How fast are the Higgs phase bubbles?

-- a comparison of two methods for calculating ultrarelativistic bubble wall velocities --

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@ Tunneling in QFT
June 25, 2026

First order cosmological phase transitions

As the Universe expands, the plasma contained within it cools down.

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(a^3 s\right) = 0 \qquad \Rightarrow \qquad T(t) \propto a(t)^{-1}$$

Cosmological phase transitions may have occurred throughout the cosmic history when the plasma temperature passed through particle physics energy scales.

- ➔ GUT scale
- ➔ SUSY scale
- ➔ electroweak scale (Higgs fields gets a vev)
- ➔ QCD scale (color confinement & chiral symmetry breaking)
- ➔ misc: B-breaking, L-breaking, PQ-breaking, dark sectors, inflaton sector, …

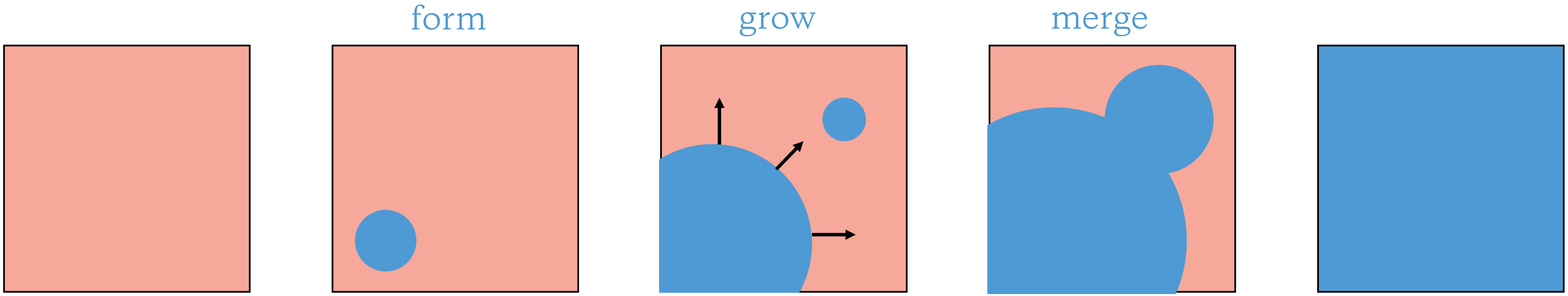
First order phase transitions are characterized by *phase coexistence*

e.g., both liquid water and water vapor are present during the condensation phase transition

SM predicts continuous crossover for EW & QCD *(at small chemical potential)* ➔ new physics is required for a first order cosmological phase transition ➔ observational probes of 1st order PT are necessarily tests of new physics

Bubble business

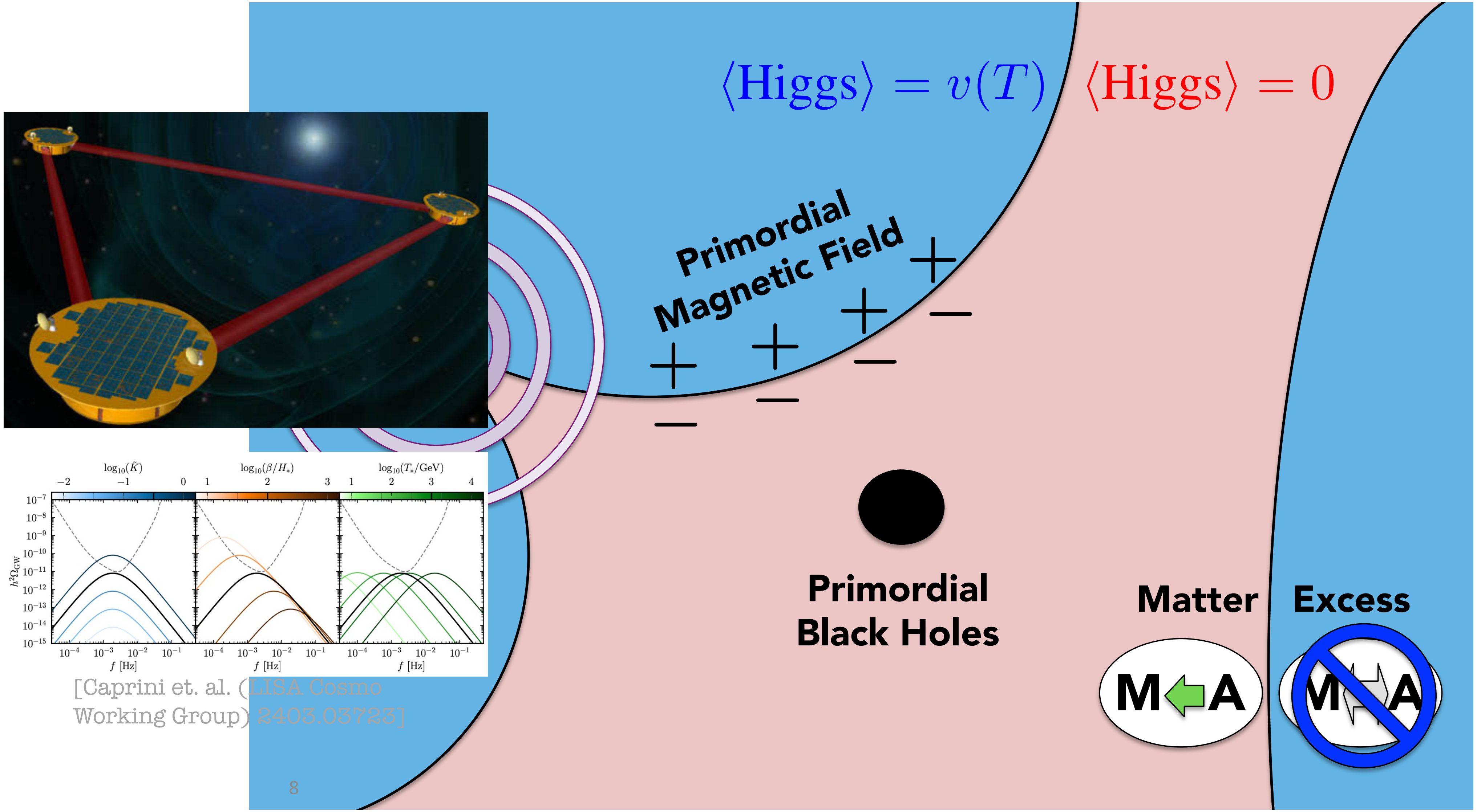
Phase coexistence means boundaries between phase domains (a.k.a., bubbles or walls)



today's talk:

How quickly do bubbles grow?
What is the speed of the bubble walls?

Why care about bubble wall speed?



[Caprini et. al. (LISA Cosmo Working Group) 2403.03723]

Equation of motion

for a nonrelativistic planar bubble wall:

$$\vec{F} = m\vec{a} \quad \Rightarrow \quad P = \sigma \dot{v}_w$$

(P = force/area = pressure)
(σ = mass/area = tension)

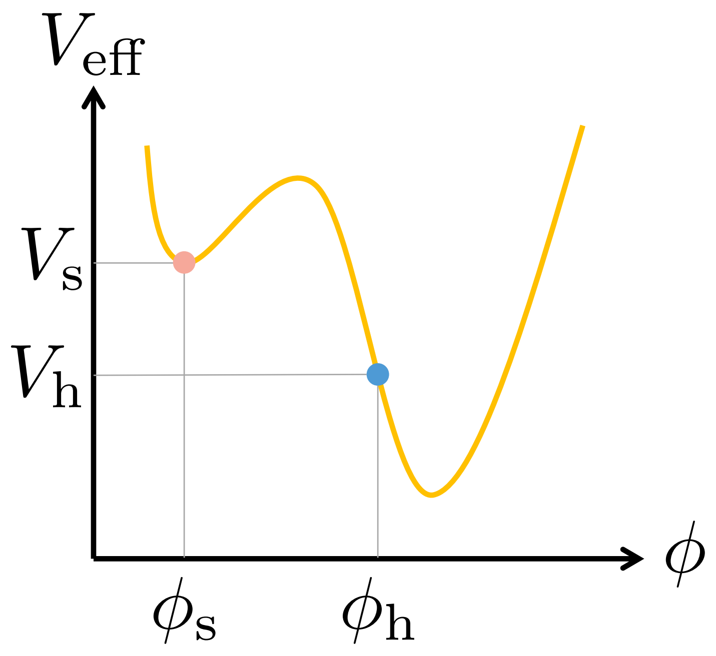
pressure on the wall

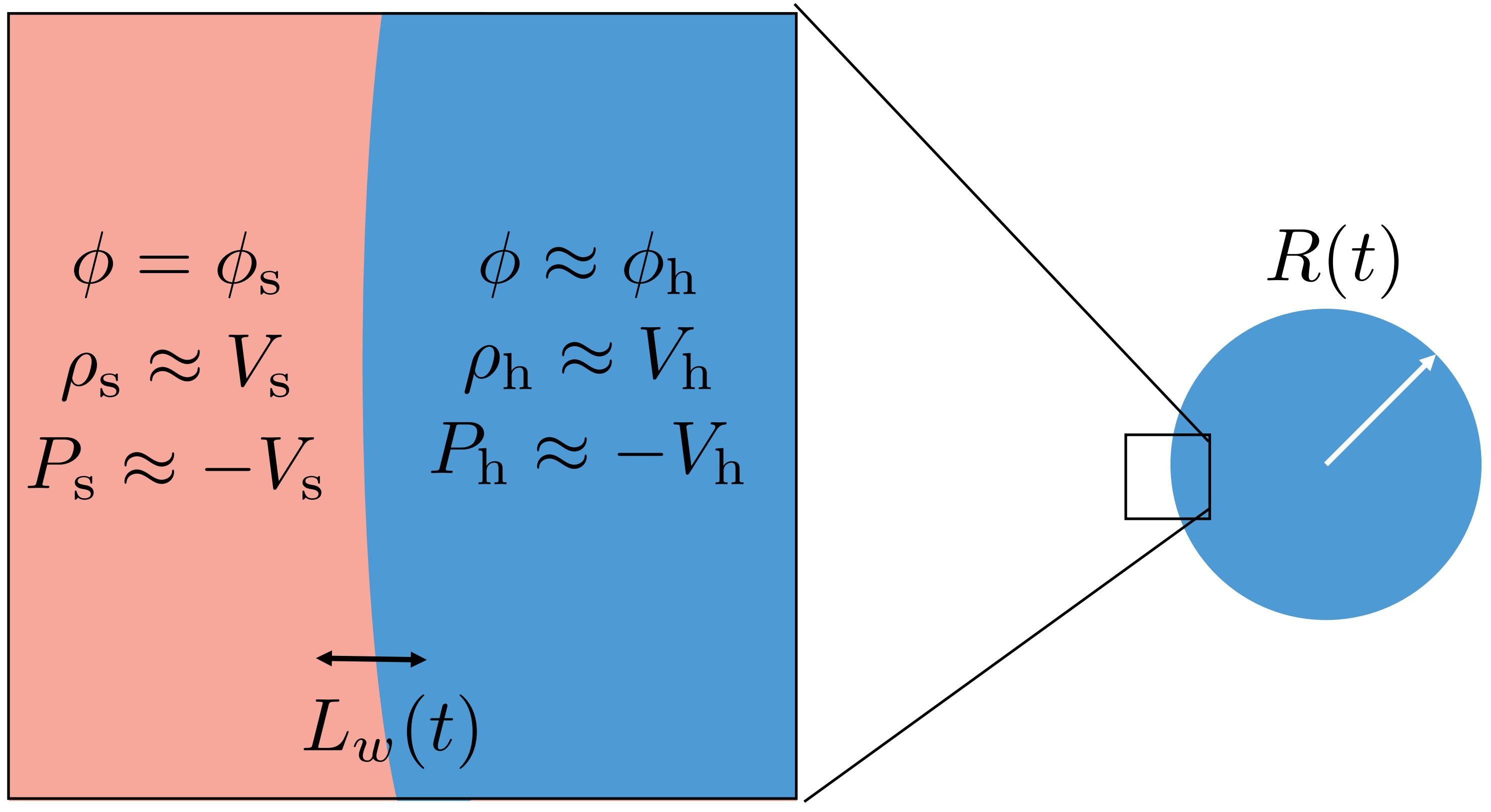
$$P = P_{\text{vac}} - \cancel{P_{\text{therm}}(v_w)}$$

to begin,
let's neglect

Starting simple: bubbles in vacuum

To begin, let’s calculate the motion of a planar bubble wall expanding in vacuum.
(This calculation applies more generally for a sufficiently dilute medium with small friction.)





$L_w \ll R \Leftrightarrow$ planar approx.

Starting simple: bubbles in vacuum

To begin, let's calculate the motion of a planar bubble wall expanding in vacuum.
(This calculation applies more generally for a sufficiently dilute medium with small friction.)

equation of motion for a planar bubble wall:

$$P_{\mathrm{vac}} = \sigma \dot{v}_w$$

differential vacuum pressure on the wall:

$$P_{\mathrm{vac}} = P_{\mathrm{h}} - P_{\mathrm{s}} = V_{\mathrm{s}} - V_{\mathrm{h}}$$

solution for the wall speed:

$$v_w(t) = v_0 + (P_{\mathrm{vac}}/\sigma)\,(t - t_0)$$

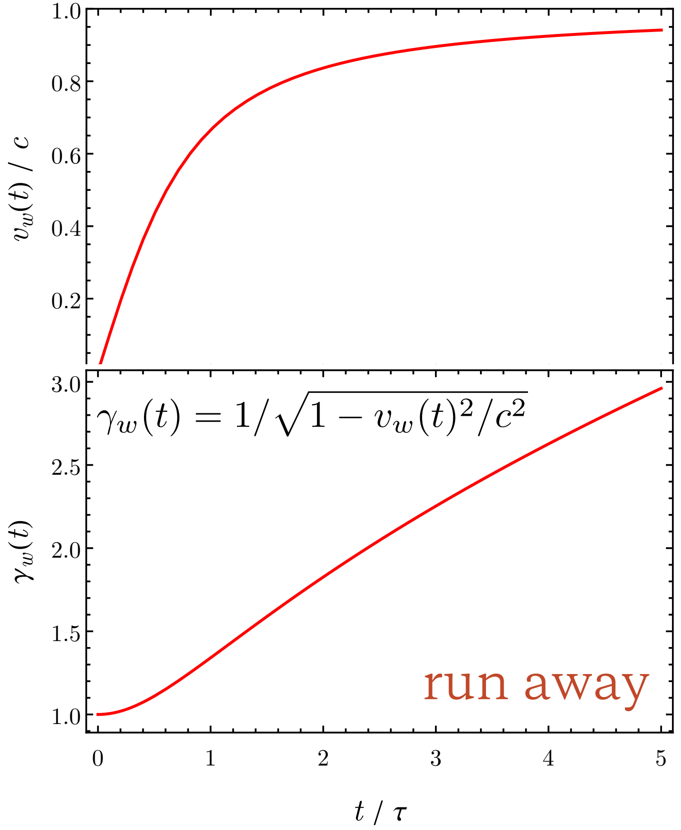
wall approaches the speed of light after a time:

$$t - t_0 \approx \tau = c\sigma / P_{\mathrm{vac}}$$

this is typically a very short time compared to Hubble:

$$\sigma \sim M^3 \quad \text{and} \quad P_{\mathrm{vac}} \sim M^4 \quad \text{and} \quad H \sim M^2/M_{\mathrm{pl}} \quad \text{so} \quad \tau H \sim M/M_{\mathrm{pl}} \ll 1$$

$$P_{\mathrm{vac}}/\sigma = \gamma_w \frac{\mathrm{d}}{\mathrm{d}t}\left(\gamma_w v_w\right)$$



now accounting for relativity

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$$P = P_{\mathrm{vac}} - \cancel{P_{\mathrm{therm}}(v_w)}$$

to begin, let's neglect

what if it's very large?

bubble wall rapidly accelerates
as $v_w \to c$ and $\gamma_w \to$ infinity
➔ "runaway" growth

More difficult: bubbles in a fluid

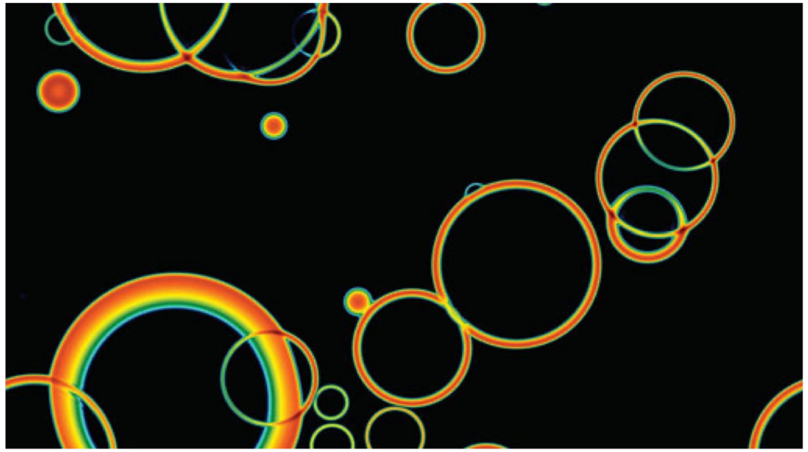
[simulation: Weir (2022)]
[WallGo: 2411.04970, 2510.27691]

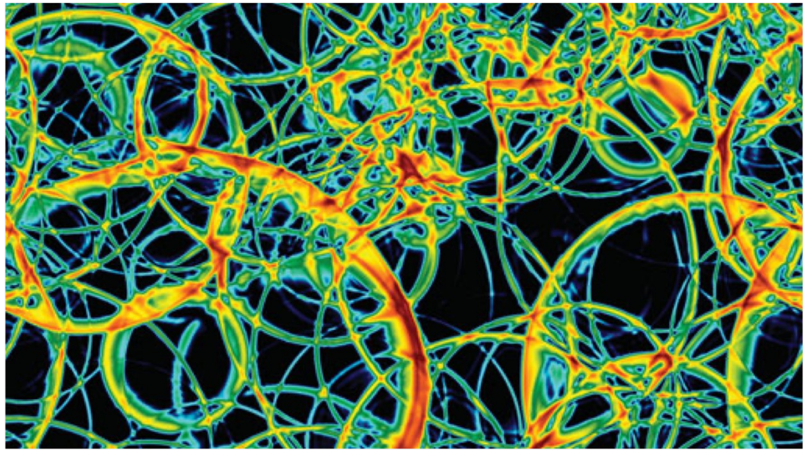
On the other extreme, bubbles can be tightly coupled to a medium.

The system is very nonlinear -- bubble moves fluid -- fluid drags bubble.

Often studied using numerical simulation (see below) or Boltzmann methods ( ).

Important to study for predicting gravitational waves and baryogenesis.





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pressure on the wall

$$P = P_{\text{vac}} - P_{\text{therm}}(v_w)$$

what if it's very large?

Is there a middle ground where the calculation is still analytically tractable?

complicated nonlinear dynamics – study with simulation / Boltzmann

Manageable middle ground: ultrarelativistic bubbles

If the bubbles are ultra-relativistic ($\gamma_w >> 1$) then they're moving much faster than the speed of sound in the medium ($c_s^2 = 1/3$). So the fluid in front of the wall doesn't "know" that the wall is coming. This makes it much easier to calculate the thermal pressure and wall speed.

$$P_{\mathrm{vac}} - P_{\mathrm{therm}} = \sigma \dot{v}_w$$

The rest of this talk is all about:
how to calculate P_{therm}

Literature

Lots of interest in ultra-relativistic bubbles, particularly at electroweak phase transition.

2009 -- Bodeker & Moore
2017 -- Bodeker & Moore
2020 -- Hoche, Kozaczuk, AL, Turner, & Wang

… lots of papers including but not limited to:

- 2020 – Azatov & Vanvlasselaer
- 2021 – Azatov, Vanvlasselaer, & Yin
- 2022 – Gouttenoire, Jinno, & Salas
- 2022 – De Curtis, Rose, Guiggiani, Muyor, & Panico
- 2022 – Laurent & Cline
- 2023 – Azatov, Barni, Petrossian-Byrne, & Vanvlasselaer
- 2024 – Ai, Laurent, & van de Vis
- …

2024 -- AL & Turner
2025 -- AL, Shakya, & Ziegler

my plan: summarize BM09, BM17, & my two papers

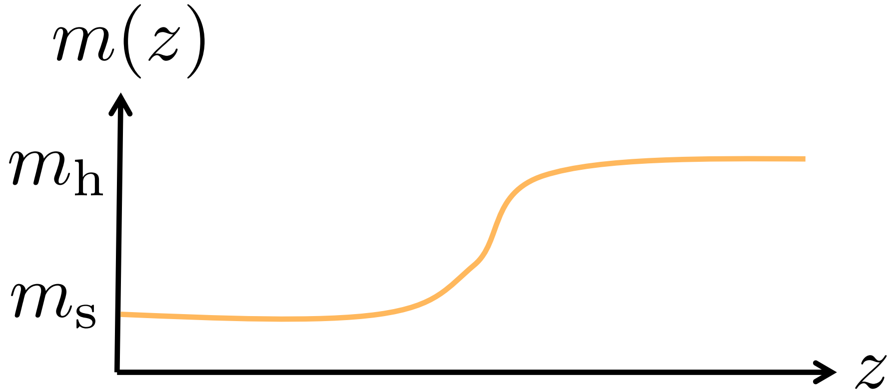
BM09

one-to-one transitions

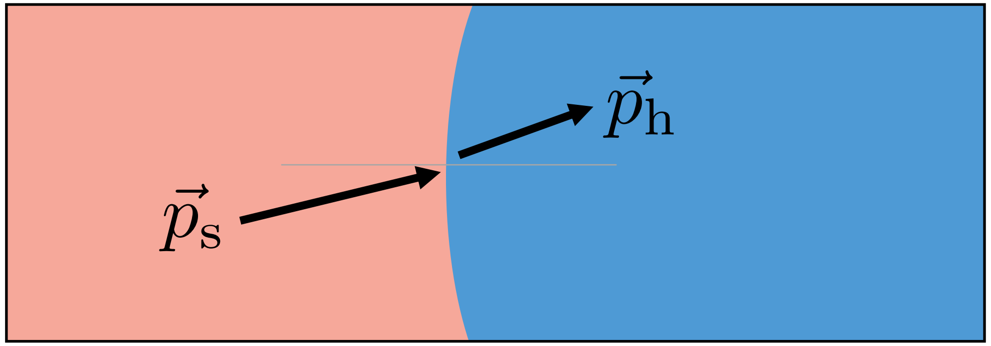
Kinematics at the wall

[Bodeker & Moore (2009)]

Suppose that there is some particle species that gains mass upon entering the bubble.



(in rest frame of the wall)



$$\left.\begin{array}{l} \Delta E = 0 \\ \Delta \vec{p}_\perp = 0 \\ \text{on-shell} \end{array}\right\} \longrightarrow \left\{ \begin{aligned} \Delta p_z &= p_{z,\mathrm{s}} - p_{z,\mathrm{h}} \\ &= \sqrt{E^2 - |\vec{p}_\perp|^2 - m_{\mathrm{s}}^2} - \sqrt{E^2 - |\vec{p}_\perp|^2 - m_{\mathrm{h}}^2} \\ &\approx \frac{m_{\mathrm{h}}^2 - m_{\mathrm{s}}^2}{2E} \\ &\sim \Delta m^2 / \gamma_w T \end{aligned} \right.$$

since $E \sim \gamma_w T >> p_{\mathrm{perp}}$ & m

Pressure on the wall

[Bodeker & Moore (2009)]

The longitudinal momentum transfer induces a force (i.e., thermal pressure) on the wall:

$$P_{\text{therm}} = \underbrace{\nu \int \frac{d^3\vec{p}}{(2\pi)^3} f(\vec{p})\, v_z}_{\sim \mathcal{F} \sim v_w \gamma_w T^3} \; \underbrace{\Delta p_z}_{\sim \Delta m^2 / \gamma_w T}$$

Observe that the dependence on γ_w cancels out!

$$\Rightarrow \quad P_{\text{therm}} \sim \Delta m^2 T^2$$

How does the thermal pressure affect the motion?

$$P_{\text{vac}} - P_{\text{therm}} = \sigma \dot{v}_w$$

Runaway is still possible despite the thermal pressure:

$$\text{if} \quad P_{\text{vac}} > P_{\text{therm}} \quad \text{then} \quad \gamma_w \to \infty \quad \text{runaway}$$

2015

Science with the space-based interferometer eLISA. II: Gravitational waves from cosmological phase transitions

Chiara Caprini[a], Mark Hindmarsh[b,c], Stephan Huber[b], Thomas Konstandin[d], Jonathan Kozaczuk[e], Germano Nardini[f], Jose Miguel No[b], Antoine Petiteau[g], Pedro Schwaller[d], Géraldine Servant[d,h], David J. Weir[i]

BM17
one-to-two transitions

Three-body kinematics

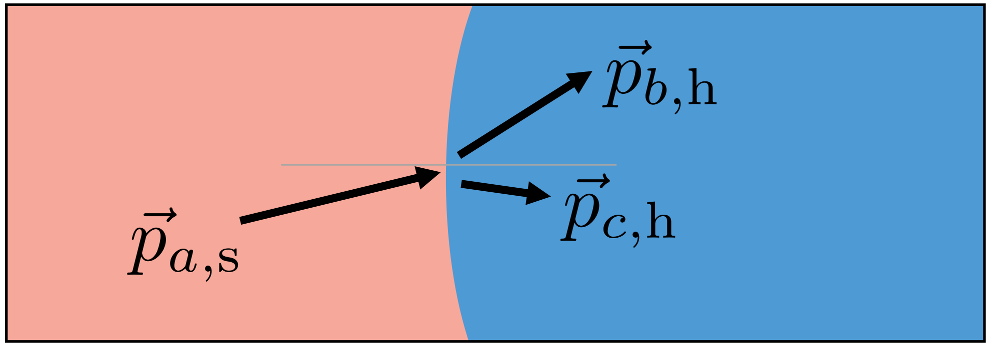
[Bodeker & Moore (2017)]

Consider a model in which a particle splits upon hitting the wall (i.e. transition radiation).

examples: $q \to q' + W$ or $e^- \to e^- + \gamma$

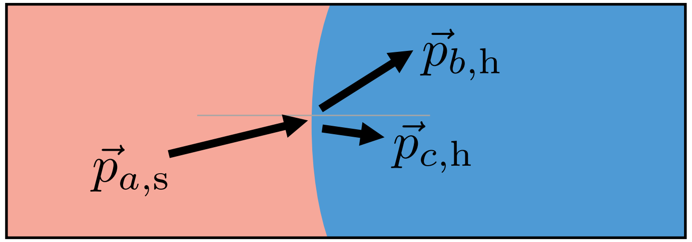
Now we add a flavor label (a,b,c) to distinguish different particle species.

(in rest frame of the wall)



Thermal pressure

[Bodeker & Moore (2017)]



Since momenta of the recoiling particles are quantum random variables, we have to calculate the *average* momentum transfer:

$$P_{\mathrm{th}} = \nu_a \int \frac{d^3\vec{p}_a}{(2\pi)^3} f_a(\vec{p}_a)\, v_{a,z} \langle \Delta p_z \rangle$$

(note angled brackets)

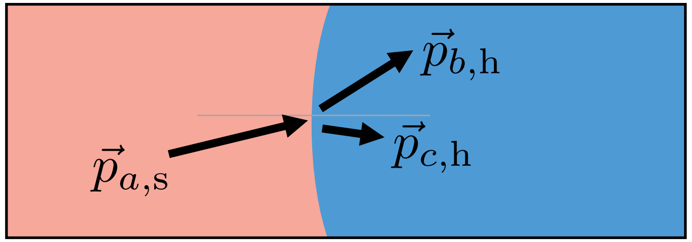
$$\langle \Delta p_z \rangle = \int d\mathbb{P}_{a\to bc}\, \Delta p_z \qquad \text{where} \qquad \Delta p_z = p_{a,z,\mathrm{s}} - p_{b,z,\mathrm{h}} - p_{c,z,\mathrm{h}}$$

How can we calculate the differential probability?

QPS = quantum particle splitting formalism → used by BM17
SCR = semiclassical current radiation formalism → used by HKLTW20 & LT24

Quantum Particle Splitting (QPS) formalism

[Bodeker & Moore (2017)]



Evaluate the probability as an S-matrix element

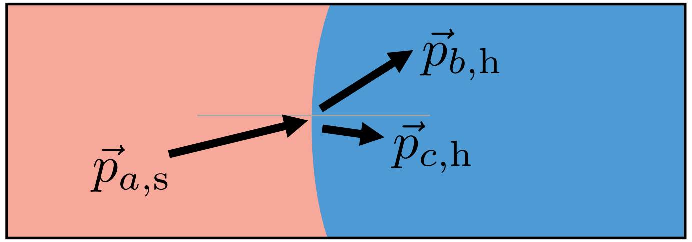
$$\mathrm{d}\mathbb{P}_{a\to bc} = \frac{1}{2E_a}\frac{\mathrm{d}^3\vec{p}_{b,\mathrm{s}}}{(2\pi)^3}\frac{1}{2E_b}\frac{\mathrm{d}^3\vec{p}_{c,\mathrm{s}}}{(2\pi)^3}\frac{1}{2E_c}(2\pi)^3\frac{p_{a,z,\mathrm{s}}}{E_a}\,\delta(\vec{p}_{a,\perp}-\vec{p}_{b,\perp}-\vec{p}_{c,\perp})\,\delta(E_a-E_b-E_c)\,|\mathcal{M}_{a\to bc}|^2$$

Use WKB approximation to calculate mode functions for particles that change mass at the wall - this affects $M_{a\text{->}bc}$

let's see
some
examples

Examples

[Bodeker & Moore (2017)]
[appendix of AL & Turner (2024)]



massless radiator / massive radiation

$$q \to q' + W$$

$$P_{\mathrm{therm}} \propto \gamma_w^1$$

$m_{\mathrm{a,s}} = m_{\mathrm{a,h}} = m_{\mathrm{b,s}} = m_{\mathrm{b,h}} = 0$

$$\langle \Delta p_z \rangle \approx \frac{g^2 C_2[R]}{4\pi^2} \left(\log \frac{m_{c,\mathrm{h}}}{m_{c,\mathrm{s}}} - \frac{m_{c,\mathrm{h}}^2 - m_{c,\mathrm{s}}^2}{2m_{c,\mathrm{h}}^2} \right) \frac{m_{c,\mathrm{h}}^2}{E_{c,\mathrm{IR}}}$$

$$\propto \gamma_w^0$$

massive radiator / massless radiation

$$e^- \to e^- + \gamma$$

$$P_{\mathrm{therm}} \propto \gamma_w^0$$

$m_{\mathrm{c,s}} = m_{\mathrm{c,h}} = 0$

$$\langle \Delta p_z \rangle \approx \frac{g^2 C_2[R]}{4\pi^2} \left(\log \frac{m_{b,\mathrm{h}}}{m_{a,\mathrm{s}}} - \frac{m_{b,\mathrm{h}}^2 - m_{a,\mathrm{s}}^2}{m_{b,\mathrm{h}}^2 + m_{a,\mathrm{s}}^2} \right) \frac{m_{b,\mathrm{h}}^2 + m_{a,\mathrm{s}}^2}{E_a} \log \frac{E_{c,\mathrm{UV}}}{E_{c,\mathrm{IR}}}$$

$$\propto \gamma_w^{-1}$$

Summary of BM17

Bodeker & Moore (2017) employs the Quantum Particle Splitting (QPS) formalism to calculate the average momentum transfer to the bubble wall, which factors into the thermal pressure.

If there exists a channel where a massless radiator emits a massive (spin-1) radiation, then they find that this channel will dominate the thermal pressure, going like:

$$q \to q' + W \quad \Rightarrow \quad P_{\text{therm}} \propto \gamma_w^1$$

Since the vacuum pressure does not grow with γ_w, they conclude that the thermal pressure will eventually win out, and the bubble wall reaches an (ultrarelativistic) terminal velocity.

For channels in which a massive radiator emits massless (spin-1) radiation, they find that the thermal pressure does not grow with increasing γ_w.

$$e^- \to e^- + \gamma \quad \Rightarrow \quad P_{\text{therm}} \propto \gamma_w^0$$

HKLTW20 & LT24
semiclassical current

Summary of HKLTW20

[HKLTW20 = 2007.10343]

- Motivated by BM17, my collaborators and I began to study EW bubble wall velocity.
- We noted the IR sensitivity of BM17's result.
- We suspected that it could be important to re-sum multiple soft emissions.
- We didn't see how to do this using BM's formalism (QPS) for calculating matrix elements.
- So we adopted a different formalism (SCR) that treats the radiator particle as a classical current and calculates the spectrum of radiation.
- Using this SCR formalism, we re-summed soft emissions to all orders.
- We concluded that the average momentum transfer and thermal pressure scale as:

$$\langle \Delta p_z \rangle \propto \gamma_w^1 \qquad \text{and} \qquad P_{\text{therm}} \propto \gamma_w^2$$

regardless of whether the radiator is massive, or the radiation is massive.

- Note that we did two things differently from BM17 - we used SCR rather than QPS - and we re-summed multiple soft emissions. It wasn't clear which change led to the different result.

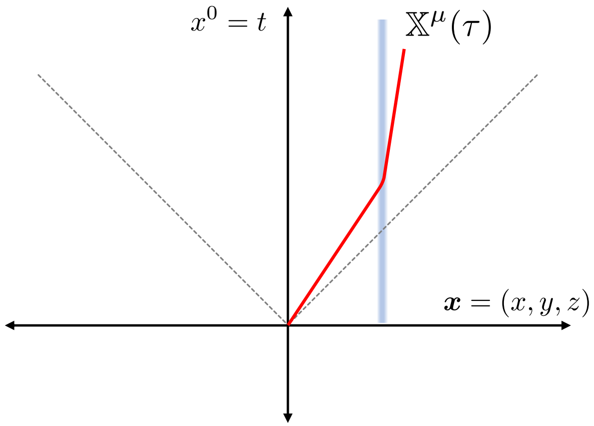
in the newer paper LT24 we clarify a subtlety about SCR formalism, which explains the different γ_w scaling

Semiclassical Current Radiation (SCR) formalism

[HKLTW20 & LT24]

Focus on channels with massive radiator / massless radiation, like: $e^- \to e^- + \gamma$

Treat the radiating particle as a classical electromagnetic current.



$$j^\mu(x) = qe \int_{-\infty}^{\infty} \mathrm{d}\tau \, \mathbb{U}^\mu(\tau) \, \delta^{(4)}\big(x - \mathbb{X}(\tau)\big)$$

Treat the electromagnetic radiation as quantum, i.e. photons.

$$\hat{A}_\mu(x) = \int \frac{\mathrm{d}^3\boldsymbol{p}}{(2\pi)^3} \frac{1}{\sqrt{2E_p}} \sum_{s=\pm 1} \left[\hat{a}_{\boldsymbol{p},s} \, \varepsilon_\mu(\boldsymbol{p}, s) \, \mathrm{e}^{-\mathrm{i}p\cdot x} + \hat{a}^\dagger_{\boldsymbol{p},s} \, \varepsilon^*_\mu(\boldsymbol{p}, s) \, \mathrm{e}^{\mathrm{i}p\cdot x} \right]$$

$$\hat{H}_{\mathrm{int}}(t) = \int \mathrm{d}^3\boldsymbol{x} \, j^\mu(t, \boldsymbol{x}) \, \hat{A}_\mu(t, \boldsymbol{x})$$

Emission probability

[HKLTW20 & LT24]

For a given current $j^\mu(x)$, calculate the probability distribution over photon momenta:

$$\mathrm{d}\mathbb{P}_{0\to 0\gamma}(\boldsymbol{p}) = \frac{\mathrm{d}^3\boldsymbol{p}}{(2\pi)^3}\,\frac{1}{2E_p}\sum_{s=\pm 1}\left|W_{0\to 0\gamma}\right|^2$$

where $\quad W_{0\to 0\gamma}(\boldsymbol{p},s) = \left\langle (\boldsymbol{p},s)_{\mathrm{OUT}} \middle| 0_{\mathrm{IN}} \right\rangle$

After a little work …

$$W_{0\to 0\gamma}(\boldsymbol{p},s) = (-\mathrm{i})\int \mathrm{d}^4x\, j(x)\cdot\varepsilon^*(\boldsymbol{p},s)\,\mathrm{e}^{\mathrm{i}p\cdot x}$$

$$= (-\mathrm{i})\,\tilde{j}(p)\cdot\varepsilon^*(\boldsymbol{p},s)$$

$$\mathrm{d}\mathbb{P}_{0\to 0\gamma} = -\frac{\mathrm{d}^3\boldsymbol{p}}{(2\pi)^3}\,\frac{1}{2E_p}\,\tilde{j}(p)^*\cdot\tilde{j}(p)$$

An Introduction to
Quantum Field Theory

Michael E. Peskin
Stanford Linear Accelerator Center

Daniel V. Schroeder
Weber State University

32 Chapter 2 The Klein-Gordon Field

Particle Creation by a Classical Source

There is one type of interaction, however, that we are already equipped to handle. Consider a Klein-Gordon field coupled to an external, classical source field $j(x)$. That is, consider the field equation

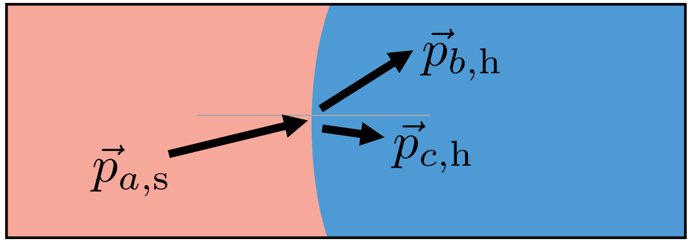
$$(\partial^2 + m^2)\phi(x) = j(x), \qquad (2.61)$$

where $|0\rangle$ still denotes the ground state of the free theory. We can interpret these results in terms of particles by identifying $|\tilde{j}(p)|^2/2E_{\mathbf{p}}$ as the probability density for creating a particle in the mode p. Then the total number of particles produced is

$$\int dN = \int \frac{d^3p}{(2\pi)^3}\frac{1}{2E_{\mathbf{p}}}|\tilde{j}(p)|^2. \qquad (2.66)$$

Comparison of QPS & SCR

In both formalisms, we calculate the average momentum transfer as



$$\langle \Delta p_z \rangle = \int \mathrm{d}\mathbb{P}_{a\to bc}\, \Delta p_z$$

where $\quad \Delta p_z = p_{a,z,\mathrm{s}} - p_{b,z,\mathrm{h}} - p_{c,z,\mathrm{h}}$

QPS Formalism: splitting probability ~~ you pick p_a to calculate probability over p_b and p_c

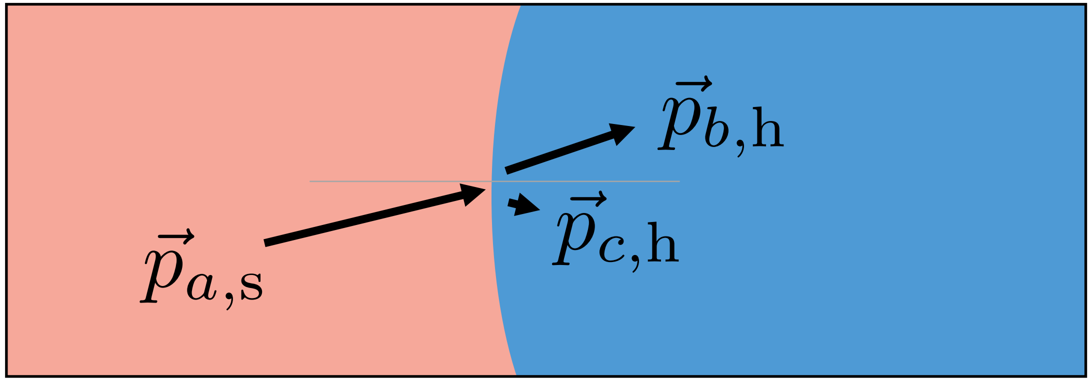
$$\mathrm{d}\mathbb{P}_{a\to bc} = \frac{1}{2E_a}\frac{\mathrm{d}^3\vec{p}_{b,\mathrm{s}}}{(2\pi)^3}\frac{1}{2E_b}\frac{\mathrm{d}^3\vec{p}_{c,\mathrm{s}}}{(2\pi)^3}\frac{1}{2E_c}(2\pi)^3\frac{p_{a,z,\mathrm{s}}}{E_a}\,\delta(\vec{p}_{a,\perp}-\vec{p}_{b,\perp}-\vec{p}_{c,\perp})\,\delta(E_a-E_b-E_c)\,|\mathcal{M}_{a\to bc}|^2$$

SCR Formalism: emission probability ~~ you pick p_a *and* p_b to calculate probability over p_c

$$\mathrm{d}\mathbb{P}_{0\to 0\gamma} = -\frac{\mathrm{d}^3\vec{p}}{(2\pi)^3}\frac{1}{2E_p}\,\tilde{j}(p)^*\cdot\tilde{j}(p)$$

How to choose p_b?

If the radiation is very soft (p_c ~ small) then the kinematics are approx. same as 1-to-1 transition:



$$|\vec{p}_{c,\mathrm{h}}| \ll |\vec{p}_{b,\mathrm{h}}|$$

$$\left.\begin{array}{c} E_b = E_a \\ \vec{p}_{b,\perp} = \vec{p}_{a,\perp} \\ \text{on-shell} \end{array}\right\} \longrightarrow \quad \langle \Delta p_z \rangle \approx \frac{g^2 C_2[R]}{4\pi^2} \left(\log \frac{m_{b,\mathrm{h}}}{m_{a,\mathrm{s}}} - \frac{m_{b,\mathrm{h}}^2 - m_{a,\mathrm{s}}^2}{m_{b,\mathrm{h}}^2 + m_{a,\mathrm{s}}^2} \right) \frac{m_{b,\mathrm{h}}^2 + m_{a,\mathrm{s}}^2}{E_a} \log \frac{E_{c,\mathrm{UV}}}{E_{c,\mathrm{IR}}}$$

SCR yields identical result as QPS formalism
(for massive radiator / massless radiation)

$$P_{\mathrm{therm}} \propto \gamma_w^0$$

How to choose p_b?

Alternatively, impose energy & transverse-momentum conservation among all 3 particles:

$$\left.\begin{array}{c} E_b = E_a - E_c \\ \vec{p}_{b,\perp} = \vec{p}_{a,\perp} - \vec{p}_{c,\perp} \\ \text{on-shell} \end{array}\right\} \longrightarrow \langle \Delta p_z \rangle = \frac{g^2}{2\pi^2} E_{c,\mathrm{UV}}$$

Now SCR yields a funny result

- does not vanish for $m_b = m_a$ (i.e., limit of no mass change)
- strong UV sensitivity

if you take $E_{c,\mathrm{UV}} \sim \gamma_w T$ then $\langle \Delta p_z \rangle \propto \gamma_w^1$ and $P_{\mathrm{therm}} \propto \gamma_w^2$

(same as HKLTW20)

Summary
& conclusion

Summary & conclusion

formalism	channel	how to choose $\boldsymbol{p}_b$	$\langle \Delta p_z \rangle$	UV cutoff $p_{\rm UV}$	$P_{\rm therm}$
C	$a \to b$	$\circ$	$\frac{m_b^2 - m_a^2}{2E_a}$	$\circ$	$\propto \gamma_w^0$
QPS	$a \to bc$	$\circ$	$\frac{q^2 e^2}{4\pi^2}\left(\log \frac{m_b}{m_a} - \frac{m_b^2 - m_a^2}{m_b^2 + m_a^2}\right)\frac{m_b^2 + m_a^2}{E_a} \log \frac{p_{\rm UV}}{p_{\rm IR}}$	$\circ$	$\propto \gamma_w^0$
SCR	$a \to bc$	1-to-1 kinematics	$\frac{q^2 e^2}{4\pi^2}\left(\log \frac{m_b}{m_a} - \frac{m_b^2 - m_a^2}{m_b^2 + m_a^2}\right)\frac{m_b^2 + m_a^2}{E_a} \log \frac{p_{\rm UV}}{p_{\rm IR}}$	$\circ$	$\propto \gamma_w^0$
	$a \to bc$	1-to-2 kinematics	$\frac{q^2 e^2}{2\pi^2}\, p_{\rm UV}$	E_a	$\propto \gamma_w^2$

For models with massive radiator / massless radiation, both QPS and SCR formalisms yield:

$$e^- \to e^- + \gamma \quad \Rightarrow \quad P_{\rm therm} \propto \gamma_w^0 \quad \Rightarrow \quad \text{runaway possible}$$

Take care to choose p_b when using the SCR formalism. Some choices gives $P_{\rm therm} \propto \gamma_w^2$

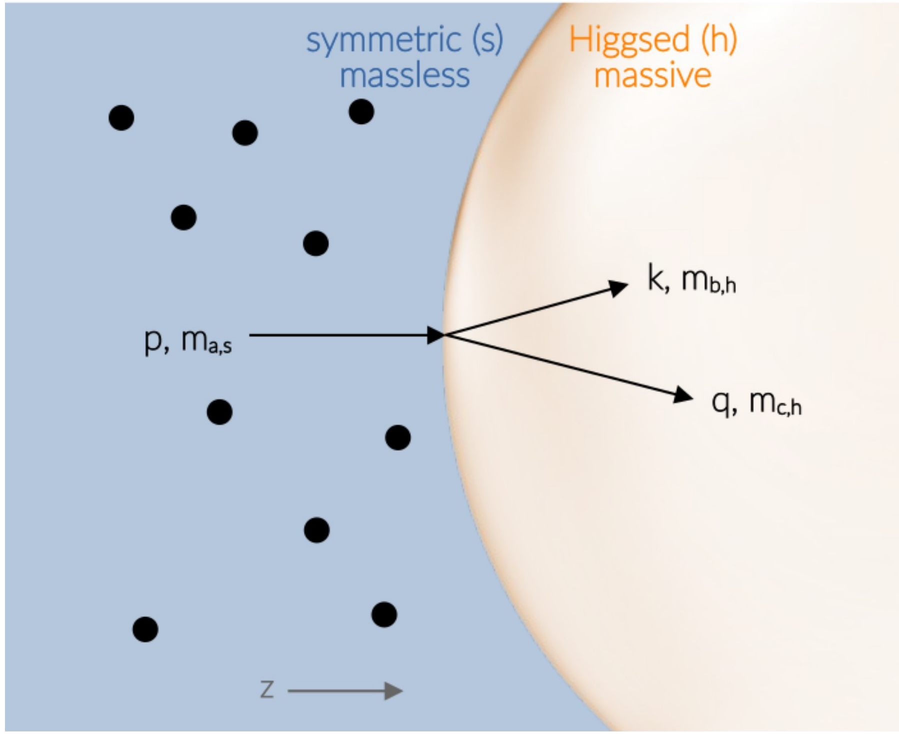
Possibly interesting directions for future studies:
- Revisit massless radiator / massive radiation using the SCR formalism.
- Consider models with no mass change, but coupling change instead.
- 1st order phase transitions with inverse symmetry breaking

Inverse symmetry breaking

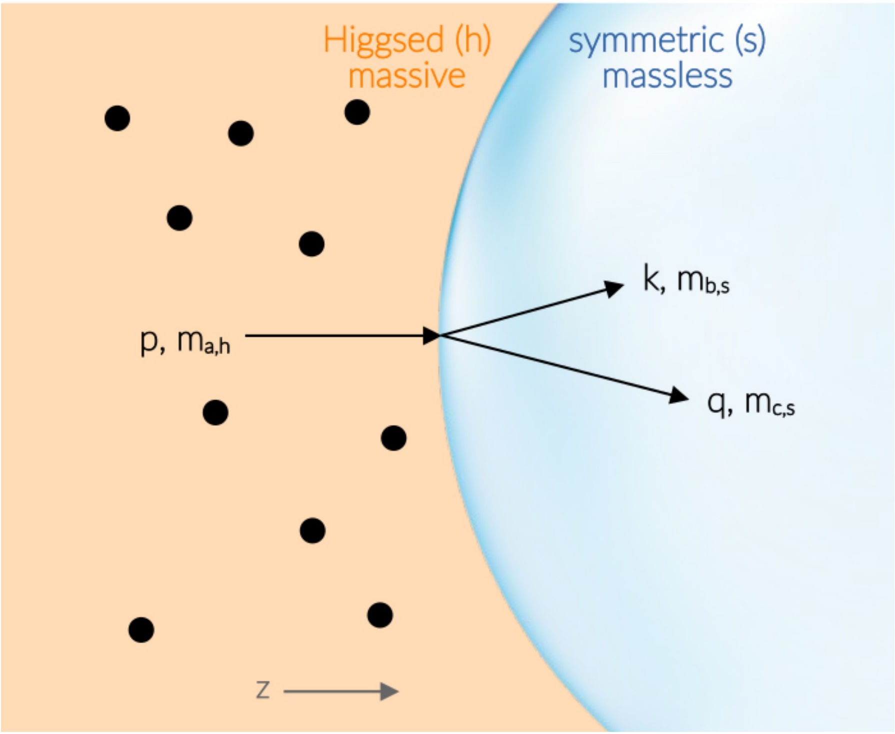
[AL, Shakya, & Ziegler 2511.10415]
(see also: Azatov, Barni, & Petrossian-Byrne 2024)

How does the thermal pressure calculation change if the mass *decreases* across the wall?

symmetry-breaking scenario



symmetry-restoring scenario



Inverse symmetry breaking

[AL, Shakya, & Ziegler 2511.10415]

For 1-to-1 transitions:

$$\Delta p_{1\to1} \equiv p_{a,z,\mathrm{out}} - p_{a,z,\mathrm{in}} \approx \frac{m_{a,\mathrm{in}}^2 - m_{a,\mathrm{out}}^2}{2E_a} = \begin{cases} +\frac{1}{2E_a} m_{a,\mathrm{h}}^2 & , \quad \text{sym-breaking scenario} \\ -\frac{1}{2E_a} m_{a,\mathrm{h}}^2 & , \quad \text{sym-restoring scenario} \end{cases}$$

	out		in
sym-breaking	$m_{a,\mathrm{s}} = 0$	$\Rightarrow$	$m_{a,\mathrm{h}} \neq 0$
sym-restoring	$m_{a,\mathrm{h}} \neq 0$	$\Rightarrow$	$m_{a,\mathrm{s}} = 0$

$$\mathcal{P}_{1\to1} = \int \frac{\mathrm{d}^3 p_{a,\mathrm{s}}}{(2\pi)^3} \frac{p_{a,z,\mathrm{s}}}{E_a} f_a(\mathbf{p}_{a,\mathrm{s}})\, \Delta p_{1\to1} \approx \begin{cases} +\frac{1}{24} m_{a,\mathrm{h}}^2 T^2 & , \quad \text{sym-breaking scenario} \\ -\frac{1}{24} m_{a,\mathrm{h}}^2 T^2 & , \quad \text{sym-restoring scenario} \end{cases}$$

sym-breaking	$m_{a,\mathrm{s}} = 0$	$\Rightarrow$	$m_{a,\mathrm{h}} \neq 0$
sym-restoring	$m_{a,\mathrm{h}} \neq 0$	$\Rightarrow$	$m_{a,\mathrm{s}} = 0$

➔ thermal pressure is negative for sym-restoring PTs

For 1-to-2 transitions:

$$\Delta p_{1\to2} \equiv p_{z,\mathrm{out}} - k_{z,\mathrm{in}} - q_{z,\mathrm{in}} \approx \frac{1}{2E_a}\left(-m_{a,\mathrm{out}}^2 + \frac{m_{b,\mathrm{in}}^2}{x} + \frac{m_{c,\mathrm{in}}^2}{1-x} + \frac{k_\perp^2}{x(1-x)}\right)$$

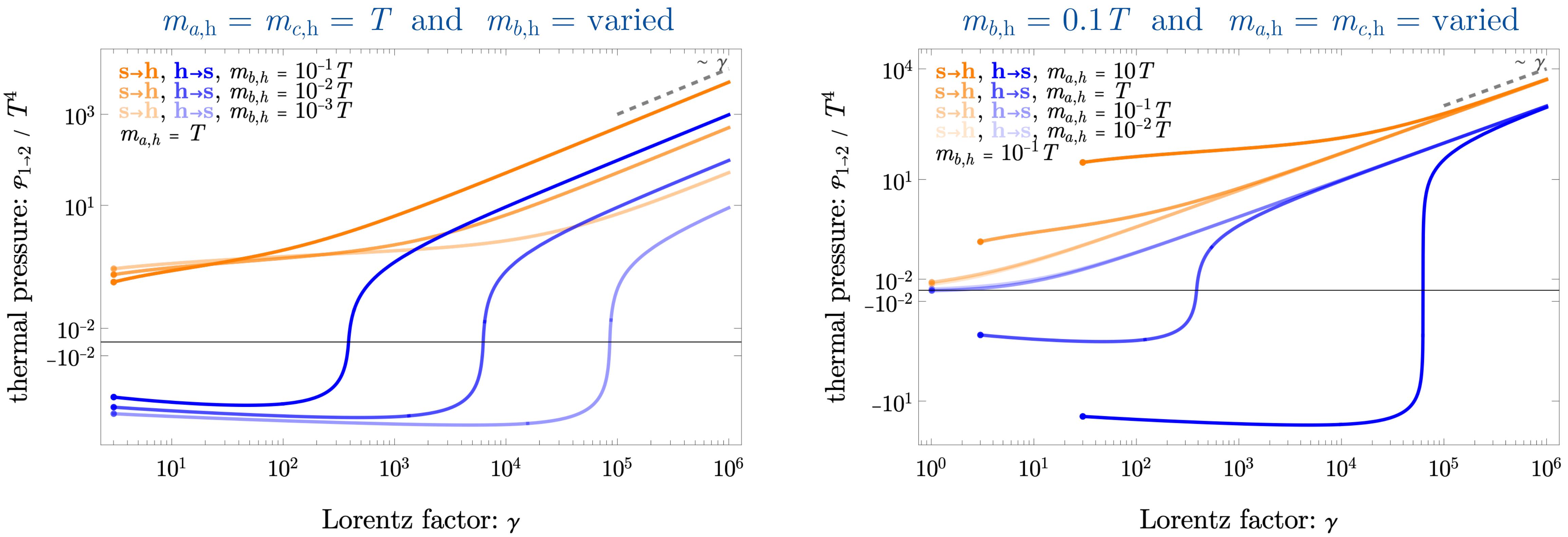
$$\Delta p_{1\to2} \approx \frac{1}{2E_a} \begin{cases} \frac{m_{b,\mathrm{h}}^2}{x} + \frac{m_{c,\mathrm{h}}^2}{1-x} + \frac{k_\perp^2}{x(1-x)} & , \quad \text{sym-breaking scenario} \\ -m_{a,\mathrm{h}}^2 + \frac{k_\perp^2}{x(1-x)} & , \quad \text{sym-restoring scenario} \end{cases}$$

	out		in
sym-breaking	$m_{a,\mathrm{s}} = 0$	$\Rightarrow$	$m_{a,\mathrm{h}} \neq 0$
sym-restoring	$m_{a,\mathrm{h}} \neq 0$	$\Rightarrow$	$m_{a,\mathrm{s}} = 0$

➔ thermal pressure may be positive or negative for sym-restoring PTs

Inverse symmetry breaking

[AL, Shakya, & Ziegler 2511.10415]



- For sym-restoring PTs … the 1-to-2 thermal pressure is positive for a values of γ
- For sym-breaking PTs … the 1-to-2 thermal pressure flips neg to pos as γ is increased

$$\Delta p_{1\to2} \approx (-m_{a,\rm h}^2 + k_\perp^2/x)/2E_a \sim \boxed{(-m_{a,\rm h}^2 + m_{b,\rm h}^2 \gamma T)}/2E_a$$

backup slides

SCR formalism

[AL & Turner (2024)]

How to choose p_b? Suppose that rather than taking 1-to-1 kinematics or 1-to-2 kinematics, you just treat p_b and p_a as separate free parameters. But suppose that they're colinear for simplicity.

differential emission probability

$$\frac{\mathrm{d}\mathbb{P}_{0\to0\gamma}}{\mathrm{d}\Omega} = \frac{q^2 e^2}{16\pi^3}\mathbb{P}_{0\to0}\frac{\mathrm{d}p}{p}\frac{(\sin^2\theta)\,(v_a - v_b)^2}{(1 - v_a\,\cos\theta)^2(1 - v_b\,\cos\theta)^2}$$

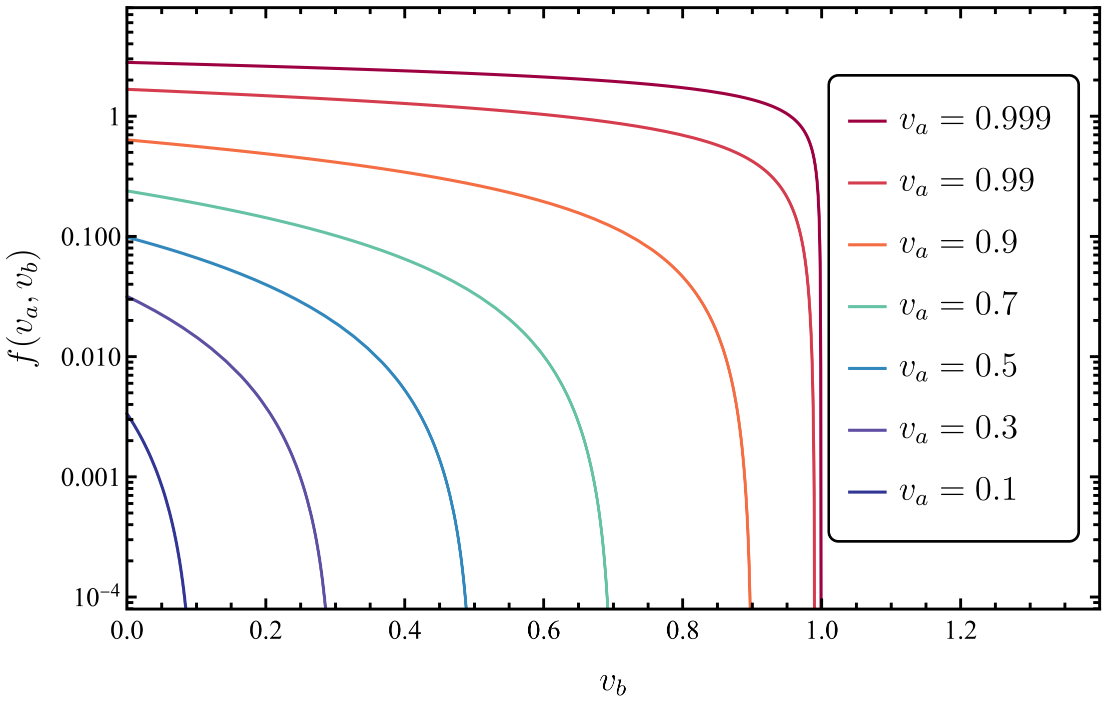
after the angular integral

$$p\frac{\mathrm{d}\mathbb{P}_{0\to0\gamma}}{\mathrm{d}p} = \left(\frac{q^2 e^2}{2\pi^2}\mathbb{P}_{0\to0}\right) f(v_a, v_b)$$

average momentum transfer $(v_a\!=\!1)$

$$\langle \Delta p_z \rangle \approx \left(\frac{q^2 e^2}{4\pi^2}\mathbb{P}_{0\to0}\right)\left(\frac{1 - v_b}{v_b^2}\right)\left(1 - \frac{1 - v_b^2}{2v_b}\log\frac{1 + v_b}{1 - v_b}\right) p_{\mathrm{UV}}$$

$$f(v_a, v_b) = \frac{1}{2}\left(\frac{1 - v_a v_b}{v_a - v_b}\right)\log\frac{(1 + v_a)(1 - v_b)}{(1 - v_a)(1 + v_b)} - 1$$

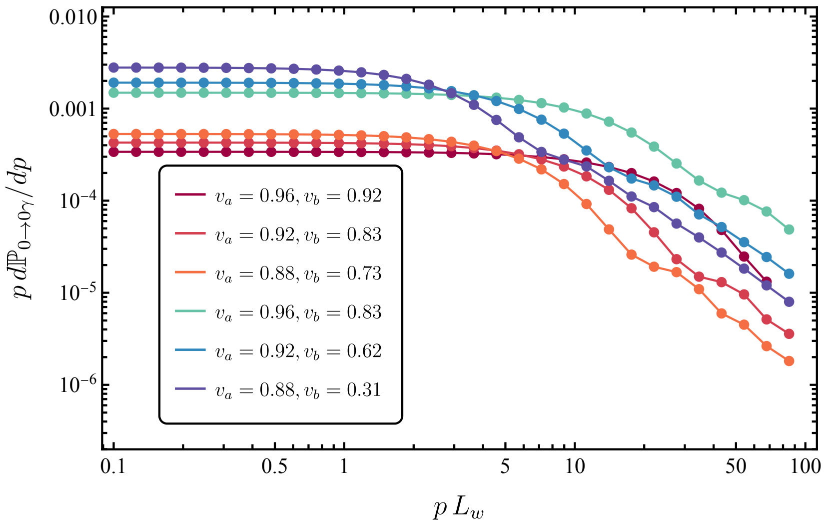


Nonzero wall thickness as a UV cutoff

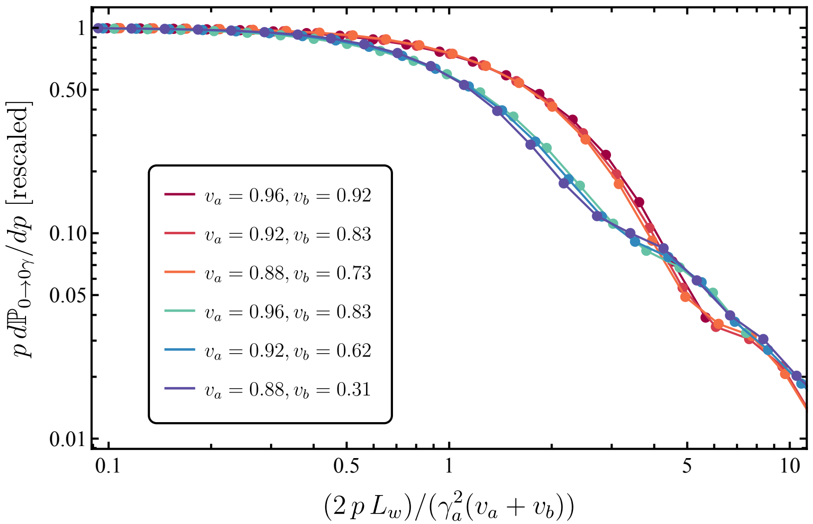
Until this point we have been neglecting the thickness of the bubble wall: L_w

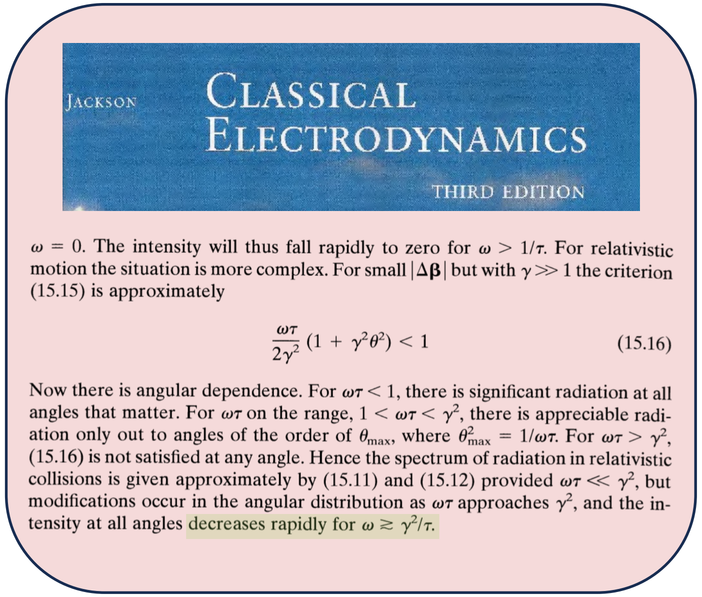
We implement nonzero L_w in SCR by modeling the momentum decrease as a gradual decline.

photon spectrum suppressed above $p \sim 1/L_w$



rescaling reveals dependence on $\gamma_a = \gamma_w$





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$\omega = 0$. The intensity will thus fall rapidly to zero for $\omega > 1/\tau$. For relativistic motion the situation is more complex. For small $|\Delta\boldsymbol{\beta}|$ but with $\gamma \gg 1$ the criterion (15.15) is approximately

$$\frac{\omega\tau}{2\gamma^2}(1 + \gamma^2\theta^2) < 1 \qquad (15.16)$$

Now there is angular dependence. For $\omega\tau < 1$, there is significant radiation at all angles that matter. For $\omega\tau$ on the range, $1 < \omega\tau < \gamma^2$, there is appreciable radiation only out to angles of the order of $\theta_{\max}$, where $\theta_{\max}^2 = 1/\omega\tau$. For $\omega\tau > \gamma^2$, (15.16) is not satisfied at any angle. Hence the spectrum of radiation in relativistic collisions is given approximately by (15.11) and (15.12) provided $\omega\tau \ll \gamma^2$, but modifications occur in the angular distribution as $\omega\tau$ approaches γ^2, and the intensity at all angles decreases rapidly for $\omega \gtrsim \gamma^2/\tau$.

We find that the photon spectrum is cutoff at: $E_{c,\mathrm{UV}} \sim v_w \gamma_w^2 L_w^{-1}$

(same as classical bremsstrahlung)

Typically this is much smaller than the incident particle energy: $E_a \sim \gamma_w T$

The distinction is relevant for the SCR calculation with 1-to-2 kinematics (b/c of UV sensitivity).

massless radiator / massive radiation

[Bodeker & Moore (2017)]
[appendix of AL & Turner (2024)]

$$d\mathbb{P}_{a\to bc} = \frac{1}{2E_a}\frac{d^3\vec{p}_{b,\mathrm{s}}}{(2\pi)^3}\frac{1}{2E_b}\frac{d^3\vec{p}_{c,\mathrm{s}}}{(2\pi)^3}\frac{1}{2E_c}(2\pi)^3\frac{p_{a,z,\mathrm{s}}}{E_a}\,\delta(\vec{p}_{a,\perp}-\vec{p}_{b,\perp}-\vec{p}_{c,\perp})\,\delta(E_a-E_b-E_c)\,|\mathcal{M}_{a\to bc}|^2$$

$$d\mathbb{P}_{a\to bc} = \frac{d^3\vec{p}_{c,\mathrm{s}}}{(2\pi)^3}\frac{1}{8E_aE_bE_c}\,|\mathcal{M}_{a\to bc}|^2\Big|_{\vec{p}_{b,\perp}=\vec{p}_{a,\perp}-\vec{p}_{c,\perp},E_b=E_a-E_c}$$

$$d\mathbb{P}_{a\to bc} = dk_\perp\,dx\,\frac{k_\perp}{32\pi^2(1-x)E_a\sqrt{x^2E_a^2-k_\perp^2-m_{c,\mathrm{s}}^2}}\,|\mathcal{M}_{a\to bc}|^2\Big|_{\vec{p}_{b,\perp}=\vec{p}_{a,\perp}-\vec{p}_{c,\perp},E_b=E_a-E_c}$$

where: $x = E_c/E_a$ and $\vec{p}_{a,\perp}=0$ and $k_\perp = |\vec{p}_{c,\perp}|$

$$|\mathcal{M}^{(0)}_{a\to bc}|^2 \approx 16g^2C_2[R]\frac{k_\perp^2\left(m_{c,\mathrm{h}}^2-m_{c,\mathrm{s}}^2\right)^2(1-x)^4}{\left(k_\perp^2+(1-x)^2m_{c,\mathrm{h}}^2\right)^2\left(k_\perp^2+(1-x)^2m_{c,\mathrm{s}}^2\right)^2}E_a^2+O(E_a^0)$$

$$x\frac{d\mathbb{P}_{a\to bc}}{dx} \approx \frac{g^2C_2[R]}{2\pi^2}\left(\frac{m_{c,\mathrm{h}}^2+m_{c,\mathrm{s}}^2}{m_{c,\mathrm{h}}^2-m_{c,\mathrm{s}}^2}\log\frac{m_{c,\mathrm{h}}}{m_{c,\mathrm{s}}}-1\right)(1-x)+O(E_a^{-2})$$

$$\Delta p_z \approx \left(\frac{k_\perp^2+m_{c,\mathrm{h}}^2}{2x}+\frac{k_\perp^2+m_{b,\mathrm{h}}^2}{2(1-x)}-\frac{m_{a,\mathrm{s}}^2}{2}\right)\frac{1}{E_a} = \left(\frac{k_\perp^2}{2x(1-x)}+\frac{m_{c,\mathrm{h}}^2}{2x}\right)\frac{1}{E_a}$$

$$\langle\Delta p_z\rangle \approx \frac{g^2C_2[R]}{4\pi^2}\left(\log\frac{m_{c,\mathrm{h}}}{m_{c,\mathrm{s}}}-\frac{m_{c,\mathrm{h}}^2-m_{c,\mathrm{s}}^2}{2m_{c,\mathrm{h}}^2}\right)\frac{m_{c,\mathrm{h}}^2}{E_{c,\mathrm{IR}}}+O(E_a^{-3})$$

$m_{a,\mathrm{s}}=0$
$m_{a,\mathrm{h}}=0$
$m_{b,\mathrm{s}}=0$
$m_{b,\mathrm{h}}=0$
$m_{c,\mathrm{s}}\neq 0$
$m_{c,\mathrm{h}}\neq 0$

massive radiator / massless radiation

[Bodeker & Moore (2017)]
[appendix of AL & Turner (2024)]

$$\mathrm{d}\mathbb{P}_{a\to bc} = \frac{1}{2E_a}\frac{\mathrm{d}^3\vec{p}_{b,\mathrm{s}}}{(2\pi)^3}\frac{1}{2E_b}\frac{\mathrm{d}^3\vec{p}_{c,\mathrm{s}}}{(2\pi)^3}\frac{1}{2E_c}\,(2\pi)^3\,\frac{p_{a,z,\mathrm{s}}}{E_a}\,\delta(\vec{p}_{a,\perp}-\vec{p}_{b,\perp}-\vec{p}_{c,\perp})\,\delta(E_a-E_b-E_c)\,|\mathcal{M}_{a\to bc}|^2$$

$$\mathrm{d}\mathbb{P}_{a\to bc} = \frac{\mathrm{d}^3\vec{p}_{c,\mathrm{s}}}{(2\pi)^3}\,\frac{1}{8E_aE_bE_c}\,|\mathcal{M}_{a\to bc}|^2\Big|_{\vec{p}_{b,\perp}=\vec{p}_{a,\perp}-\vec{p}_{c,\perp},\,E_b=E_a-E_c}$$

$$\mathrm{d}\mathbb{P}_{a\to bc} = \mathrm{d}k_\perp\,\mathrm{d}x\,\frac{k_\perp}{32\pi^2(1-x)E_a\sqrt{x^2E_a^2-k_\perp^2-m_{c,\mathrm{s}}^2}}\,|\mathcal{M}_{a\to bc}|^2\Big|_{\vec{p}_{b,\perp}=\vec{p}_{a,\perp}-\vec{p}_{c,\perp},\,E_b=E_a-E_c}$$

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$$|\mathcal{M}^{(0)}_{a\to bc}|^2 \approx 16g^2C_2[R]\frac{k_\perp^2\left(m_{b,\mathrm{h}}^2-m_{a,\mathrm{s}}^2\right)^2(1-x)^2x^4}{\left(k_\perp^2+x^2\,m_{b,\mathrm{h}}^2\right)^2\left(k_\perp^2+x^2\,m_{a,\mathrm{s}}^2\right)^2}\,E_a^2+O(E_a^0)$$

$$x\frac{\mathrm{d}\mathbb{P}_{a\to bc}}{\mathrm{d}x} \approx \frac{g^2C_2[R]}{2\pi^2}\left(\frac{m_{b,\mathrm{h}}^2+m_{a,\mathrm{s}}^2}{m_{b,\mathrm{h}}^2-m_{a,\mathrm{s}}^2}\log\frac{m_{b,\mathrm{h}}}{m_{a,\mathrm{s}}}-1\right)(1-x)+O(E_a^{-2})$$

$$\Delta p_z \approx \left(\frac{k_\perp^2+m_{c,\mathrm{h}}^2}{2x}+\frac{k_\perp^2+m_{b,\mathrm{h}}^2}{2(1-x)}-\frac{m_{a,\mathrm{s}}^2}{2}\right)\frac{1}{E_a} = \left(\frac{k_\perp^2}{2x(1-x)}+\frac{m_{b,\mathrm{h}}^2}{2(1-x)}-\frac{m_{a,\mathrm{s}}^2}{2}\right)\frac{1}{E_a}$$

$$\langle\Delta p_z\rangle \approx \frac{g^2C_2[R]}{4\pi^2}\left(\log\frac{m_{b,\mathrm{h}}}{m_{a,\mathrm{s}}}-\frac{m_{b,\mathrm{h}}^2-m_{a,\mathrm{s}}^2}{m_{b,\mathrm{h}}^2+m_{a,\mathrm{s}}^2}\right)\frac{m_{b,\mathrm{h}}^2+m_{a,\mathrm{s}}^2}{E_a}\log\frac{E_{c,\mathrm{UV}}}{E_{c,\mathrm{IR}}}+O(E_a^{-3})$$

$m_{a,\mathrm{s}} \neq 0$
$m_{a,\mathrm{h}} = m_{b,\mathrm{h}}$
$m_{b,\mathrm{s}} = m_{a,\mathrm{s}}$
$m_{b,\mathrm{h}} \neq 0$
$m_{c,\mathrm{s}} = 0$
$m_{c,\mathrm{h}} = 0$