Tunneling in QFT

Production of particles from expanding bubble walls

Miguel Vanvlasselaer
miguel.vanvlasselaer@ub.edu

University of Barcelona, ICCUB and COST Action

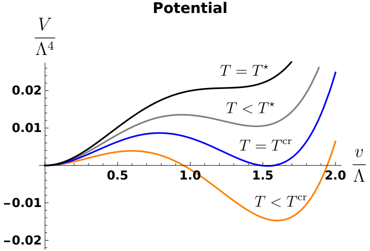
UNIVERSITAT DE BARCELONA

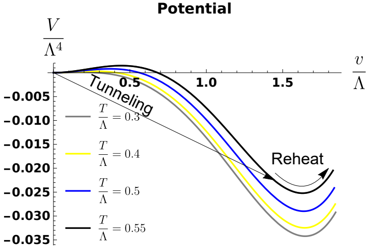
cost
EUROPEAN COOPERATION IN SCIENCE & TECHNOLOGY

ICCUB
Institut de Ciències del Cosmos
UNIVERSITAT DE BARCELONA

First order phase transition (FOPT) in the early universe

Universe high-T after inflation:cooling of primordial soup



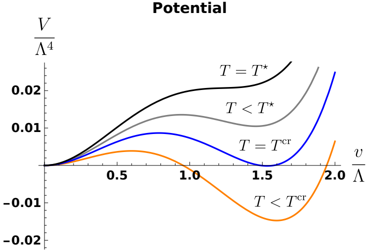


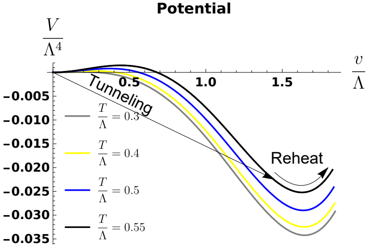
- QFT $=$ landscape of minima $\Rightarrow$ PT

First order phase transition (FOPT) in the early universe

Universe high-T after inflation:cooling of primordial soup

- QFT $=$ landscape of minima $\Rightarrow$ PT
- Potential $V(T,\phi) = V_0(\phi) + V_{\text{thermal}}(T,\phi)$

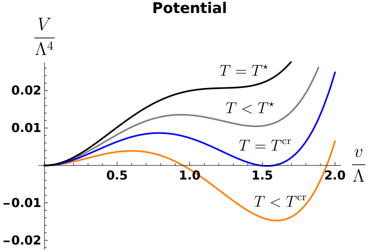


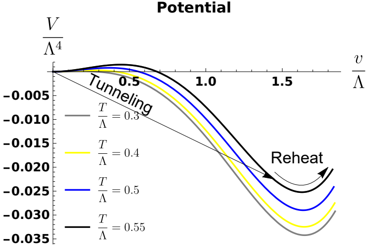


First order phase transition (FOPT) in the early universe

Universe high-T after inflation:cooling of primordial soup

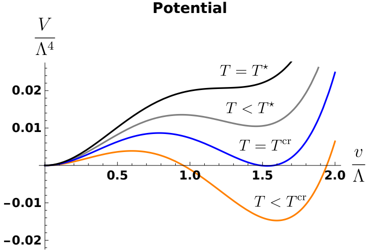
- QFT $=$ landscape of minima $\Rightarrow$ PT
- Potential $V(T,\phi) = V_0(\phi) + V_{\text{thermal}}(T,\phi)$
- High T: $V \supset \phi^2 T^2 \quad \Rightarrow$ sym restoration

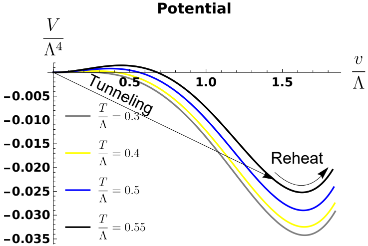




First order phase transition (FOPT) in the early universe

Universe high-T after inflation:cooling of primordial soup

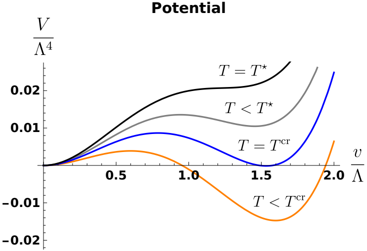




- QFT $=$ landscape of minima $\Rightarrow$ PT
- Potential $V(T, \phi) = V_0(\phi) + V_{\rm thermal}(T, \phi)$
- High T: $V \supset \phi^2 T^2 \quad \Rightarrow$ sym restoration
- FOPT feature a barrier between two vacua and cooling $(T_{\rm nuc} < T_{\rm cr})$

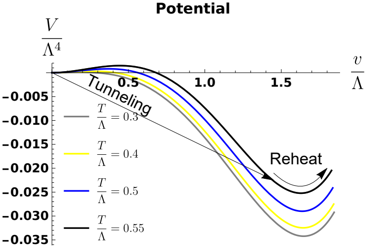
First order phase transition (FOPT) in the early universe

Universe high-T after inflation:cooling of primordial soup



- QFT $=$ landscape of minima $\Rightarrow$ PT
- Potential $V(T,\phi) = V_0(\phi) + V_{\mathrm{thermal}}(T,\phi)$
- High T: $V \supset \phi^2 T^2 \quad \Rightarrow$ sym restoration
- FOPT feature a barrier between two vacua and cooling ($T_{\mathrm{nuc}} < T_{\mathrm{cr}}$)
- Nucleation controlled by bounce solution

$$\Gamma \sim T^4 Exp\left[-\frac{S_3}{T}\right]$$

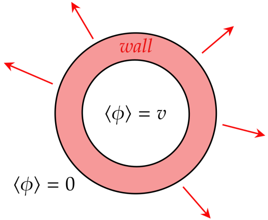


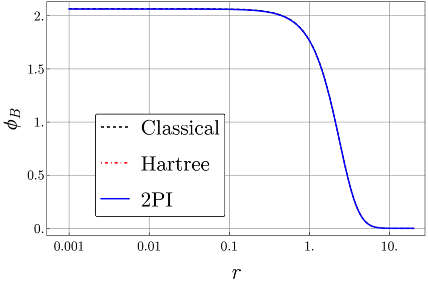
Nucleation and early expansion

- Energy released $\Delta V \Rightarrow$ Driving energy:

$$E_{\text{driving}} = -\frac{4}{3}\pi \Delta V R^3 \ \textcolor{red}{\text{VS}} \ \Delta \mathcal{P}_{\text{tension}} = 4\pi\sigma R^2$$

Expansion when $R_{\text{initial}} > R_c \sim \sigma/\Delta V$





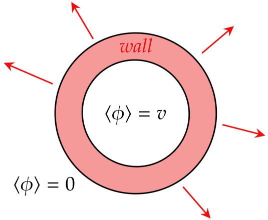
Nucleation and early expansion

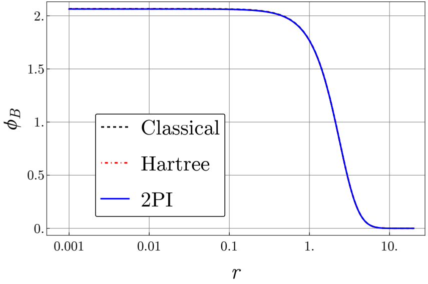
- Energy released ΔV $\Rightarrow$ Driving energy:

$$E_{\text{driving}} = -\frac{4}{3}\pi \Delta V R^3 \text{ VS } \Delta\mathcal{P}_{\text{tension}} = 4\pi\sigma R^2$$

  Expansion when $R_{\text{initial}} > R_c \sim \sigma/\Delta V$

- Transition starts approximately when one bubble per Hubble volume, $T = T_{\text{nuc}}$





Nucleation and early expansion

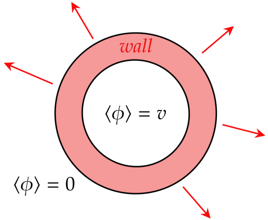
- Energy released $\Delta V \Rightarrow$ Driving energy:

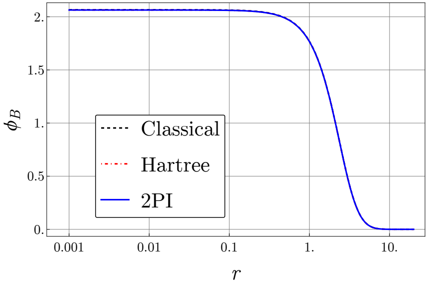
$$E_{\text{driving}} = -\frac{4}{3}\pi \Delta V R^3 \text{ VS } \Delta \mathcal{P}_{\text{tension}} = 4\pi\sigma R^2$$

  Expansion when $R_{\text{initial}} > R_c \sim \sigma/\Delta V$

- Transition starts approximately when one bubble per Hubble volume, $T = T_{\text{nuc}}$
- Strength of the FOPT

$$\alpha_n \equiv \frac{\Delta V}{\rho_r(T_{\text{nuc}})} \propto 1/T^4_{\text{nuc}}$$





Nucleation and early expansion

- Energy released ΔV $\Rightarrow$ Driving energy:

$$E_{\text{driving}} = -\frac{4}{3}\pi\Delta V R^3 \ \text{VS}\ \Delta\mathcal{P}_{\text{tension}} = 4\pi\sigma R^2$$

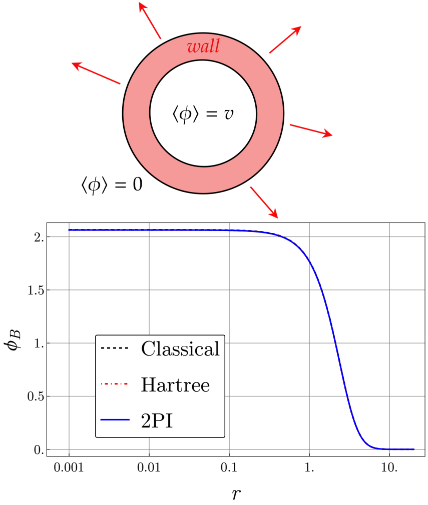
Expansion when $R_{\text{initial}} > R_c \sim \sigma/\Delta V$

- Transition starts approximately when one bubble per Hubble volume, $T = T_{\text{nuc}}$
- Strength of the FOPT

$$\alpha_n \equiv \frac{\Delta V}{\rho_r(T_{\text{nuc}})} \propto 1/T^4_{\text{nuc}}$$

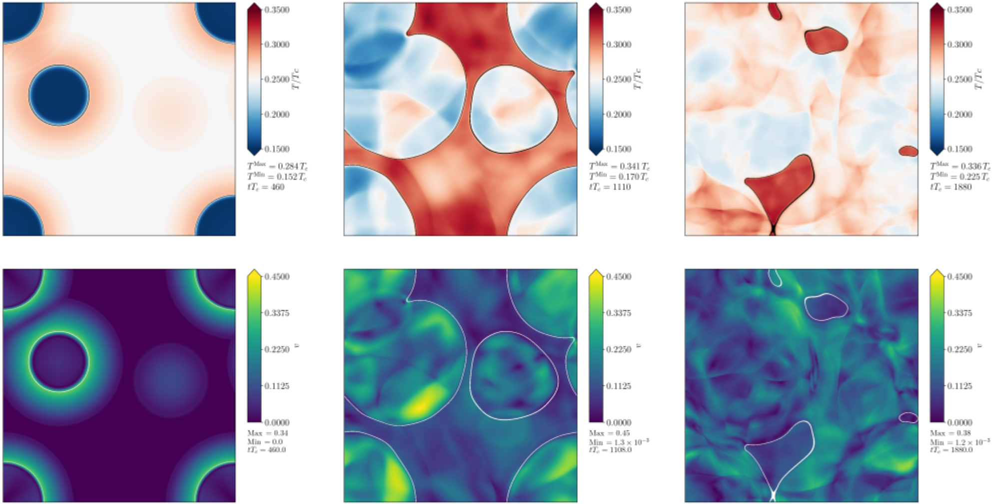
- Duration of the FOPT

$$\beta \equiv \frac{t_{\text{exp}}}{t_{PT}} = \frac{1}{t_{PT}H} \propto R^{-1}_{\text{collision}}$$

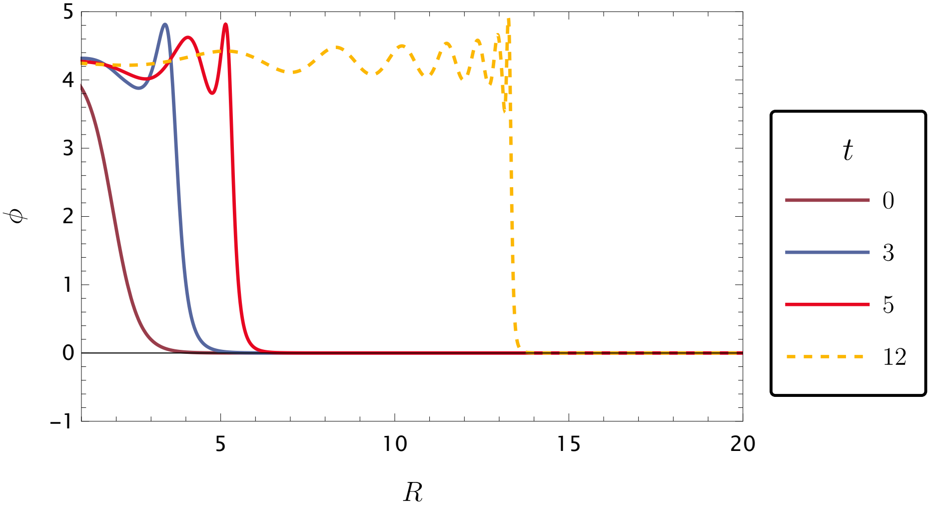


FOPT pictorially

Cutting, Hindmarsh and Weir:[1906.00480]: video in https://vimeo.com/showcase/5968055



The bubble wall in time



Pheno consequences of FOPTs

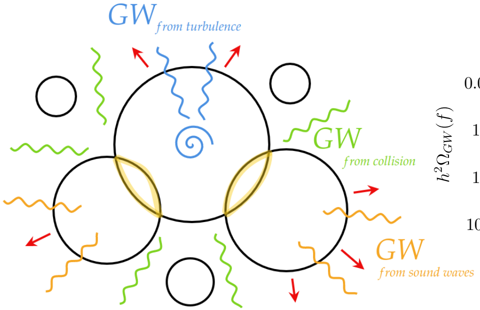
Pheno consequences of FOPTs

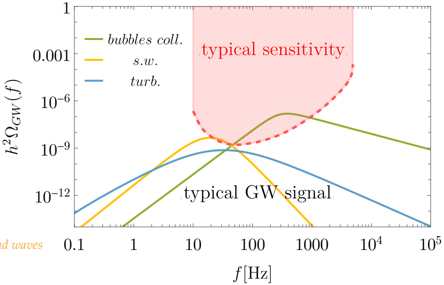
FOPT and Gravitational waves

1. Bubbles can produce a stochastic GW background from

FOPT and Gravitational waves

1. Bubbles can produce a stochastic GW background from
   - bubble collision
   - sound waves
   - turbulence





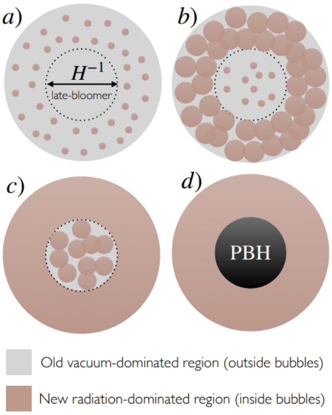
Primordial GWs could be observed soon: Frequency $\Rightarrow$ information about $T_{\rm reh}$: $f_{\rm peak} \propto T_{\rm reh}$

FOPT and PBH

1. FOPT can create a large abundance of PBH

FOPT and PBH

1. FOPT can create a large abundance of PBH
   K. Sato, M. Sasaki, H. Kodama, and K.-i. Maeda (1981-82), M. Lewicki, P. Toczek, and V. Vaskonen (2023), Gouttenoire, Volansky (2023)

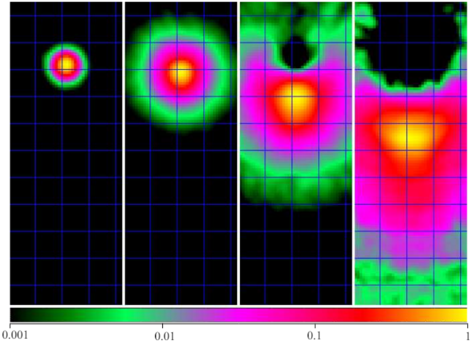


FOPT and Magnetic fields

1. FOPT can create Magnetic Fields

FOPT and Magnetic fields

1. FOPT can create Magnetic Fields
   Vachaspati, Durrer, Neronov, Balaji, Fairbairn, Olea-Romacho, Vaskonen, Lewicki ...
   Suggested by Fermi-LAT/HESS (2306.05132) $B \sim 10^{-15}$ G and $\lambda \sim 1$ Mpc



In the SM from dumbbells ? 2412.00641: Brandenburg, Vachaspati

FOPT and Baryogenesis

1. Bubbles can create baryon anti-baryon asymmetry

FOPT and Baryogenesis

1. Bubbles can create baryon anti-baryon asymmetry

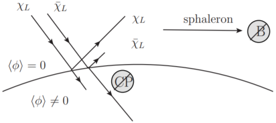
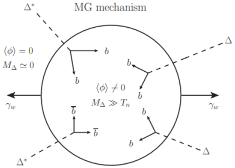
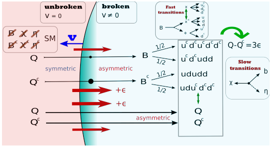


Figure: Credit:T.Konstandin [1302.6713]

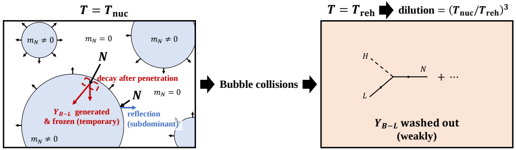


Traditional EWBG: $v_w \lesssim c_s$



Yin, Azatov, MV 21':
$\gamma_w \equiv 1/\sqrt{1-v_w^2} \gtrsim M_B^2/vT_{\rm nuc}$

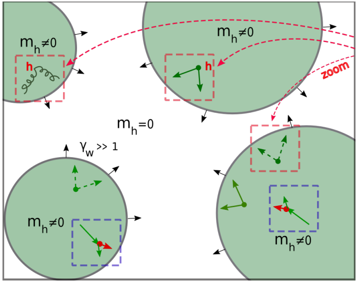
Baldes, Blasi, Mariotti, Sevrin, Turbang 21' $\gamma_w \gtrsim M_\Delta^2/vT_{\rm nuc}$



Eung,Dutka,Jung, Nagels, MV, 23': $\gamma_w \gtrsim M_{\rm RHN}/T_{\rm nuc}$

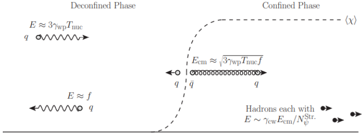
FOPT and DM

Supercool DM: Hambye, Strumia, Tesi 18'

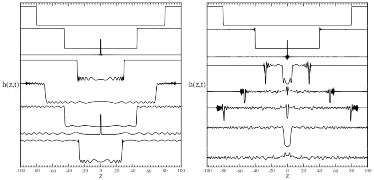


Azatov, Yin, MV 21' ,
Baldes, Sala, Gouttenoire, 22'
Azatov, Yin, Nagels, MV 24'

$$\gamma_w \equiv 1/\sqrt{1-v_w^2} \gtrsim M_{\rm DM}^2/vT_{\rm nuc}$$



Baldes,Gouttenoire, Sala, Servant, 22' : $\gamma_w \gg 1$



Falkowski, No, 12' Giudice, Pomarol, Lee, Shakya 24' :
Runaway walls: $\gamma_w \sim M_{\rm pl}/v \to 10^{15}$

Production of heavy particles in PT

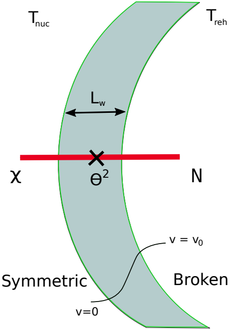
Production of heavy particles in PT

[arXiv:2010.02590] with *Aleksandr Azatov*

Production of out-of-equilibrium heavy states via wall[2010.02590]:Idea

Out-of-equilibrium heavy states

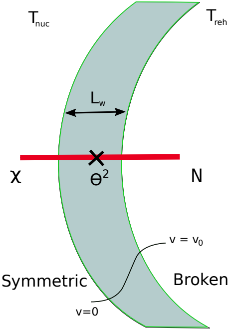
ϕ scalar, χ light fermion, N heavy fermion: $\mathcal{L}_{int} = Y\phi\bar{\chi}N + M\bar{N}N$
$n_N(M \gg T) \sim (MT)^{3/2} e^{-M/T} \to 0.$



Production of out-of-equilibrium heavy states via wall[2010.02590]:Idea

Out-of-equilibrium heavy states

ϕ scalar, χ light fermion, N heavy fermion: $\mathcal{L}_{int} = Y\phi\bar{\chi}N + M\bar{N}N$
$n_N(M \gg T) \sim (MT)^{3/2}e^{-M/T} \to 0.$

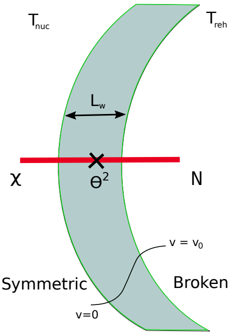


- ϕ undergoes FOPT with relativistic walls

Production of out-of-equilibrium heavy states via wall[2010.02590]:Idea

Out-of-equilibrium heavy states

ϕ scalar, χ light fermion, N heavy fermion: $\mathcal{L}_{int} = Y\phi\bar{\chi}N + M\bar{N}N$
$n_N(M \gg T) \sim (MT)^{3/2}e^{-M/T} \to 0.$



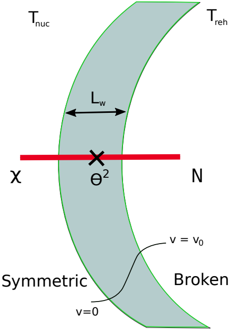
- ϕ undergoes FOPT with relativistic walls
- $p_\chi = (E,0,0,E), p_N = (E,0,0,\sqrt{E^2-M^2}) : \Delta p_z = p_N^z - p_\chi^z \neq 0$

Q: $\chi \to N$ possible?

Production of out-of-equilibrium heavy states via wall[2010.02590]:Idea

Out-of-equilibrium heavy states

ϕ scalar, χ light fermion, N heavy fermion: $\mathcal{L}_{int} = Y\phi\bar{\chi}N + M\bar{N}N$
$n_N(M \gg T) \sim (MT)^{3/2} e^{-M/T} \to 0.$



- ϕ undergoes FOPT with relativistic walls
- $p_\chi = (E, 0, 0, E), p_N = (E, 0, 0, \sqrt{E^2 - M^2}) : \Delta p_z = p_N^z - p_\chi^z \neq 0$

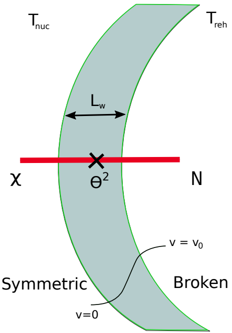
Q: $\chi \to N$ possible?

- No wall: $(2\pi)^4 \delta^4(p_\chi - p_N)$: $\Rightarrow \cancel{\chi \to N}$

Production of out-of-equilibrium heavy states via wall[2010.02590]:Idea

Out-of-equilibrium heavy states

ϕ scalar, χ light fermion, N heavy fermion: $\mathcal{L}_{int} = Y\phi\bar{\chi}N + M\bar{N}N$
$n_N(M \gg T) \sim (MT)^{3/2}e^{-M/T} \to 0.$



- ϕ undergoes FOPT with relativistic walls
- $p_\chi = (E,0,0,E), p_N = (E,0,0,\sqrt{E^2-M^2}) : \Delta p_z = p_N^z - p_\chi^z \neq 0$

Q: $\chi \to N$ possible?

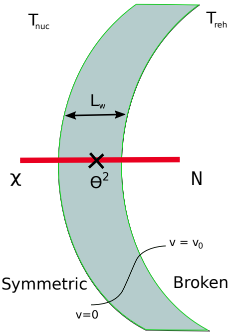
- No wall: $(2\pi)^4\delta^4(p_\chi - p_N)$: $\Rightarrow \cancel{\chi \to N}$
- With wall: if $E > M$, $\chi \to N$ allowed

$$\int d^3x_\perp e^{i\Delta p_\perp \cdot x_\perp} \int \langle\phi\rangle(z)e^{iz\Delta p_z}dz \propto (2\pi)^3\delta^2(\Delta p_\perp)\delta(\Delta E)\frac{\sin \Delta p_z L_w}{\Delta p_z L_w}$$

Production of out-of-equilibrium heavy states via wall[2010.02590]:Idea

Out-of-equilibrium heavy states

ϕ scalar, χ light fermion, N heavy fermion: $\mathcal{L}_{int} = Y\phi\bar{\chi}N + M\bar{N}N$
$n_N(M \gg T) \sim (MT)^{3/2} e^{-M/T} \to 0.$



- ϕ undergoes FOPT with relativistic walls
- $p_\chi = (E, 0, 0, E), p_N = (E, 0, 0, \sqrt{E^2 - M^2}) : \Delta p_z = p_N^z - p_\chi^z \neq 0$

Q: $\chi \to N$ possible?

- No wall: $(2\pi)^4 \delta^4(p_\chi - p_N)$: $\Rightarrow \cancel{\chi \to N}$
- With wall: if $E > M$, $\chi \to N$ allowed

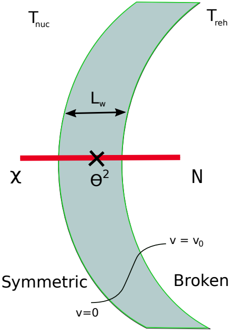
$$\int d^3x_\perp e^{i\Delta p_\perp \cdot x_\perp} \int \langle\phi\rangle(z) e^{iz\Delta p_z} dz \propto (2\pi)^3 \delta^2(\Delta p_\perp)\delta(\Delta E) \frac{\sin \Delta p_z L_w}{\Delta p_z L_w}$$

- Exchange momentum bound: $\Delta p_z \lesssim 1/L_w \sim v_\phi$ $\cancel{\Delta p_z \propto E}$

Production of out-of-equilibrium heavy states via wall[2010.02590]: computation

Out-of-equilibrium heavy states

ϕ scalar, χ light fermion, N heavy fermion: $\mathcal{L}_{int} = Y\phi\bar{\chi}N + M\bar{N}N$
$n_N(M \gg T) \sim (MT)^{3/2}e^{-M/T} \to 0.$

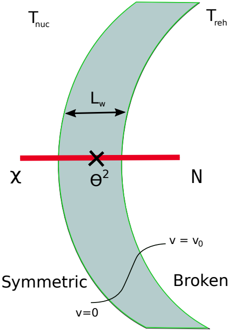


- Assume very fast wall: $\gamma_{wp} \equiv \frac{1}{\sqrt{1-v_w^2}} \gg 1$

Production of out-of-equilibrium heavy states via wall[2010.02590]: computation

Out-of-equilibrium heavy states

ϕ scalar, χ light fermion, N heavy fermion: $\mathcal{L}_{int} = Y\phi\bar{\chi}N + M\bar{N}N$
$n_N(M \gg T) \sim (MT)^{3/2}e^{-M/T} \to 0.$

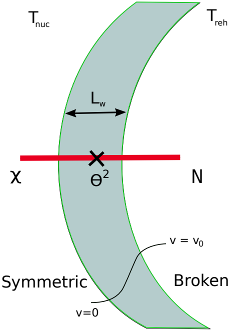


- Assume very fast wall: $\gamma_{wp} \equiv \frac{1}{\sqrt{1-v_w^2}} \gg 1$
- In the wall frame: $E \sim p_z \sim \gamma_{wp}T_{\rm nuc} \gg v_\phi$

Production of out-of-equilibrium heavy states via wall[2010.02590]: computation

Out-of-equilibrium heavy states

ϕ scalar, χ light fermion, N heavy fermion: $\mathcal{L}_{int} = Y\phi\bar{\chi}N + M\bar{N}N$
$n_N(M \gg T) \sim (MT)^{3/2}e^{-M/T} \to 0.$

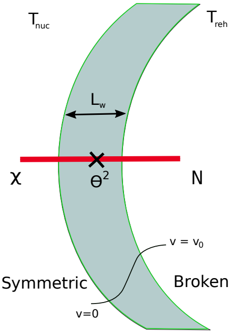


- Assume very fast wall: $\gamma_{wp} \equiv \frac{1}{\sqrt{1-v_w^2}} \gg 1$
- In the wall frame: $E \sim p_z \sim \gamma_{wp} T_{\rm nuc} \gg v_\phi$
- $|\mathcal{M}|^2 \approx Y^2 v_\phi^2 \times \frac{E}{\Delta p_z}\left(\frac{\sin \Delta p_z L_w}{\Delta p_z L_w}\right)^2 \qquad \Delta p_z = E - \sqrt{E^2 - M^2} \to \frac{M^2}{2E}$

Production of out-of-equilibrium heavy states via wall[2010.02590]: computation

Out-of-equilibrium heavy states

ϕ scalar, χ light fermion, N heavy fermion: $\mathcal{L}_{int} = Y\phi\bar{\chi}N + M\bar{N}N$
$n_N(M \gg T) \sim (MT)^{3/2}e^{-M/T} \to 0.$



- Assume very fast wall: $\gamma_{wp} \equiv \frac{1}{\sqrt{1-v_w^2}} \gg 1$
- In the wall frame: $E \sim p_z \sim \gamma_{wp} T_{\text{nuc}} \gg v_\phi$
- $|\mathcal{M}|^2 \approx Y^2 v_\phi^2 \times \frac{E}{\Delta p_z}\left(\frac{\sin \Delta p_z L_w}{\Delta p_z L_w}\right)^2 \qquad \Delta p_z = E - \sqrt{E^2 - M^2} \to \frac{M^2}{2E}$
-

$$P(\chi \to N) \approx \quad \theta^2 \times \Theta(\gamma_{wp} T_{\text{nuc}} - M^2 L_w), \qquad \theta \equiv \frac{Y v_\phi}{M}$$

Creation of out-of-equilibrium states: consequences

- Each production induces a kick[2010.02590]: $\Delta p_z \sim \frac{M^2}{2E}$

$$\Rightarrow \qquad \mathcal{P} \propto n_\chi \times \Delta p_z \times P(\chi \to N) \sim \frac{Y^2 v^2 T^2}{48} \Theta(\gamma_{wp} T_{\rm nuc} - M^2 L_w) \quad \text{(pressure on the wall)}$$

Creation of out-of-equilibrium states: consequences

- Each production induces a kick[2010.02590]: $\Delta p_z \sim \frac{M^2}{2E}$

$$\Rightarrow \qquad \mathcal{P} \propto n_\chi \times \Delta p_z \times P(\chi \to N) \sim \frac{Y^2 v^2 T^2}{48} \Theta(\gamma_{wp} T_{\rm nuc} - M^2 L_w) \quad \text{(pressure on the wall)}$$

- Production of stable states (Dark Matter φ) via portal $\varphi^2\phi^2$[2101.05721]: $\phi \to \varphi\varphi$

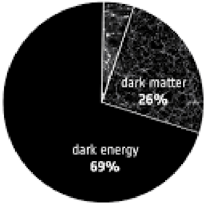
Creation of out-of-equilibrium states: consequences

- Each production induces a kick[2010.02590]: $\Delta p_z \sim \frac{M^2}{2E}$

$$\Rightarrow \qquad \mathcal{P} \propto n_\chi \times \Delta p_z \times P(\chi \to N) \sim \frac{Y^2 v^2 T^2}{48} \Theta(\gamma_{wp} T_{\rm nuc} - M^2 L_w) \quad \text{(pressure on the wall)}$$

- Production of stable states (Dark Matter φ) via portal $\varphi^2\phi^2$[2101.05721]: $\phi \to \varphi\varphi$
- out-of-equilibrium abundance of N via $\chi \to N$ [2106.14913]: baryogenesis

Baryogenesis with relativistic walls

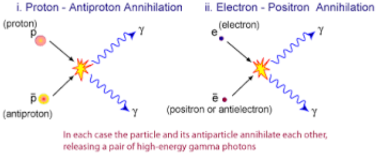


Production of matter

[arXiv:2106.14913] with *Aleksandr Azatov* and *Wen Yin*

Problem of baryogenesis and Sakharov conditions

- Inflation: erase the initial conditions
- Inflation produces $n_B = n_{\bar{B}}$ (likely...)



- $\frac{n_B - n_{\bar{B}}}{n_\gamma} \to 0(10^{-20}...)$ (naive expectation): empty universe
- $\frac{n_B - n_{\bar{B}}}{n_\gamma} \to 10^{-10}$ (observation): need a process that turns one out of 10^{10} anti-baryon $\to$ baryon

Idea of baryogenesis with relativistic bubble walls

- **Out-of-equilibrium situation**; Automatically embedded via the process of production in $\chi \to N$
- **CP-violation**: $\Gamma(\chi \to N_I) \neq \Gamma(\bar{\chi} \to \bar{N}_I)$
- **B-number violation**

Possible baryogenesis in our production setting?

Idea of baryogenesis with relativistic bubble walls

- **Out-of-equilibrium situation**: $\chi \to N$ via relativistic bubble expansion

Possible baryogenesis in our production setting?

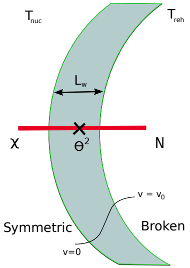
Idea of baryogenesis with relativistic bubble walls

- **Out-of-equilibrium situation**: $\chi \to N$ via relativistic bubble expansion
- **CP-violation**: $\Gamma(\chi \to N_I) \neq \Gamma(\bar{\chi} \to \bar{N}_I)$??

CP violation inside the bubble wall [2101.05721]: Azatov, MV, Yin

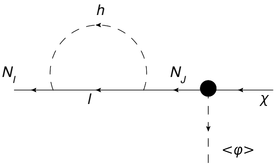
Ingredients: Higgs field H, ϕ scalar, 2 heavy N_I, SM $SU(2)_L$-fermions L_α, and χ_i light fermions

$$\mathcal{L} = -M_I \bar{N}_I N_I - Y_{iI} \phi \bar{N}_I P_R \chi_i - y_{I\alpha} (H \bar{L}_\alpha) P_R N_I + h.c.$$



$$A(\chi_i \to N_I)_{\text{tree}} \propto Y_{iI}$$

$+$



$$A(\chi_i \to N_I)_{1-\text{loop}} \propto$$

$$+ \sum_{\alpha, J} Y_{iJ} y^*_{\alpha J} y_{\alpha I} \times f^{(HL)}_{IJ}$$

$\Rightarrow$

$$\frac{\Gamma(\chi \to N_I) - \Gamma(\bar{\chi} \to \bar{N}_I)}{\Gamma(\chi \to N_I) + \Gamma(\bar{\chi} \to \bar{N}_I)} =$$

$$\frac{2 \sum_{\alpha, J, i} \mathrm{Im}(Y_{iI} Y^*_{iJ} y_{\alpha J} y^*_{\alpha I}) \mathrm{Im} f^{(HL)}_{IJ}}{\sum_i |Y_{iI}|^2}.$$

and

$$\mathrm{Im}[f^{(HL)}_{IJ}(x)] = \frac{1}{16\pi} \frac{\sqrt{x}}{1-x}, x = \frac{M_J^2}{M_I^2}$$

Possible baryogenesis in our production setting?

Idea of baryogenesis with relativistic bubble walls

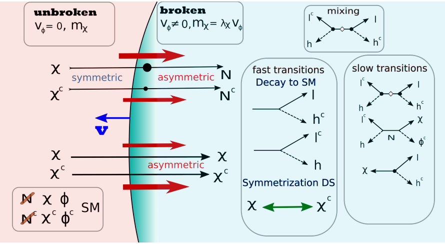
- **Out-of-equilibrium situation**: $\chi \to N$ via relativistic bubble expansion
- **CP-violation**: CP-violating phase in the yukawas $y_{I\alpha}$ from interference tree-loop level
- **B-number violation**: B or L-violating interactions

Possible baryogenesis ? [2101.05721]: Azatov, MV, Yin

Idea of baryogenesis with relativistic bubble walls

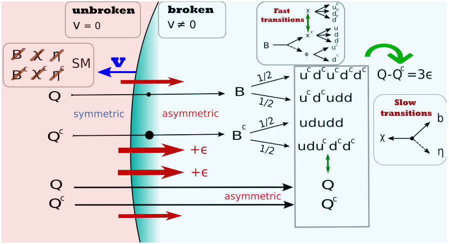
- **Out-of-equilibrium situation**: $\chi \to N$ via relativistic bubble expansion
- **CP-violation**: CP-violating phase in the yukawas $y_{I\alpha}$ from interference tree-loop level
- **B-number violation**: B or L-violating interactions

Phase transition induced leptogenesis



Or

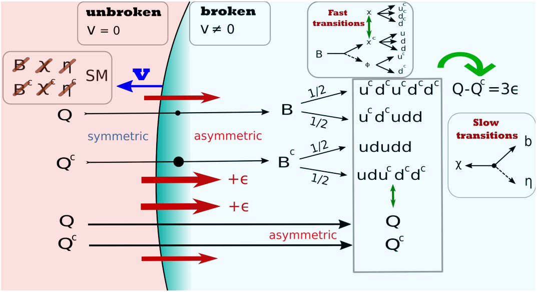
EWPT Baryogenesis with relativistic walls



Low energy baryogenesis [2101.05721]: Azatov, MV, Yin

$$\mathcal{L}_{SM} + \sum_{I=1,2} \underbrace{Y_I(\bar{B}_I H) P_L Q}_{\text{production}} + M_I \bar{B}_I B_I + \underbrace{y_I \eta \chi^c P_L B_I + \kappa \eta^c b t}_{\text{decay dark sector}} + \underbrace{\frac{1}{2} m_\chi \bar{\chi}^c \chi}_{\text{B-violating}} + m_\eta^2 |\eta|^2 .$$

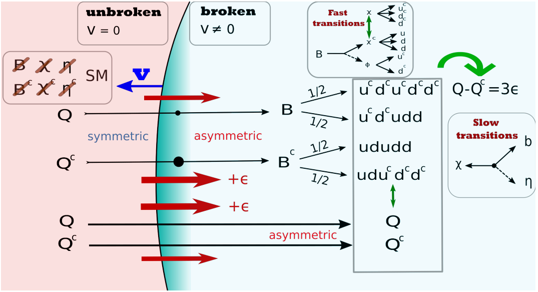
- χ_i Massive majorana, η diquark, B_I heavy vectorlike b-like quarks. $B(\eta) = 2/3, B(\chi) = 1$.



Low energy baryogenesis [2101.05721]: Azatov, MV, Yin

$$\mathcal{L}_{SM} + \sum_{I=1,2} \underbrace{Y_I(\bar{B}_I H)P_L Q}_{\text{production}} + M_I \bar{B}_I B_I + \underbrace{y_I \eta \chi^c P_L B_I + \kappa \eta^c b t}_{\text{decay dark sector}} + \underbrace{\frac{1}{2} m_\chi \bar{\chi}^c \chi}_{\text{B-violating}} + m_\eta^2 |\eta|^2 .$$

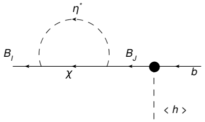
- χ_i Massive majorana, η diquark, B_I heavy vectorlike b-like quarks. $B(\eta) = 2/3, B(\chi) = 1$.
- B broken by two units: $m_\chi \bar{\chi}^c \chi$



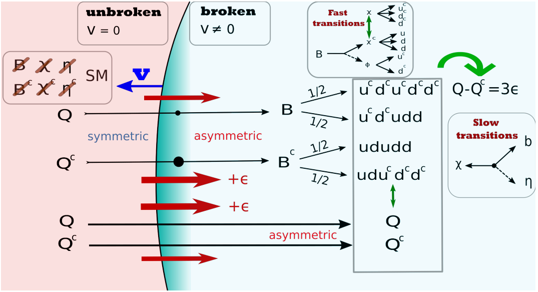
Low energy baryogenesis [2101.05721]: Azatov, MV, Yin

$$\mathcal{L}_{SM} + \sum_{I=1,2} \underbrace{Y_I(\bar{B}_I H)P_L Q}_{\text{production}} + M_I \bar{B}_I B_I + \underbrace{y_I \eta \chi^c P_L B_I + \kappa \eta^c b t}_{\text{decay dark sector}} + \underbrace{\frac{1}{2} m_\chi \bar{\chi}^c \chi}_{\text{B-violating}} + m_\eta^2 |\eta|^2.$$

- χ_i Massive majorana, η diquark, B_I heavy vectorlike b-like quarks. $B(\eta) = 2/3, B(\chi) = 1$.
- B broken by two units: $m_\chi \bar{\chi}^c \chi$



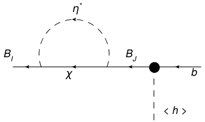
- CP violation:



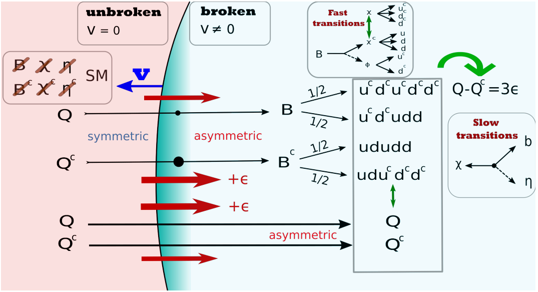
Low energy baryogenesis [2101.05721]: Azatov, MV, Yin

$$\mathcal{L}_{SM} + \sum_{I=1,2} \underbrace{Y_I(\bar{B}_I H)P_L Q}_{\text{production}} + M_I \bar{B}_I B_I + \underbrace{y_I \eta \chi^c P_L B_I + \kappa \eta^c b t}_{\text{decay dark sector}} + \underbrace{\frac{1}{2} m_\chi \bar{\chi}^c \chi}_{\text{B-violating}} + m_\eta^2 |\eta|^2 .$$

- χ_i Massive majorana, η diquark, B_I heavy vectorlike b-like quarks. $B(\eta) = 2/3, B(\chi) = 1$.
- B broken by two units: $m_\chi \bar{\chi}^c \chi$



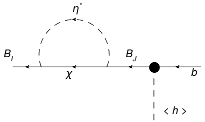
- CP violation:
- Production: $\mathcal{P}(Q \to B_I) \neq \mathcal{P}(Q^c \to B_I^c)$



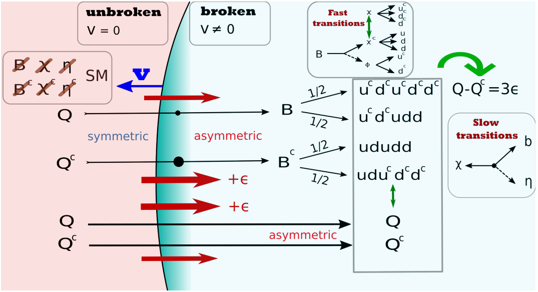
Low energy baryogenesis [2101.05721]: Azatov, MV, Yin

$$\mathcal{L}_{SM} + \sum_{I=1,2} \underbrace{Y_I(\bar{B}_I H)P_L Q}_{\text{production}} + M_I \bar{B}_I B_I + \underbrace{y_I \eta \chi^c P_L B_I + \kappa \eta^c bt}_{\text{decay dark sector}} + \underbrace{\frac{1}{2} m_\chi \bar{\chi}^c \chi}_{\text{B-violating}} + m_\eta^2 |\eta|^2.$$

- χ_i Massive majorana, η diquark, B_I heavy vectorlike b-like quarks. $B(\eta) = 2/3, B(\chi) = 1$.
- B broken by two units: $m_\chi \bar{\chi}^c \chi$



- CP violation:
- Production: $\mathcal{P}(Q \to B_I) \neq \mathcal{P}(Q^c \to B_I^c)$
- $\Delta n_B \equiv n_{SM-q} - n_{SM-\bar{q}} \approx 3 n_b^0 \sum_I \theta_I^2 \epsilon_I$



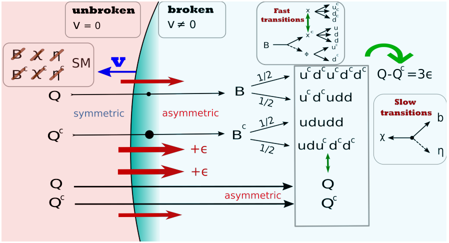
IF : $\gamma_w \equiv \sqrt{1/(1-v_w^2)} > M_B^2/(T v_{EW})$

Low energy baryogenesis: constraints [2101.05721]: Azatov, MV, Yin

$$\mathcal{L}_{SM} + \sum_{I=1,2} \underbrace{Y_I(\bar{B}_I H)P_L Q}_{\text{production}} + M_I \bar{B}_I B_I + \underbrace{y_I \eta \chi^c P_L B_I + \kappa \eta^c b t}_{\text{decay dark sector}} + \underbrace{\frac{1}{2} m_\chi \bar{\chi}^c \chi}_{\text{B-violating}} + m_\eta^2 |\eta|^2 .$$

- $N \leftrightarrow \bar{N}$: $\frac{(\sum \kappa \theta_I y_I)^2}{M_\eta^4 M_\chi} \overline{u^c}\,\overline{d^c} u^c d^c u d d$

$$\delta m_{\not{n}-\bar{n}} \sim \frac{\Lambda_{QCD}^6}{M_\eta^4 m_\chi} \left(\sum \kappa \theta_I y_I\right)^2 < 10^{-33}\,\text{GeV}$$



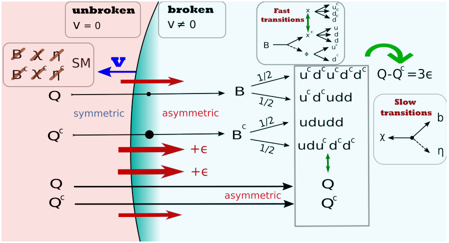
Low energy baryogenesis: constraints [2101.05721]: Azatov, MV, Yin

$$\mathcal{L}_{SM} + \sum_{I=1,2} \underbrace{Y_I(\bar{B}_I H)P_L Q}_{\text{production}} + M_I \bar{B}_I B_I + \underbrace{y_I \eta \chi^c P_L B_I + \kappa \eta^c b t}_{\text{decay dark sector}} + \underbrace{\frac{1}{2} m_\chi \bar{\chi}^c \chi}_{\text{B-violating}} + m_\eta^2 |\eta|^2 .$$

- $N \leftrightarrow \bar{N}$: $\frac{(\sum \kappa\theta_I y_I)^2}{M_\eta^4 M_\chi} \overline{u^c}\,\overline{d^c} u^c d^c u d d$

$$\delta m_{\not{n}-\bar{n}} \sim \frac{\Lambda_{QCD}^6}{M_\eta^4 m_\chi} \left(\sum \kappa\theta_I y_I\right)^2 < 10^{-33}\,\text{GeV}$$

- Flavor and collider: 2 TeV $\lesssim m_\chi \sim m_\eta \sim M_B$



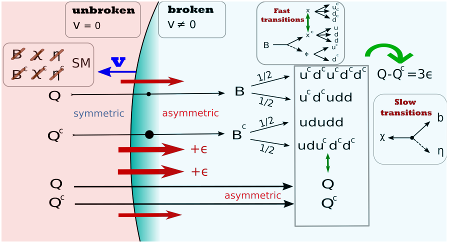
Low energy baryogenesis: constraints [2101.05721]: Azatov, MV, Yin

$$\mathcal{L}_{SM} + \sum_{I=1,2} \underbrace{Y_I(\bar{B}_I H) P_L Q}_{\text{production}} + M_I \bar{B}_I B_I + \underbrace{y_I \eta \chi^c P_L B_I + \kappa \eta^c b t}_{\text{decay dark sector}} + \underbrace{\frac{1}{2} m_\chi \bar{\chi}^c \chi}_{\text{B-violating}} + m_\eta^2 |\eta|^2 .$$

- $N \leftrightarrow \bar{N}$: $\frac{(\sum \kappa \theta_I y_I)^2}{M_\eta^4 M_\chi} \overline{u^c} \overline{d^c} u^c d^c u d d$

$$\delta m_{\not{n}-\bar{n}} \sim \frac{\Lambda_{QCD}^6}{M_\eta^4 m_\chi} \left(\sum \kappa \theta_I y_I\right)^2 < 10^{-33} \text{GeV}$$

- Flavor and collider: 2 TeV $\lesssim m_\chi \sim m_\eta \sim M_B$



Low energy baryogenesis: constraints [2101.05721]: Azatov, MV, Yin

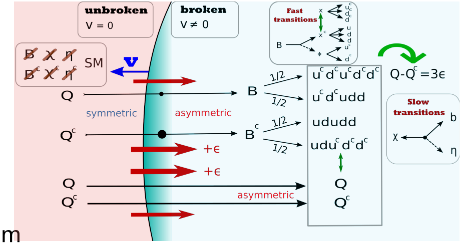
$$\mathcal{L}_{SM} + \sum_{I=1,2} \underbrace{Y_I(\bar{B}_I H) P_L Q}_{\text{production}} + M_I \bar{B}_I B_I + \underbrace{y_I \eta \chi^c P_L B_I + \kappa \eta^c b t}_{\text{decay dark sector}} + \underbrace{\frac{1}{2} m_\chi \bar{\chi}^c \chi}_{\text{B-violating}} + m_\eta^2 |\eta|^2 .$$

- $N \leftrightarrow \bar{N}$: $\frac{(\sum \kappa\theta_I y_I)^2}{M_\eta^4 M_\chi}\overline{u^c}\,\overline{d^c} u^c d^c udd$

$$\delta m_{\cancel{n}-\bar{n}} \sim \frac{\Lambda_{QCD}^6}{M_\eta^4 m_\chi}\left(\sum \kappa\theta_I y_I\right)^2 < 10^{-33}\,\text{GeV}$$

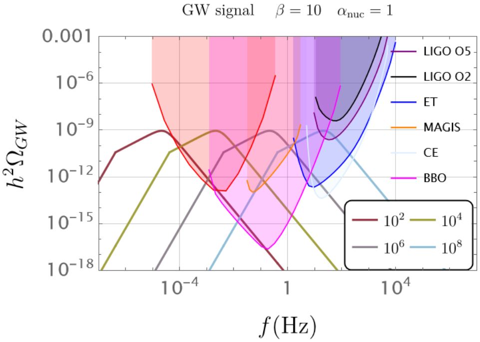
- Flavor and collider: 2 TeV $\lesssim m_\chi \sim m_\eta \sim M_B$
- EDM bounds

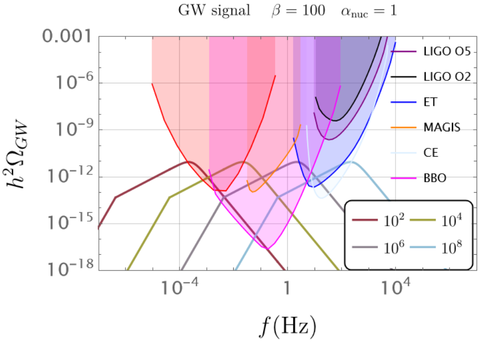
$$\frac{d_e}{e} \sim \frac{m_e (y_Y Y_e)^2}{(4\pi)^6}\left(\frac{1}{\Lambda_{EDM}^2}\right) \sim 3\times 10^{-33} \times \left(\frac{10\text{ TeV}}{\Lambda_{EDM}}\right)^2 \text{ cm}$$



Observation prospects of GW

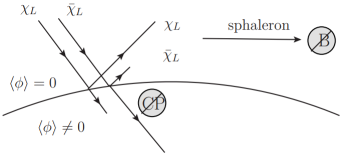
$$\left(T_{\rm reh}, \quad \beta, \quad \alpha_n, \quad v_w\right) \quad \Rightarrow \quad (\Omega^{GW}_{\rm peak}, \quad f^{GW}_{\rm peak})$$





Using Bulk flow model: Konstandin, Jinno, Takimoto, ...

EW bubble and EWBG



Relation BAU and velocity:

$$\boxed{\text{BAU} \to 0, \qquad v_w \to 1}$$

Credit:T.Konstandin [1302.6713]

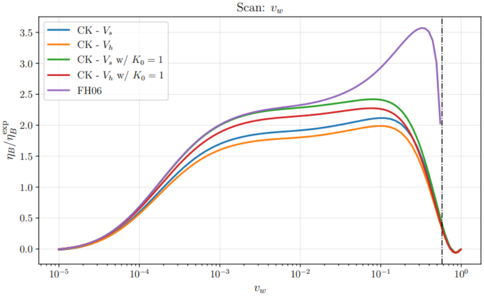
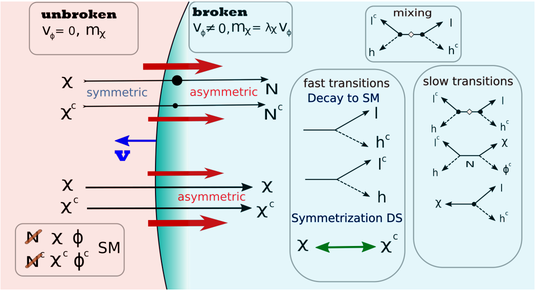


Figure: Credit: Giulio Barni

Phase transition induced leptogenesis

$$\sum_{iI}\underbrace{\left(Y_{iI}(\phi\bar{\chi}_i)P_L N_I + Y^\star_{iI}\bar{N}_I P_R(\phi^\dagger\chi_i)\right)}_{\text{production sector}} - V(\phi) + \underbrace{\sum_i \lambda_\chi \phi \bar{\chi}^c_i \chi_i}_{\text{Majorana mass}} + \sum_I M_I \bar{N}_I N_I + \underbrace{\sum_{\alpha I} y_{\alpha I}(H\bar{L}_{\alpha,SM})P_R N_I}_{\text{CP-violation}}$$

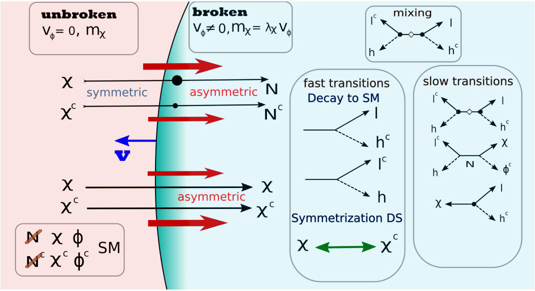
- χ_i Majorana, ϕ scalar, N_I Heavy Dirac RHN: $U_L(1): L(\chi) = -1, L(N) = 1, L(\phi) = 2.$



Phase transition induced leptogenesis

$$\sum_{iI}\underbrace{\Big(Y_{iI}(\phi\bar{\chi}_i)P_L N_I + Y^\star_{iI}\bar{N}_I P_R(\phi^\dagger\chi_i)\Big)}_{\text{production sector}} - V(\phi) + \underbrace{\sum_i \lambda_\chi \phi \bar{\chi}^c_i \chi_i}_{\text{Majorana mass}} + \sum_I M_I \bar{N}_I N_I + \underbrace{\sum_{\alpha I} y_{\alpha I}(H\bar{L}_{\alpha,SM})P_R N_I}_{\text{CP-violation}}$$

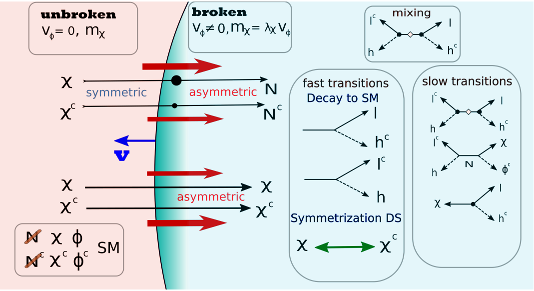
- χ_i Majorana, ϕ scalar, N_I Heavy Dirac RHN: $U_L(1) : L(\chi) = -1, L(N) = 1, L(\phi) = 2.$
- Production $\mathcal{P}(\chi_i \to N_I) \neq \mathcal{P}(\chi^c_i \to N^c_I) \propto \theta^2_{iI}$



Phase transition induced leptogenesis

$$\sum_{iI}\underbrace{\Big(Y_{iI}(\phi\bar{\chi}_i)P_L N_I + Y^{\star}_{iI}\bar{N}_I P_R(\phi^{\dagger}\chi_i)\Big)}_{\text{production sector}} - V(\phi) + \underbrace{\sum_i \lambda_\chi \phi \bar{\chi}^c_i \chi_i}_{\text{Majorana mass}} + \sum_I M_I \bar{N}_I N_I + \underbrace{\sum_{\alpha I} y_{\alpha I}(H\bar{L}_{\alpha,SM})P_R N_I}_{\text{CP-violation}}$$

- χ_i Majorana, ϕ scalar, N_I Heavy Dirac RHN: $U_L(1): L(\chi) = -1, L(N) = 1, L(\phi) = 2.$
- Production $\mathcal{P}(\chi_i \to N_I) \neq \mathcal{P}(\chi^c_i \to N^c_I) \propto \theta^2_{iI}$
- $\frac{\Delta n_L}{s} \sim \sum_{iI\alpha} \epsilon_{iI} \times \frac{\theta^2_{iI}}{g_*}\left(\frac{T_{nuc}}{T_{reh}}\right)^3 \times \frac{|y_{I\alpha}|^2}{|y_{I\alpha}|^2+|Y_{iI}|^2}$

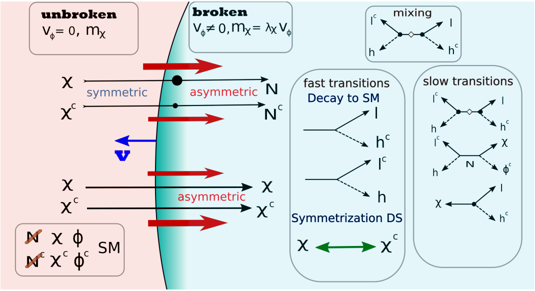


Phase transition induced leptogenesis

$$\sum_{iI}\underbrace{\Big(Y_{iI}(\phi\bar{\chi}_i)P_L N_I + Y^{\star}_{iI}\bar{N}_I P_R(\phi^\dagger\chi_i)\Big)}_{\text{production sector}} - V(\phi) + \underbrace{\sum_i \lambda_\chi \phi \bar{\chi}^c_i \chi_i}_{\text{Majorana mass}} + \sum_I M_I \bar{N}_I N_I + \underbrace{\sum_{\alpha I} y_{\alpha I}(H\bar{L}_{\alpha,SM})P_R N_I}_{\text{CP-violation}}$$

- χ_i Majorana, ϕ scalar, N_I Heavy Dirac RHN: $U_L(1): L(\chi) = -1, L(N) = 1, L(\phi) = 2$.
- Production $\mathcal{P}(\chi_i \to N_I) \neq \mathcal{P}(\chi^c_i \to N^c_I) \propto \theta^2_{iI}$
- $\frac{\Delta n_L}{s} \sim \sum_{iI\alpha} \epsilon_{iI} \times \frac{\theta^2_{iI}}{g_*}\left(\frac{T_{nuc}}{T_{reh}}\right)^3 \times \frac{|y_{I\alpha}|^2}{|y_{I\alpha}|^2+|Y_{iI}|^2}$
- Wash-outs impose:

$$10^9\text{GeV} \lesssim v \lesssim 10^{13}\text{GeV}, \qquad \frac{\lambda_\chi v}{T_{\text{reh}}} \gtrsim 15$$



Conclusion baryogenesis

- Production of heavy states far away from equilibrium when fast walls

Conclusion baryogenesis

- Production of heavy states far away from equilibrium when fast walls
- Interference between tree-loop level diagrams induces CP violation

Conclusion baryogenesis

- Production of heavy states far away from equilibrium when fast walls
- Interference between tree-loop level diagrams induces CP violation
- fast wall EWBG via upscattering:

$$2\ \text{TeV} \lesssim M_B \sim m_\chi \sim m_\eta$$

.

Conclusion baryogenesis

- Production of heavy states far away from equilibrium when fast walls
- Interference between tree-loop level diagrams induces CP violation
- fast wall EWBG via upscattering:

$$2\text{ TeV} \lesssim M_B \sim m_\chi \sim m_\eta$$

.

- EDM bounds are relaxed

Conclusion baryogenesis

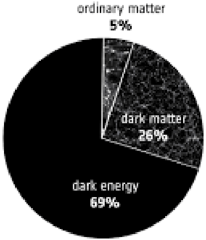
- Production of heavy states far away from equilibrium when fast walls
- Interference between tree-loop level diagrams induces CP violation
- fast wall EWBG via upscattering:

$$2\ \text{TeV} \lesssim M_B \sim m_\chi \sim m_\eta$$

.

- EDM bounds are relaxed
- Typical GW signal from this scenario is enhanced: likely observable in LISA

Production of Dark Matter

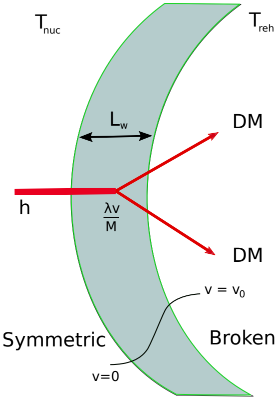


Production of Dark Matter

[arXiv:2101.05721] with *Aleksandr Azatov* and *Wen Yin*

Production mechanism[2101.05721]

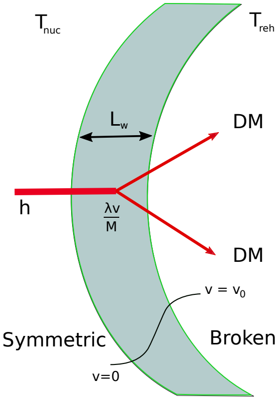
- Portal Dark Matter: $\mathcal{L} \supset -\frac{\lambda}{2} h^2\phi^2 - M_\phi^2\phi^2 - V(h)$



Production mechanism[2101.05721]

- Portal Dark Matter: $\mathcal{L} \supset -\frac{\lambda}{2} h^2 \phi^2 - M_\phi^2 \phi^2 - V(h)$
- Non-vanishing VEV:

$h \to h + v$ trilinear coupling $\boxed{\mathcal{L} \supset -\lambda v h \phi^2}$

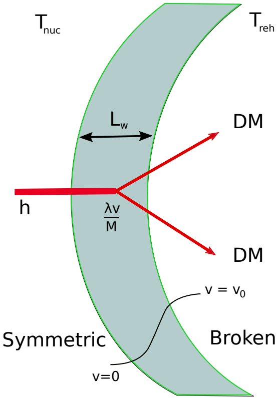


Production mechanism[2101.05721]

- Portal Dark Matter: $\mathcal{L} \supset -\frac{\lambda}{2}h^2\phi^2 - M_\phi^2\phi^2 - V(h)$
- Non-vanishing VEV:

  $h \to h + v$ trilinear coupling $\boxed{\mathcal{L} \supset -\lambda v h \phi^2}$
- In the *wall frame*: $E_h \sim p_{z,h} \sim \gamma_{wp} T \gg T$

$$P_{h\to\phi\phi} \sim \frac{\lambda^2 v^2}{M_\phi^2}\Theta(\gamma_{wp}T - M_\phi^2 L_w)$$



Production mechanism[2101.05721]

- Portal Dark Matter: $\mathcal{L} \supset -\frac{\lambda}{2} h^2\phi^2 - M_\phi^2 \phi^2 - V(h)$
- Non-vanishing VEV:

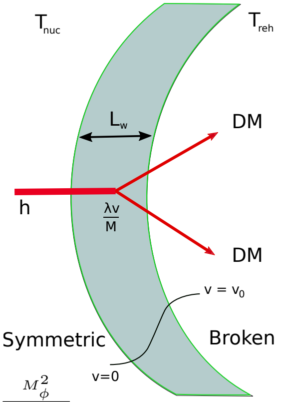
  $h \to h + v$ trilinear coupling $\boxed{\mathcal{L} \supset -\lambda v h \phi^2}$
- In the *wall frame*: $E_h \sim p_{z,h} \sim \gamma_{wp} T \gg T$

$$P_{h\to\phi\phi} \sim \frac{\lambda^2 v^2}{M_\phi^2}\Theta(\gamma_{wp}T - M_\phi^2 L_w)$$

- Behind the wall, accumulation of relics ϕ

$$n_\phi^{\text{BE}} \approx \frac{T^3}{12\pi^4}\frac{\lambda^2 v^2}{M_\phi^2} e^{-\frac{M_\phi^2}{2vT\gamma_{wp}}} + \mathcal{O}(1/\gamma_w)$$

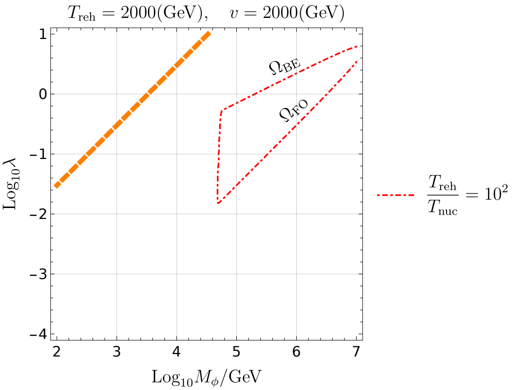
$$\Rightarrow \Omega_{\phi,\text{BE}}^{\text{today}} h^2 \approx 5.4\times 10^5 \times \left(\frac{1}{g_{\star S}(T_{\text{reh}})}\right)\left(\frac{\lambda^2 v}{M_\phi}\right)\left(\frac{v}{\text{GeV}}\right)\left(\frac{T_{\text{nuc}}}{T_{\text{reh}}}\right)^3 e^{-\frac{M_\phi^2}{2vT\gamma_{wp}}}$$



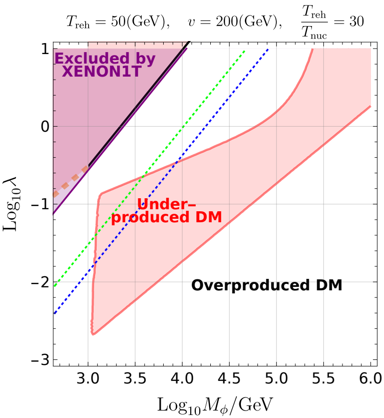
The DM is Heavy ...

[2101.05721]: Azatov, Yin, MV

h is a dark Higgs



h is a the Higgs



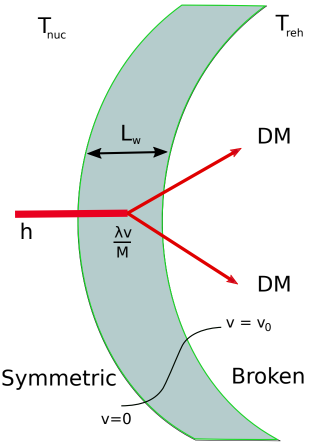
... and fast !

[2207.05096]: Baldes, Gouttenoire, Sala

- Warm Dark Matter

$$V_{\rm eq} \lesssim 4.2 \times 10^{-5} \qquad \text{From Lyman-}\alpha$$

$$\Rightarrow M_{\rm WDM} \sim \text{keV}$$



… and fast !

[2207.05096]: Baldes, Gouttenoire, Sala

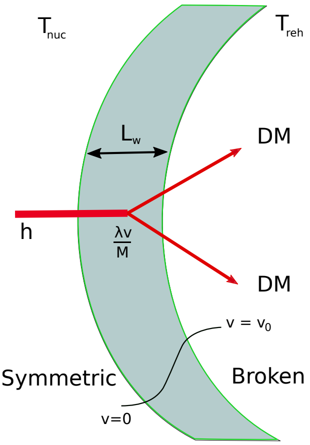
- Warm Dark Matter

$$V_{\rm eq} \lesssim 4.2 \times 10^{-5} \qquad \text{From Lyman-}\alpha$$

$$\Rightarrow M_{\rm WDM} \sim \text{keV}$$

- At production

$$\bar{E}_{\phi,\text{plasma}} \approx \frac{1}{2}\frac{M_\phi^2}{T_{\rm nuc}}$$



... and fast !

[2207.05096]: Baldes, Gouttenoire, Sala

- Warm Dark Matter

$$V_{\rm eq} \lesssim 4.2 \times 10^{-5} \qquad \text{From Lyman-}\alpha$$

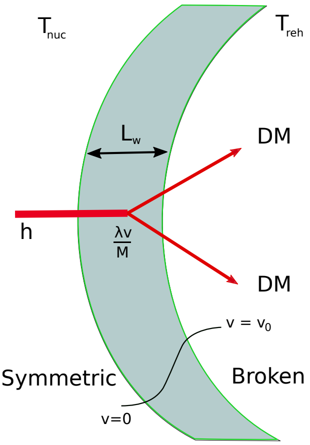
$$\Rightarrow M_{\rm WDM} \sim \text{keV}$$

- At production

$$\bar{E}_{\phi,\text{plasma}} \approx \frac{1}{2}\frac{M_\phi^2}{T_{\rm nuc}}$$

- If free streaming

$$V_{\rm eq} \approx 0.3\frac{T_{\rm eq}\bar{E}_\phi}{T_{\rm reh}M_\phi} \approx 10^{-10}\frac{\text{GeV} \times M_\phi}{T_{\rm reh}T_{\rm nuc}}$$



… and fast !

[2207.05096]: Baldes, Gouttenoire, Sala

- Warm Dark Matter

$$V_{\rm eq} \lesssim 4.2 \times 10^{-5} \qquad \text{From Lyman-}\alpha$$

$$\Rightarrow M_{\rm WDM} \sim \text{keV}$$

- At production

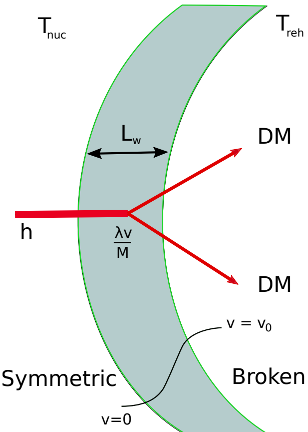
$$\bar{E}_{\phi,\text{plasma}} \approx \frac{1}{2}\frac{M_\phi^2}{T_{\rm nuc}}$$

- If free streaming

$$V_{\rm eq} \approx 0.3\frac{T_{\rm eq}\bar{E}_\phi}{T_{\rm reh}M_\phi} \approx 10^{-10}\frac{\text{GeV} \times M_\phi}{T_{\rm reh}T_{\rm nuc}}$$

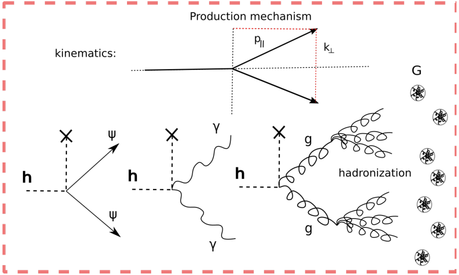
- Heavy WDM for

$$\boxed{V_{\rm eq} \sim 10^{-5}, \qquad v \sim 10^2\text{GeV}, \qquad M_\phi \sim 10^8\text{GeV}}$$



Secluded sectors ?

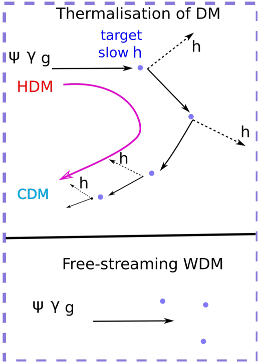
[2101.05721]: Azatov, Yin, MV, [2406.12554]:Azatov, Yin, Nagels, MV



$$\frac{\lambda}{2}h^2\phi^2, \qquad \frac{h^2\psi\bar{\psi}}{\Lambda}, \qquad \frac{h^2 FF}{\Lambda^2}$$

$$P_{h\to\psi\bar{\psi}}(p_0) \approx |V|^2\Theta(1-\Delta p_z L_w)\Theta(\Lambda^2 - 2p_0 v)$$

$$h \to \phi\phi, \psi\bar{\psi}, \gamma\gamma,$$



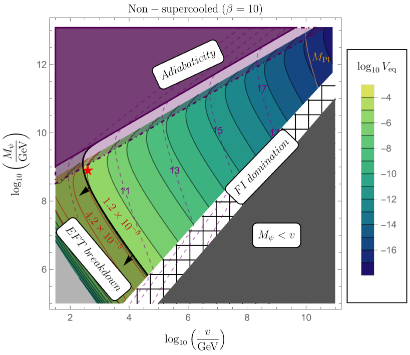
- Free-streaming regime
- Thermalised regime
- Transition between the regimes

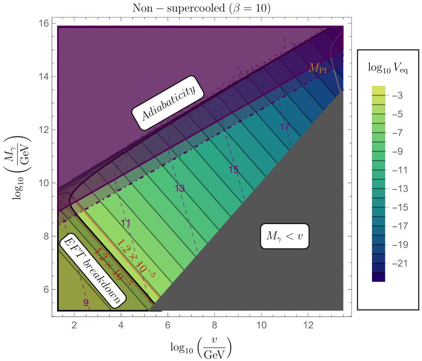
Secluded sectors ?

[2101.05721]: Azatov, Yin, MV, [2101.05721]:Azatov, Yin, Nagels, MV

$$\gamma_w(R) \approx \frac{2R}{3R_{\rm nuc}} \qquad \to \qquad \gamma_w^{\rm coll} \approx 0.06\frac{M_{\rm pl}T_{\rm nuc}}{\beta v^2}$$

EFT bound : $s \approx \gamma_w T v < \Lambda^2$, adiabaticity : $s > M^2$

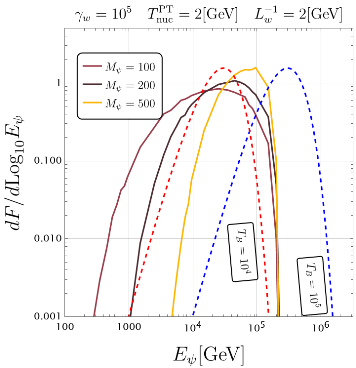




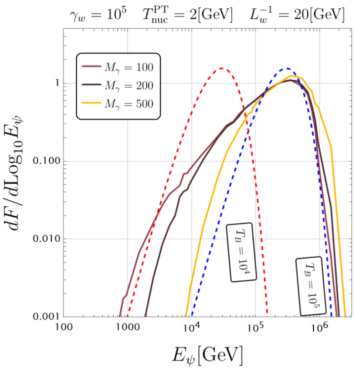
DM from bubble wall

[2101.05721]: Azatov, Yin, MV, [2406.12554]:Azatov, Yin, Nagels, MV

$hh\psi\bar{\psi}$



$hhFF$



About the pressure on the bubble wall

About the velocity of the bubble wall

The problem at hand

In vacuum: $\gamma_w \propto \frac{R}{R_{\text{initial}}} \approx RT_{\text{nuc}} \to \frac{M_p}{\beta v}$ (Runaway) $\boxed{\gamma_w \equiv \frac{1}{\sqrt{1-\xi_w^2}}}$

In plasma: $\Delta V = \underbrace{\Delta\mathcal{P}(\gamma_w = \gamma_w^{MAX})}_{??}$ (Runaway or terminal velocity)

Code for v_w, L_w: [2411.04970]: Ekstedt, Gould, Hirvonen, Laurent, Niemi, Van de Vis

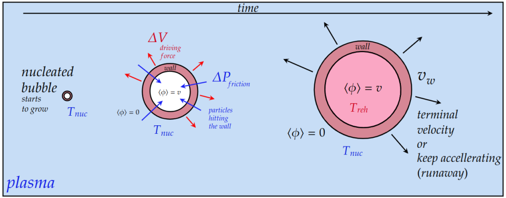


Figure: Credit: Giulio Barni, thanks to him

Kinetic vs. kick (big thanks to Matthias Carosi for the slides)

Two main approaches exist for studying the dynamics of a single bubble

Kinetic vs. kick (big thanks to Matthias Carosi for the slides)

Two main approaches exist for studying the dynamics of a single bubble

Kinetic picture
[Moore and Propokopec '95]

Set of dynamical equations

$$\begin{cases} \Box\varphi + U'(\varphi) + \sum_i \frac{\partial m_i^2}{\partial\varphi}\int_{\vec{p}}\delta f_i(\vec{p},x) = \ 0 \\ \frac{\partial f_i}{\partial t} = \ -\mathcal{C}_i[f,\varphi] \end{cases}$$

Kinetic vs. kick (big thanks to Matthias Carosi for the slides)

Two main approaches exist for studying the dynamics of a single bubble

Kinetic picture
[Moore and Propokopec '95]

Set of dynamical equations

$$\begin{cases} \Box\varphi + U'(\varphi) + \sum_i \frac{\partial m_i^2}{\partial\varphi} \int_{\vec{p}} \delta f_i(\vec{p}, x) = 0 \\ \frac{\partial f_i}{\partial t} = -\mathcal{C}_i[f, \varphi] \end{cases}$$

✔ includes nonequilibrium effects

✘ misses production processes

Kinetic vs. kick (big thanks to Matthias Carosi for the slides)

Two main approaches exist for studying the dynamics of a single bubble

Kinetic picture
[Moore and Propokopec '95]

Set of dynamical equations

$$\begin{cases} \Box\varphi + U'(\varphi) + \sum_i \frac{\partial m_i^2}{\partial\varphi} \int_{\vec{p}} \delta f_i(\vec{p}, x) = 0 \\ \frac{\partial f_i}{\partial t} = -\mathcal{C}_i[f, \varphi] \end{cases}$$

- ✔ includes nonequilibrium effects
- ✘ misses production processes

Kick picture
[Dine et al. '92, Bodeker and Moore '09, '17]

Pressure from the flux of particles

$$\mathcal{P}_{\text{kick}} = \sum_{i,X} \int_{\vec{p}} \partial\mathbb{P}_{i\to X}(\vec{p})\, f_i(\vec{p})\, \Delta p_{\perp\,,i\to X}$$

Kinetic vs. kick (big thanks to Matthias Carosi for the slides)

Two main approaches exist for studying the dynamics of a single bubble

Kinetic picture
[Moore and Propokopec '95]

Set of dynamical equations

$$\begin{cases} \Box\varphi + U'(\varphi) + \sum_i \frac{\partial m_i^2}{\partial\varphi} \int_{\vec{p}} \delta f_i(\vec{p}, x) = 0 \\ \frac{\partial f_i}{\partial t} = -\mathcal{C}_i[f, \varphi] \end{cases}$$

✔ includes nonequilibrium effects

✘ misses production processes

Kick picture
[Dine et al. '92, Bodeker and Moore '09, '17]

Pressure from the flux of particles

$$\mathcal{P}_{\text{kick}} = \sum_{i,X} \int_{\vec{p}} \partial\mathbb{P}_{i\to X}(\vec{p})\, f_i(\vec{p})\, \Delta p_{\perp\, ,i\to X}$$

✔ includes all production processes

✘ only valid for ultrarelativistic walls

Kinetic vs. kick (big thanks to Matthias Carosi for the slides)

Two main approaches exist for studying the dynamics of a single bubble

Kinetic picture
[Moore and Propokopec '95]

Set of dynamical equations

$$\begin{cases} \Box\varphi + U'(\varphi) + \sum_i \frac{\partial m_i^2}{\partial\varphi} \int_{\vec{p}} \delta f_i(\vec{p}, x) = 0 \\ \frac{\partial f_i}{\partial t} = -\mathcal{C}_i[f, \varphi] \end{cases}$$

✔ includes nonequilibrium effects

✘ misses production processes

Kick picture
[Dine et al. '92, Bodeker and Moore '09, '17]

Pressure from the flux of particles

$$\mathcal{P}_{\text{kick}} = \sum_{i,X} \int_{\vec{p}} \partial\mathbb{P}_{i\to X}(\vec{p})\, f_i(\vec{p})\, \Delta p_{\perp\,,i\to X}$$

✔ includes all production processes

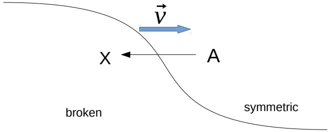
✘ only valid for ultrarelativistic walls

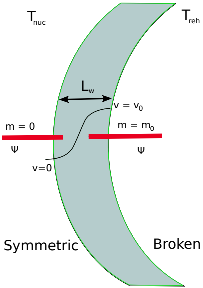
Why would we care about processes which are higher order in the couplings?

Pressure from 1 to 1 (Relativistic regime)

- $1 \to 1, A = X$ with $m_h > m_s$:

$$\frac{dE}{dz} = 0, \qquad E = \sqrt{p_\perp^2 + p_z^2 + m^2(z)}$$





Pressure from 1 to 1 (Relativistic regime)

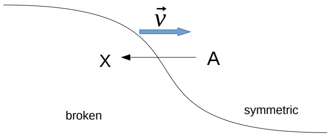
- $1 \to 1, A = X$ with $m_h > m_s$:

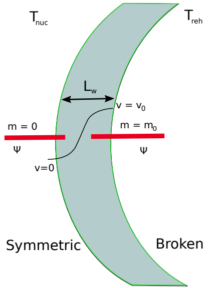
$$\frac{dE}{dz} = 0, \qquad E = \sqrt{p_\perp^2 + p_z^2 + m^2(z)}$$

- LO relativistic pressure [0903.4099]:

$$\int dP_{A\to A} \to 1, \qquad (p_h^Z - p_s^Z) \approx -\frac{\Delta m_i^2}{2E}$$

$$\Rightarrow \boxed{\mathcal{P}_{1\to 1} \to \sum_i \frac{\Delta m_i^2 T^2}{24}}, \qquad \Delta m_i^2 \equiv m_{bro,i}^2 - m_{sym,i}^2$$





Pressure from 1 to 1 (Relativistic regime)

- $1 \to 1, A = X$ with $m_h > m_s$:

$$\frac{dE}{dz} = 0, \qquad E = \sqrt{p_\perp^2 + p_z^2 + m^2(z)}$$

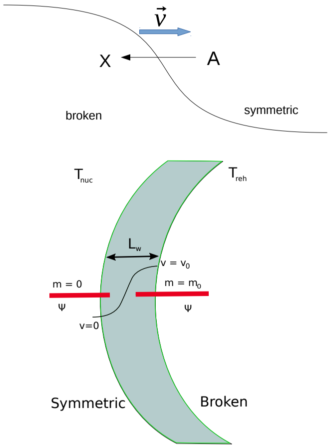
- LO relativistic pressure [0903.4099]:

$$\int dP_{A \to A} \to 1, \qquad (p_h^Z - p_s^Z) \approx -\frac{\Delta m_i^2}{2E}$$

$$\Rightarrow \boxed{\mathcal{P}_{1\to 1} \to \sum_i \frac{\Delta m_i^2 T^2}{24}}, \qquad \Delta m_i^2 \equiv m_{bro,i}^2 - m_{sym,i}^2$$

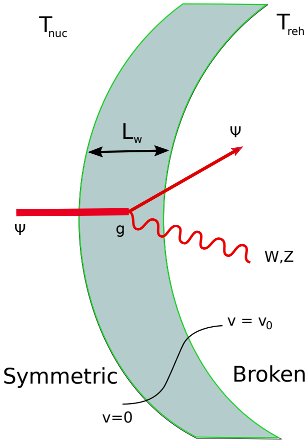
- Entering in the wall ? need $p_{z,s} \sim \gamma T > \Delta m$

$$\boxed{\gamma T \gg \Delta m}$$



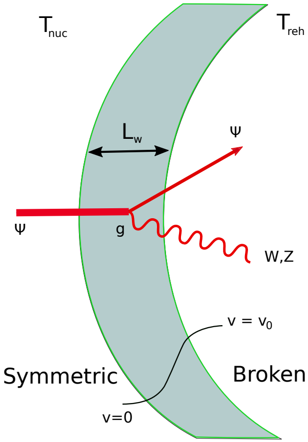
Pressure by splitting

- NLO relativistic pressure; gauge bosons V [1703.08215]



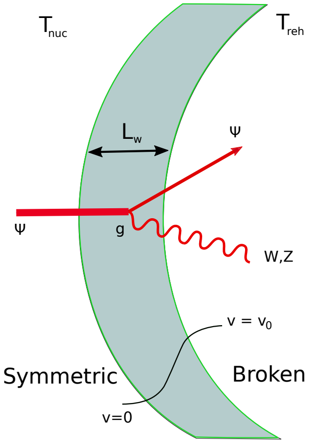
Pressure by splitting

- NLO relativistic pressure; gauge bosons V [1703.08215]
- $$\int dP_{A\to VA} \sim \begin{cases} 0 & \text{if TS} \\ \frac{g^3 v}{16\pi^2} & \text{Wall breaks TS} \end{cases}$$



Pressure by splitting

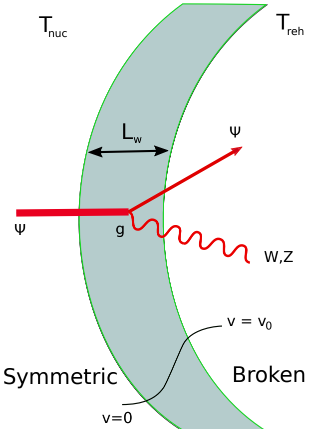
- NLO relativistic pressure; gauge bosons V [1703.08215]
- $$\int dP_{A\to VA} \sim \begin{cases} 0 & \text{if TS} \\ \frac{g^3 v}{16\pi^2} & \text{Wall breaks TS} \end{cases}$$
- Soft bosons emission $\Delta p \sim m_V$



Pressure by splitting

- NLO relativistic pressure; gauge bosons V [1703.08215]
-
$$\int dP_{A\to VA} \sim \begin{cases} 0 & \text{if TS} \\ \frac{g^3 v}{16\pi^2} & \text{Wall breaks TS} \end{cases}$$
- Soft bosons emission $\Delta p \sim m_V$
- Pressure induced

$$\Rightarrow \boxed{\mathcal{P}_{1\to 2} \sim \sum_i g_i \frac{g^3 v}{16\pi^2} \gamma T^3}$$

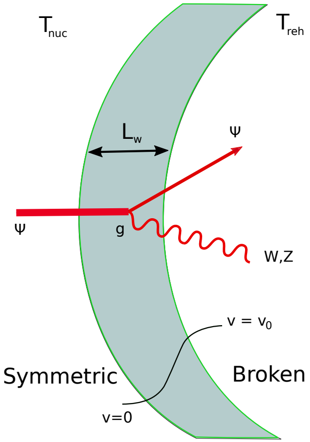


Pressure by splitting

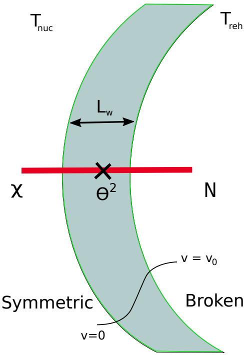
- NLO relativistic pressure; gauge bosons V [1703.08215]
-
$$\int dP_{A\to VA} \sim \begin{cases} 0 & \text{if TS} \\ \frac{g^3 v}{16\pi^2} & \text{Wall breaks TS} \end{cases}$$
- Soft bosons emission $\Delta p \sim m_V$
- Pressure induced

$$\Rightarrow \boxed{\mathcal{P}_{1\to 2} \sim \sum_i g_i \frac{g^3 v}{16\pi^2} \gamma T^3}$$

- What about $1 \to N$??: claim $\mathcal{P}_{1\to N} \sim \alpha\gamma^2 T^4$ in [arXiv:2007.10343] but disputed in [arXiv:2010.02590] and in [arXiv:2112.07686]: $\mathcal{P}_{1\to N} \approx \mathcal{P}_{1\to 2} \times \log \frac{v}{T}$

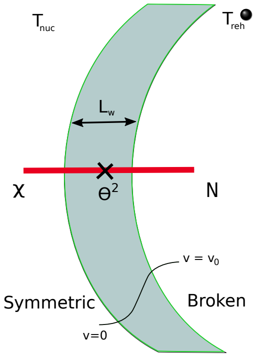


Pressure by mixing with heavy states[2010.02590]



- Toy model; $\mathcal{L}_{int} = Y\phi\bar{\chi}N + M_N\bar{N}N, \quad M_N \gg T_{nuc}$

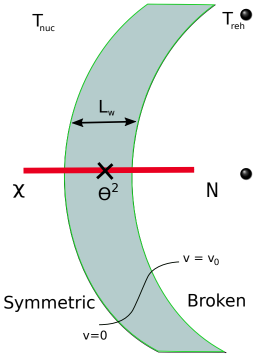
Pressure by mixing with heavy states[2010.02590]



- Toy model; $\mathcal{L}_{int} = Y\phi\bar{\chi}N + M_N\bar{N}N, \quad M_N \gg T_{nuc}$
- Probability of transition

$$\Delta P_{\chi\to N} \sim \frac{Y^2 v^2}{M_N^2}, \qquad \langle\phi\rangle \equiv v$$

Pressure by mixing with heavy states[2010.02590]

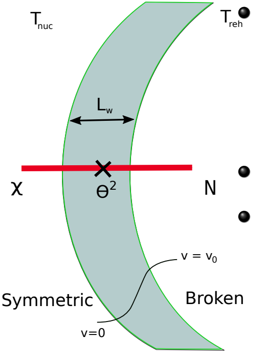


- Toy model; $\mathcal{L}_{int} = Y\phi\bar{\chi}N + M_N\bar{N}N, \quad M_N \gg T_{nuc}$
- Probability of transition

$$\Delta P_{\chi\to N} \sim \frac{Y^2v^2}{M_N^2}, \qquad \langle\phi\rangle \equiv v$$

- $p_\chi = (E,0,0,E) \qquad p_N = (E,0,0,\sqrt{E^2 - M_N^2}) \qquad \Rightarrow \qquad \Delta p \sim \frac{M_N^2}{2\gamma T}$

Pressure by mixing with heavy states[2010.02590]

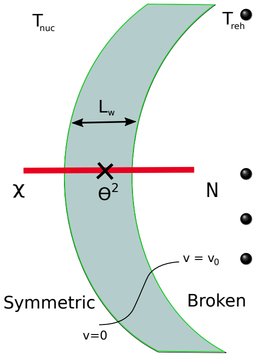


- Toy model; $\mathcal{L}_{int} = Y\phi\bar{\chi}N + M_N\bar{N}N, \quad M_N \gg T_{nuc}$
- Probability of transition

$$\Delta P_{\chi\to N} \sim \frac{Y^2v^2}{M_N^2}, \qquad \langle\phi\rangle \equiv v$$

- $p_\chi = (E,0,0,E) \qquad p_N = (E,0,0,\sqrt{E^2 - M_N^2}) \qquad \Rightarrow \qquad \Delta p \sim \frac{M_N^2}{2\gamma T}$
- Non-adiabatic regime; $\Delta p L_w < 1 \quad \Rightarrow$ multiply by $\Theta(\gamma T - M_N^2 L_w)$

Pressure by mixing with heavy states[2010.02590]



- Toy model; $\mathcal{L}_{int} = Y\phi\bar{\chi}N + M_N\bar{N}N, \quad M_N \gg T_{nuc}$
- Probability of transition

$$\Delta P_{\chi\to N} \sim \frac{Y^2v^2}{M_N^2}, \qquad \langle\phi\rangle \equiv v$$

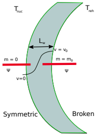
- $p_\chi = (E,0,0,E) \qquad p_N = (E,0,0,\sqrt{E^2 - M_N^2}) \qquad \Rightarrow \quad \Delta p \sim \frac{M_N^2}{2\gamma T}$
- Non-adiabatic regime; $\Delta p L_w < 1 \quad \Rightarrow$ multiply by $\Theta(\gamma T - M_N^2 L_w)$
- Pressure on the wall

$$\Delta\mathcal{P}_{mixing} \approx \frac{Y^2T^2v^2}{48}\Theta(\gamma T - M^2L_w)$$

Pressure depends on M only in the Θ-function

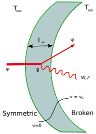
How to compute the velocity of the bubble wall ? Contributions

Mass Gain pressure [0903.4099], BM



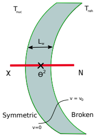
$$\Delta\mathcal{P}_{LO} \approx \sum_i \frac{\Delta m_i^2 T^2}{24}, \quad \Delta m_i^2 \equiv m^2_{bro,i} - m^2_{sym,i}$$

Soft bosons emission pressure [1703.08215], BM



$$\Delta\mathcal{P}_{NLO} \sim \sum_i g_i \frac{g^3_{\text{gauge}} v}{16\pi^2} \gamma_{wp} T^3$$

Mixing pressure [2010.02590] $\Delta p_z \sim \frac{M^2}{2E}$



$$\Delta\mathcal{P}_{\text{mixing}} \sim \frac{Y^2 v^2 T^2}{48} \Theta(\gamma_{wp} T_{\text{nuc}} - M^2 L_w)$$

Regime of expansion:

$\boxed{\Delta V > \Delta\mathcal{P}_{\text{tot}} : \quad \text{Runaway}}$ VS $\boxed{\Delta V = \Delta\mathcal{P}_{\text{tot}}(\gamma = \gamma_{\text{ter}}) : \quad \text{Terminal velocity}}$

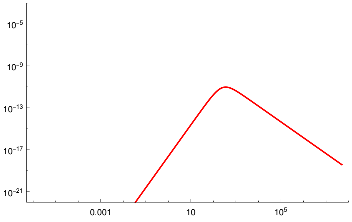
Different regimes give different GW signal

Transition strong enough for relativistic walls : $\boxed{\Delta V > \Delta \mathcal{P}_{LO}}$

Runaway regime: acceleration until collision

$\Downarrow$

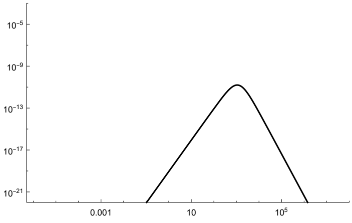
Energy in the thin bubble wall: *scalar field*



Terminal velocity regime: $\gamma \to \gamma^{MAX}$

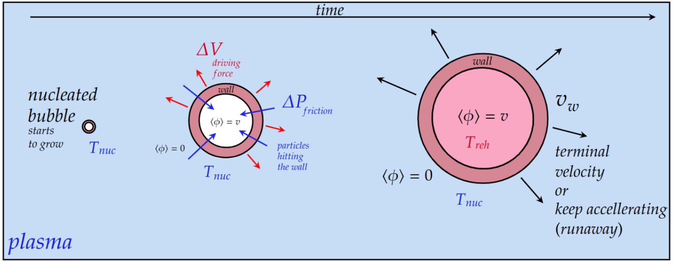
$\Downarrow$

Energy in the plasma through *sound waves*



Observable effects ??

Bubble dynamics from first principles



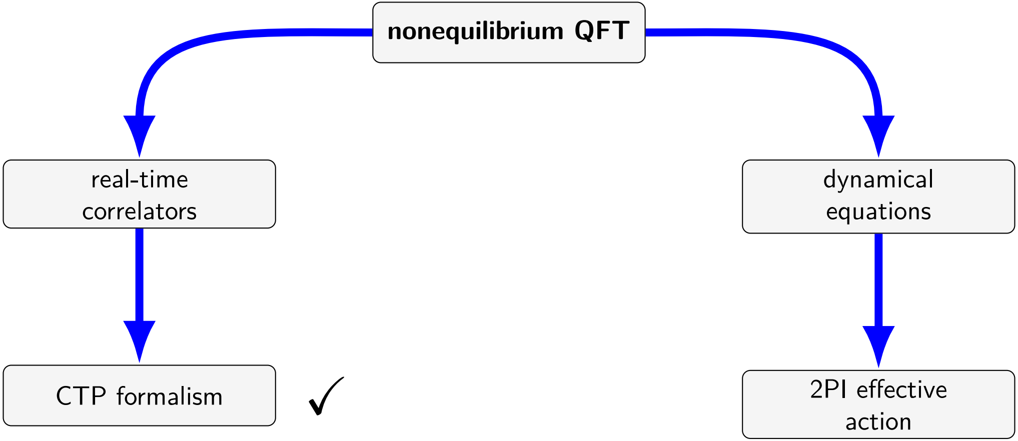
Bubble dynamics from first principles

[arXiv: 2504.13725] with *Wen-Yuan Ai*, *Matthias Carosi*, *Bjorn Garbrecht* and *Carlos Tamarit*

Bubble dynamics from first principles

Goal: develop a description of the bubble expansion from first principles using *nonequilibrium quantum field theory* and bridge the gap between kinetic and kick approaches.

The tools of nonequilibrium QFT



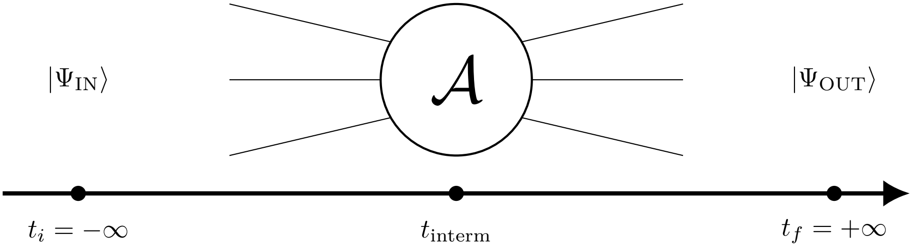
The in-out formalism

The in-out formalism

The path integral formulation of QFT is built to study transition rates: the *in-out formalism*

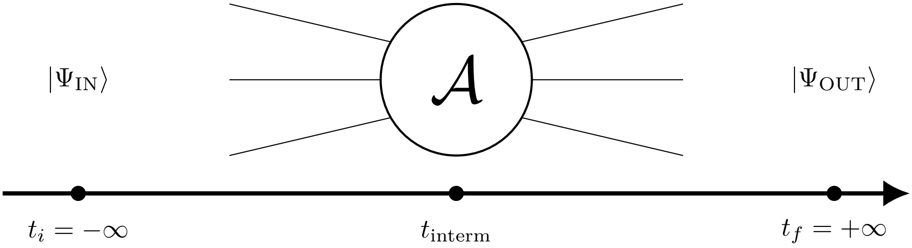
The in-out formalism

The path integral formulation of QFT is built to study transition rates: the *in-out formalism*



The in-out formalism

The path integral formulation of QFT is built to study transition rates: the *in-out formalism*

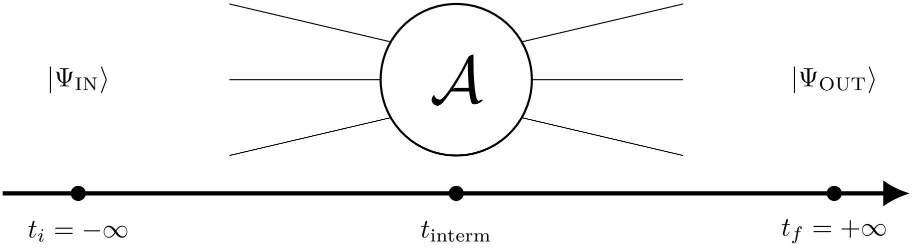


Using it, we compute transition amplitudes between **asymptotic states**

$$\mathcal{A} = \langle \Psi_{\mathrm{OUT}} | \, \mathcal{O}(\hat{\phi}) \, | \Psi_{\mathrm{IN}} \rangle = \mathcal{N} \int [\mathcal{D}\phi] \, \Psi^*_{\mathrm{OUT}}(\phi) \mathcal{O}(\phi) \Psi_{\mathrm{IN}}(\phi) e^{iS[\phi]}$$

The in-out formalism

The path integral formulation of QFT is built to study transition rates: the *in-out formalism*



Using it, we compute transition amplitudes between **asymptotic states**

$$\mathcal{A} = \langle \Psi_{\rm OUT} | \, \mathcal{O}(\hat{\phi}) \, | \Psi_{\rm IN} \rangle = \mathcal{N} \int [\mathcal{D}\phi] \, \Psi^*_{\rm OUT}(\phi) \mathcal{O}(\phi) \Psi_{\rm IN}(\phi) e^{iS[\phi]}$$

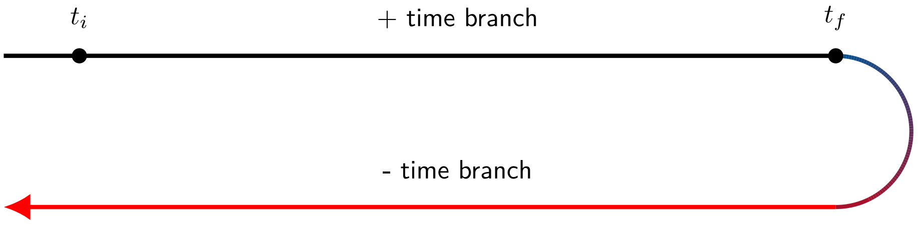
But how can we compute time (and space) dependent correlators?

The CTP formalism

We obtain the in-in, or *closed time path* (CTP) formalism

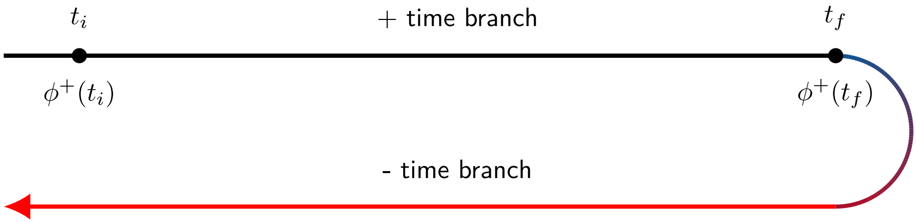
The CTP formalism

We obtain the in-in, or *closed time path* (CTP) formalism



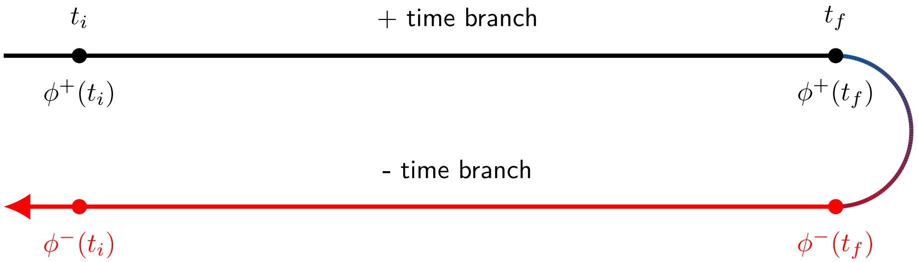
The CTP formalism

We obtain the in-in, or *closed time path* (CTP) formalism



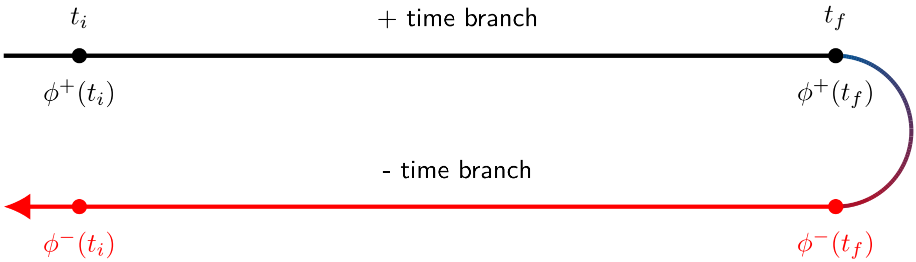
The CTP formalism

We obtain the in-in, or *closed time path* (CTP) formalism



The CTP formalism

We obtain the in-in, or *closed time path* (CTP) formalism



We introduce the label $\pm$ for the time branch, double our degrees of freedom, and can now use all the tools from the path integral formalism.

The CTP formalism

The CTP formalism

Time ordering is replaced by ordering on the CTP $\mathcal{T} \to \mathcal{T}_{\mathrm{CTP}}$.

The CTP formalism

Time ordering is replaced by ordering on the CTP $\mathcal{T} \to \mathcal{T}_{\mathrm{CTP}}$. We then have four two-point functions

The CTP formalism

Time ordering is replaced by ordering on the CTP $\mathcal{T} \to \mathcal{T}_{\rm CTP}$. We then have four two-point functions

$$\langle \mathcal{T}_{\rm CTP}\phi^+(x)\phi^+(y)\rangle = \langle \mathcal{T}\phi(x)\phi(y)\rangle = \Delta^T(x,y)$$

The CTP formalism

Time ordering is replaced by ordering on the CTP $\mathcal{T} \to \mathcal{T}_{\mathrm{CTP}}$. We then have four two-point functions

$$\langle \mathcal{T}_{\mathrm{CTP}} \phi^{+}(x)\phi^{+}(y)\rangle = \langle \mathcal{T}\phi(x)\phi(y)\rangle = \Delta^{T}(x,y)$$

$$\langle \mathcal{T}_{\mathrm{CTP}} \phi^{-}(x)\phi^{-}(y)\rangle = \langle \overline{\mathcal{T}}\phi(x)\phi(y)\rangle = \Delta^{\overline{T}}(x,y)$$

The CTP formalism

Time ordering is replaced by ordering on the CTP $\mathcal{T} \to \mathcal{T}_{\mathrm{CTP}}$. We then have four two-point functions

$$\langle \mathcal{T}_{\mathrm{CTP}} \phi^+(x)\phi^+(y)\rangle = \langle \mathcal{T} \phi(x)\phi(y)\rangle = \Delta^{T}(x,y)$$

$$\langle \mathcal{T}_{\mathrm{CTP}} \phi^-(x)\phi^-(y)\rangle = \langle \overline{\mathcal{T}} \phi(x)\phi(y)\rangle = \Delta^{\overline{T}}(x,y)$$

$$\langle \mathcal{T}_{\mathrm{CTP}} \phi^+(x)\phi^-(y)\rangle = \langle \phi(y)\phi(x)\rangle = \Delta^{<}(x,y)$$

The CTP formalism

Time ordering is replaced by ordering on the CTP $\mathcal{T} \to \mathcal{T}_{\rm CTP}$. We then have four two-point functions

$$\langle \mathcal{T}_{\rm CTP} \phi^+(x)\phi^+(y)\rangle = \langle \mathcal{T}\phi(x)\phi(y)\rangle = \Delta^T(x,y)$$

$$\langle \mathcal{T}_{\rm CTP} \phi^-(x)\phi^-(y)\rangle = \langle \overline{\mathcal{T}}\phi(x)\phi(y)\rangle = \Delta^{\overline{T}}(x,y)$$

$$\langle \mathcal{T}_{\rm CTP} \phi^+(x)\phi^-(y)\rangle = \langle \phi(y)\phi(x)\rangle = \Delta^<(x,y)$$

$$\langle \mathcal{T}_{\rm CTP} \phi^-(x)\phi^+(y)\rangle = \langle \phi(x)\phi(y)\rangle = \Delta^>(x,y)$$

The CTP formalism

Time ordering is replaced by ordering on the CTP $\mathcal{T} \to \mathcal{T}_{\text{CTP}}$. We then have four two-point functions

$$\langle \mathcal{T}_{\text{CTP}} \phi^+(x)\phi^+(y)\rangle = \langle \mathcal{T}\phi(x)\phi(y)\rangle = \Delta^T(x,y)$$

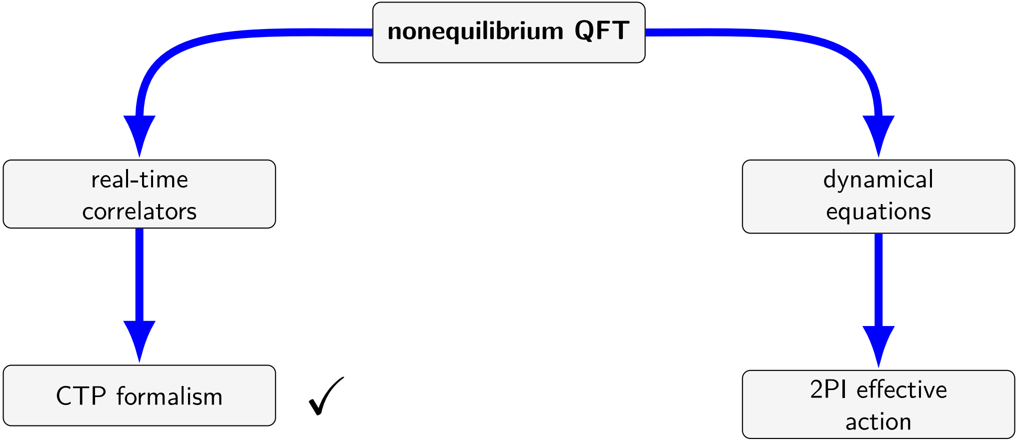
$$\langle \mathcal{T}_{\text{CTP}} \phi^-(x)\phi^-(y)\rangle = \langle \overline{\mathcal{T}}\phi(x)\phi(y)\rangle = \Delta^{\overline{T}}(x,y)$$

$$\langle \mathcal{T}_{\text{CTP}} \phi^+(x)\phi^-(y)\rangle = \langle \phi(y)\phi(x)\rangle = \Delta^<(x,y)$$

$$\langle \mathcal{T}_{\text{CTP}} \phi^-(x)\phi^+(y)\rangle = \langle \phi(x)\phi(y)\rangle = \Delta^>(x,y)$$

We can then do perturbation theory, but in particular, we will be interested in generating equations of motion. For this, we work with the effective action.

The tools of nonequilibrium QFT



Introducing the 2PI effective action

We introduce the generator of connected one- point functions

$$e^{W[J\quad]} = Z[J\quad] = \int [\mathcal{D}\phi]\, e^{iS[\phi]+\int_x J(x)\phi(x)}$$

Introducing the 2PI effective action

We introduce the generator of connected one- point functions

$$e^{W[J]} = Z[J] = \int [\mathcal{D}\phi]\, e^{iS[\phi]+\int_x J(x)\phi(x)}$$

and define the one-particle-irreducible (1PI) effective action

$$\Gamma_{1\mathrm{PI}}[\varphi] = \max_J \; -W[J] + \int_x J(x)\varphi(x)$$

Introducing the 2PI effective action

We introduce the generator of connected one- and two-point functions

$$e^{W[J,R]} = Z[J,R] = \int [\mathcal{D}\phi]\, e^{iS[\phi]+\int_x J(x)\phi(x)+\frac{1}{2}\int_{x,y}\phi(x)R(x,y)\phi(y)}$$

and define the one-particle-irreducible (1PI) effective action

$$\Gamma_{1\mathrm{PI}}[\varphi] = \max_J \; -W[J] + \int_x J(x)\varphi(x)$$

Introducing the 2PI effective action

We introduce the generator of connected one- and two-point functions

$$e^{W[J,R]} = Z[J,R] = \int [\mathcal{D}\phi]\, e^{iS[\phi]+\int_x J(x)\phi(x)+\frac{1}{2}\int_{x,y}\phi(x)R(x,y)\phi(y)}$$

and define the two-particle-irreducible (2PI) effective action

$$\Gamma_{2\mathrm{PI}}[\varphi,\Delta] = \max_{J,R} \; -W[J,R] + \int_x J(x)\varphi(x) + \frac{1}{2}\int_{x,y}\Delta(x,y)R(x,y)$$

Introducing the 2PI effective action

We introduce the generator of connected one- and two-point functions

$$e^{W[J,R]} = Z[J,R] = \int [\mathcal{D}\phi]\, e^{iS[\phi]+\int_x J(x)\phi(x)+\frac{1}{2}\int_{x,y}\phi(x)R(x,y)\phi(y)}$$

and define the two-particle-irreducible (2PI) effective action

$$\Gamma_{2\mathrm{PI}}[\varphi,\Delta] = \max_{J,R} \; -W[J,R] + \int_x J(x)\varphi(x) + \frac{1}{2}\int_{x,y}\Delta(x,y)R(x,y)$$

Equations for the one- and two-point functions are then easily generated

$$\frac{\delta\Gamma_{2\mathrm{PI}}}{\delta\varphi(x)} = 0 \qquad\qquad \frac{\delta\Gamma_{2\mathrm{PI}}}{\delta\Delta(x,y)} = 0$$

The tools of nonequilibrium QFT

Perturbative expansion

2PI effective action $\quad + \quad$ CTP formalism $\quad \Longrightarrow \quad$ out-of-equilibrium dynamical equations

Perturbative expansion

2PI effective action $+$ CTP formalism $\Longrightarrow$ out-of-equilibrium dynamical equations

Loop expansion of the effective action:

$$\Gamma_{2\mathrm{PI}}[\varphi^a, \Delta^{ab}] = S[\varphi^+] - S[\varphi^-] + \frac{i}{2}\,\mathrm{tr}\log\Delta^{-1} + \frac{i}{2}\,\mathrm{tr}\,G_\varphi^{-1}\Delta + \Gamma_2[\varphi^a, \Delta^{ab}]$$

and the trace runs over the CTP indices as well.

Perturbative expansion

2PI effective action $+$ CTP formalism $\Longrightarrow$ out-of-equilibrium dynamical equations

Loop expansion of the effective action:

$$\Gamma_{2\mathrm{PI}}[\varphi^a, \Delta^{ab}] = S[\varphi^+] - S[\varphi^-] + \frac{i}{2}\operatorname{tr}\log \Delta^{-1} + \frac{i}{2}\operatorname{tr} G_\varphi^{-1}\Delta + \Gamma_2[\varphi^a, \Delta^{ab}]$$

and the trace runs over the CTP indices as well. The inverse tree-level propagator reads

$$G_\varphi^{ab,-1}(x,y) = i\delta^{(4)}(x-y)a\delta^{ab}\left(\Box + U''(\varphi^a)\right)$$

Perturbative expansion

2PI effective action $+$ CTP formalism $\Longrightarrow$ out-of-equilibrium dynamical equations

Loop expansion of the effective action:

$$\Gamma_{2\mathrm{PI}}[\varphi^a, \Delta^{ab}] = S[\varphi^+] - S[\varphi^-] + \frac{i}{2}\,\mathrm{tr}\log\Delta^{-1} + \frac{i}{2}\,\mathrm{tr}\,G_\varphi^{-1}\Delta + \Gamma_2[\varphi^a, \Delta^{ab}]$$

and the trace runs over the CTP indices as well. The inverse tree-level propagator reads

$$G_\varphi^{ab,-1}(x,y) = i\delta^{(4)}(x-y)a\delta^{ab}\left(\Box + U''(\varphi^a)\right)$$

All terms of loop order larger than two are inside Γ_2

$$\Gamma_2 \supset \text{ two-particle-irreducible vacuum diagrams with two or more loops}$$

The full dynamical equations

The full dynamical equations

$$\Gamma_{2\mathrm{PI}}[\varphi^a, \Delta^{ab}] = S[\varphi^+] - S[\varphi^-] + \frac{i}{2}\,\mathrm{tr}\log\Delta^{-1} + \frac{i}{2}\,\mathrm{tr}\,G_0^{-1}\Delta + \Gamma_2[\varphi^a, \Delta^{ab}]$$

The full dynamical equations

$$\Gamma_{2\mathrm{PI}}[\varphi^a, \Delta^{ab}] = S[\varphi^+] - S[\varphi^-] + \frac{i}{2}\,\mathrm{tr}\log\Delta^{-1} + \frac{i}{2}\,\mathrm{tr}\,G_0^{-1}\Delta + \Gamma_2[\varphi^a, \Delta^{ab}]$$

$$\boxed{\frac{\delta\Gamma_{2\mathrm{PI}}}{\delta\varphi^+(x)} = 0}$$

$$\Big\downarrow$$

bubble EoM

The full dynamical equations

$$\Gamma_{2\mathrm{PI}}[\varphi^a, \Delta^{ab}] = S[\varphi^+] - S[\varphi^-] + \frac{i}{2}\,\mathrm{tr}\log\Delta^{-1} + \frac{i}{2}\,\mathrm{tr}\,G_0^{-1}\Delta + \Gamma_2[\varphi^a, \Delta^{ab}]$$

$$\boxed{\frac{\delta\Gamma_{2\mathrm{PI}}}{\delta\varphi^+(x)} = 0} \quad\longrightarrow\quad \text{bubble EoM}$$

$$\boxed{\frac{\delta\Gamma_{2\mathrm{PI}}}{\delta\Delta^{ab}(x,y)} = 0} \quad\longrightarrow\quad \text{Schwinger-Dyson eqs. / Boltzmann eqs.}$$

The full dynamical equations

$$\Gamma_{2\mathrm{PI}}[\varphi^a, \Delta^{ab}] = S[\varphi^+] - S[\varphi^-] + \frac{i}{2}\,\mathrm{tr}\log\Delta^{-1} + \frac{i}{2}\,\mathrm{tr}\,G_0^{-1}\Delta + \Gamma_2[\varphi^a, \Delta^{ab}]$$

$$\boxed{\frac{\delta\Gamma_{2\mathrm{PI}}}{\delta\varphi^+(x)} = 0} \qquad \boxed{\frac{\delta\Gamma_{2\mathrm{PI}}}{\delta\Delta^{ab}(x,y)} = 0}$$

$$\Downarrow \qquad\qquad\qquad\qquad \Downarrow$$

bubble EoM

Schwinger-Dyson eqs.
Boltzmann eqs.

We obtain a system of coupled equations that include all **thermal**, **quantum** and **nonequilibrium** effects at the given perturbative order.

The bubble wall equation of motion

The bubble wall equation of motion

Having solved for the two-point function at leading order in the gradients, we have the EoM for the bubble wall

$$\Box\varphi(x) + U_0'(\varphi(x)) + \frac{1}{2}\frac{\partial m_\varphi^2}{\partial\varphi(x)}\int\frac{\partial^4 k}{(2\pi)^4}\overline{\Delta}^T(k,x) + \int_y \underbrace{\Pi^{\mathrm{R}}(x,y)}_{\sim\frac{\delta^2\Gamma_2}{\delta\varphi(x)\delta\varphi(y)}}\varphi(y) = 0$$

The bubble wall equation of motion

Having solved for the two-point function at leading order in the gradients, we have the EoM for the bubble wall

$$\Box\varphi(x) + U_0'(\varphi(x)) + \frac{1}{2}\frac{\partial m_\varphi^2}{\partial\varphi(x)}\int\frac{\partial^4 k}{(2\pi)^4}\overline{\Delta}^T(k,x) + \int_y \Pi^{\mathrm{R}}(x,y)\varphi(y) = 0$$

- One-loop term

$$\frac{1}{2}\frac{\partial m_\varphi^2}{\partial\varphi(x)}\int\frac{\partial^4 k}{(2\pi)^4}\overline{\Delta}^T(k,x) = \frac{\lambda}{2}\varphi(x)\int\frac{\partial^4 k}{(2\pi)^4}\frac{i}{k^2-m^2+i\epsilon} + \frac{\partial m_\varphi^2}{\partial\varphi(x)}\int\frac{\partial^3\vec{k}}{(2\pi)^3 2E_{\vec{k}}}f(\vec{k},x)$$

The bubble wall equation of motion

Having solved for the two-point function at leading order in the gradients, we have the EoM for the bubble wall

$$\Box\varphi(x) + U_0'(\varphi(x)) + \frac{1}{2}\frac{\partial m_\varphi^2}{\partial\varphi(x)}\int\frac{\partial^4 k}{(2\pi)^4}\overline{\Delta}^T(k,x) + \int_y \Pi^{\mathrm{R}}(x,y)\varphi(y) = 0$$

- One-loop term

$$\frac{1}{2}\frac{\partial m_\varphi^2}{\partial\varphi(x)}\int\frac{\partial^4 k}{(2\pi)^4}\overline{\Delta}^T(k,x) = \underbrace{\frac{\lambda}{2}\varphi(x)\int\frac{\partial^4 k}{(2\pi)^4}\frac{i}{k^2-m^2+i\epsilon}}_{\text{T=0 correction to } U_0'} + \frac{\partial m_\varphi^2}{\partial\varphi(x)}\int\frac{\partial^3\vec{k}}{(2\pi)^3 2E_{\vec{k}}}f(\vec{k},x)$$

The bubble wall equation of motion

Having solved for the two-point function at leading order in the gradients, we have the EoM for the bubble wall

$$\Box\varphi(x) + U_0'(\varphi(x)) + \frac{1}{2}\frac{\partial m_\varphi^2}{\partial\varphi(x)}\int\frac{\partial^4 k}{(2\pi)^4}\overline{\Delta}^T(k,x) + \int_y \Pi^{\mathrm{R}}(x,y)\varphi(y) = 0$$

- One-loop term

$$\frac{1}{2}\frac{\partial m_\varphi^2}{\partial\varphi(x)}\int\frac{\partial^4 k}{(2\pi)^4}\overline{\Delta}^T(k,x) = \underbrace{\frac{\lambda}{2}\varphi(x)\int\frac{\partial^4 k}{(2\pi)^4}\frac{i}{k^2-m^2+i\epsilon}}_{\text{T=0 correction to } U_0'} + \underbrace{\frac{\partial m_\varphi^2}{\partial\varphi(x)}\int\frac{\partial^3\vec{k}}{(2\pi)^3 2E_{\vec{k}}}f(\vec{k},x)}_{f=f_{\mathrm{eq}}+\delta f=(T\neq 0)\text{ corr. } + \text{ off-eq.}}$$

The bubble wall equation of motion

Having solved for the two-point function at leading order in the gradients, we have the EoM for the bubble wall

$$\Box\varphi(x) + U_0'(\varphi(x)) + \frac{1}{2}\frac{\partial m_\varphi^2}{\partial\varphi(x)}\int\frac{\partial^4 k}{(2\pi)^4}\overline{\Delta}^T(k,x) + \int_y \Pi^{\mathrm{R}}(x,y)\varphi(y) = 0$$

- One-loop term

$$\frac{1}{2}\frac{\partial m_\varphi^2}{\partial\varphi(x)}\int\frac{\partial^4 k}{(2\pi)^4}\overline{\Delta}^T(k,x) = \frac{\lambda}{2}\varphi(x)\int\frac{\partial^4 k}{(2\pi)^4}\frac{i}{k^2 - m^2 + i\epsilon} + \frac{\partial m_\varphi^2}{\partial\varphi(x)}\int\frac{\partial^3\vec{k}}{(2\pi)^3 2E_{\vec{k}}} f(\vec{k},x)$$

- Packing it into a more familiar form

$$\Box\varphi(x) + U'(\varphi(x)) + \sum_i \frac{\partial m_i^2}{\partial\varphi(x)}\int_{\mathbf{p}}\delta f_i(\mathbf{p},x) + \int_y \Pi^{\mathrm{R}}(x,y)\varphi(y) = 0$$

The bubble wall equation of motion

Having solved for the two-point function at leading order in the gradients, we have the EoM for the bubble wall

$$\Box\varphi(x) + U_0'(\varphi(x)) + \frac{1}{2}\frac{\partial m_\varphi^2}{\partial\varphi(x)}\int\frac{\partial^4 k}{(2\pi)^4}\overline{\Delta}^T(k,x) + \int_y \Pi^{\mathrm{R}}(x,y)\varphi(y) = 0$$

- One-loop term

recall kinetic approach: $\Box\varphi + U'(\varphi) + \sum_i \frac{\partial m_i^2}{\partial\varphi}\int_{\vec{p}}\delta f_i(\vec{p},x) = 0$

$$\frac{1}{2}\frac{\partial m_\varphi^2}{\partial\varphi(x)}\int\frac{}{(2\pi)^4}\Delta(k,x) = \frac{}{2}\varphi(x)\int\frac{}{(2\pi)^4}\frac{}{k^2-m^2+i\epsilon} + \frac{}{\partial\varphi(x)}\int\frac{\vec{k}}{(2\pi)^3 2E_{\vec{k}}}f(\vec{k},x)$$

- Packing it into a more familiar form

$$\Box\varphi(x) + U'(\varphi(x)) + \sum_i \frac{\partial m_i^2}{\partial\varphi(x)}\int_{\mathbf{p}}\delta f_i(\mathbf{p},x) + \int_y \Pi^{\mathrm{R}}(x,y)\varphi(y) = 0$$

The bubble wall equation of motion

Having solved for the two-point function at leading order in the gradients, we have the EoM for the bubble wall

$$\Box\varphi(x) + U_0'(\varphi(x)) + \frac{1}{2}\frac{\partial m_\varphi^2}{\partial\varphi(x)}\int\frac{\partial^4 k}{(2\pi)^4}\overline{\Delta}^T(k,x) + \int_y \Pi^{\mathrm{R}}(x,y)\varphi(y) = 0$$

- One-loop term

recall kinetic approach: $\Box\varphi + U'(\varphi) + \sum_i \frac{\partial m_i^2}{\partial\varphi}\int_{\vec{p}}\delta f_i(\vec{p},x) = 0$

$$\frac{1}{2}\frac{\partial m_\varphi^2}{\partial\varphi(x)}\int\frac{}{(2\pi)^4}\Delta\ (k,x) = \frac{}{2}\varphi(x)\int\frac{}{(2\pi)^4}\frac{}{k^2-m^2+i\epsilon} + \frac{}{\partial\varphi(x)}\int\frac{\vec{k}}{(2\pi)^3 2E_{\vec{k}}}f(\vec{k},x)$$

- Packing it into a more familiar form

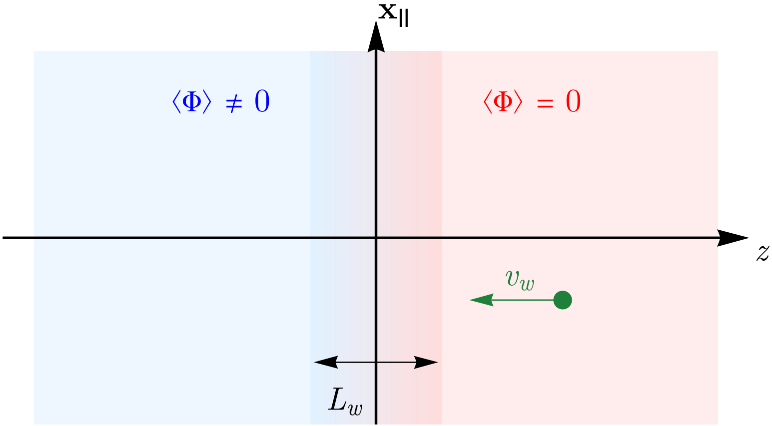
$$\Box\varphi(x) + U'(\varphi(x)) + \sum_i \frac{\partial m_i^2}{\partial\varphi(x)}\int_{\mathbf{p}}\delta f_i(\mathbf{p},x) + \int_y NewTerm\,\Pi^{\mathrm{R}}(x,y)\varphi(y) = 0$$

NEW TERM!

Identifying sources of friction

Identifying sources of friction

In the *stationary planar* wall limit $\varphi(x) = \varphi(z)$, with the wall centered at $z = 0$ **in the wall frame**



Identifying sources of friction

In the *stationary planar* wall limit $\varphi(x) = \varphi(z)$, with the wall centered at $z = 0$ **in the wall frame**

$$\frac{\partial^2}{\partial z^2}\varphi(z) + U'(\varphi(z), T) + \frac{\partial m_\varphi^2}{\partial \varphi(z)} \int_{\vec{k}} \delta f(\vec{k}, z) + \int \partial z' \underbrace{\pi^R(z, z')}_{\int \partial \vec{x}_\parallel \partial \vec{x'}_\parallel \Pi^R(\vec{x}, \vec{x}')} \varphi(z') \quad = 0$$

Identifying sources of friction

In the *stationary planar* wall limit $\varphi(x) = \varphi(z)$, with the wall centered at $z = 0$ **in the wall frame**

$$\int_{-\delta}^{\delta} \partial z \, \frac{\partial}{\partial z}\varphi(z) \left(\frac{\partial^2}{\partial z^2}\varphi(z) + U'(\varphi(z), T) + \frac{\partial m_\varphi^2}{\partial \varphi(z)} \int_{\vec{k}} \delta f(\vec{k}, z) + \int \partial z' \, \pi^R(z, z')\varphi(z') \right) = 0$$

Identifying sources of friction

In the *stationary planar* wall limit $\varphi(x) = \varphi(z)$, with the wall centered at $z = 0$ **in the wall frame**

$$\int_{-\delta}^{\delta} \partial z \, \frac{\partial}{\partial z}\varphi(z) \left(\frac{\partial^2}{\partial z^2}\varphi(z) + U'(\varphi(z), T) + \frac{\partial m_\varphi^2}{\partial \varphi(z)} \int_{\vec{k}} \delta f(\vec{k}, z) + \int \partial z' \, \pi^R(z, z')\varphi(z') \right) = 0$$

$$\Rightarrow \int_{-\delta}^{\delta} \partial z \, \frac{\partial}{\partial z}\left[\frac{1}{2}\left(\frac{\partial \varphi(z)}{\partial z}\right)^2 + U(\varphi(z), T)\right] = \int_{-\delta}^{\delta} \partial z \left[\frac{\partial U}{\partial T}\frac{\partial T}{\partial z} - \frac{\partial m_\varphi^2}{\partial z} \int_{\vec{k}} \delta f(\vec{k}, z) - \frac{\partial \varphi(z)}{\partial z} \int \partial z' \, \pi^R(z, z')\varphi(z') \right]$$

Identifying sources of friction

In the *stationary planar* wall limit $\varphi(x) = \varphi(z)$, with the wall centered at $z = 0$ **in the wall frame**

$$\int_{-\delta}^{\delta} \partial z \, \frac{\partial}{\partial z}\varphi(z) \left(\frac{\partial^2}{\partial z^2}\varphi(z) + U'(\varphi(z), T) + \frac{\partial m_\varphi^2}{\partial \varphi(z)} \int_{\vec{k}} \delta f(\vec{k}, z) + \int \partial z' \, \pi^R(z, z')\varphi(z') \right) = 0$$

$$\Rightarrow \int_{-\delta}^{\delta} \partial z \, \frac{\partial}{\partial z} \left[\frac{1}{2} \left(\frac{\partial \varphi(z)}{\partial z} \right)^2 + U(\varphi(z), T) \right] = \int_{-\delta}^{\delta} \partial z \left[\frac{\partial U}{\partial T} \frac{\partial T}{\partial z} - \frac{\partial m_\varphi^2}{\partial z} \int_{\vec{k}} \delta f(\vec{k}, z) - \frac{\partial \varphi(z)}{\partial z} \int \partial z' \, \pi^R(z, z')\varphi(z') \right]$$

Identifying sources of friction

In the *stationary planar* wall limit $\varphi(x)=\varphi(z)$, with the wall centered at $z=0$ **in the wall frame**

$$\int_{-\delta}^{\delta}\partial z\,\frac{\partial}{\partial z}\varphi(z)\left(\frac{\partial^2}{\partial z^2}\varphi(z)+U'(\varphi(z),T)+\frac{\partial m_\varphi^2}{\partial\varphi(z)}\int_{\vec{k}}\delta f(\vec{k},z)+\int\partial z'\,\pi^R(z,z')\varphi(z')\right)=0$$

$$\Rightarrow \underbrace{\int_{-\delta}^{\delta}\partial z\,\frac{\partial}{\partial z}\left[\frac{1}{2}\left(\frac{\partial\varphi(z)}{\partial z}\right)^2+U(\varphi(z),T)\right]}_{\Delta U\equiv\mathcal{P}_{\text{driving}}}=\int_{-\delta}^{\delta}\partial z\left[\frac{\partial U}{\partial T}\frac{\partial T}{\partial z}-\frac{\partial m_\varphi^2}{\partial z}\int_{\vec{k}}\delta f(\vec{k},z)-\frac{\partial\varphi(z)}{\partial z}\int\partial z'\,\pi^R(z,z')\varphi(z')\right]$$

Identifying sources of friction

In the *stationary planar* wall limit $\varphi(x) = \varphi(z)$, with the wall centered at $z = 0$ **in the wall frame**

$$\int_{-\delta}^{\delta} \partial z \, \frac{\partial}{\partial z}\varphi(z) \left(\frac{\partial^2}{\partial z^2}\varphi(z) + U'(\varphi(z), T) + \frac{\partial m_\varphi^2}{\partial \varphi(z)} \int_{\vec{k}} \delta f(\vec{k}, z) + \int \partial z' \, \pi^R(z, z')\varphi(z') \right) = 0$$

$$\Rightarrow \underbrace{\int_{-\delta}^{\delta} \partial z \, \frac{\partial}{\partial z}\left[\frac{1}{2}\left(\frac{\partial \varphi(z)}{\partial z}\right)^2 + U(\varphi(z), T) \right]}_{\Delta U \equiv \mathcal{P}_{\text{driving}}} = \int_{-\delta}^{\delta} \partial z \left[\frac{\partial U}{\partial T}\frac{\partial T}{\partial z} - \frac{\partial m_\varphi^2}{\partial z} \int_{\vec{k}} \delta f(\vec{k}, z) - \frac{\partial \varphi(z)}{\partial z} \int \partial z' \, \pi^R(z, z')\varphi(z') \right] \equiv \mathcal{P}_{\text{LTE}}$$

Identifying sources of friction

In the *stationary planar* wall limit $\varphi(x) = \varphi(z)$, with the wall centered at $z = 0$ **in the wall frame**

$$\int_{-\delta}^{\delta} \partial z \, \frac{\partial}{\partial z}\varphi(z) \left(\frac{\partial^2}{\partial z^2}\varphi(z) + U'(\varphi(z), T) + \frac{\partial m_\varphi^2}{\partial \varphi(z)} \int_{\vec{k}} \delta f(\vec{k}, z) + \int \partial z' \, \pi^R(z, z')\varphi(z') \right) = 0$$

$$\Rightarrow \underbrace{\int_{-\delta}^{\delta} \partial z \, \frac{\partial}{\partial z} \left[\frac{1}{2} \left(\frac{\partial \varphi(z)}{\partial z} \right)^2 + U(\varphi(z), T) \right]}_{\Delta U \equiv \mathcal{P}_{\text{driving}}} = \int_{-\delta}^{\delta} \partial z \left[\frac{\partial U}{\partial T} \frac{\partial T}{\partial z} \right. \qquad \equiv \mathcal{P}_{\text{LTE}}$$

$$- \frac{\partial m_\varphi^2}{\partial z} \int_{\vec{k}} \delta f(\vec{k}, z) \qquad \equiv \mathcal{P}_{\text{dissipative}}$$

$$\left. - \frac{\partial \varphi(z)}{\partial z} \int \partial z' \, \pi^R(z, z')\varphi(z') \right]$$

Identifying sources of friction

In the *stationary planar* wall limit $\varphi(x) = \varphi(z)$, with the wall centered at $z = 0$ **in the wall frame**

$$\int_{-\delta}^{\delta} \partial z \, \frac{\partial}{\partial z}\varphi(z) \left(\frac{\partial^2}{\partial z^2}\varphi(z) + U'(\varphi(z), T) + \frac{\partial m_\varphi^2}{\partial \varphi(z)} \int_{\vec{k}} \delta f(\vec{k}, z) + \int \partial z' \, \pi^R(z, z')\varphi(z') \right) = 0$$

$$\begin{aligned}
\Rightarrow \underbrace{\int_{-\delta}^{\delta} \partial z \, \frac{\partial}{\partial z}\left[\frac{1}{2}\left(\frac{\partial \varphi(z)}{\partial z}\right)^2 + U(\varphi(z), T) \right]}_{\Delta U \equiv \mathcal{P}_{\text{driving}}} = \int_{-\delta}^{\delta} \partial z \, \Bigg[& \frac{\partial U}{\partial T}\frac{\partial T}{\partial z} && \equiv \mathcal{P}_{\text{LTE}} \\
& - \frac{\partial m_\varphi^2}{\partial z} \int_{\vec{k}} \delta f(\vec{k}, z) && \equiv \mathcal{P}_{\text{dissipative}} \\
& - \frac{\partial \varphi(z)}{\partial z} \int \partial z' \, \pi^R(z, z')\varphi(z') \Bigg] && \equiv \mathcal{P}_{\text{vertex}}
\end{aligned}$$

Particle production

Particle production

- Introduce a heavy scalar field χ in the Lagrangian

$$\mathcal{L}_{\rm int} \supset -\frac{g}{4}\phi^2\chi^2 \,, \qquad m_\chi \gg m_\phi \,, T \implies f_\chi \sim 0$$

Particle production

- Introduce a heavy scalar field χ in the Lagrangian

$$\mathcal{L}_{\rm int} \supset -\frac{g}{4}\phi^2\chi^2\,, \qquad m_\chi \gg m_\phi\,, T \implies f_\chi \sim 0$$

- The **new term** leads to a new contribution to the friction, encoding higher-order effects

$$\mathcal{P}_{\rm vertex} \approx -\int \frac{\partial q_\perp}{2\pi}\, q_\perp \left|\widetilde{\varphi}(q_\perp)\right|^2 \, \mathrm{Im}\, \widetilde{\pi}^{\rm R}(q_\perp)$$

$\gamma_w \gg 1$

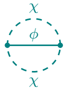
Particle production

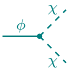
- Introduce a heavy scalar field χ in the Lagrangian

$$\mathcal{L}_{\rm int} \supset -\frac{g}{4}\phi^2\chi^2\,, \qquad m_\chi \gg m_\phi\,, T \implies f_\chi \sim 0$$

- The **new term** leads to a new contribution to the friction, encoding higher-order effects

$$\mathcal{P}_{\rm vertex} \approx -\int \frac{\partial q_\perp}{2\pi}\, q_\perp \left|\widetilde{\varphi}(q_\perp)\right|^2 \,{\rm Im}\,\widetilde{\pi}^{\rm R}(q_\perp)\Big\} \sim {\rm Im}\left[\quad\right] = \Big|\quad\Big|^2$$





$\gamma_w \gg 1$

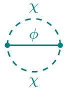
Particle production

- Introduce a heavy scalar field χ in the Lagrangian

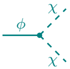
$$\mathcal{L}_{\text{int}} \supset -\frac{g}{4}\phi^2\chi^2 \,, \qquad m_\chi \gg m_\phi \,, T \implies f_\chi \sim 0$$

- The **new term** leads to a new contribution to the friction, encoding higher-order effects

$$\mathcal{P}_{\text{vertex}} \approx -\int \frac{\partial q_\perp}{2\pi}\, q_\perp \left|\widetilde{\varphi}(q_\perp)\right|^2 \operatorname{Im}\widetilde{\pi}^{\mathrm{R}}(q_\perp) \Big\} \sim \operatorname{Im}\Bigg[$$



$$\Bigg] = \Bigg|$$



$$\Bigg|^2$$

$$= \frac{g^2}{2}\int_{\vec{p},\vec{k}_1,\vec{k}_2} (2\pi)^2\delta^{(2)}(\vec{p}_\parallel - \vec{k}_{1,\parallel} - \vec{k}_{2,\parallel})(2\pi)\delta(E^{(\phi)}_{\vec{p}} - E^{(\chi)}_{\vec{k}_1} - E^{(\chi)}_{\vec{k}_2})\times$$
$$\times\, f_\phi(\vec{p}) \quad \Delta p_\perp \quad \left|\tilde{\varphi}(\Delta p_\perp)\right|^2$$

$\gamma_w \gg 1$

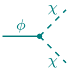
Particle production

- Introduce a heavy scalar field χ in the Lagrangian

$$\mathcal{L}_{\rm int} \supset -\frac{g}{4}\phi^2\chi^2\,, \qquad m_\chi \gg m_\phi\,, T \implies f_\chi \sim 0$$

- The **new term** leads to a new contribution to the friction, encoding higher-order effects

$$\mathcal{P}_{\rm vertex} \approx -\int \frac{\partial q_\perp}{2\pi}\, q_\perp\, |\widetilde{\varphi}(q_\perp)|^2\ {\rm Im}\,\widetilde{\pi}^{\rm R}(q_\perp)\Big\} \sim {\rm Im}\Big[\ \Big] = \Big|\ \Big|^2$$



$$= \frac{g^2}{2}\int_{\vec p,\vec k_1,\vec k_2} (2\pi)^2\delta^{(2)}(\vec p_\parallel - \vec k_{1,\parallel} - \vec k_{2,\parallel})(2\pi)\delta(E^{(\phi)}_{\vec p} - E^{(\chi)}_{\vec k_1} - E^{(\chi)}_{\vec k_2})\times$$

$$\times\ f_\phi(\vec p)\quad \Delta p_\perp \quad |\tilde\varphi(\Delta p_\perp)|^2$$

$\gamma_w \gg 1$

Particle production

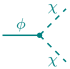
- Introduce a heavy scalar field χ in the Lagrangian

$$\mathcal{L}_{\rm int} \supset -\frac{g}{4}\phi^2\chi^2\,, \qquad m_\chi \gg m_\phi\,, T \implies f_\chi \sim 0$$

- The **new term** leads to a new contribution to the friction, encoding higher-order effects

$$\mathcal{P}_{\rm vertex} \approx -\int \frac{\partial q_\perp}{2\pi}\, q_\perp\, |\widetilde{\varphi}(q_\perp)|^2\; \mathrm{Im}\, \widetilde{\pi}^{\rm R}(q_\perp) \Big\} \sim \mathrm{Im}\Bigg[$$

$$\Bigg] = \Bigg|$$



$$\Bigg|^2$$

$$= \frac{g^2}{2}\int_{\vec{p},\vec{k}_1,\vec{k}_2} (2\pi)^2\delta^{(2)}(\vec{p}_{\parallel} - \vec{k}_{1,\parallel} - \vec{k}_{2,\parallel})(2\pi)\delta(E^{(\phi)}_{\vec{p}} - E^{(\chi)}_{\vec{k}_1} - E^{(\chi)}_{\vec{k}_2})\times$$

$$\times\, f_\phi(\vec{p}) \quad \Delta p_\perp \quad |\tilde{\varphi}(\Delta p_\perp)|^2$$

$\gamma_w \gg 1$

Particle production

- Introduce a heavy scalar field χ in the Lagrangian

$$\mathcal{L}_{\rm int} \supset -\frac{g}{4}\phi^2\chi^2\,, \qquad m_\chi \gg m_\phi\,, T \implies f_\chi \sim 0$$

- The **new term** leads to a new contribution to the friction, encoding higher-order effects

$$\mathcal{P}_{\rm vertex} \approx -\int \frac{\partial q_\perp}{2\pi}\, q_\perp\, |\widetilde{\varphi}(q_\perp)|^2\, {\rm Im}\, \widetilde{\pi}^{\rm R}(q_\perp) \Big\} \sim {\rm Im}\Big[$$

$$\Big] = \Big|$$

$$\Big|^2$$

$$= \frac{g^2}{2}\int_{\vec{p},\vec{k}_1,\vec{k}_2} (2\pi)^2\delta^{(2)}(\vec{p}_{||} - \vec{k}_{1,||} - \vec{k}_{2,||})(2\pi)\delta(E^{(\phi)}_{\vec{p}} - E^{(\chi)}_{\vec{k}_1} - E^{(\chi)}_{\vec{k}_2})\times$$

$$\times f_\phi(\vec{p}) \quad \Delta p_\perp \quad |\tilde{\varphi}(\Delta p_\perp)|^2$$

$\gamma_w \gg 1$

Fourier tf. of the wall

Particle production

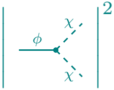
- Introduce a heavy scalar field χ in the Lagrangian

$$\mathcal{L}_{\text{int}} \supset -\frac{g}{4}\phi^2\chi^2\,, \qquad m_\chi \gg m_\phi\,, T \implies f_\chi \sim 0$$

- The **new term** leads to a new contribution to the friction, encoding higher-order effects

recall: $\mathcal{P}_{\text{kick}} = \sum_{i,X} \int_{\vec{p}} \partial\mathbb{P}_{i\to X}(\vec{p})\, f_i(\vec{p})\, \Delta p_{\perp\,,i\to X}$

$\mathcal{P}_{\text{vertex}} \approx$ … χ … $=$



$$= \frac{g^2}{2}\int_{\vec{p},\vec{k}_1,\vec{k}_2}(2\pi)^2\delta^{(2)}(\vec{p}_{\|}-\vec{k}_{1,\|}-\vec{k}_{2,\|})(2\pi)\delta(E^{(\phi)}_{\vec{p}}-E^{(\chi)}_{\vec{k}_1}-E^{(\chi)}_{\vec{k}_2})\times$$
$$\times\, f_\phi(\vec{p}) \quad \Delta p_\perp \quad |\tilde{\varphi}(\Delta p_\perp)|^2$$

$\gamma_w \gg 1$

Fourier tf. of the wall

The ultrarelativistic regime

The ultrarelativistic regime

Analytic formula for an ultrarelativistic (tanh) wall in the limit of light ϕ-particles

$$\mathcal{P}_{\phi\to\chi\chi}^{\gamma_w\to\infty} \approx \frac{g^2 v_b^2 T^2}{24\times 32\pi^2}\log\left(\frac{\gamma_w T}{2\pi L_w m_\chi^2}\right)$$

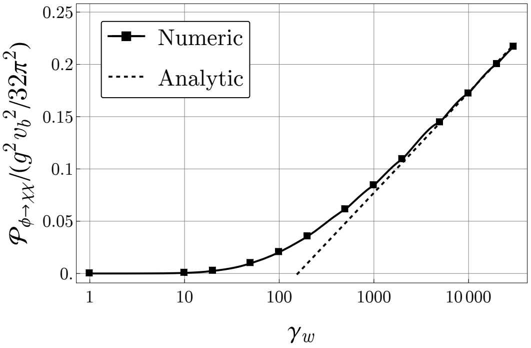
which approaches the result from the kick picture.

The ultrarelativistic regime

Analytic formula for an ultrarelativistic (tanh) wall in the limit of light ϕ-particles

$$\mathcal{P}_{\phi\to\chi\chi}^{\gamma_w\to\infty} \approx \frac{g^2 v_b^2 T^2}{24\times 32\pi^2}\log\left(\frac{\gamma_w T}{2\pi L_w m_\chi^2}\right)$$

which approaches the result from the kick picture.



The ultrarelativistic regime

Analytic formula for an ultrarelativistic (tanh) wall in the limit of light ϕ-particles

$$\mathcal{P}_{\phi\to\chi\chi}^{\gamma_w\to\infty} \approx \frac{g^2 v_b^2 T^2}{24\times 32\pi^2} \log\left(\frac{\gamma_w T}{2\pi L_w m_\chi^2}\right)$$

which approaches the result from the kick picture. Similarly, we show in our work that particle mixing and transition radiation are also captured within this framework.