Self-consistent Bounce in False Vacuum Decay

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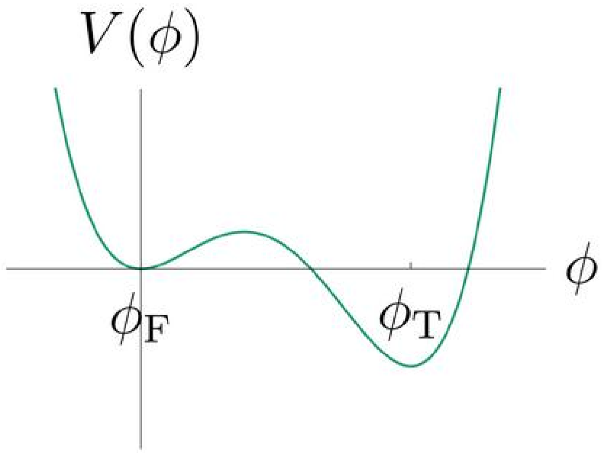

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Collaborators: Jean Alexandre, Juan S. Cruz, Björn Garbrecht, Peter Millington, Sarben Sarkar, Carlos Tamarit

Outline

- A brief introduction to FOPT
- Bounce formalism on FVD
- 1PI effective action and quantum-corrected bounce
  (main part of the talk)
- Backreactions to the plasma and 2PI
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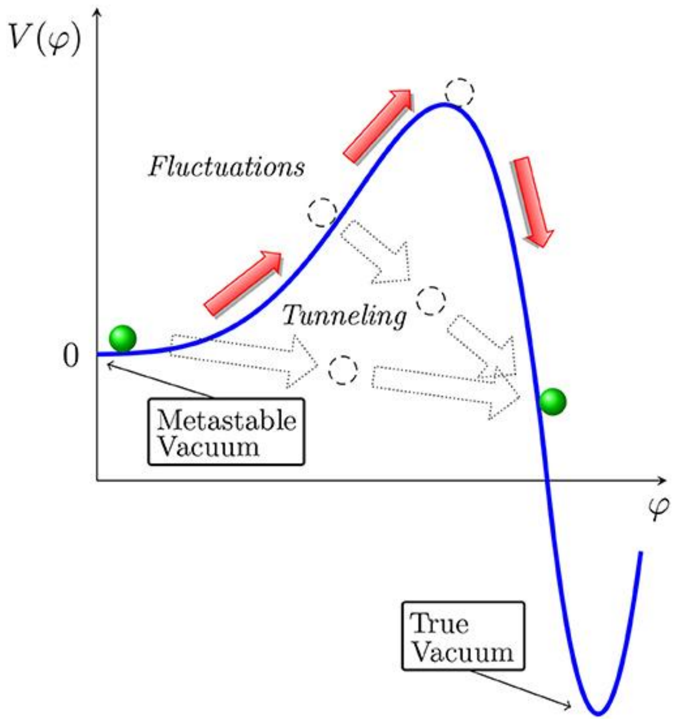
FOPTs



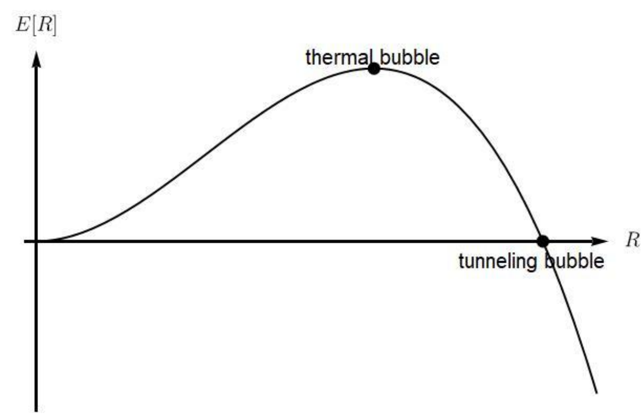
[Markkanen, Rajantie and Stopyra 2018]

Bubble nucleation caused by

1. Quantum fluctuations: energy is conserved in the scalar field sector, O(4) bounce

2. Thermal fluctuations: energy not conserved, O(3) bounce, backreactions to the plasma is supposed to exist

Thin-wall picture:



Why interesting?

Theory

- Real-time picture (Langevin vs Picard-Lefschetz)
- Precision calculation of bubble nucleation
- BH-seeded bubble nucleation (Hartle-Hawking vs Unruh)
- Perturbative vs non-perturbative (Resurgence)
- Bubble wall dynamics (at finite T)
- Higgs instability
- ...

Phenomenology

- Gravitational waves production
- Baryogenesis/Leptongenesis
- Dark Matter production
- Magnetic field production
- PBH formation
- ...

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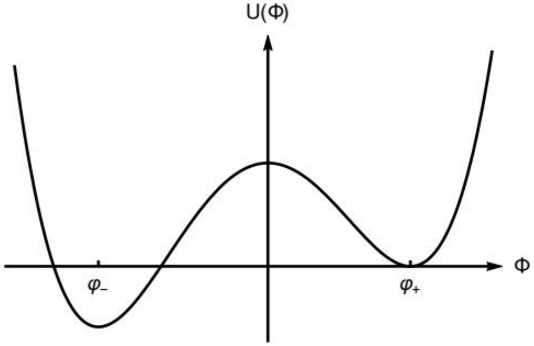
Bounce formalism

Central idea: [Callan & Coleman, 1977]

False Vacuum ⟶ Unstable ⟶ Complex Energy



$$E_0 = \mathrm{Re}E_0 + i\mathrm{Im}E_0$$

Decay rate: $\gamma = -2\mathrm{Im}E_0$

Consider the Euclidean transition amplitude $\langle\varphi_+|\mathrm{e}^{-H\mathcal{T}}|\varphi_+\rangle$. Insert a complete basis of energy eigenstates

$$\langle\varphi_+|\mathrm{e}^{-H\mathcal{T}}|\varphi_+\rangle = \sum_n \mathrm{e}^{-E_n\mathcal{T}}|\langle\varphi_+|n\rangle|^2$$

Taking $\mathcal{T} \to +\infty$,

$$\left.\begin{aligned} \langle\varphi_+|\mathrm{e}^{-H\mathcal{T}}|\varphi_+\rangle &\overset{\mathcal{T}\to+\infty}{=\!=\!=} \mathrm{e}^{-E_0\mathcal{T}}|\langle\varphi_+|0\rangle|^2 \\ \langle\varphi_+|\mathrm{e}^{-H\mathcal{T}}|\varphi_+\rangle &= \int \mathcal{D}\Phi\ \mathrm{e}^{-S_E[\Phi]} \equiv Z^E \end{aligned}\right\} \Rightarrow \boxed{\gamma = -2\mathrm{Im}E_0 = \lim_{\mathcal{T}\to+\infty}\frac{2}{\mathcal{T}}\mathrm{Im}(\ln Z^E)}$$

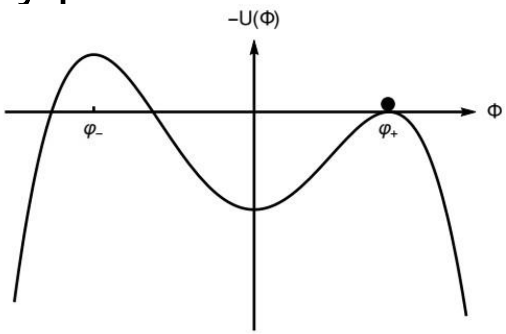
Calculating the partition function

Method of steepest descent:

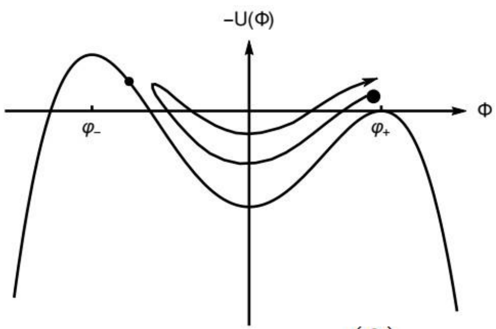
1. Find stationary points: $\delta S_E\big|_{\varphi_a} = 0$
2. Expand about the stationary points: $\Phi = \varphi_a + \Delta\Phi_a$

$$\int \mathcal{D}\Phi \; \mathrm{e}^{-S_E[\Phi]} = \sum_a \mathrm{e}^{-S_E[\varphi_a]} \int \mathcal{D}\Delta\Phi_a \; \mathrm{e}^{-\frac{1}{2}\Delta\Phi_a \left(\left.\frac{\delta^2 S_E[\Phi]}{\delta\Phi^2}\right|_{\varphi_a} \right) \Delta\Phi_a + \ldots}$$

In Euclidean space, the potential is upside-down. There are two types of stationary points.



Trivial false vacuum



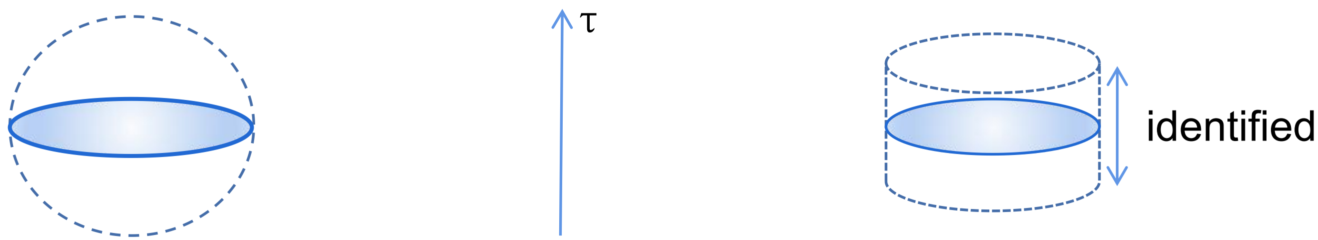
Bounce $\varphi_b^{(0)}$

Callan-Coleman: $$\boxed{\frac{\gamma}{V} = \left(\frac{B^{(0)}}{2\pi}\right)^2 \left| \frac{\det'[-\partial^2 + U''(\varphi_b^{(0)})]}{\det[-\partial^2 + U''(\varphi_+)]} \right|^{-1/2} \mathrm{e}^{-B^{(0)}}}$$

The critical bubble from the bounce

O(4) bounce gives critical bubble at $\tau = 0$

O(3) bounce is the critical bubble



How to compute the quantum-corrected bounce/critical bubble? Or even further, what if we do not have a bounce solution at the classical level? (e.g., like the standard model!)

Method 1: classical potential to effective potential in the Callan-Coleman formula ⟶ double counting ⟶ separating the scales ⟶ nucleation EFT (more useful for finite-T PTs) [Gould & Hirvonen, 2021]

Method 2: nPI effective action formalism ⟶ self-consistent bounce over all scales

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Introduce an effective action that gives:
(1) self-consistent bounce EoM;
(2) generalized decay rate formula

$\Downarrow$

Taylor expand the one-loop effective action about the classical bounce $\longrightarrow$ functional derivative of functional determinants

$\Downarrow$

Regularisation of one-loop effective action $\longrightarrow$ known expressions

$\Downarrow$

Take functional derivative

1PI effective action

With a quantum field theory, we start with the generating functional

$$Z[J] = \int \mathcal{D}\Phi \, \mathrm{e}^{-S[\Phi] + \int \mathrm{d}^d x \, J(x)\Phi(x)}$$

Then, a macroscopic configuration is described by the one-point function

$$\varphi_J(x) = \langle 0|\Phi(x)|0\rangle_J = \frac{\delta \ln Z[J]}{\delta J(x)}$$

Legendre transform: $\Gamma[\varphi_J] = -\ln Z[J] + \int \mathrm{d}^d x \, J(x)\varphi_J(x)$

$$\Rightarrow \frac{\delta\Gamma[\varphi_J]}{\delta\varphi_J(x)} = J(x) \xrightarrow{\text{in the absence of external source}} \frac{\delta\Gamma[\varphi]}{\delta\varphi(x)} = 0$$

quantum bounce EoM, when applied in real time, quantum bubble wall EoM

With the 1PI effective action, the decay rate can be formulated as

$$\frac{\gamma}{V} = \frac{2\left|\mathrm{Im}\,\mathrm{e}^{-(\Gamma[\varphi_b]-\Gamma[\varphi_{\mathrm{FV}}])}\right|}{V\mathcal{T}}$$

[Garbrecht & Millington, 2015; Plascencia & Tamarit, 2015]

Quantum corrections to the classical bounce

Truncate at the one-loop, the effective action reads

$$\Gamma^{(1)}[\varphi] = S[\varphi] + \frac{1}{2}\boxed{\ln\frac{\det\mathcal{M}[\varphi]}{\det\widehat{\mathcal{M}}}} \equiv S[\varphi] + \frac{1}{2}\boxed{D[\varphi]}$$

where $\mathcal{M}[\varphi] = -\partial^2 + U''(\varphi)$ and $\widehat{\mathcal{M}} \equiv \mathcal{M}[\varphi_{\mathrm{FV}}]$.

$$\text{EoM: } \left.\frac{\delta S[\varphi]}{\delta\varphi(x)}\right|_{\varphi_b^{(1)}} + \frac{1}{2}\left.\frac{\delta D[\varphi]}{\delta\varphi(x)}\right|_{\varphi_b^{(1)}} = 0$$

Expanding this equation abount $\varphi_b^{(0)} \equiv \varphi_b^{(1)} - \Delta\varphi_b$ gives

$$[-\partial^2 + U''(\varphi_b^{(0)})]\Delta\varphi_b + \frac{1}{2}\left.\frac{\delta D[\varphi]}{\delta\varphi(x)}\right|_{\varphi_b^{(0)}} = 0$$

Substituting the same expansion into the effective action, one obtains

$$\Gamma^{(1)}[\varphi_b^{(1)}] = S[\varphi_b^{(0)}] + \frac{1}{2}D[\varphi_b^{(0)}] + \frac{1}{4}\int \mathrm{d}^d x \boxed{\left.\frac{\delta D[\varphi]}{\delta\varphi(x)}\right|_{\varphi_b^{(0)}}}\Delta\varphi_b(x)$$

new quantity to compute: functional derivative of the functional determinant

Making use of O(d) symmetry

With O(d) symmetry, we only need to consider $D[\varphi]$ for O(d)-symmetric configurations.

EoM: $$\frac{2\pi^{d/2}r^{d-1}}{\Gamma(d/2)}\left[-\frac{\mathrm{d}^2}{\mathrm{d}r^2}-\frac{d-1}{r}\frac{\mathrm{d}}{\mathrm{d}r}+U''(\varphi_b^{(0)})\right]\Delta\varphi_b(r)+\frac{1}{2}\left.\frac{\delta D[\varphi]}{\delta\varphi(r)}\right|_{\varphi_b^{(0)}}=0$$

Effective action:

$$\Gamma^{(1)}[\varphi_b^{(1)}]=S[\varphi_b^{(0)}]+\frac{1}{2}D[\varphi_b^{(0)}]+\frac{1}{4}\int\mathrm{d}r\left.\frac{\delta D[\varphi]}{\delta\varphi(r)}\right|_{\varphi_b^{(0)}}\Delta\varphi_b(r)$$

Now, we just need to compute $D[\varphi(r)]$ and its functional derivative, both evaluated at the classical bounce!

divergent, needs renormalisation

- Method 1: Dunne-Min [Dunne & Min, 2005; Dunne & Kirsten, 2006]
- Method 2: Baacke-Lavrelashvili [Baacke & Kiselev, 1993; Baacke & Lavrelashvili, 2004]

Gel'fand-Yaglom theorem

For O(d) configurations, (hyper)spherical harmonic decomposition gives

$$D[\varphi] = \sum_{l=0}^{\infty} \deg(l;d) \ln\left(\frac{\det \mathcal{M}_l[\varphi]}{\det \widehat{\mathcal{M}}_l}\right)$$

where

$$\mathcal{M}_l[\varphi] = -\frac{\mathrm{d}^2}{\mathrm{d}r^2} - \frac{d-1}{r}\frac{\mathrm{d}}{\mathrm{d}r} + \frac{l\,(l+d-2)}{r^2} + U''(\varphi(r))$$

$$\widehat{\mathcal{M}}_l = -\frac{\mathrm{d}^2}{\mathrm{d}r^2} - \frac{d-1}{r}\frac{\mathrm{d}}{\mathrm{d}r} + \frac{l\,(l+d-2)}{r^2} + m^2$$

Then (Gel'fand-Yaglom theorem)

$$\frac{\det \mathcal{M}_l[\varphi]}{\det \widehat{\mathcal{M}}_l} = \frac{\psi_l(\infty;\varphi)}{\hat{\psi}_l(\infty)} \equiv T_l(\infty;\varphi)$$

where $\mathcal{M}_l[\varphi]\psi_l(r;\varphi) = 0$, $\widehat{\mathcal{M}}_l\hat{\psi}_l(r) = 0$ (the solutions are required to have the same regular behavior at $\mathrm{r} = 0$), equivalently

$$\left[-\frac{\mathrm{d}^2}{\mathrm{d}r^2} - \left(2m\frac{I_{l+d/2-2}(mr)}{I_{l+d/2-1}(mr)} - \frac{2l+d-3}{r}\right)\frac{\mathrm{d}}{\mathrm{d}r} + W(\varphi(r))\right] T_l(r;\varphi) = 0$$

Comparing DM and BL

Formally, we have

n!)

Dunne-M

$\frac{1}{2}(D[\varphi])$ … $2m^2)$ $\}$

where W

Baacke-

$\frac{1}{2}(D[\varphi$ … $\varphi))^2]\}$

une

divergent result (valid only "on-shell")

Two regularisation approaches

Large-l and WKB: [Ekstedt, Gould & Hirvonen, 2023]

$$\frac{1}{2}\sum_{l=2}^{l_{\max}} \underbrace{\deg(d;l)\left(\log T_l - \log T_l^{(\mathrm{WKB})}\right)}_{\text{finite and converges faster}} + \underbrace{\frac{1}{2}\sum_{l=2}^{\infty}\deg(d;l)\log T_l^{(\mathrm{WKB})} + S_{\mathrm{ct}}}_{\text{finite and known}}$$

$$+\underbrace{\frac{1}{2}\sum_{l=l_{\max}+1}^{\infty}\deg(d;l)\left(\log T_l - \log T_l^{(\mathrm{WKB})}\right)}_{\approx 0}.$$

Interaction picture: [WA, Alexandre, & Sarkar, 2023]

(divergent part) $\left(\frac{1}{2}D[\varphi]\right)^{(2)} = \frac{1}{2}\left[\ln\det(-\partial^2 + m^2 + W(\varphi)) - \ln\det(-\partial^2+m^2)\right]_{\mathcal{O}(W^2)}$

$$\left(\frac{1}{2}D[\varphi]\right)^{\mathrm{reg}} = \underbrace{\left[\left(\frac{1}{2}D[\varphi]\right) - \left(\frac{1}{2}D[\varphi]\right)^{(2)}\right]}_{\text{numerical part}} + \underbrace{\left[\left(\frac{1}{2}D[\varphi]\right)^{(2)} - \left(\frac{1}{2}D[\varphi]\right)_{\mathrm{pole}}\right]}_{\text{analytical part}}$$

Taking functional derivative of T_l

Start with

$$\mathcal{H}_l[\varphi]T_l(r;\varphi) = 0 \qquad (1)$$

Taking functional derivative of this equation gives

$$\mathcal{H}_l[r';\varphi]\frac{\delta T_l(r';\varphi)}{\delta\varphi(r)} = -\frac{\delta\mathcal{H}_l[r';\varphi]}{\delta\varphi(r)}T_l(r';\varphi) = \delta(r'-r)U'''(\varphi(r'))T_l(r';\varphi)$$

Then we get

$$\frac{\delta T_l(r';\varphi)}{\delta\varphi(r)} = \int \mathrm{d}r''\, G_l(r',r'';\varphi)\delta(r''-r)U'''(\varphi(r''))T_l(r'';\varphi) = G_l(r',r;\varphi)U'''(\varphi(r))T_l(r;\varphi)$$

(B.cs for the Greens function depends on Eq. (1) above)

$$\Rightarrow \boxed{\frac{\delta T_l(\infty;\varphi)}{\delta\varphi(r)} = G_l(\infty,r;\varphi)U'''(\varphi(r))T_l(r;\varphi)}$$

[WA, Alexandre, & Sarkar, 2023]

The same procedure applies for calculating the function derivative of

$h_l^{(1)}$ and $h_l^{(2)}$

in the Baacke-Lavrelashvili formula

Back to the Dunne-Min formula

$$\frac{1}{2}(D[\varphi])^{\mathrm{reg}}_{\mathrm{DM}} = \frac{1}{2}\sum_{l=0}^{\infty}(l+1)^2\left\{\ln T_l(\infty;\varphi) - \frac{\int_0^\infty \mathrm{d}r\, rW(\varphi)}{2(l+1)} + \frac{\int_0^\infty \mathrm{d}r\, r^3 W(\varphi)(W(\varphi)+2m^2)}{8(l+1)^3}\right\}$$
$$-\frac{1}{16}\int_0^\infty \mathrm{d}r\, r^3 W(\varphi)\left(W(\varphi)+2m^2\right)\left[\log\left(\frac{\mu r}{2}\right)+\gamma_{\mathrm{E}}+1\right]$$

We can show that

$$\left.\frac{\delta T_l(\infty;\varphi)}{\delta\varphi(r)}\right|_{r=\infty} = G_l(\infty,\infty;\varphi)U'''(\varphi(r))T_l(r;\varphi) = 0$$

$$G(r,r') = \theta(r-r')\left[-\frac{f_1(r)f_2(r')}{\mathcal{W}[f_1(r'),f_2(r')]} + \frac{f_1(r')f_2(r)}{\mathcal{W}[f_1(r'),f_2(r')]}\right]$$

But taking the functional derivative of other terms inside the sum does not give a vanishing result (recall $W(\varphi) = U''(\varphi) - m^2$) !

A summary of this “section”

- We show how to compute the functional derivative of a functional determinant efficiently, using either the Dunne-Min and Baacke-Lavrelashvili formulae
- Need to compute several Green’s functions for each l
- The functional derivative of the Dunne-Min formula is divergent and cutoff-dependent, meanding that this formula is valid only “on-shell”
- The Baacke-Lavrelashvili formula is FINE

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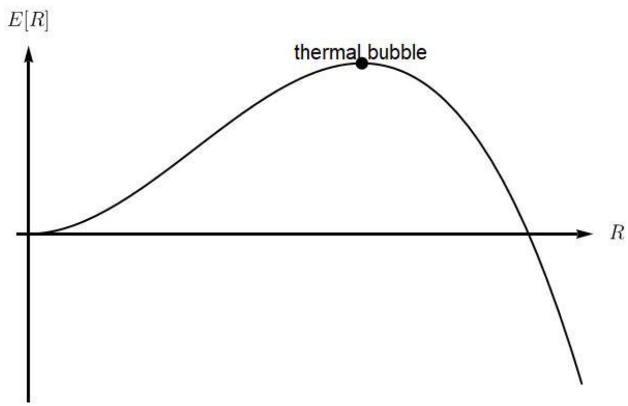
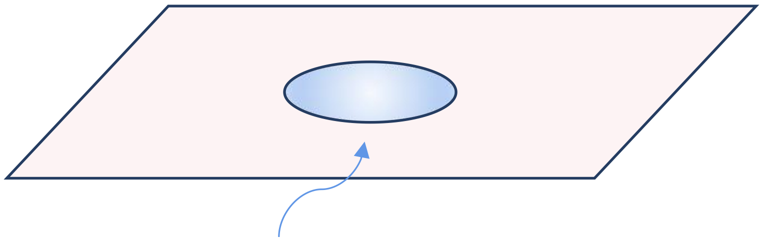
Back-reaction to the plasma





The total energy of the bubble configuration is not zero

- ❑ Energy is not conserved in canonical ensemble!
- ❑ But still, is any way to study back-reactions from the bubble to the plasma?

Bubble: one-point function
Plasma/particles: two-point functions

⇨ Need to construct coupled EoMs between one and two-point functions

2PI effective action formalism

Perturbation calculations of bubble nucleation rates suffers from many uncertainties! [Croon, Gould, Schicho, Tenkanen & White, 2020]

Introduce local and non-local sources

$$Z_{\text{eff}}[J,K] = \int \mathcal{D}\Phi \, \exp\left\{ -S_{\text{eff}}[\Phi] + \int \mathrm{d}^3x \, J(x)\Phi(x) + \frac{1}{2}\int \mathrm{d}^3x_1 \, \mathrm{d}^3x_2 \, K(x_1,x_2)\Phi(x_1)\Phi(x_2) \right\}$$

From it, one has

$$\varphi(x) \equiv \langle \Phi(x) \rangle = \frac{\delta \ln Z_{\text{eff}}[J,K]}{\delta J(x)}$$

$$\Delta_\phi(x_1,x_2) \equiv \langle \Phi(x_1)\Phi(x_2) \rangle_c = 2\frac{\delta \ln Z_{\text{eff}}[J,K]}{\delta K(x_1,x_2)} - \varphi(x_1)\varphi(x_2)$$

Legendre transform:

$$\Gamma_{2\text{PI}}[\varphi,\Delta_\phi] = -\ln Z_{\text{eff}}[J,K] + \int \mathrm{d}^3x \, J(x)\varphi(x) + \frac{1}{2}\iint \mathrm{d}^3x_1 \, \mathrm{d}^3x_2 \, K(x_1,x_2) \left[\Delta_\phi(x_1,x_2) + \varphi(x_1)\varphi(x_2)\right]$$

$$\Rightarrow \left\{ \begin{aligned} \frac{\delta \Gamma_{2\text{PI}}[\varphi,\Delta_\phi]}{\delta \varphi(x)} &= J(x) \\ \frac{\delta \Gamma_{2\text{PI}}[\varphi,\Delta_\phi]}{\delta \Delta_\phi(x_1,x_2)} &= \frac{1}{2}K(x_1,x_2) \end{aligned} \right. \quad \Rightarrow \quad \left\{ \begin{aligned} \frac{\delta \Gamma_{2\text{PI}}[\varphi,\Delta_\phi]}{\delta \varphi(x)} &= 0 \\ \frac{\delta \Gamma_{2\text{PI}}[\varphi,\Delta_\phi]}{\delta \Delta_\phi(x_1,x_2)} &= 0 \end{aligned} \right.$$

In the absence of external sources

2PI effective action formalism

The 2PI effective action reads

$$\Gamma_{2\mathrm{PI}}[\varphi, \Delta_\phi] = S_{\mathrm{eff}}[\varphi] + \frac{1}{2}\mathrm{Tr}\ln\Delta_\phi^{-1} + \frac{1}{2}\mathrm{Tr}\left(\mathcal{M}[\varphi]\Delta_\phi\right) + \underbrace{\Gamma_2[\varphi, \Delta_\phi]}_{\text{represented by 2PI diagrams}}$$

Together with a localization procedure, one might arrive at coupled scalar EoM and Boltzmann equations [WA, Beniwal, Maggi & Marsh, 2023]

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Summary

- False vacuum decay can be formulated in terms of the 1PI effective action
- We show how efficiently compute self-consistent bounce, and related quantum corrections to the decay rate using existing formulae
- The Dunne-Min formula may be valid only "on-shell"; when taking functional derivatives, it gives a divergent, cutoff-dependent result
- 2PI effective action may be used for studying backreaction effects in bubble nucleation

Thank you!

Two regularisation approaches

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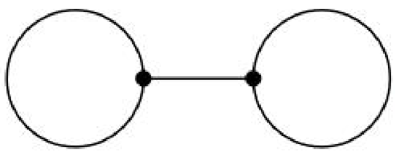
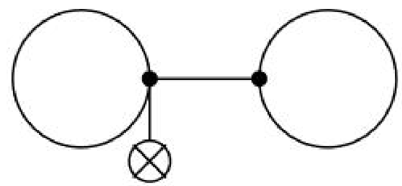
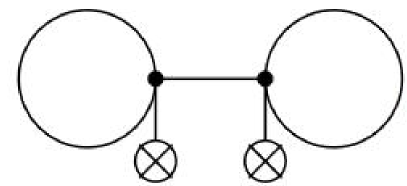
Diagrams computed at the one-loop effective action

 ,
 ,
 ,