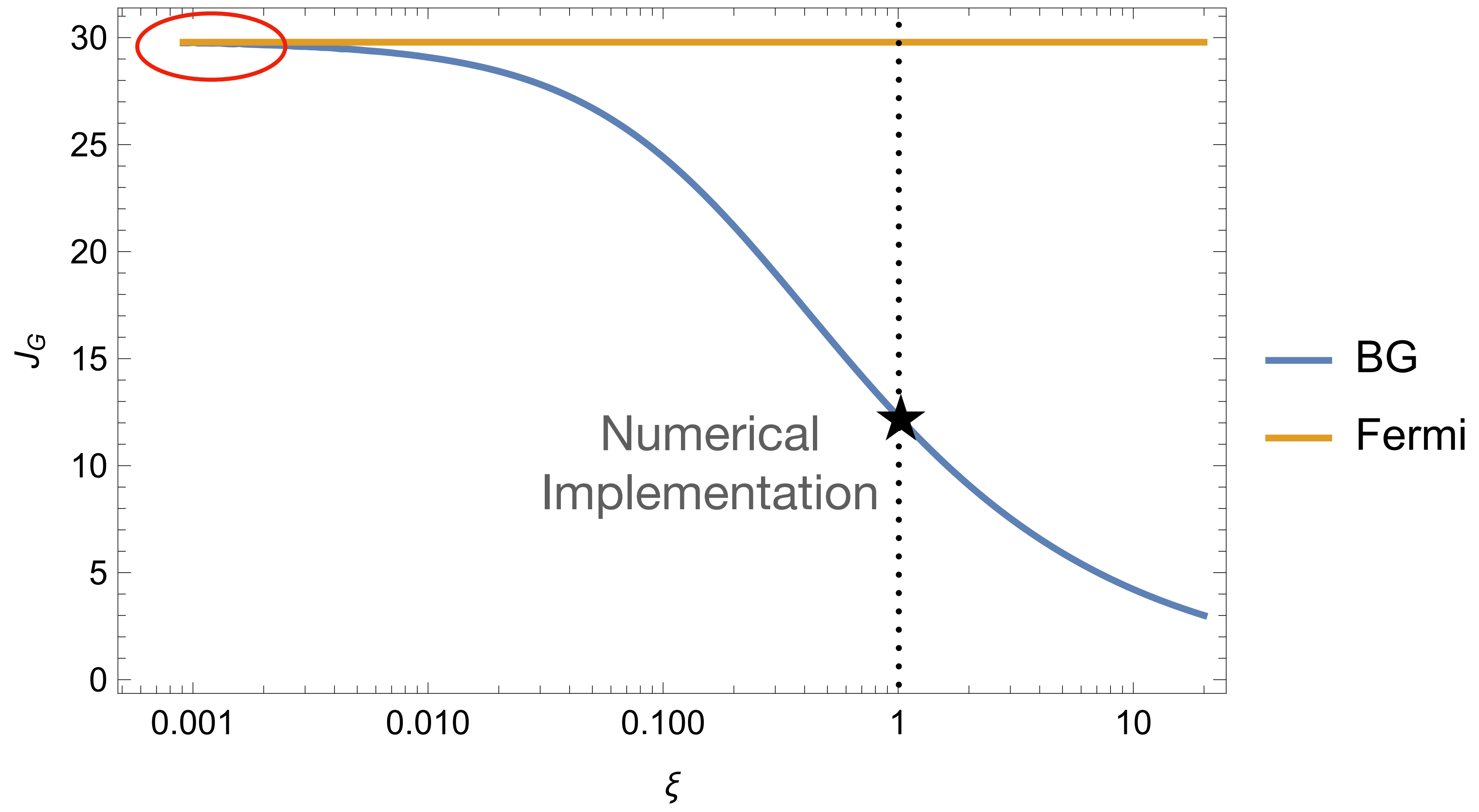
$$V_G J_G^{\mathrm{BG}} \left(\frac{\det' \mathscr{M}_0^{(Sa)}}{\det \widehat{\mathscr{M}}_0^{(Sa)}} \right)^{-1/2} \left(\frac{\det \mathscr{M}_0^{(c\bar{c})}}{\det \widehat{\mathscr{M}}_0^{(c\bar{c})}} \right) = V_G \sqrt{2\pi m_h \bar{\phi}(0)} \lim_{r\to\infty} \sqrt{r^3 \bar{\phi}(r) f_0^{(h)}(r)}\, J_G^{\mathrm{BG}} \sqrt{\frac{\xi g^2 f_0^{(c\bar{c})}(r)}{\pi r^3 \partial_r f_0^{(c\bar{c})}(r)}}$$

Fermi gauge

$$V_G J_G \left(\frac{\det' \mathscr{M}_0^{(Sa)}}{\det \widehat{\mathscr{M}}_0^{(Sa)}} \right)^{-1/2} \left(\frac{\det \mathscr{M}_0^{(c\bar{c})}}{\det \widehat{\mathscr{M}}_0^{(c\bar{c})}} \right) = V_G \sqrt{2\pi m_h \bar{\phi}(0)} \lim_{r\to\infty} \sqrt{r^3 \bar{\phi}(r) f_0^{(h)}(r)}$$

$$\boxed{J_G^{\mathrm{BG}} = \sqrt{\frac{\pi r^3 \partial_r f_0^{(c\bar{c})}(r)}{\xi g^2 f_0^{(c\bar{c})}(r)}}}$$

Massless would-be NG boson
(Same flat direction)



Reduction of computational time

BG gauge with $\xi = 1$

$$\mathscr{A} = J_T \left[\prod_{l=0}^{\infty} \left| \frac{\det' \mathscr{M}_l^{(h)}}{\det \widehat{\mathscr{M}}_l^{(h)}} \right|^{-(l+1)^2/2} \right]$$

Gauge sector

$$\times \left[\prod_{l=1}^{\infty} \left(\frac{\det \mathscr{M}_l^{(SLa)}}{\det \widehat{\mathscr{M}}_l^{(SLa)}} \right)^{-(l+1)^2/2} \right]$$

$$\times \cancel{\left[\prod_{l=1}^{\infty} \left(\frac{\det \mathscr{M}_l^{(T)}}{\det \widehat{\mathscr{M}}_l^{(T)}} \right)^{-(l+1)^2} \right]}$$

Cancel out

$$\times \cancel{\left[\prod_{l=1}^{\infty} \left(\frac{\det \mathscr{M}_l^{(c\bar{c})}}{\det \widehat{\mathscr{M}}_l^{(c\bar{c})}} \right)^{(l+1)^2} \right]}$$

$$\times \begin{cases} \left(\dfrac{\det \mathscr{M}^{(Sa)_0}}{\det \widehat{\mathscr{M}}_0^{(Sa)}} \right)^{-1/2} \left(\dfrac{\det \mathscr{M}_0^{(c\bar{c})}}{\det \widehat{\mathscr{M}}_0^{(c\bar{c})}} \right) = \sqrt{\dfrac{\bar{\phi}(0)}{\bar{\phi}(\infty)}} & \text{Case 1 (w/o gauge zero mode)} \\ V_G J_G \left(\dfrac{\det' \mathscr{M}^{(Sa)_0}}{\det \widehat{\mathscr{M}}_0^{(Sa)}} \right)^{-1/2} \left(\dfrac{\det \mathscr{M}_0^{(c\bar{c})}}{\det \widehat{\mathscr{M}}_0^{(c\bar{c})}} \right) = V_G \sqrt{2\pi m_h \bar{\phi}(0)} \lim_{r\to\infty} \sqrt{r^3 \bar{\phi}(r) f_0^{(h)}(r)} & \text{Case 2 (w/ gauge zero mode)} \end{cases}$$

Known analytically

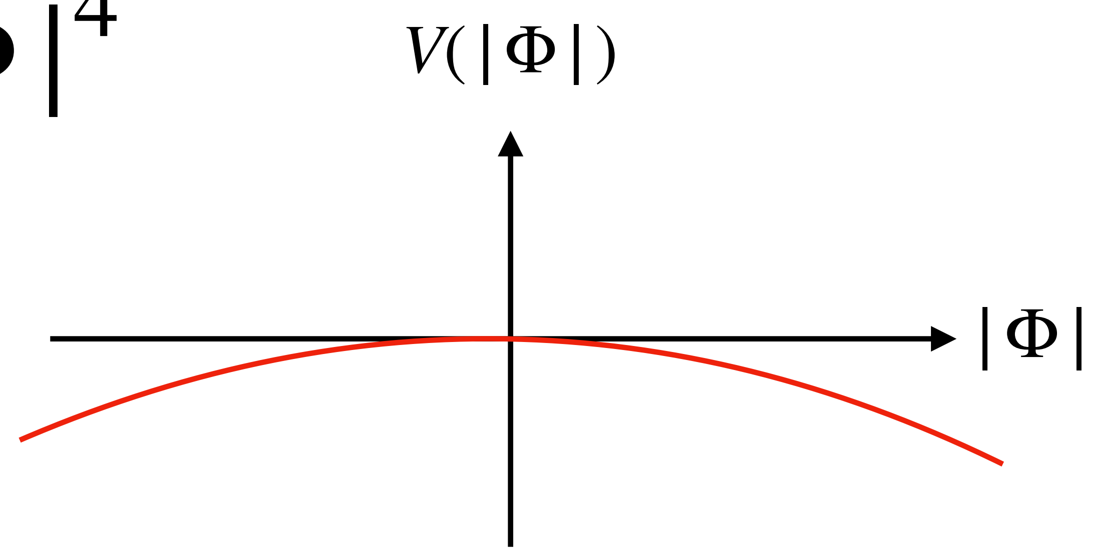
Fubini instanton

Fubini instanton and decay rate

Potential

$$V(|\Phi|) = \lambda |\Phi|^4$$

$$(\lambda < 0)$$



Bounce

$$\bar{\phi}(r) = \sqrt{\frac{8}{|\lambda|}} \frac{R}{R^2 + r^2}$$

R: Arbitrary constant

Decay rate

$$\gamma = \int \mathrm{d}R \mathscr{A} e^{-\mathscr{B}} \qquad \mathscr{B} = \frac{8\pi^2}{3|\lambda|}$$

$$\mathscr{A} = J_D J_T \left[\prod_{l=0}^{\infty} \left| \frac{\det' \mathscr{M}_l^{(h)}}{\det \widehat{\mathscr{M}}_l^{(h)}} \right|^{-(l+1)^2/2} \right]$$

Translational+dilatational zero modes

$$\times \left[\prod_{l=1}^{\infty} \left(\frac{\det \mathscr{M}_l^{(T)}}{\det \widehat{\mathscr{M}}_l^{(T)}} \right)^{-(l+1)^2} \right]$$

$$\times \left[\prod_{l=0}^{\infty} \left(\frac{\det \mathscr{M}_l^{(c\bar{c})}}{\det \widehat{\mathscr{M}}_l^{(c\bar{c})}} \right)^{(l+1)^2} \right]$$

$$\times V_G J_G \left(\frac{\det' \mathscr{M}^{(Sa)_0}}{\det \widehat{\mathscr{M}}_0^{(Sa)}} \right)^{-1/2} \left[\prod_{l=1}^{\infty} \left(\frac{\det \mathscr{M}_l^{(SLa)}}{\det \widehat{\mathscr{M}}_l^{(SLa)}} \right)^{-(l+1)^2/2} \right]$$

Gauge zero mode

Gauge independence

Fermi gauge

[A. Andreassen, W. Frost, M. D. Schwartz, ’17; S. Chigusa, T. Moroi, YS, ’17 & ’18]

$$\frac{\det \mathscr{M}_l^{(c\bar{c})}}{\det \widehat{\mathscr{M}}_l^{(c\bar{c})}} = 1$$

$$\frac{\det \mathscr{M}_l^{(SLa)}}{\det \widehat{\mathscr{M}}_l^{(SLa)}} = \frac{l}{l+2}\frac{\Gamma(l+1)\Gamma(l+2)}{\Gamma(l+1-z_g)\Gamma(l+2+z_g)}$$

Gauge independent

$$\frac{\det' \mathscr{M}_0^{(Sa)}}{\det \widehat{\mathscr{M}}_0^{(Sa)}} = \frac{|\lambda|}{16\pi}J_G^2$$

$$\frac{\det \mathscr{M}_l^{(T)}}{\det \widehat{\mathscr{M}}_l^{(T)}} = \frac{\Gamma(l+1)\Gamma(l+2)}{\Gamma(l+1-z_g)\Gamma(l+2+z_g)}$$

$$z_g = -\frac{1}{2}\left(1-\sqrt{1-\frac{8g^2}{|\lambda|}}\right)$$

$$J_G = \sqrt{\pi\int \mathrm{d}r r^3 \bar{\phi}^2} = \lim_{r\to\infty}\sqrt{\frac{8\pi R^2}{|\lambda|}\ln r}$$

Jacobian itself is divergent
(gauge zero mode is not normalisable)
But, it is canceled in the prefactor

Multi-field bounce

Symmetries

Gauge charges of bounce fields

$$D_\mu \phi = (\partial_\mu + g_a T^a A_\mu^a)\phi$$

$$(T^a)^T = -T^a \qquad [T^a, T^b] = -f^{abc}T^c$$

$$\phi = \begin{pmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \\ \vdots \end{pmatrix}$$: Vector of real scalars

$$a : \overbrace{1, \cdots, n_B}^{\text{Massive at false vacuum}}, \underbrace{n_B + 1, \ldots, n_G}_{\text{Massless at false vacuum}} = n_B + n_U$$

$$A : 1, \cdots, n_U$$

is the index for these

$$M_{ia} = -\sum_k g_a T^a_{ik} \bar{\phi}_k$$

* $M^T M$ is the gauge boson mass matrix

* We ignore the gauge bosons that do not couple to the bounce

Symmetries broken by bounce

Symmetry of the action (Fermi gauge)

$$S_E[\bar{\phi}] = S_E[e^{\theta^A \tilde{T}^A + \theta^\alpha_G \tilde{T}^\alpha_G}\bar{\phi}]$$

$$\alpha : 1, \cdots, n_{NG}$$

Pure NG bosons

$$\tilde{T}^A = \sum_B \kappa_{AB} T^B$$

False vacuum is symmetric, bounce is not

$$\bar{\phi}(\infty) = e^{\theta^A \tilde{T}^A + \theta^\alpha_G \tilde{T}^\alpha_G}\bar{\phi}(\infty)$$

$$\bar{\phi}(r) \neq e^{\theta^A \tilde{T}^A + \theta^\alpha_G \tilde{T}^\alpha_G}\bar{\phi}(r)$$

Orthogonal (would-be) NG bosons

$$\int d^4x (\tilde{T}^A \bar{\phi})^T (\tilde{T}^B \bar{\phi}) = 0 \quad (A \neq B)$$

$$\int d^4x (\tilde{T}^\alpha_G \bar{\phi})^T (\tilde{T}^\beta_G \bar{\phi}) = 0 \quad (\alpha \neq \beta)$$

$$\int d^4x (\tilde{T}^A \bar{\phi})^T (\tilde{T}^\beta_G \bar{\phi}) = 0$$

(This is automatic)

Jacobian for BG gauge with multi-field bounce

After the subtraction of gauge zero modes in the BG gauge, the Jacobian is given by

$$J_G^{\mathrm{BG}} = \underbrace{\sqrt{\det \mathscr{K}^G}}_{\text{Global symmetry}} \underbrace{\sqrt{\det \mathscr{K}}\, \det \kappa \left(\prod_A g_A^2 \right)^{-1/2}}_{\text{Gauge symmetry}}$$

$$\mathscr{K}^G_{\alpha\beta} = \pi \int \mathrm{d}r r^3 \left(\tilde{T}^\alpha_G \bar{\phi}\right)^T \left(\tilde{T}^\beta_G \bar{\phi}\right) \qquad \mathscr{K}_{AB} = \lim_{r\to\infty} \frac{\pi r^3}{\xi} \left[\left(\partial_r \Psi_0^{(c\bar{c})}(r)\right) \left(\Psi_0^{(c\bar{c})}(r)\right)^{-1} \right]_{AB}$$

$$(\Delta_0 - \xi M^T M)\Psi_0^{(c\bar{c})} = 0$$

V_G : Group volume measured with $\tilde{T}^A, \tilde{T}^\alpha$

Multi-field Fubini instanton

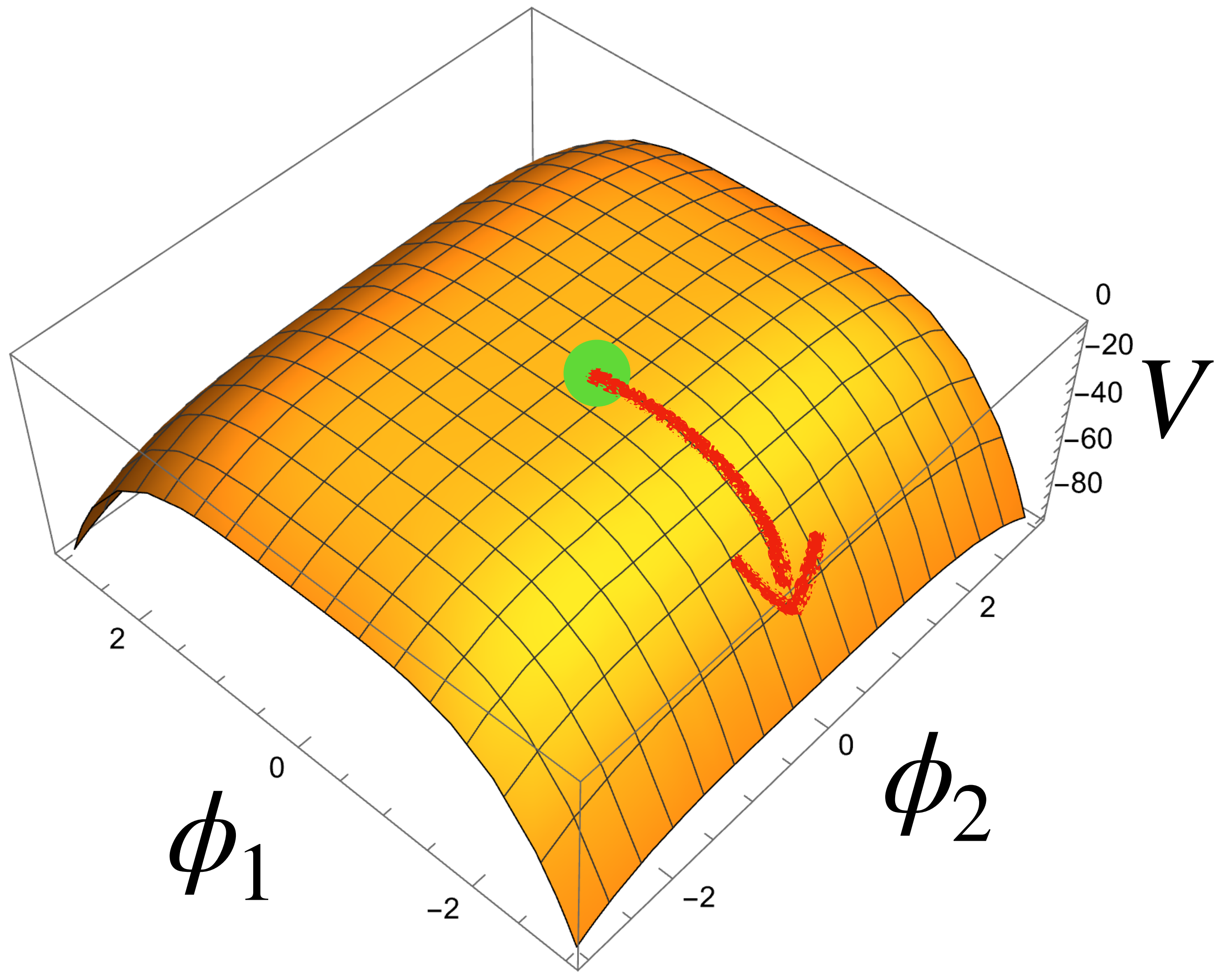
[S. Oda, YS, D. Takahashi, ’20]

Multi-field quartic potential

Renormalisation scale (=1/R)

$$V = \lambda_1(\mu)|\Phi_1|^4 + \lambda_2(\mu)|\Phi_2|^4 + \lambda_3(\mu)|\Phi_1|^2|\Phi_2|^2 + \cdots$$

(No dimensionful parameters)



Reduced to a single field bounce

$$V_{\rm eff} = \lambda_{\rm eff}(\mu)|\Phi|^4$$ (Steepest direction for given μ)

The rest is the same with the single field Fubini instanton calculation

(If there are more than one coupling with a similar size, we need to add their decay rates)

Phenomenological applications

Standard Model

[A. Andreassen, W. Frost, M. D. Schwartz, '17; S. Chigusa, T. Moroi, YS, '17 & '18]

Particle Data Group 2023

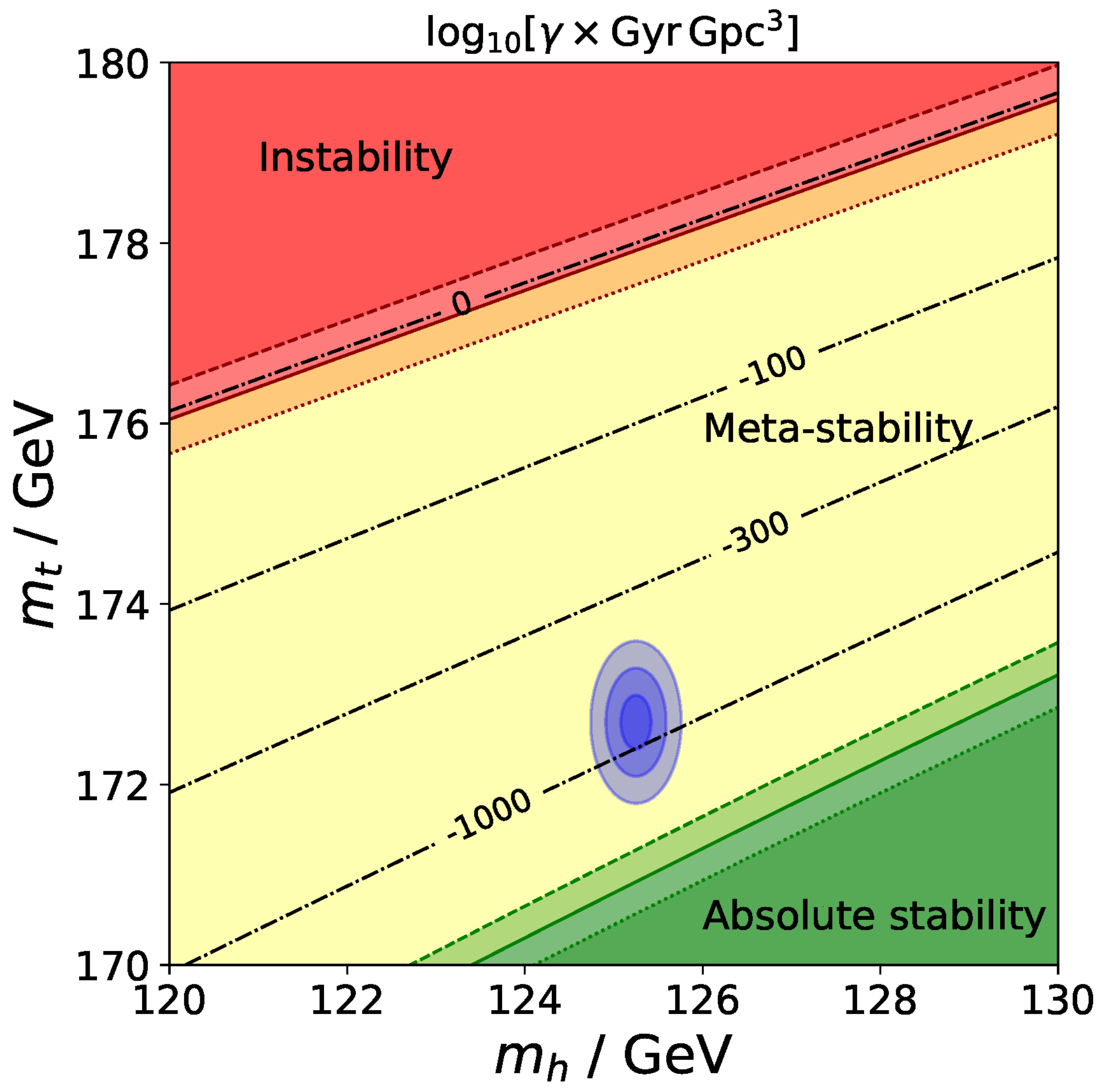
$$m_h^{\text{pole}} = 125.25 \pm 0.17 \ \text{GeV}$$
$$m_t^{\text{pole}} = 172.69 \pm 0.30 \ \text{GeV}$$
$$\alpha_s(m_Z) = 0.1179 \pm 0.0009$$

Vacuum decay rate in the standard model

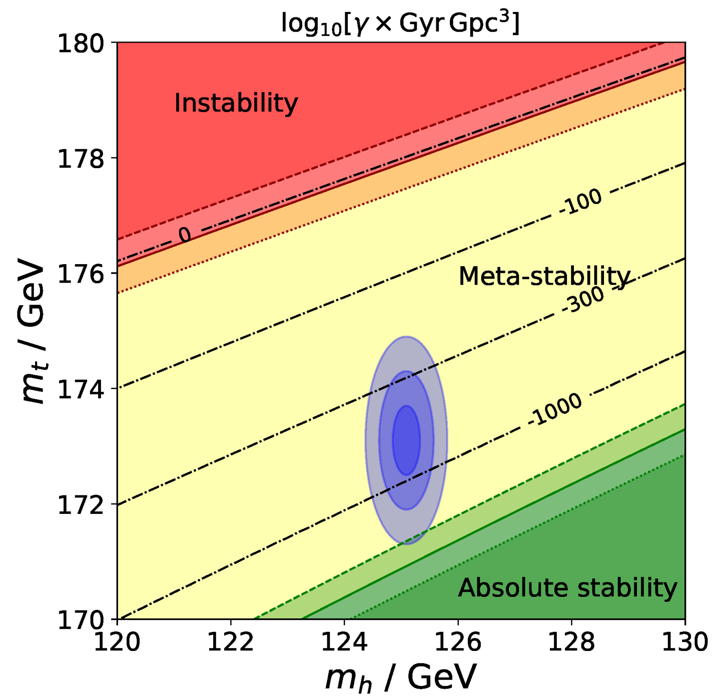
$$\log_{10} \frac{\gamma}{\text{Gyr} \cdot \text{Gpc}^3} = -785^{+45+155+182}_{-50-222-277}$$

$\Delta m_h \ \ \Delta m_t \ \ \Delta \alpha_s$ (1σ)

(Renormalization scale uncertainty: ±2)



[S. Chigusa, T. Moroi, YS, '17 (Updated 2023)]



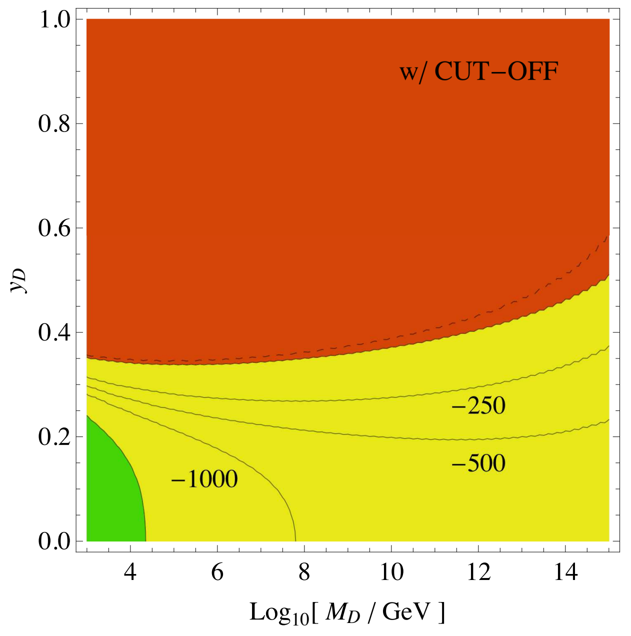
W/ PDG 2017

Fubini instanton

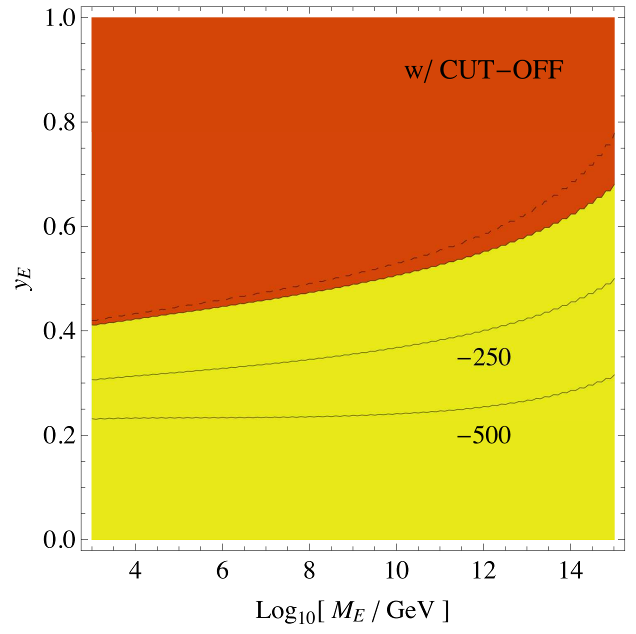
BSMs with Fubini instanton

[S. Chigusa, T. Moroi, YS, ’18]

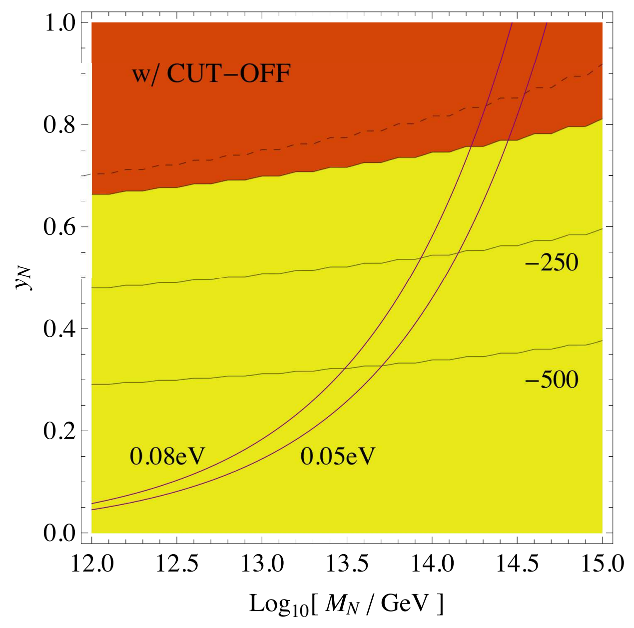
SM + VL quarks



SM + VL leptons

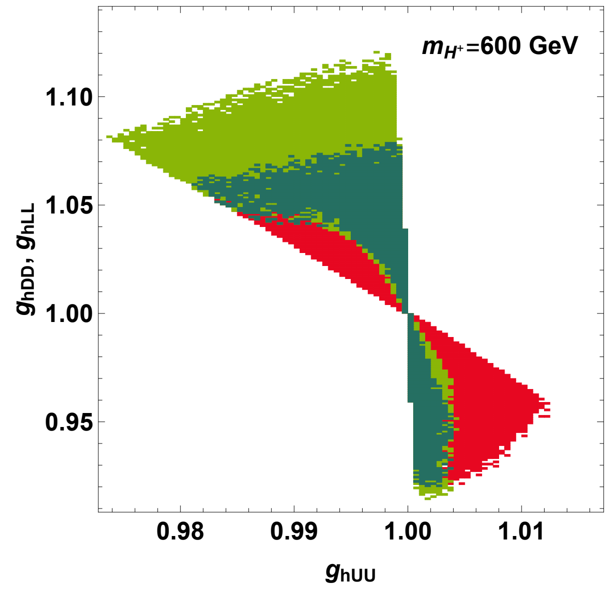


SM + RH neutrinos



DFSZ axion (2HDM)

[S. Oda, YS, D. Takahashi, ’20]



Public code

https://github.com/YShoji-HEP/ELVAS

README MIT license

ELVAS

C++ Package for ELectroweak VAcuum Stability

Introduction

ELVAS is a C++ package for the calculation of the decay rate of a false vacuum at the one-loop level, based on the formulae developed in [1, 2]. ELVAS is applicable to models with the following features:

- Only one scalar boson is responsible for the vacuum decay.
- Classical scale invariance (approximately) holds. In particular, the potential of the scalar field responsible for the vacuum decay should be well approximated by the quartic form for the calculation of the bounce solution. (Thus, the bounce is nothing but the so-called Fubini instanton.)
- The instability of the scalar potential occurs due to RG effects; thus, the quartic coupling constant becomes negative at a high scale.

If you use ELVAS in scholarly work, please cite [1] and [2].

Download and Install

ELVAS requires a c++ compiler that supports C++14 and the *boost* library (http://www.boost.org/users/download/#live). The required version of *boost* is 1.59.0 or higher.

Unix

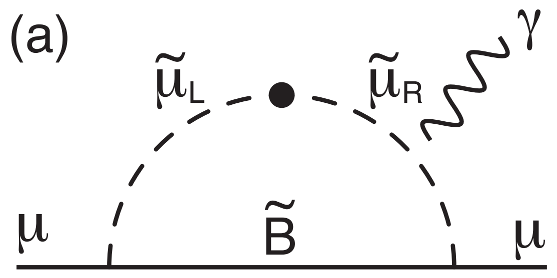
For UNIX systems, follow the instruction below.

Multifield bounce + gauge zero mode

SM + bino + sleptons

[S. Chigusa, T. Moroi, YS; ’22 & ’23]

(Minimal supersymmetric particles to explain the muon g-2 anomaly)



Bounce

$$h(r), \tilde{\ell}_L(r), \tilde{\ell}_R(r)$$

(smuon or stau)

Prefactor

size of $\mathscr{M}_l$

3x3

$$\mathscr{A} = A^{(\Re H^0, \Re \tilde{\ell}_L, \Re \tilde{\ell}_R)}$$ scalars

4x4 7x7

$$\times A^{(W^\pm, H^\pm, \tilde{\nu}_L)} A^{(Z, \gamma, \Im H^0, \Im \tilde{\ell}_L, \Im \tilde{\ell}_R)}$$ gauge+scalars

Gauge zero mode from the U(1) EM breaking

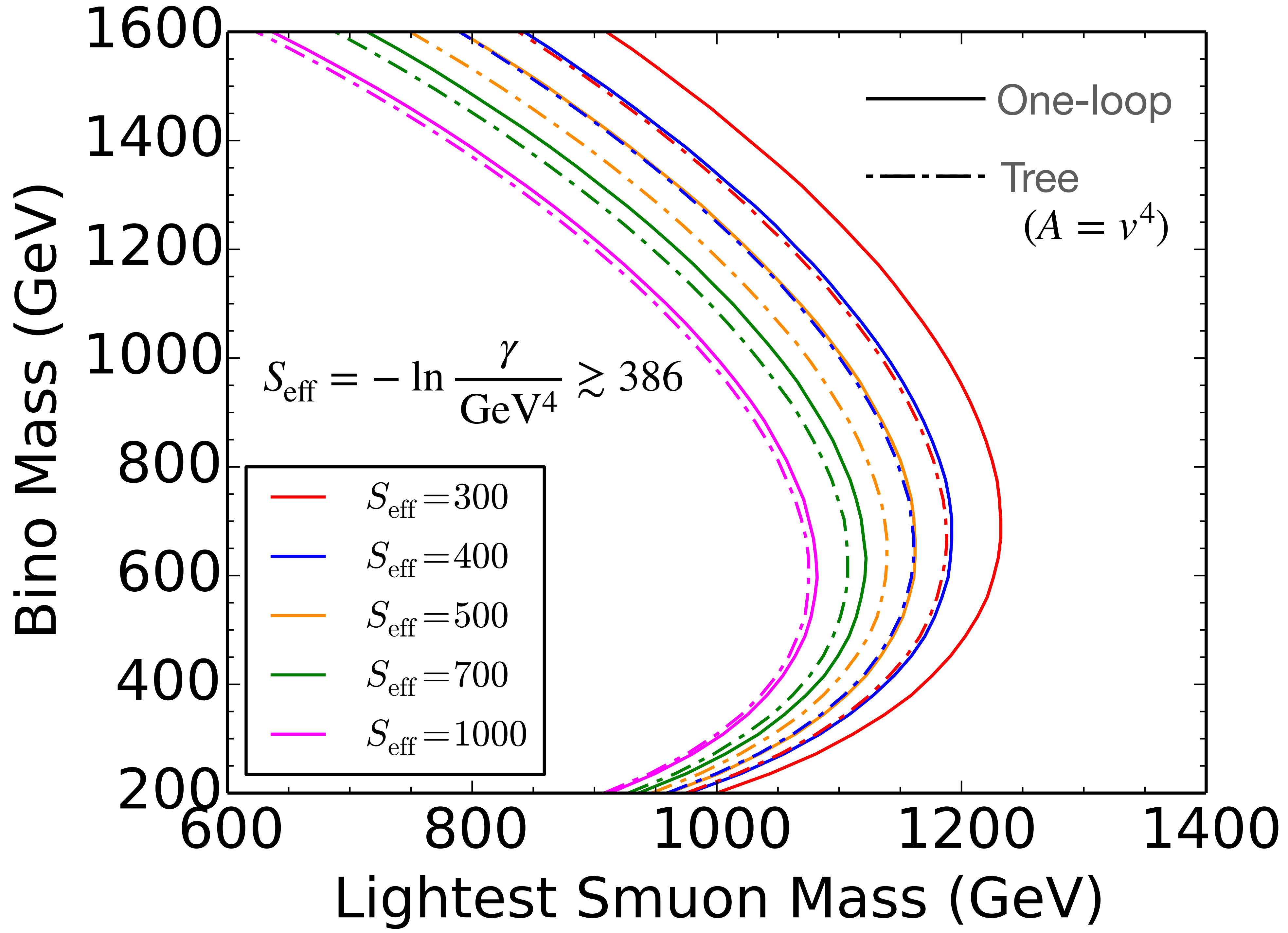
4x4 6x6

$$\times A^{(t_L, t_R)} A^{(\tilde{B}, \ell_L, \ell_R)}$$ fermions

1x1 2x2

$$\times A^{(\tilde{\nu}_L)} A^{(\Im \tilde{\ell}_L, \Im \tilde{\ell}_R)}$$ other generations of sleptons if they are light

Smuon instability (heavy staus)



(The trig-linear coupling is determined to explain the muon g-2 anomaly)

Summary

- Precise determination of vacuum decay rate requires computation of the prefactor, and contributions from gauge bosons are not negligible
- It was not clear whether the prefactor is gauge independent because the functional determinant depends the gauge parameter non-trivially
- We have explicitly shown that the prefactor is gauge-independent and is reduced to one physical functional determinant
- We derived the Jacobian for the gauge zero modes, which enables the computation of the gauge determinants for any scalar potential
- We have applied our prescription to standard model and several models beyond the standard model

Semi-analytic solution

BG gauge

$$\psi = \begin{pmatrix} \partial_r \chi \\ \frac{L}{r}\chi \\ g\bar{\phi}\chi \end{pmatrix} + \begin{pmatrix} \frac{1}{g^2\bar{\phi}^2}\frac{L}{r}\eta \\ \frac{1}{g^2\bar{\phi}^2}\frac{1}{r^2}\partial_r r^2 \eta \\ 0 \end{pmatrix} + \begin{pmatrix} -\frac{2\bar{\phi}'}{g^2\bar{\phi}^3}\zeta \\ 0 \\ \frac{1}{g\bar{\phi}}\zeta \end{pmatrix}$$

$$\underbrace{\left(-\Delta_l + \xi g^2\bar{\phi}^2\right)}_{\textbf{FP}}\chi = -\frac{2\bar{\phi}'}{g^2\bar{\phi}^3}\frac{L}{r}\eta - \frac{1}{r^3}\partial_r r^3 \frac{2\bar{\phi}'}{g^2\bar{\phi}^3}\zeta$$

$$\underbrace{\left(-\Delta_l + \xi g^2\bar{\phi}^2\right)}_{\textbf{FP}}\zeta = 0$$

$$\left(-\Delta_l + g^2\bar{\phi}^2 + \frac{2\bar{\phi}'}{\bar{\phi}}\frac{1}{r^2}\partial_r r^2\right)\eta = \frac{L}{r}\frac{2\bar{\phi}'}{\bar{\phi}}\zeta$$

Semi-analytic solution
Fermi gauge

$$\psi = \begin{pmatrix} \partial_r \chi \\ \frac{L}{r}\chi \\ g\bar{\phi}\chi \end{pmatrix} + \begin{pmatrix} \frac{1}{g^2\bar{\phi}^2}\frac{L}{r}\eta \\ \frac{1}{g^2\bar{\phi}^2}\frac{1}{r^2}\partial_r r^2 \eta \\ 0 \end{pmatrix} + \begin{pmatrix} -\frac{2\bar{\phi}'}{g^2\bar{\phi}^3}\zeta \\ 0 \\ \frac{1}{g\bar{\phi}}\zeta \end{pmatrix}$$

$$\underbrace{-\Delta_l}_{\text{FP}}\chi = -\frac{2\bar{\phi}'}{g^2\bar{\phi}^3}\frac{L}{r}\eta - \frac{1}{r^3}\partial_r r^3 \frac{2\bar{\phi}'}{g^2\bar{\phi}^3}\zeta - \xi\zeta$$

$$\underbrace{-\Delta_l}_{\text{FP}}\zeta = 0$$

$$\left(-\Delta_l + g^2\bar{\phi}^2 + \frac{2\bar{\phi}'}{\bar{\phi}}\frac{1}{r^2}\partial_r r^2\right)\eta = \frac{L}{r}\frac{2\bar{\phi}'}{\bar{\phi}}\zeta$$