TROUBLE IN THE BUBBLE

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University of Helsinki & Helsinki Institute of Physics

Based on arXiv:2404.01876

With Oliver Gould & David Weir

TESTING BUBBLE NUCLEATION CALCULATIONS FOR STRONG PHASE TRANSITIONS

Tunneling in QFT
19th of June 2025

PHASE TRANSITIONS

- Grand Unified Theories, Electroweak, QCD...
- In the Standard Model (SM) the electroweak PT is a crossover
- SM is incomplete → beyond SM (BSM) physics
- Things to look for: topological defects, bubbles from EWPT, … ?

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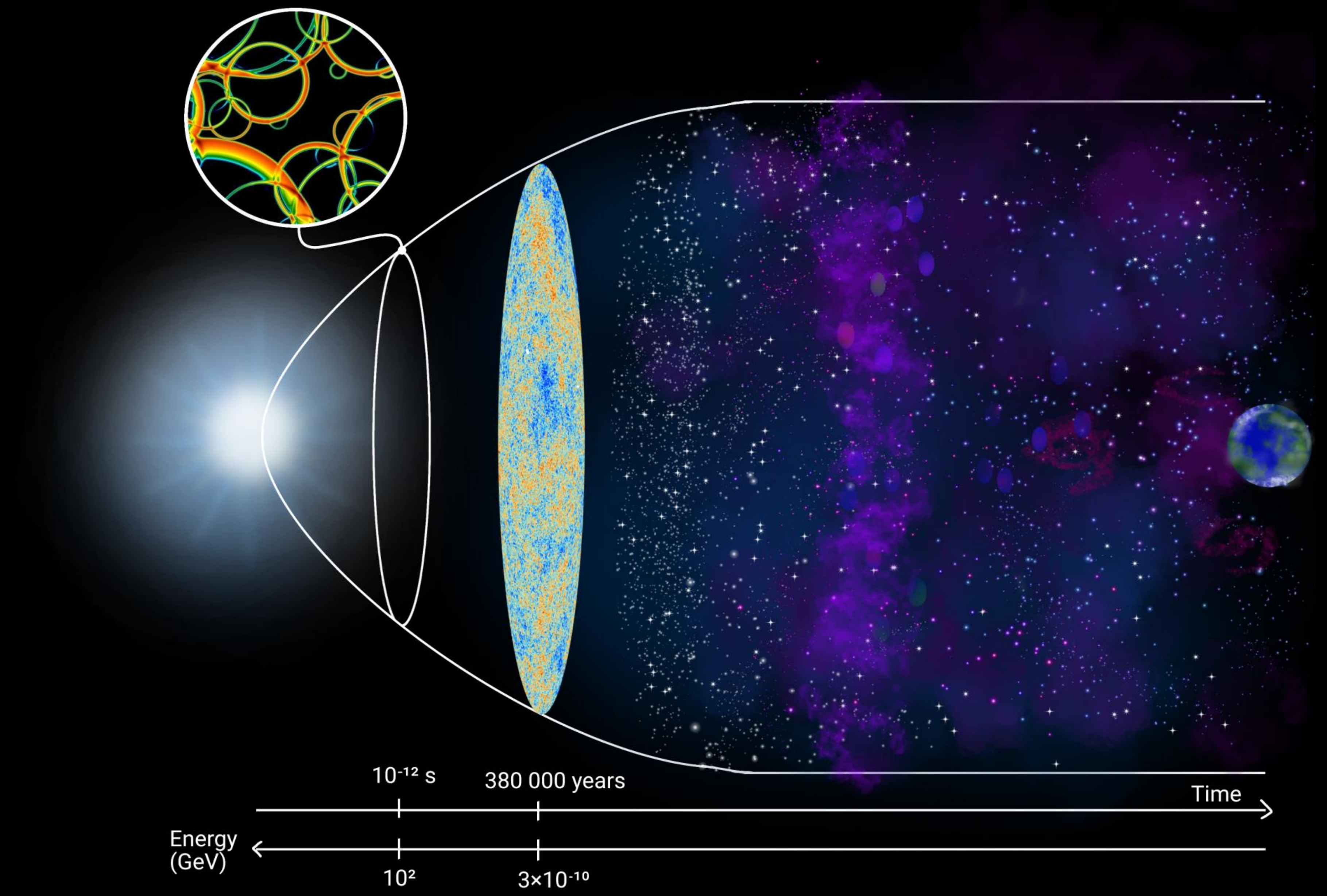
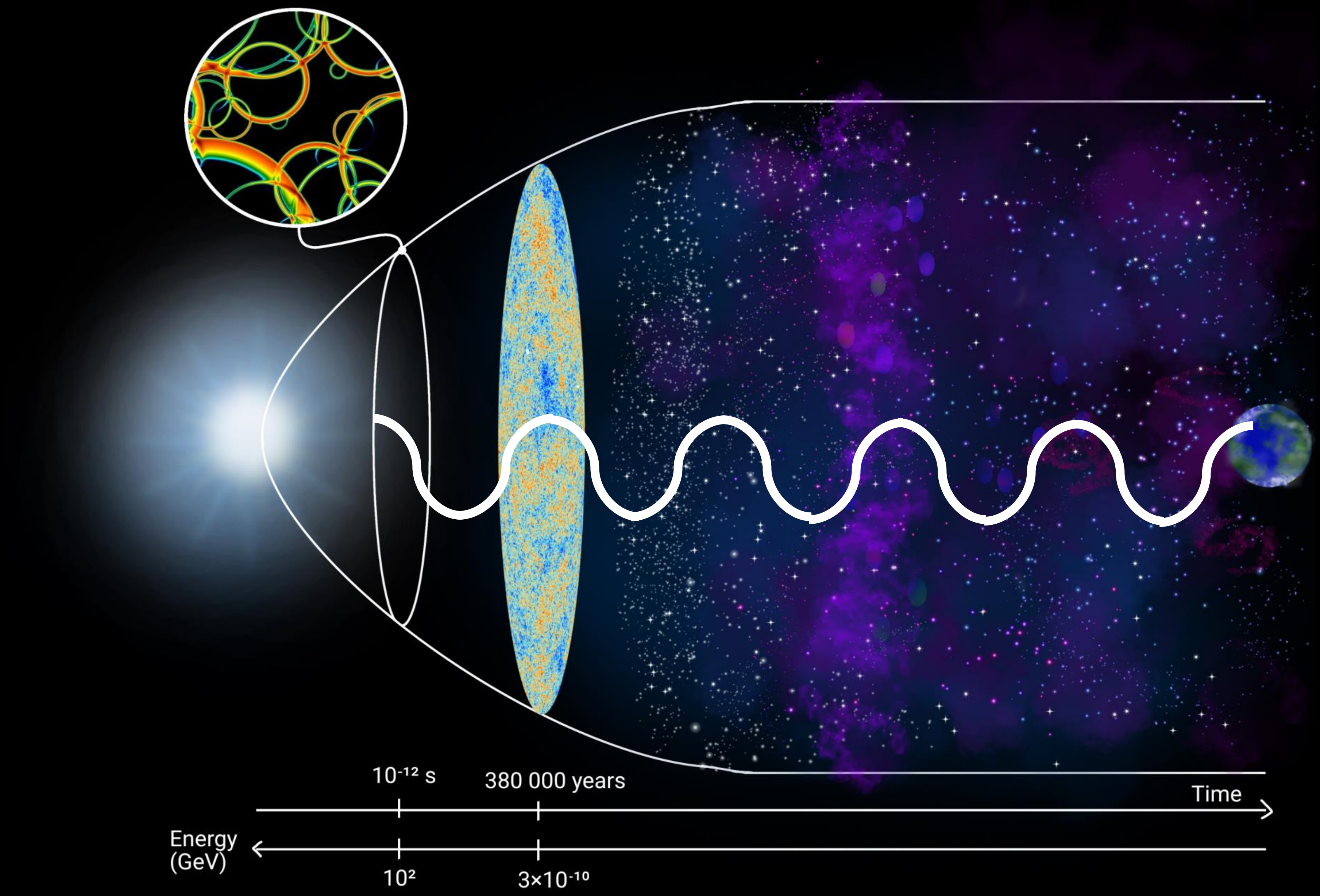

10-12 s
380 000 years
Time
Energy
(GeV)
10²
3×10-10



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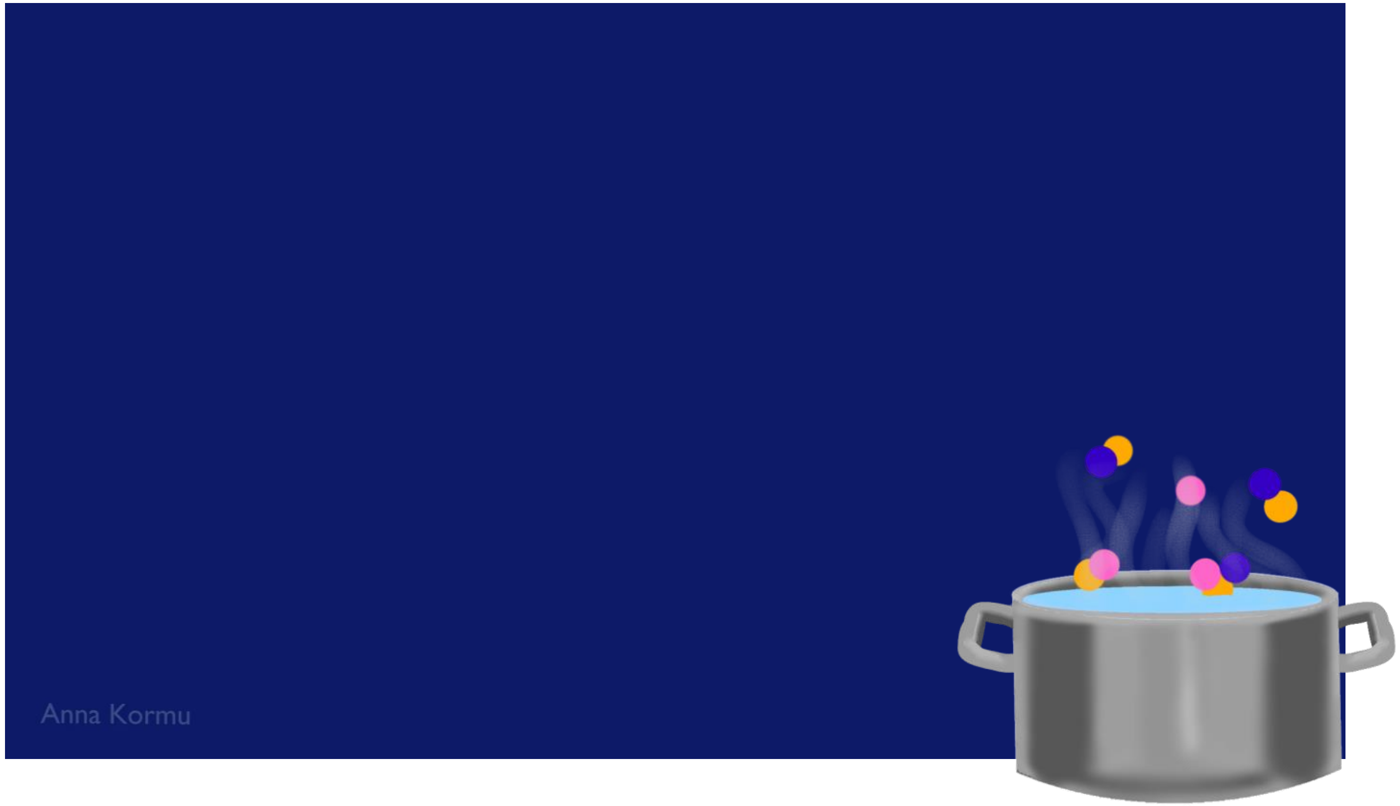
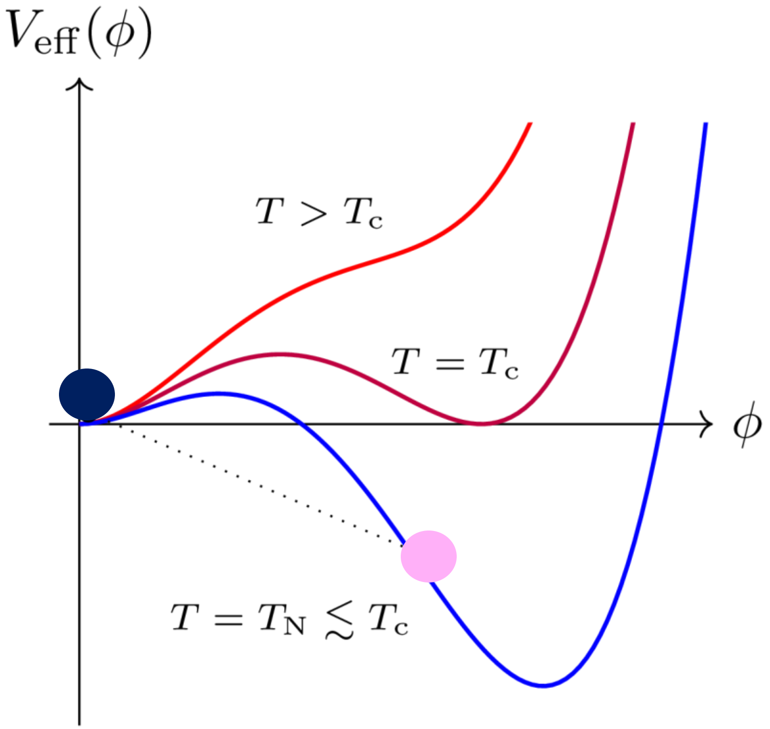
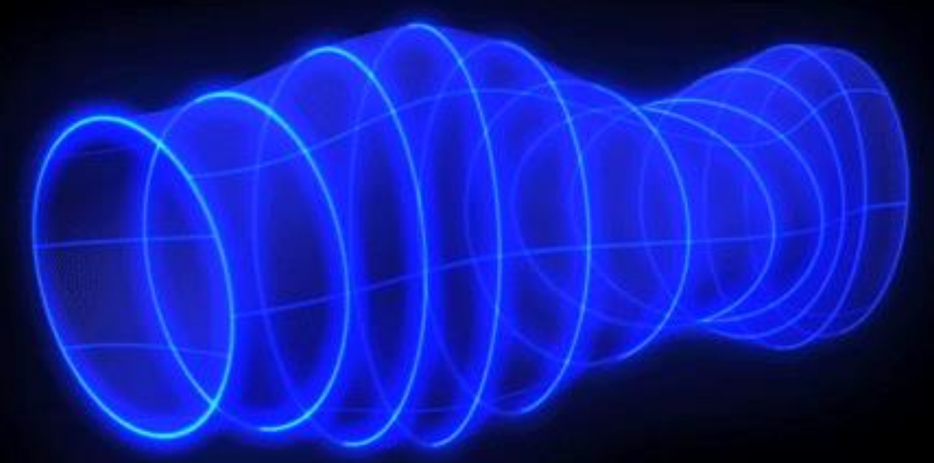




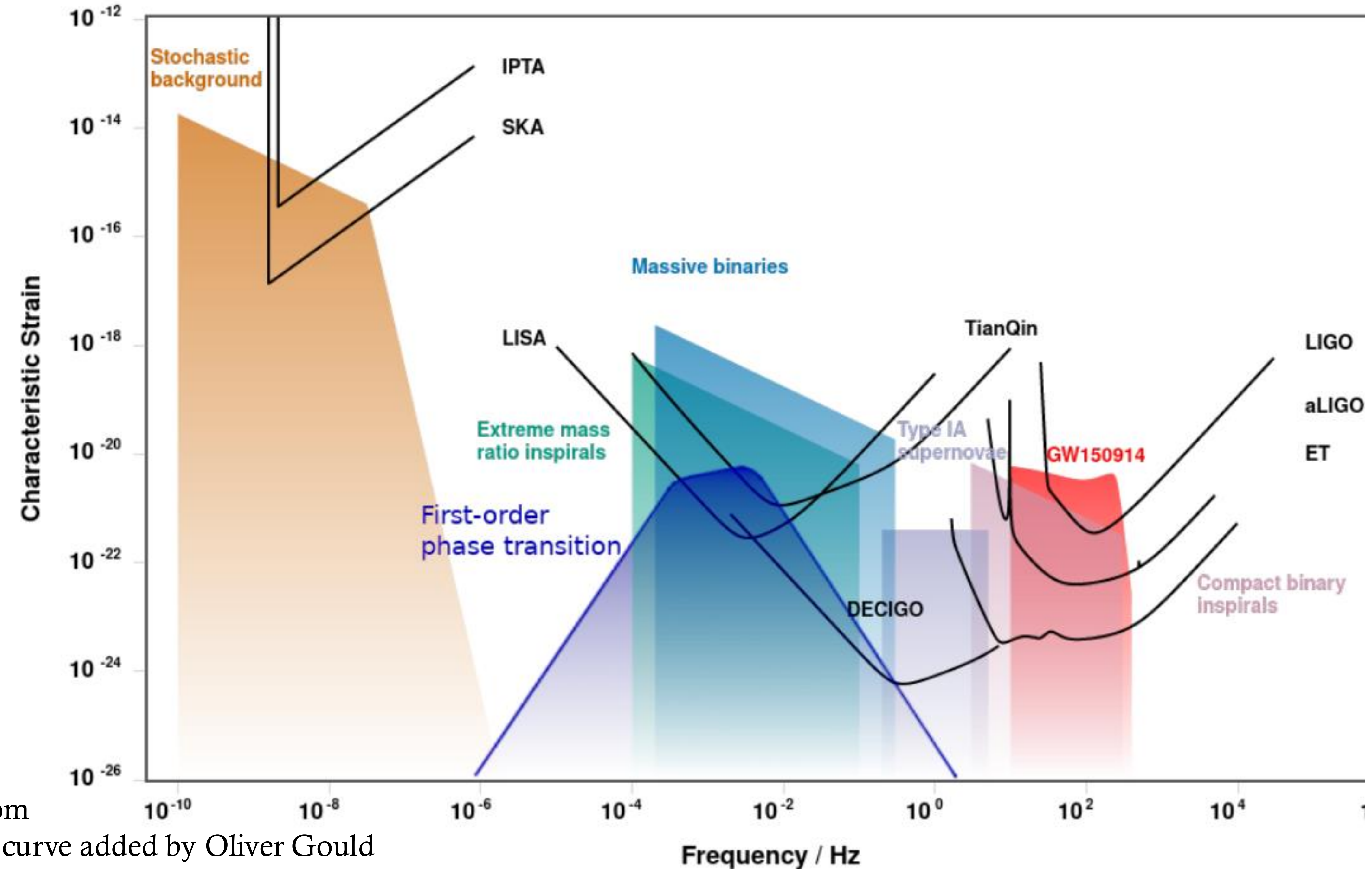
FIRST ORDER PHASE TRANSITIONS

GRAVITATIONAL WAVES

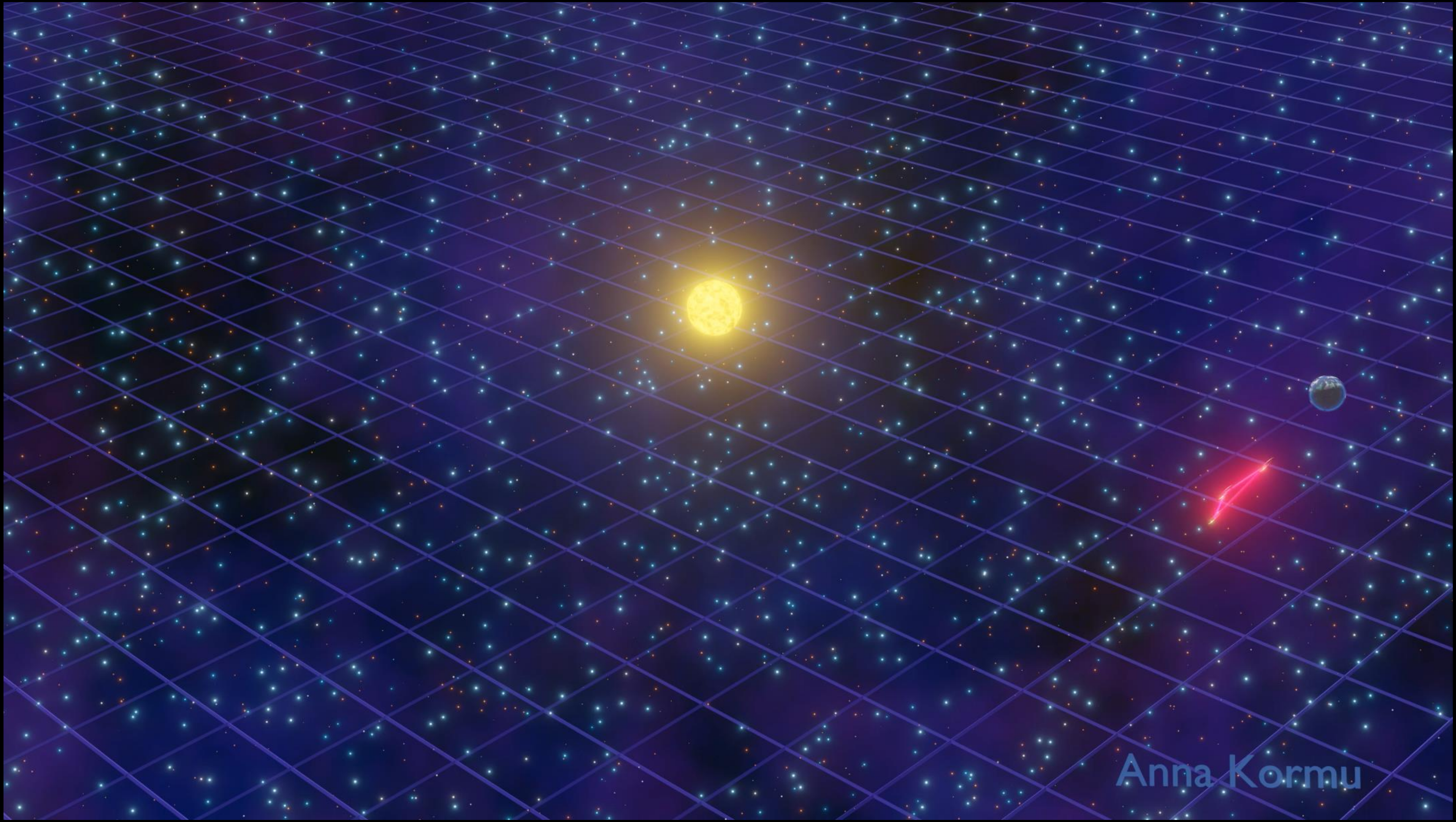
- Perturbations in the space-time geometry $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$
- Stochastic gravitational wave background $\sum \text{planar wave} \times \text{amplitude} \times \text{polarisation}$
- GW spectrum usually modelled by broken power laws
- Main contributions: bubble wall collisions, sound waves in the surrounding plasma and turbulence

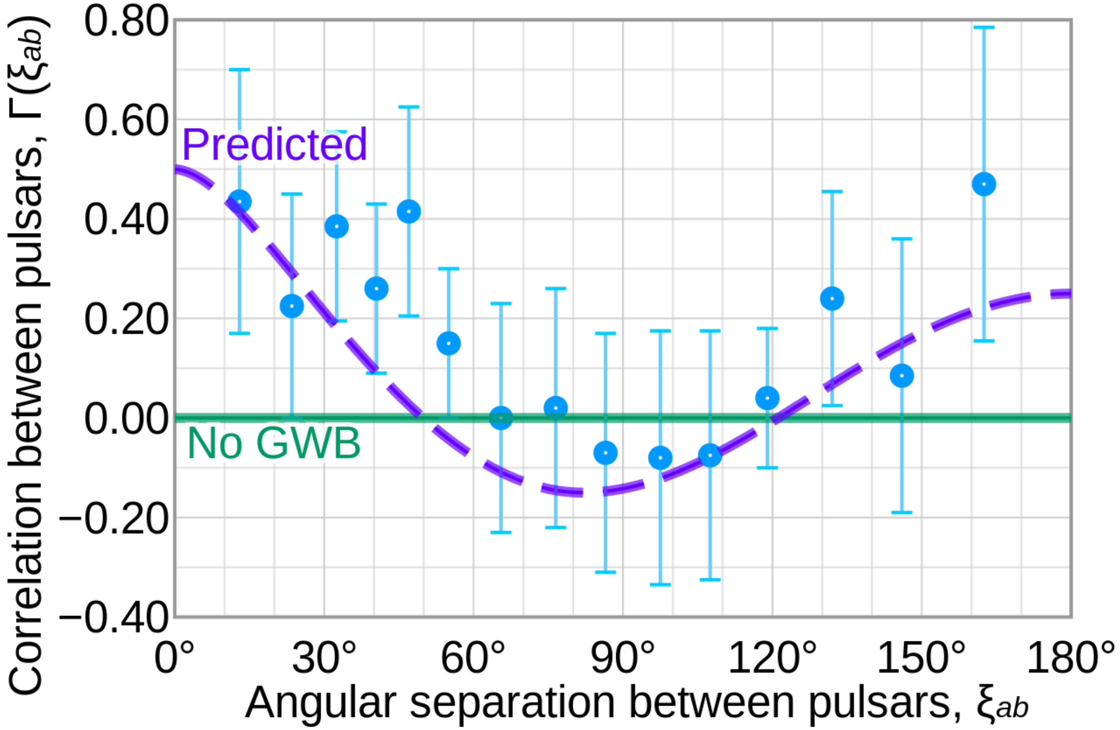


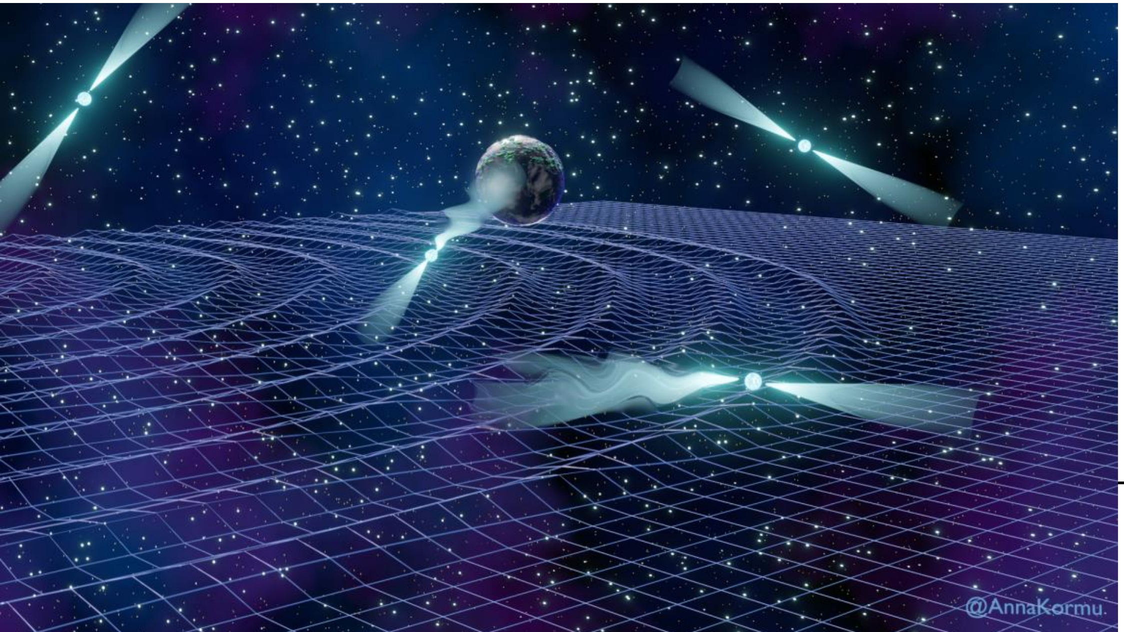
ESA / C. Carreau



Gwplotter.com
1st order PT curve added by Oliver Gould


Anna Kormu





PULSAR TIMING ARRAYS

- Hints of stochastic GW background - June 2023
  (European PTA, Indian PTA, NANOGrav, Parkes PTA ’23)

- Mostly likely supermassive black holes, but new physics cannot be ruled out yet

STOCHASTIC GW BACKGROUND?

Dimensional reduction $\mathcal{L}_{4d} \rightarrow \mathcal{L}_{3d}$

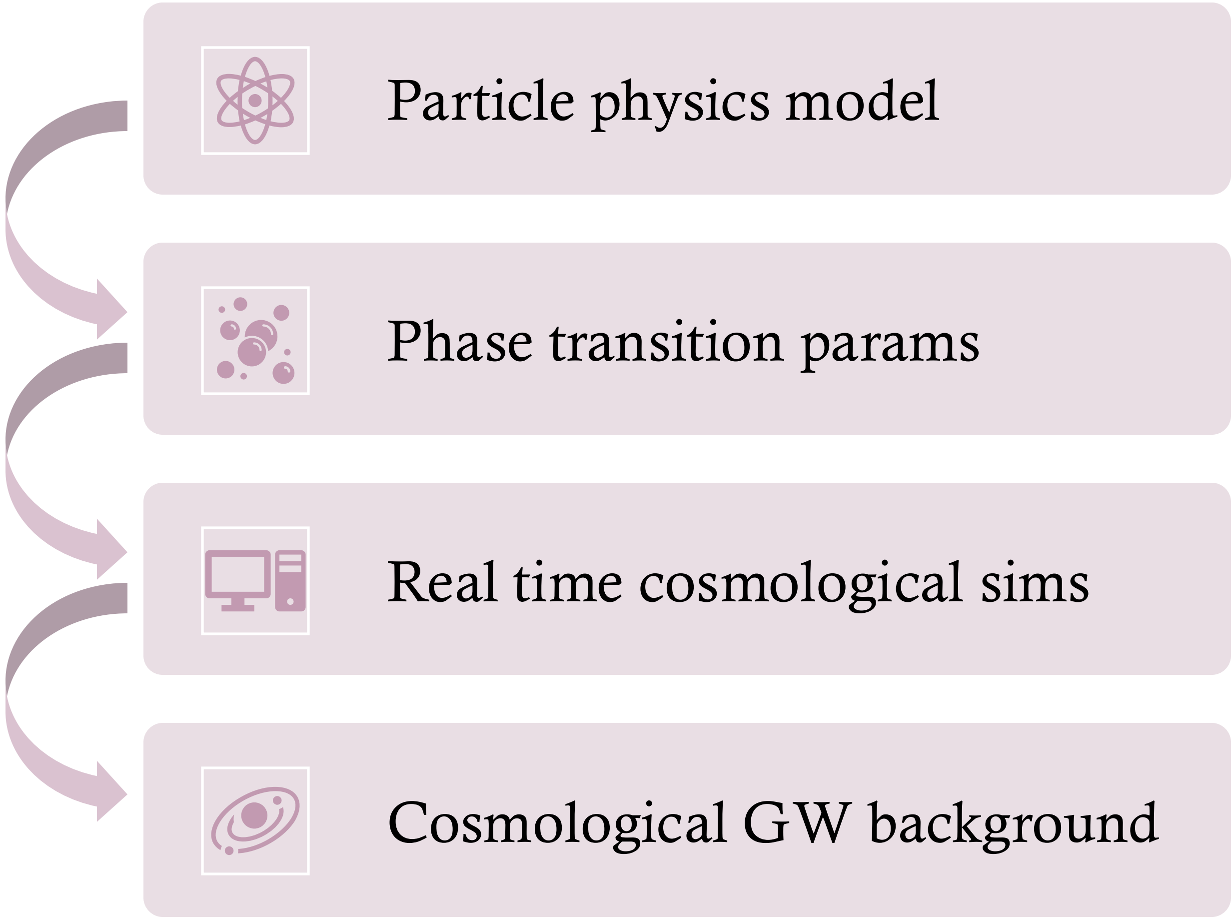
T_* : transition temperature

β_* : inverse duration of the transition

α_* : transition strenght

v_w : bubble wall speed

$$\Omega_{\mathrm{gw}} = F(T_*, R_*, \alpha_*, v_w)$$



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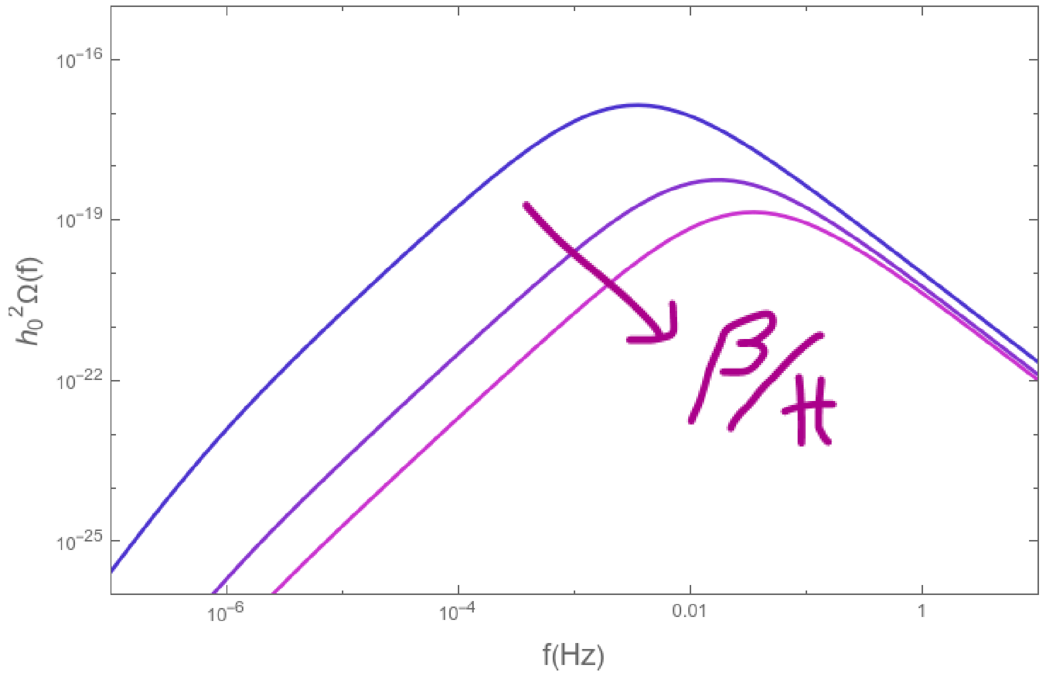

- Particle physics model
- Phase transition params
- Real time cosmological sims
- Cosmological GW background

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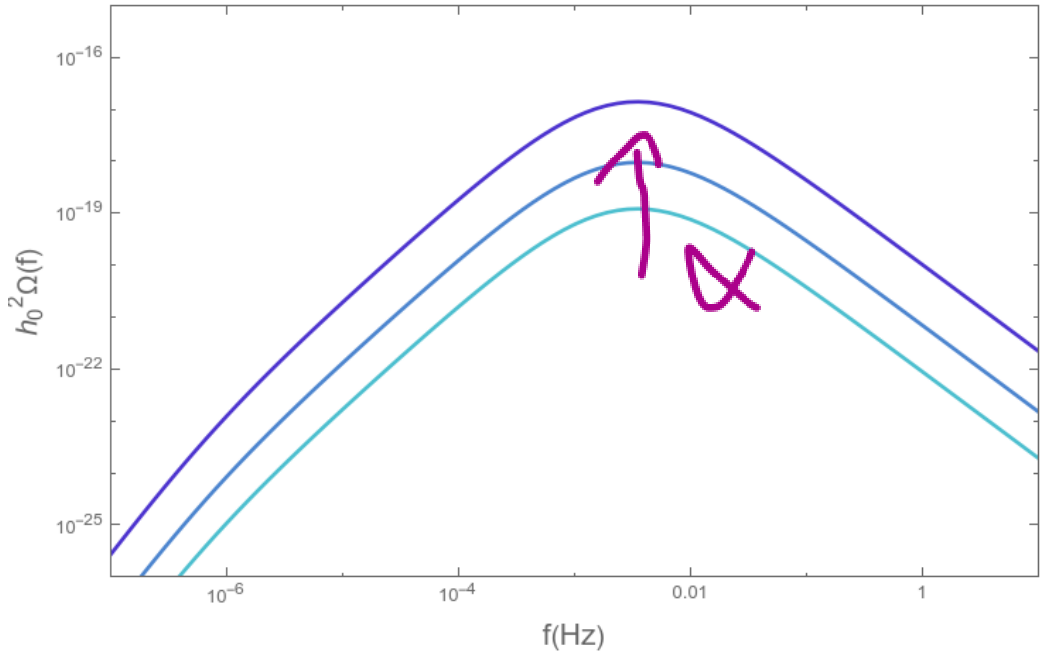

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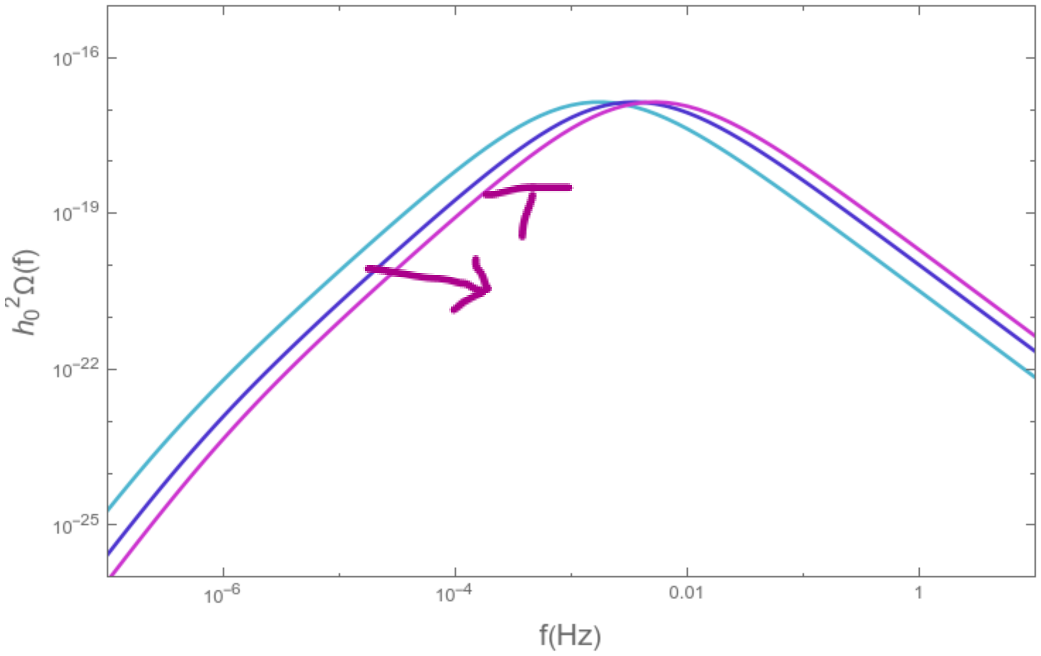

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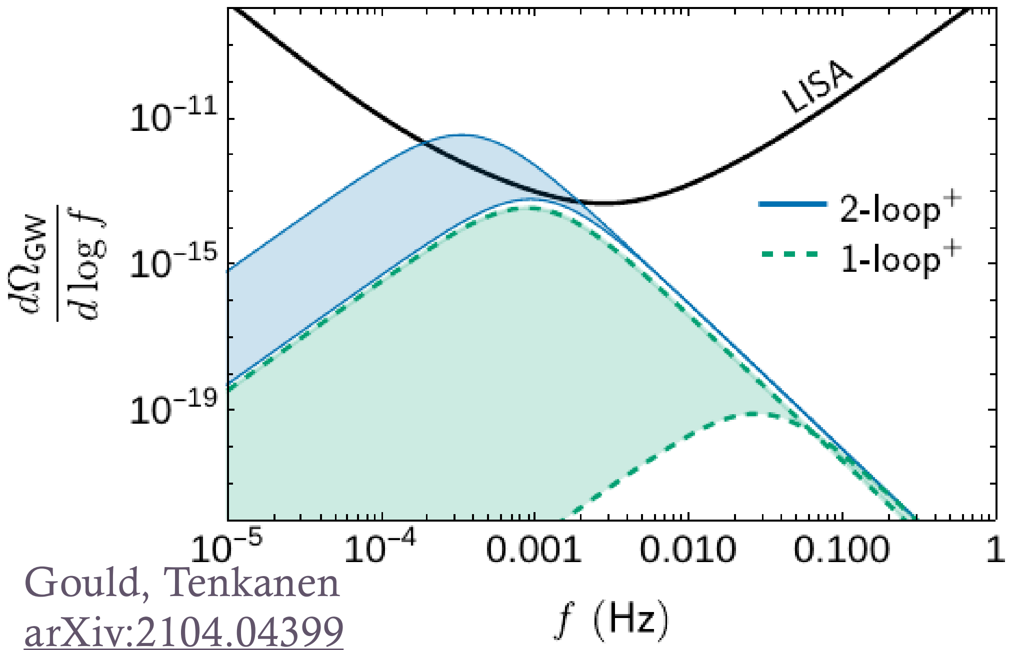

Gould, Tenkanen
arXiv:2104.04399

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- Particle physics model
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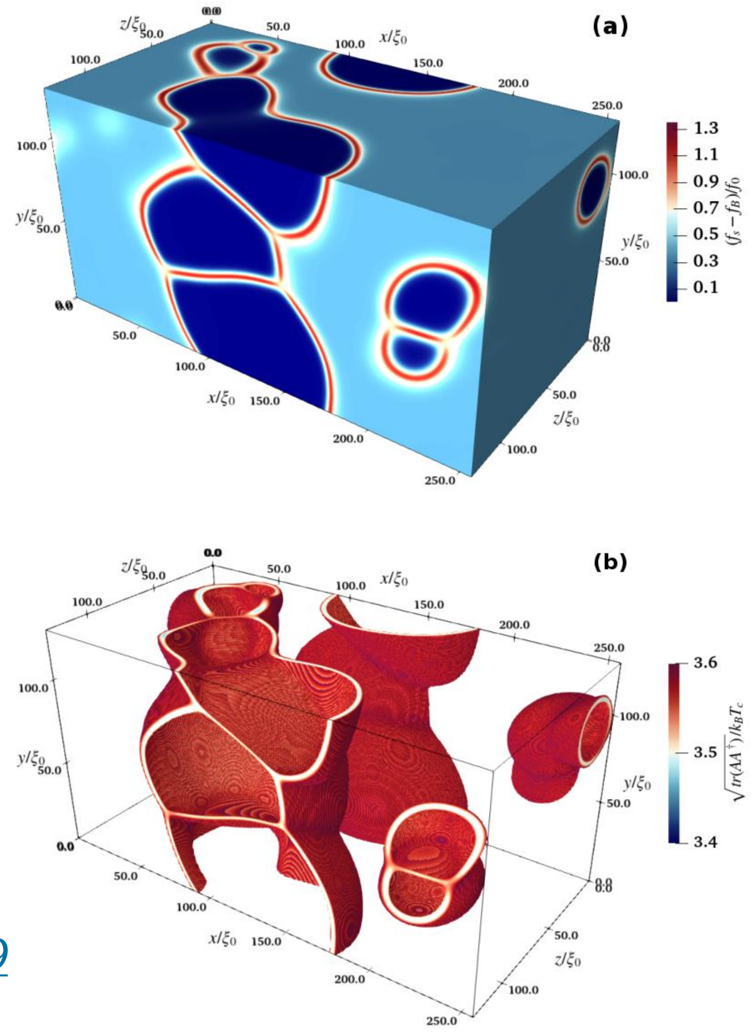
ANALOGUE EXPERIMENTS

- Testing cosmology in laboratory: nucleation theory essentially the same in laboratory and in cosmology
- Superfluid Helium-3 Hindmarsh et al. arXiv:2401.07878
- Ferromagnetic superfluids Zenesini et al. arXiv:2305.05225
- Proposals to test nucleation in (other) ultracold atomic gases arXiv:1408.1163 arXiv:2212.03621 arXiv:2307.02549



Hindmarsh et al.
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FATE OF THE FALSE VACUUM

- At zero temperature: relativistic field theory generalisation by Callan & Coleman (Phys. Rev. D 16, 1762 (1977))
- Classical field theory (a complete one-loop calculation) by Langer (Annals Phys. 41, 108 (1967), Annals Phys. 54, 258 (1969))
- Finite temperature approach in quantum field theory introduced later by Affleck & Linde (Phys. Rev. Lett. 46, 388 (1981), Phys. Lett. B 100, 37 (1981))
- More recent studies, crucially showing that the nucleation rate factorises, Gould, Hirvonen arXiv:2108.04377, Hirvonen arXiv:2403.07987, Ekstedt arXiv:2201.07331

$$\Gamma = A_{\text{dyn}} \times A_{\text{stat}} \qquad\qquad \Gamma = A_{\text{dyn}} \times \sqrt{\left|\frac{\det(S''[\phi_0]/2\pi)}{\det'(S''[\phi_{\text{b}}]/2\pi)}\right|} \left(\frac{\Delta S[\phi_{\text{b}}]}{2\pi}\right)^{3/2} e^{-\Delta S[\phi_{\text{b}}]}$$

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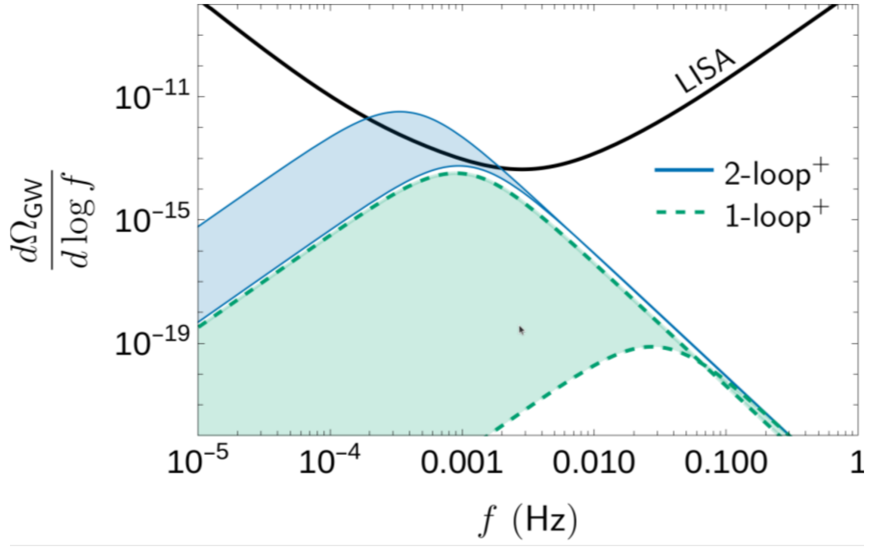
Usually not computed!

FATE OF THE FALSE VACUUM

- Determining the prefactor difficult, even in perturbation theory
- Perturbation theory depends on the details of the semiclassical approximation

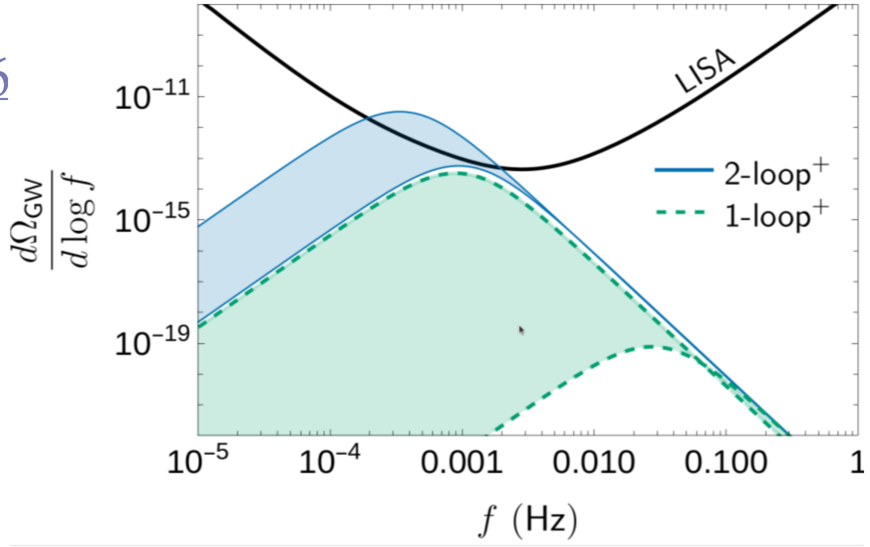
Gould, Tenkanen arXiv:2104.04399



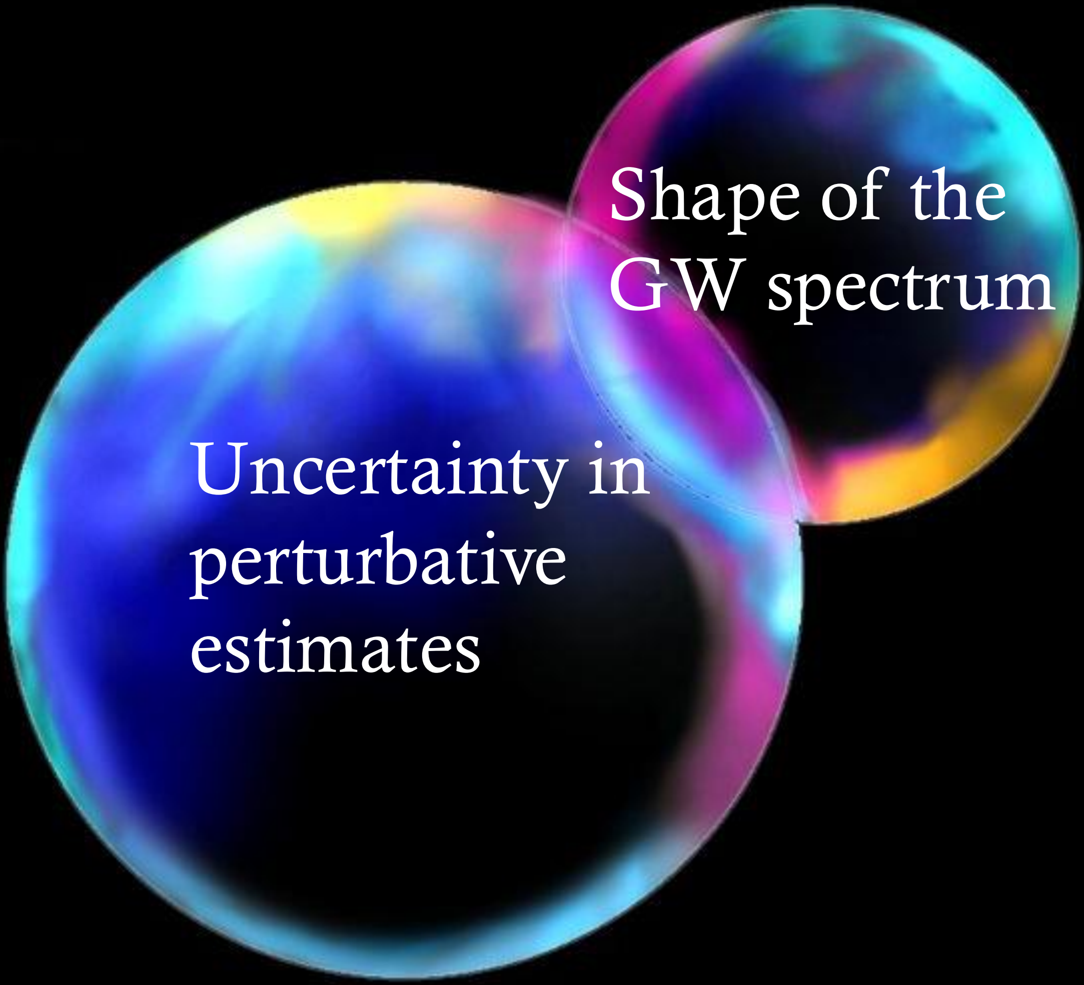
FATE OF THE FALSE VACUUM

- Determining the prefactor difficult, even in perturbation theory
- Perturbation theory depends on the details of the semiclassical approximation
- Introduces uncertainty! How accurate are our cosmological predictions?
- Non-perturbatively: real time (wait&see) approaches, recently eg. Pîrvu et al. arXiv:2408.06411, Batini et al. arXiv:2310.04206
- Moore & Rummukainen introduce a simulation method (hep-lat/0103036, hep-ph/0009132)

Gould, Tenkanen arXiv:2104.04399



KEY POINTS –WHY BUBBLE NUCLEATION CALCULATIONS ARE IMPORTANT



REAL SCALAR THEORY

For equilibrium dynamics, see Gould, arXiv:2101.05528

- Toy model possessing key features of BSM models
  - Potential has a tree-level barrier
  - Strong phase transition
  - Perturbative expansion simpler (we understand the dynamics)
- Dimensional reduction (imaginary time, high temp)

$$\mathscr{L} = -\frac{1}{2}\partial_\mu\varphi\partial^\mu\varphi - V(\varphi) - J_1\varphi - J_2\varphi^2$$

Interaction terms

$$V(\varphi) = \sigma\varphi + \frac{1}{2}m^2\varphi^2 + \frac{1}{3!}g\varphi^3 + \frac{1}{4!}\lambda\varphi^4,$$

Model parameters

DIMENSIONAL REDUCTION

Kajantie et al. hep-ph/9508379

- At high temperatures system looks 3d
- Dimensional reduction 4d cont $\rightarrow$ 3d cont
- Integrate out heavy modes, match correlation functions
- (3d cont $\rightarrow$ 3d lattice)

$$\mathcal{L}_{4d}$$
$$\downarrow$$
$$\mathcal{L}_{3d}$$
$$\downarrow$$
$$\mathcal{L}_{\mathrm{3d,lattice}}$$

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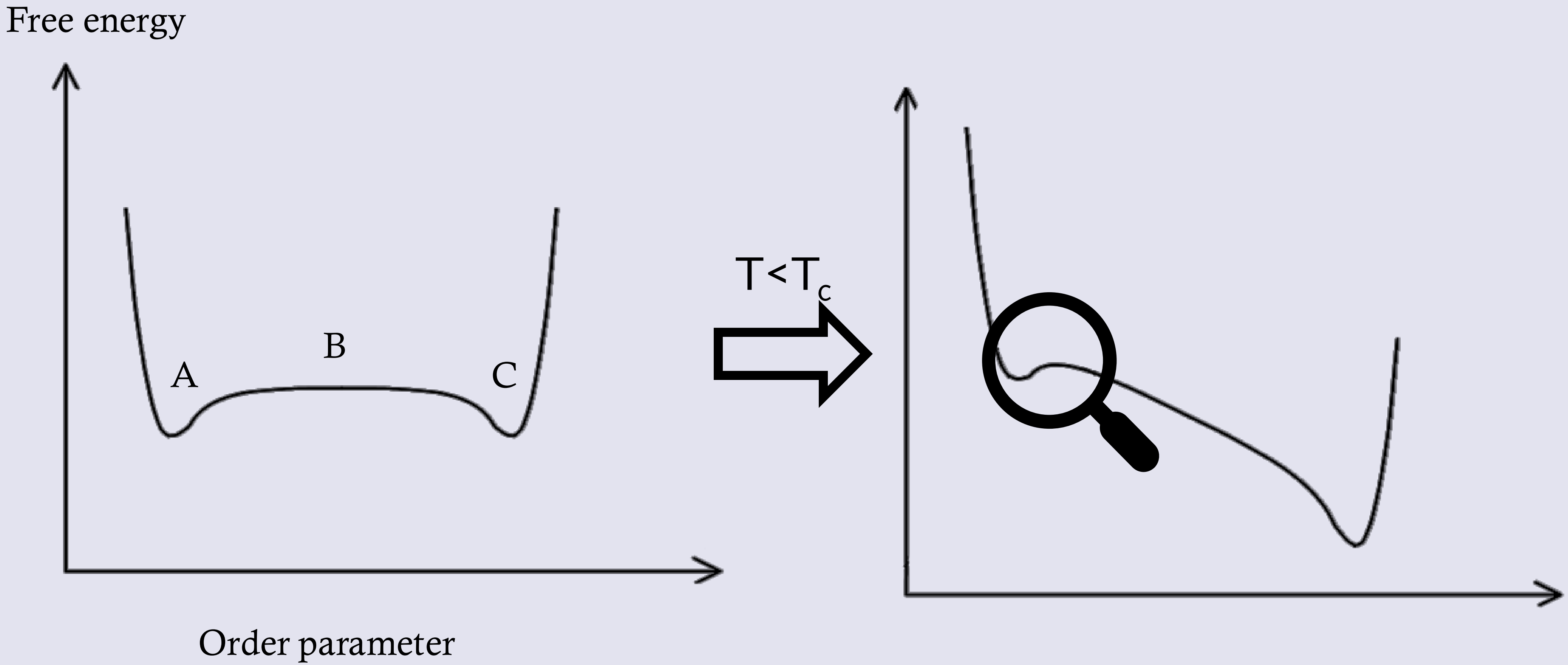
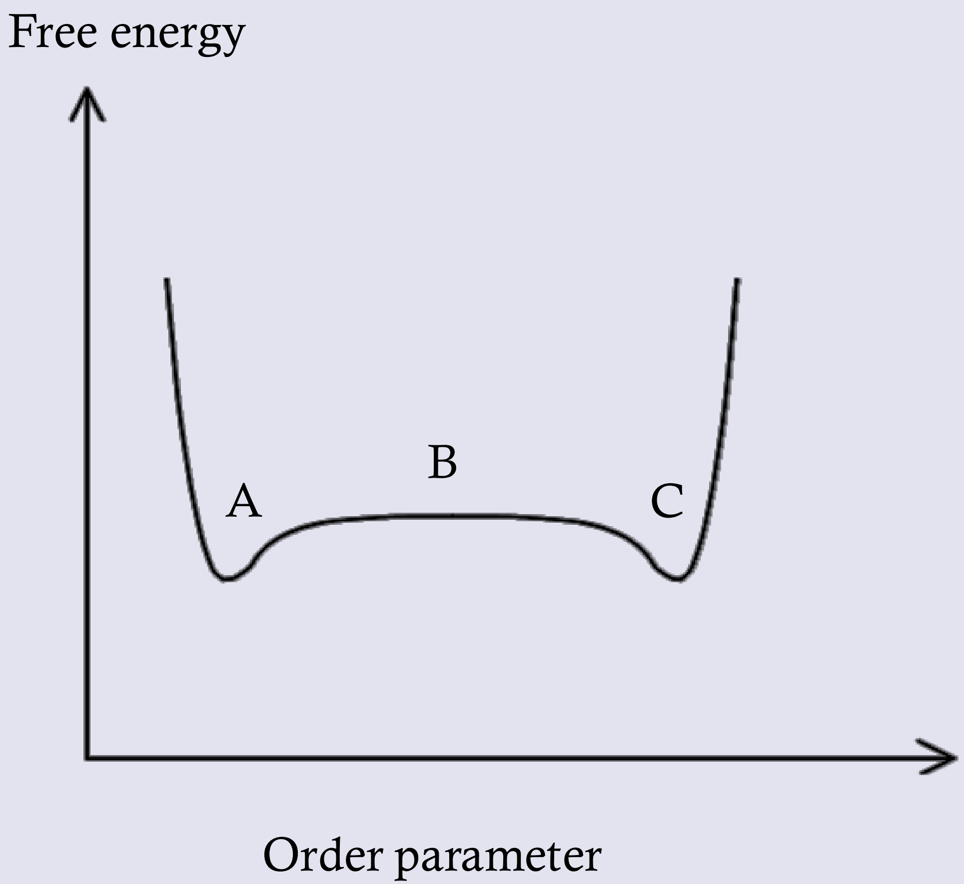
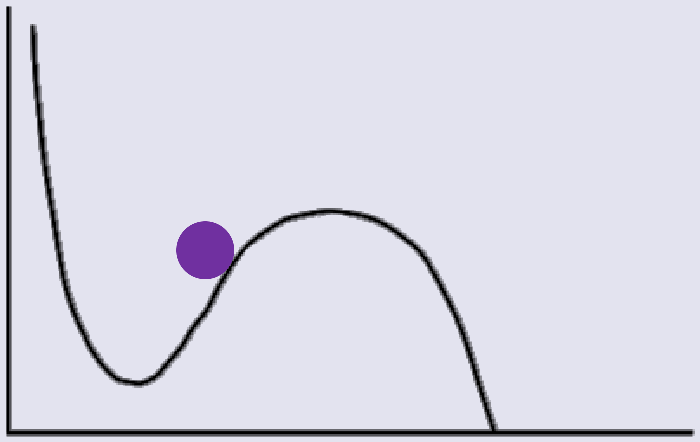
Lattice spacing **Renormalisation counterterms** **Lattice parameters**

$$S_{\mathrm{lat}} = \sum_x a^3 \left[-\frac{1}{2} Z_\phi \phi_x (\nabla^2_{\mathrm{lat}} \phi)_x + \sigma_{\mathrm{lat}} \phi_x + \frac{1}{2} Z_\phi Z_m m^2_{\mathrm{lat}} \phi_x^2 + \frac{1}{3!} g_{\mathrm{lat}} \phi_x^3 + \frac{1}{4!} Z_\phi^2 \lambda_{\mathrm{lat}} \phi_x^4 \right]$$


Free energy
B
A
C
T<T$_c$
Order parameter





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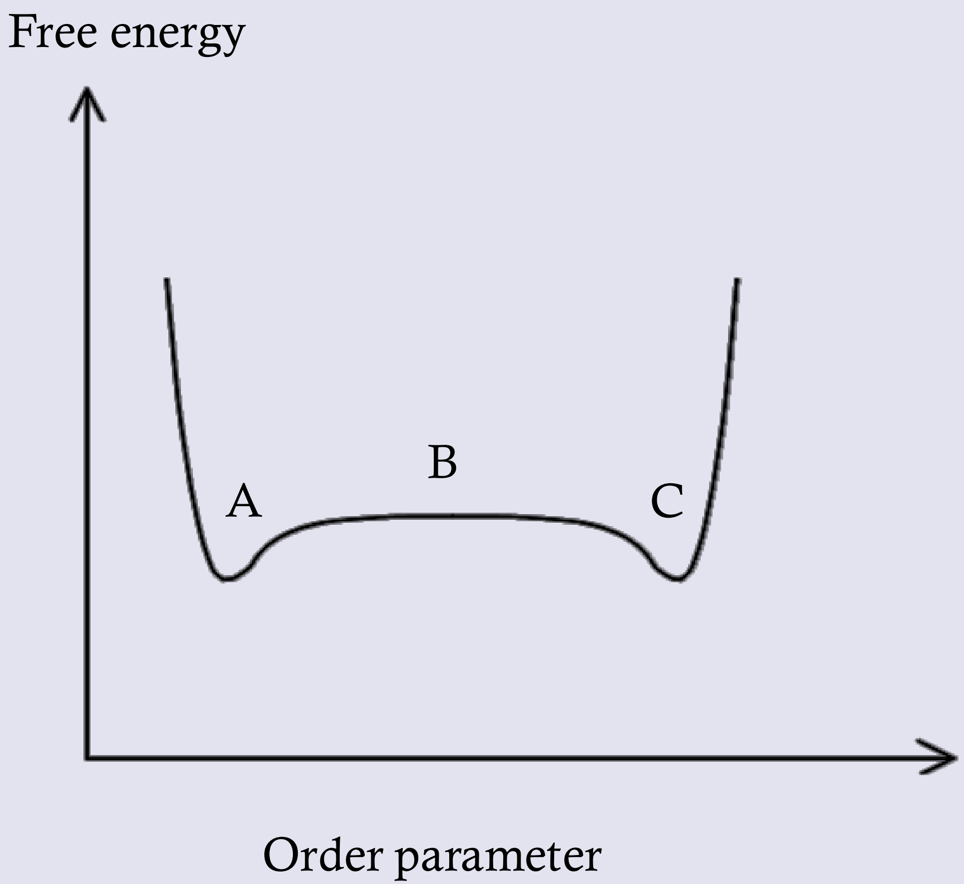

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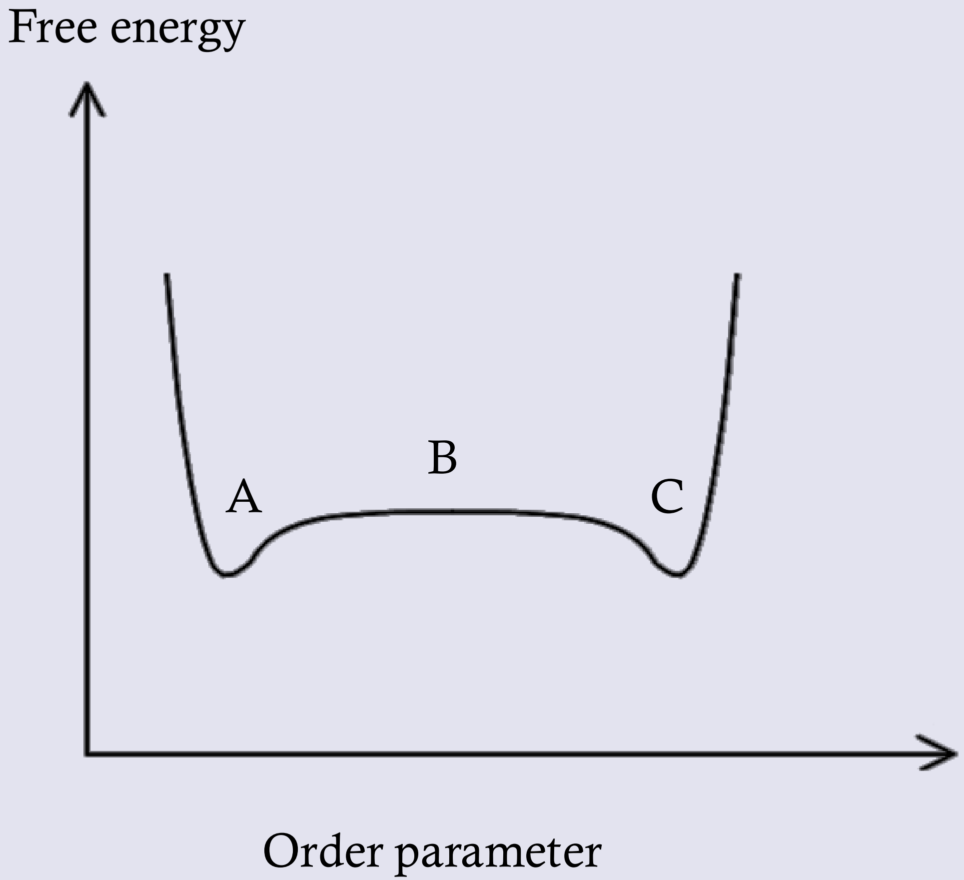

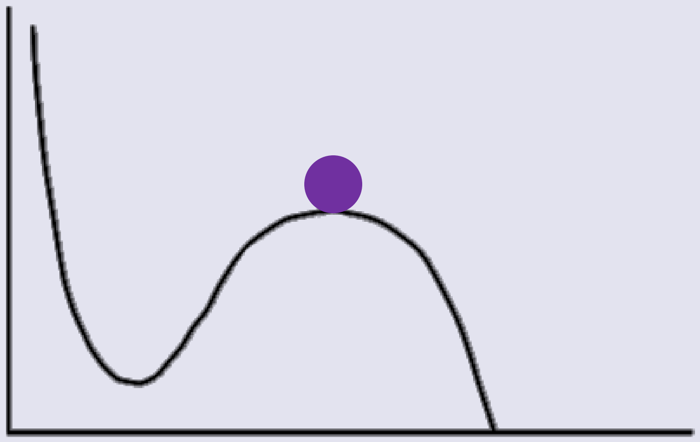

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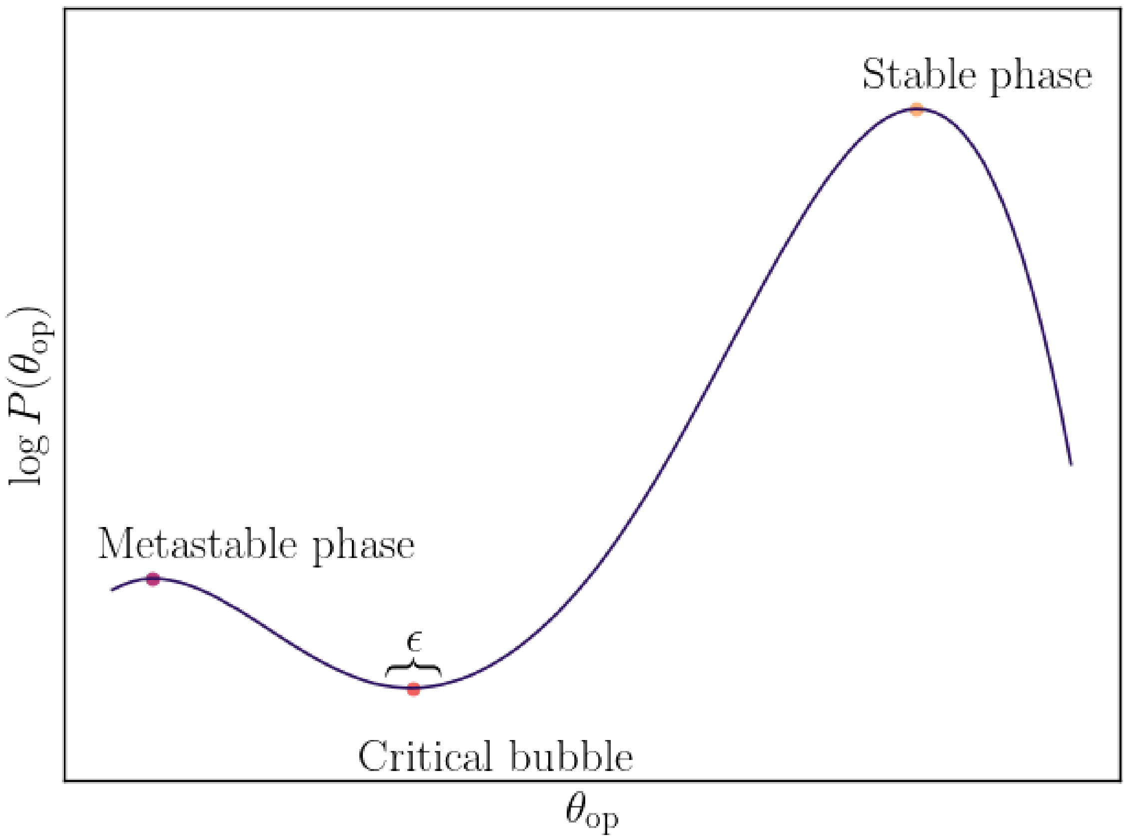
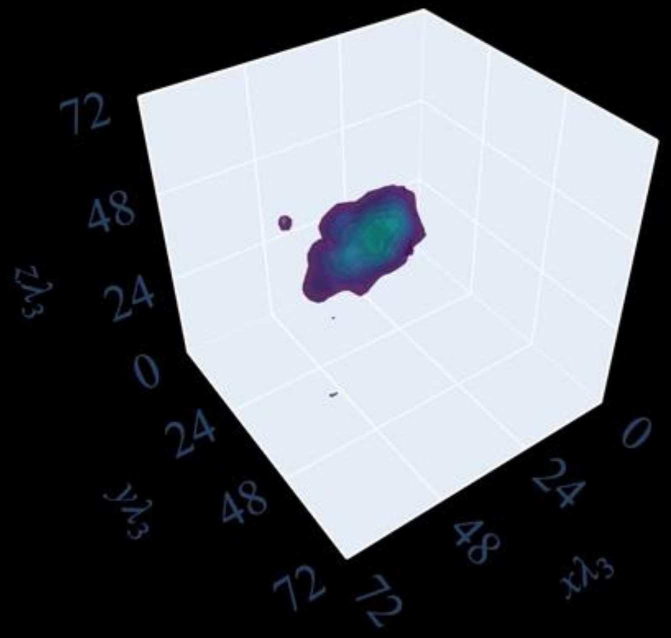
CALCULATING BUBBLE NUCLEATION RATES NON-PERTURBATIVELY

1. Simulate probability distribution (multicanonical MC) P_c
2. Calculate the probability of critical bubbles relative to the metastable state
3. Draw separatrix configurations from a small area ϵ around the critical bubble peak





$t = \Delta t$

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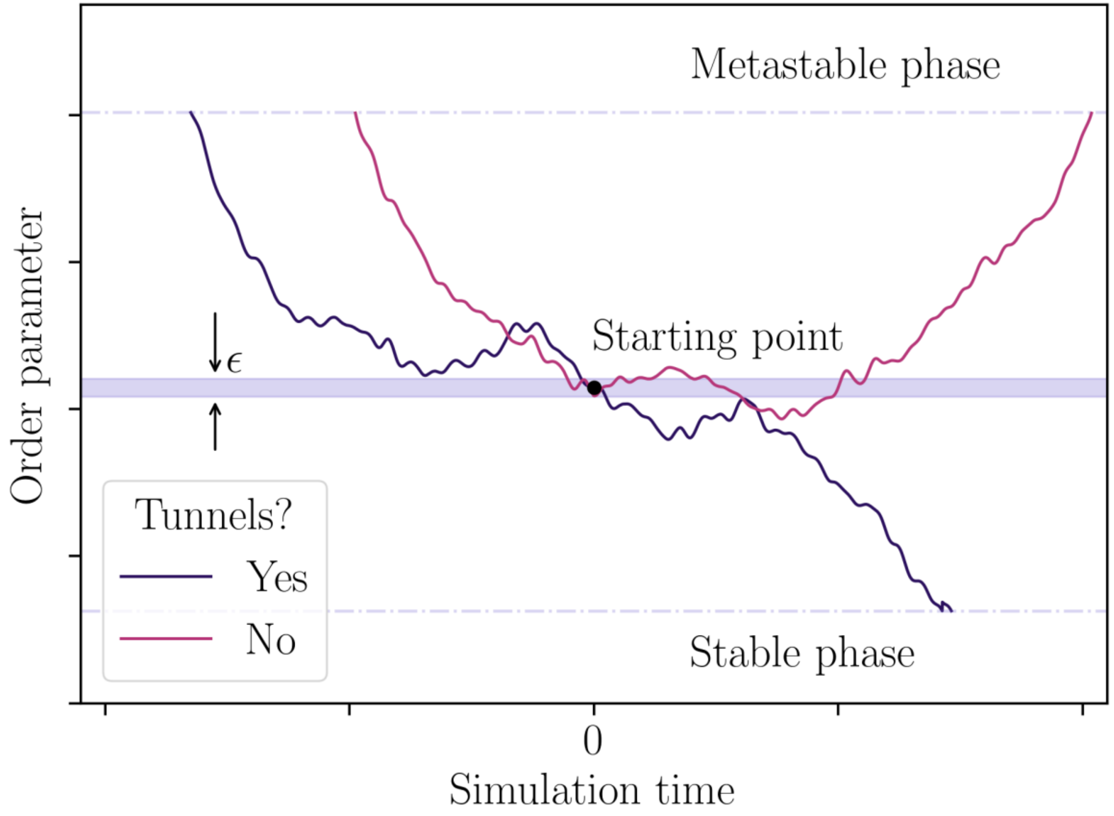
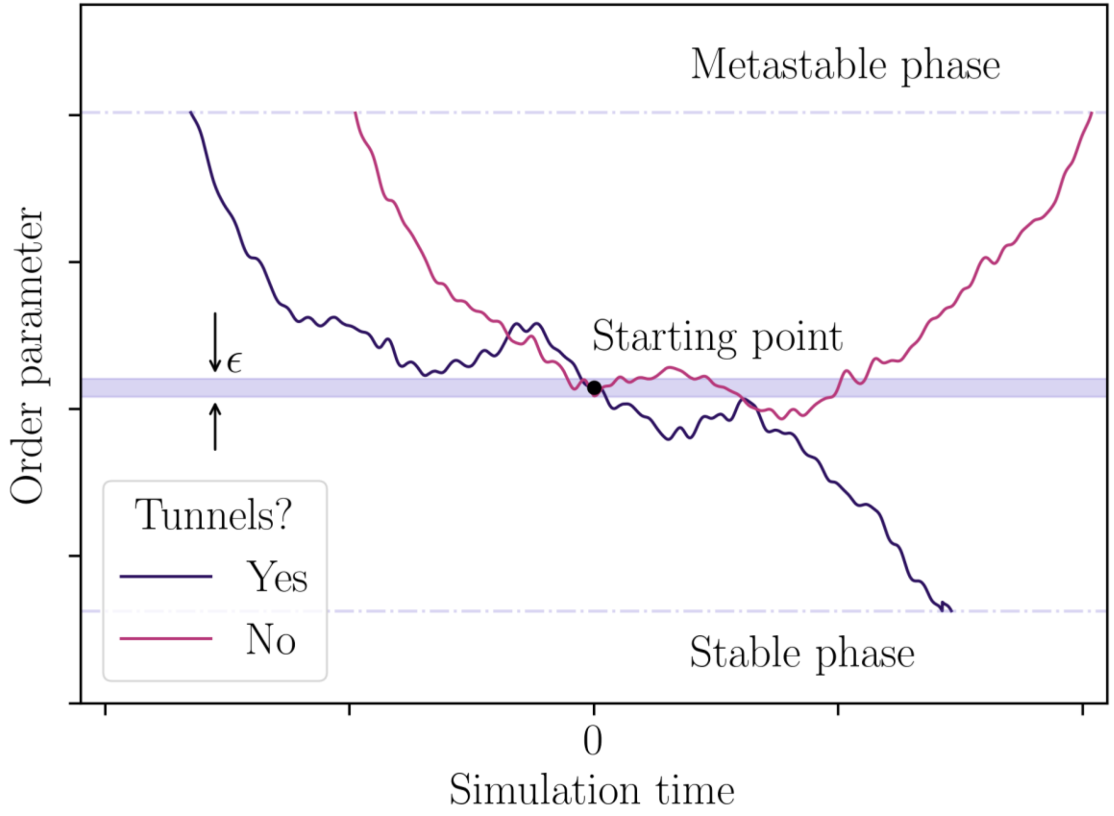


CALCULATING BUBBLE NUCLEATION RATES NON-PERTURBATIVELY

4. Evolve trajectories backwards and forwards in time to calculate the tunnelling fraction

$$\mathbf{d} = \frac{\delta_{\text{tunnel}}}{N_{\text{crossings}}}$$



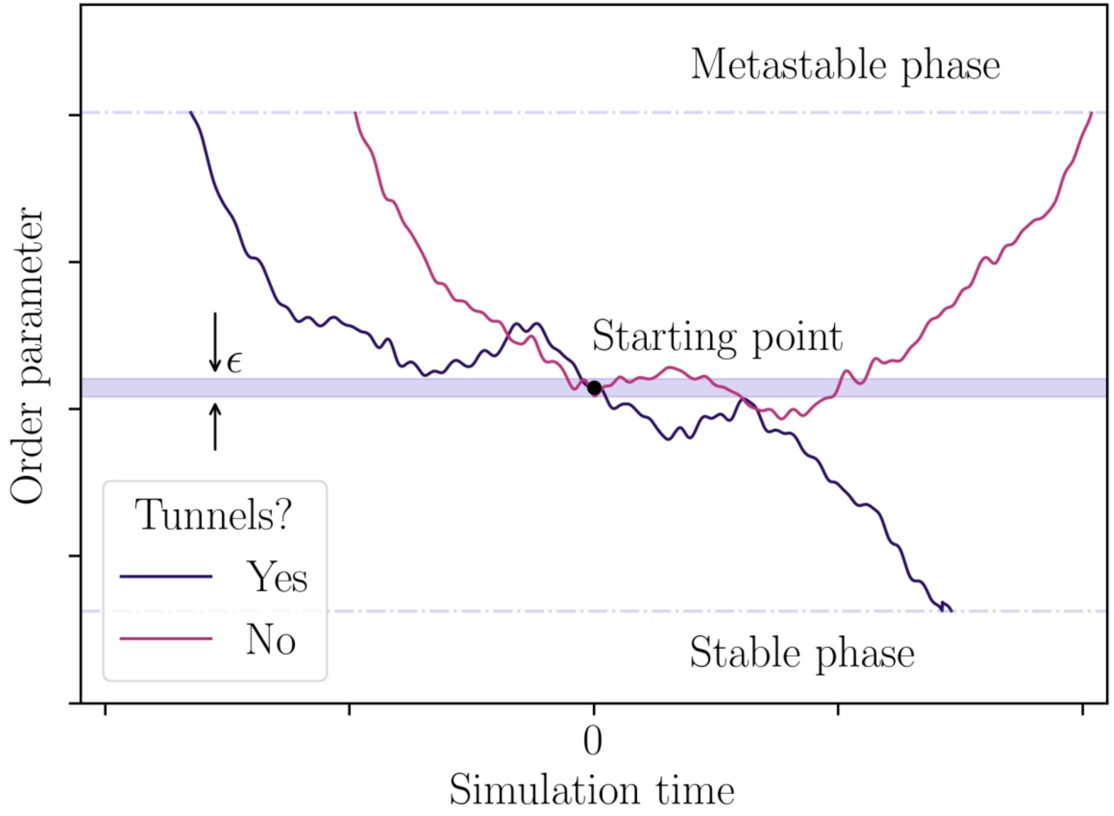
CALCULATING BUBBLE NUCLEATION RATES NON-PERTURBATIVELY

5. Calculate the flux – the rate of change of the order parameter

$$\langle \text{flux} \rangle = \left\langle \left| \frac{\Delta\theta}{\Delta t} \right|_{\theta_c} \right\rangle = \sqrt{\frac{8}{\pi\mathcal{V}}(\theta_c + A^2)}$$

6. Compute the full rate

$$\Gamma\mathcal{V} \approx \frac{1}{2} P_c \langle \text{flux} \rangle \langle \mathbf{d} \rangle$$

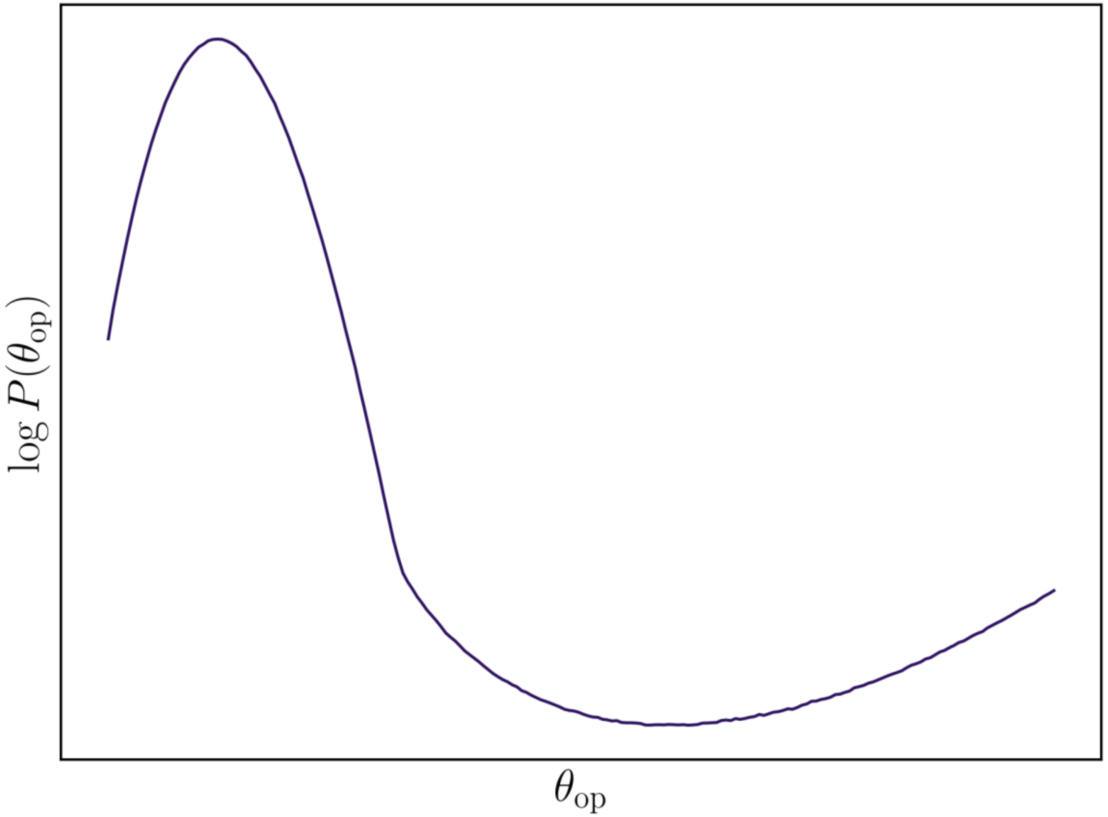


CALCULATING BUBBLE NUCLEATION RATES NON-PERTURBATIVELY

- Order parameter you pick should be able to distinguish between the different phases
- However, the bulk fluctuations can bury under the critical bubble branch

$$\theta_{\text{lin}} = \bar{\phi}$$

$$\theta = \bar{\phi}^2 - 2A\bar{\phi}$$



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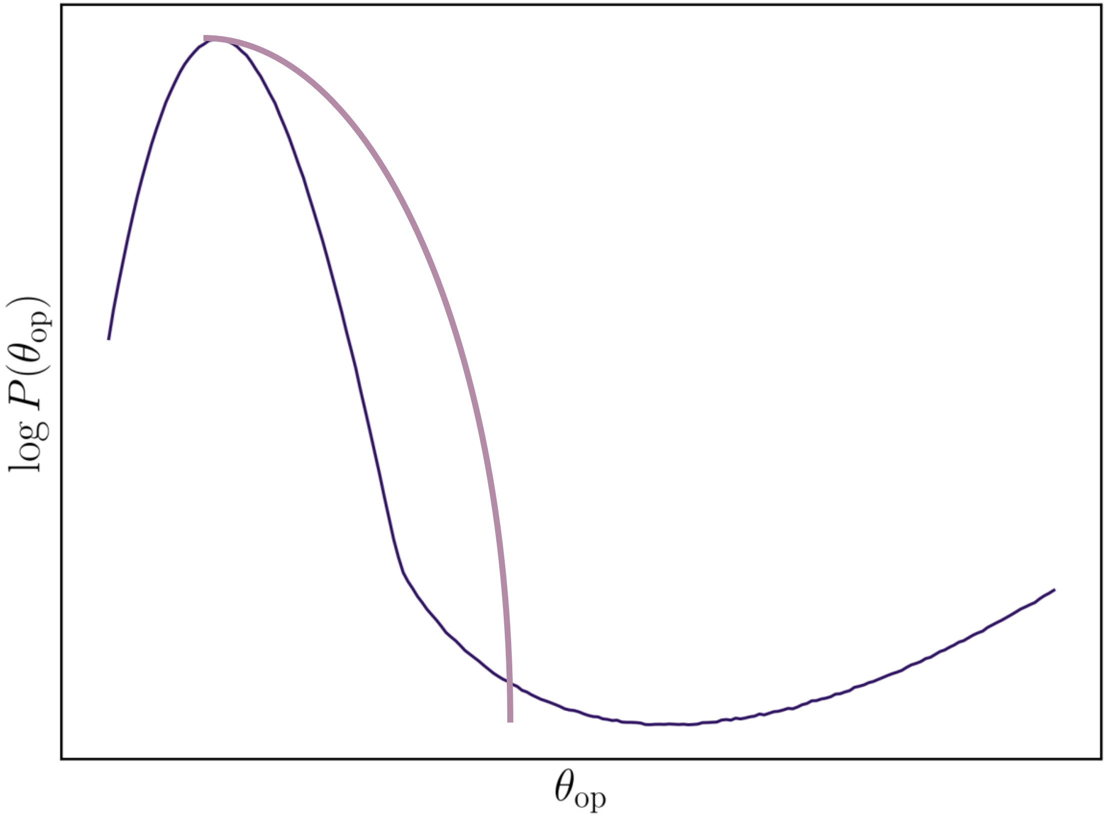

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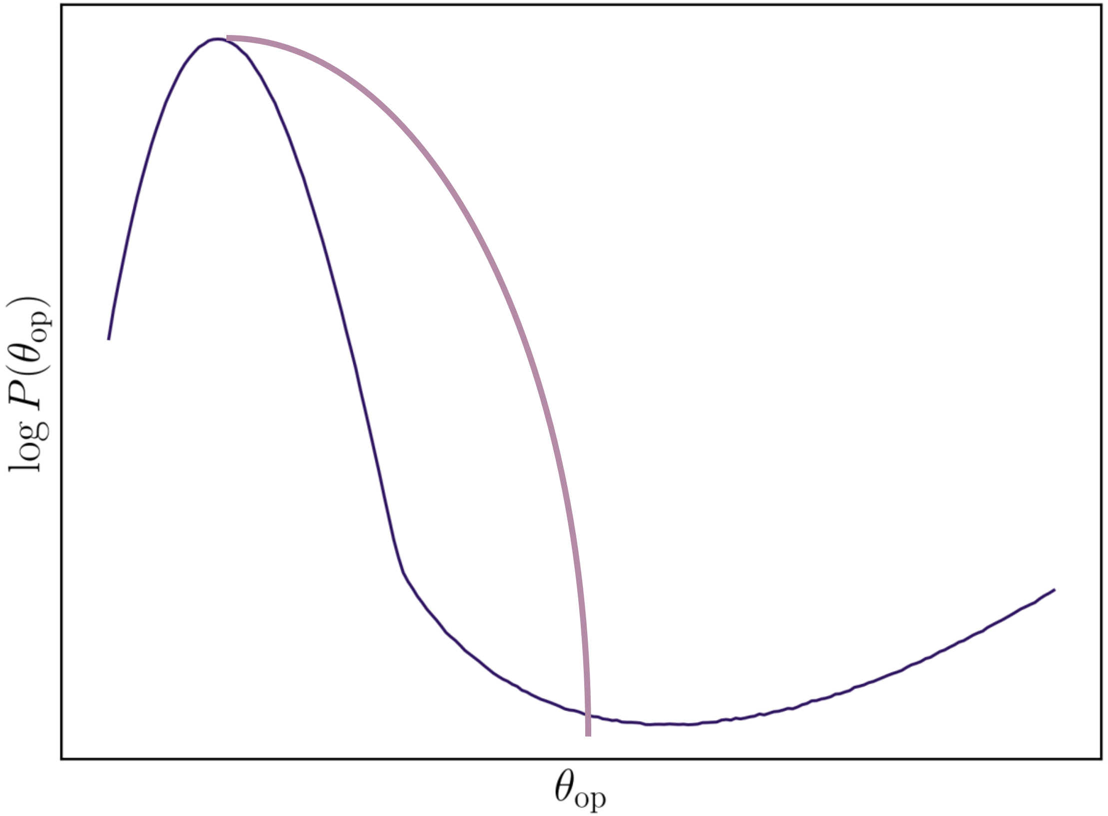
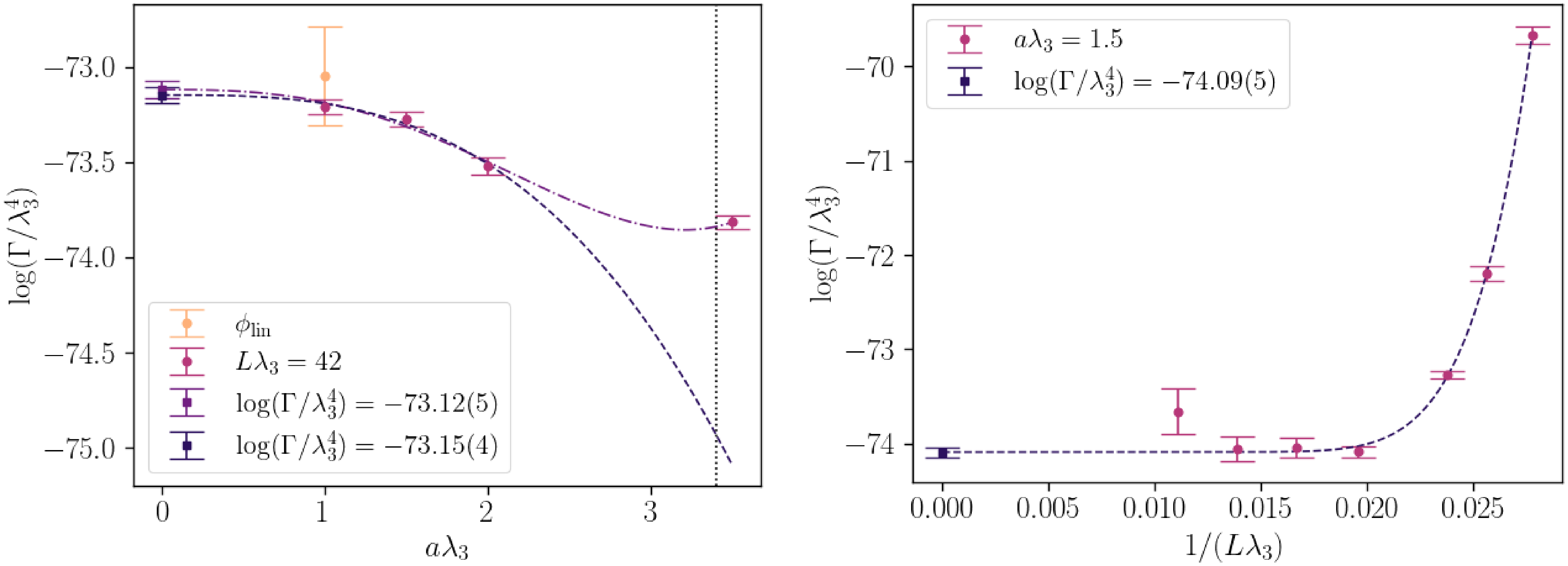


CONTINUUM & VOLUME LIMIT



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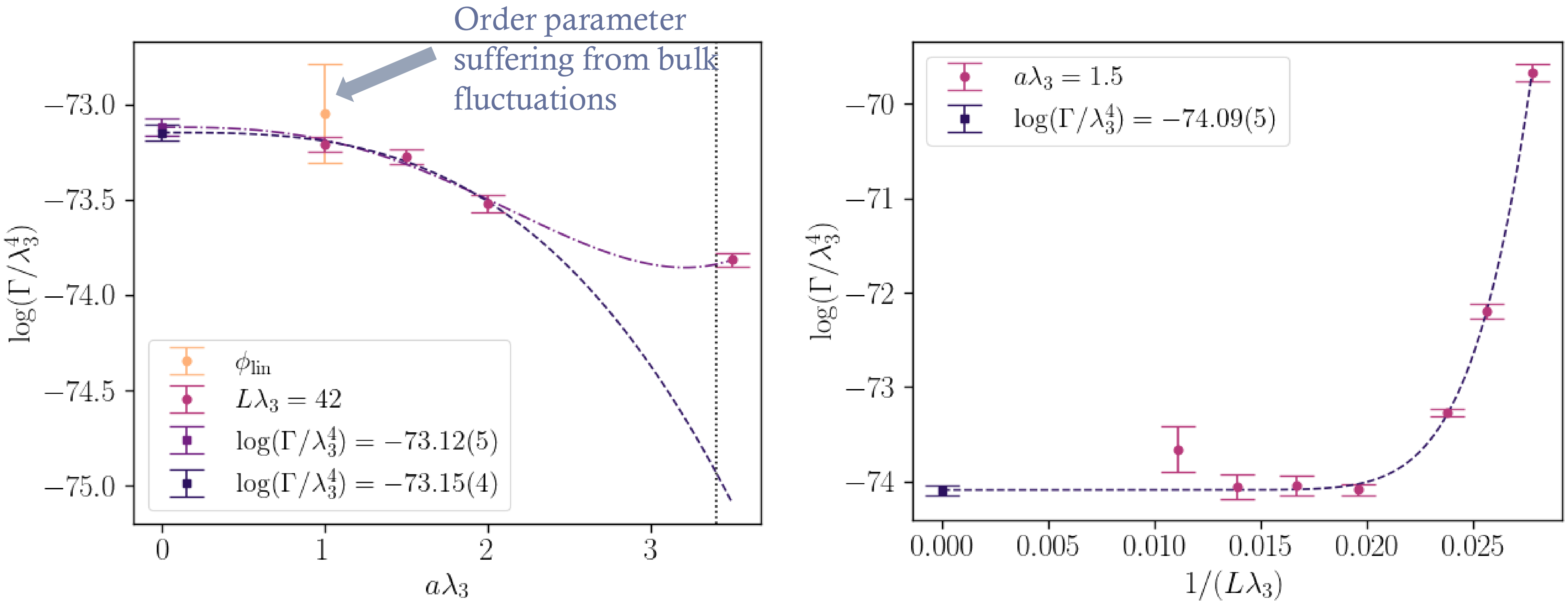
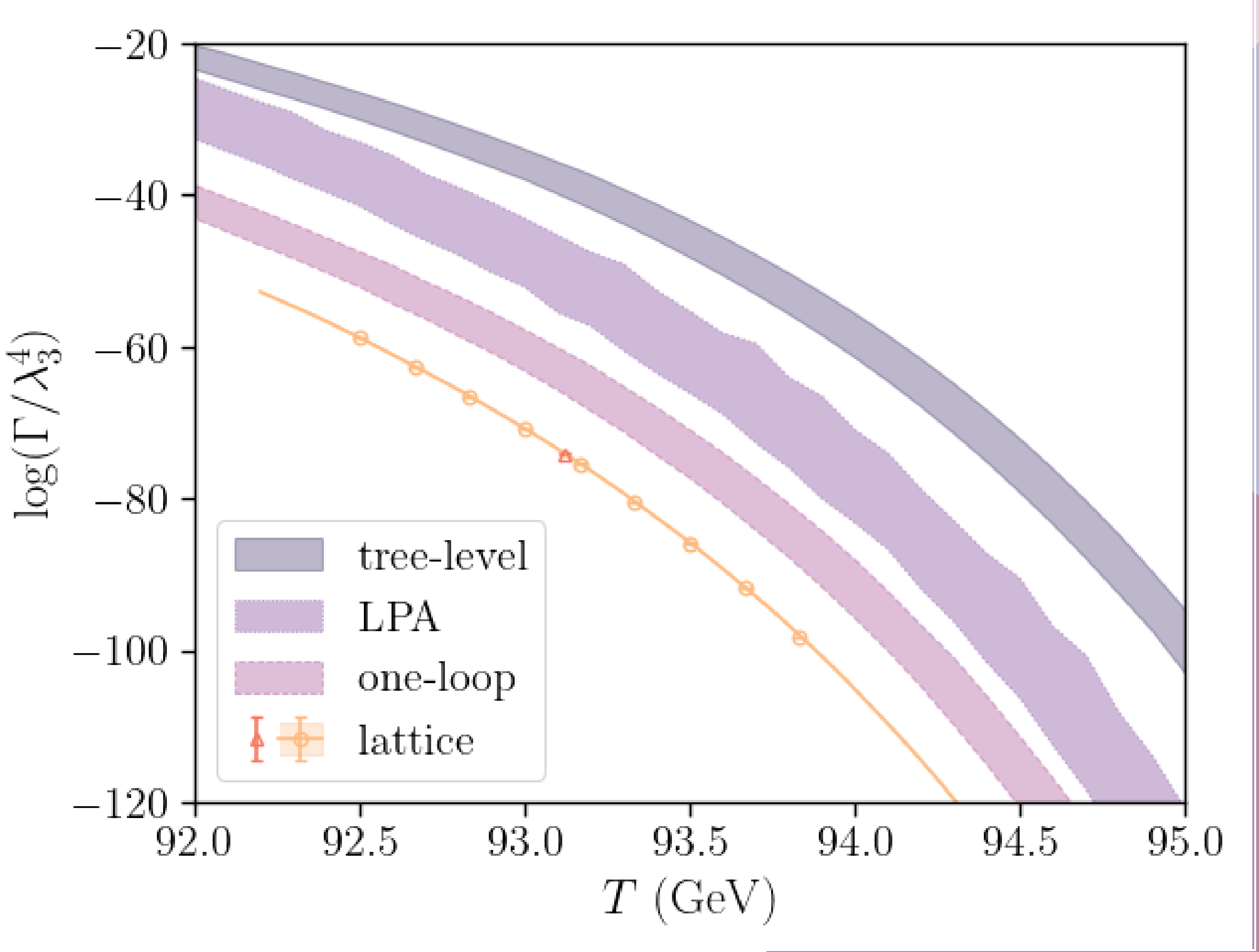
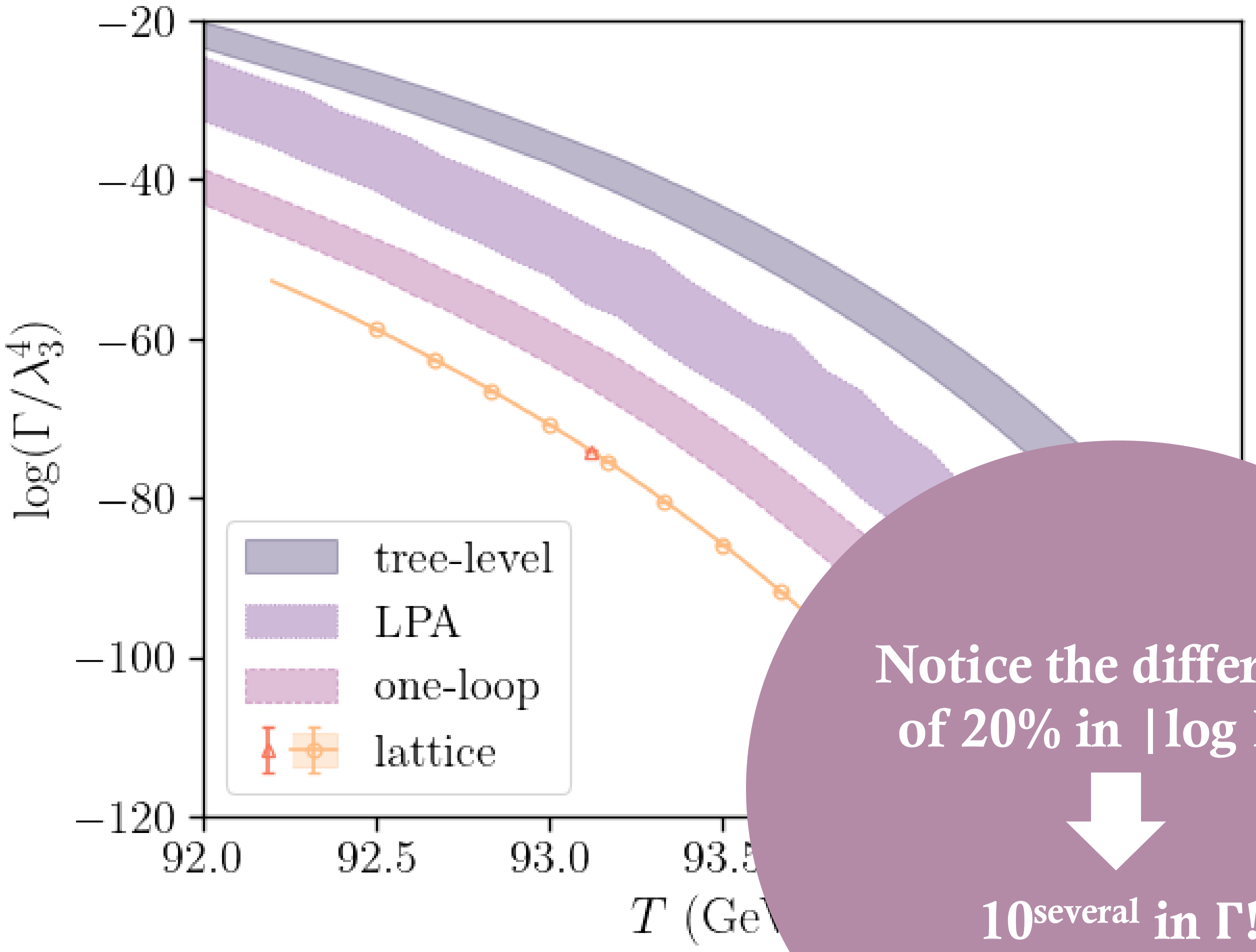


COMPARISON WITH PERTURBATION THEORY



- Tree-level = functional det and dynamic prefactor approx as T^4
- LPA = Local potential approx
- One-loop = full one-loop calculation from BubbleDet
- Lattice = one simulated param point, reweighted to other temps

COMPARISON WITH PERTURBATION THEORY



Notice the difference of 20% in |log Γ| !

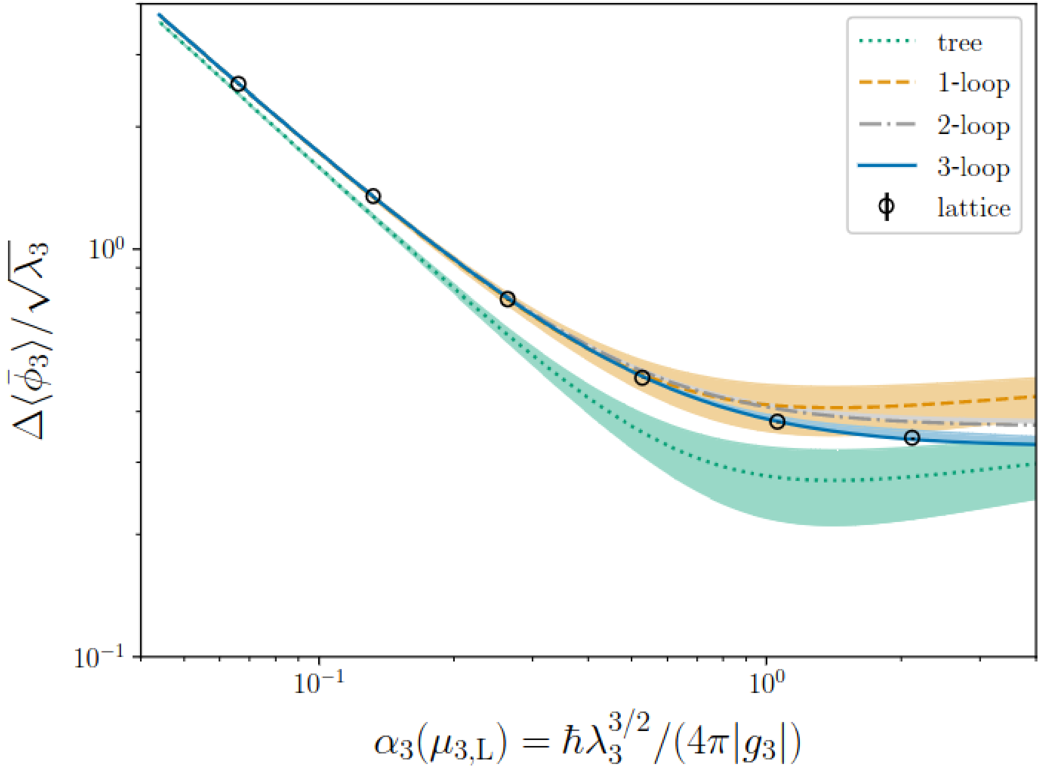
10^{several} in Γ!

- Tree-level = functional det and dynamic prefactor approx as T^4
- LPA = Local potential approx
- One-loop = full one-loop calculation from BubbleDet
- Lattice = one simulated param point, reweighted to other temps

DO WE FULLY UNDERSTAND BUBBLE NUCLEATION?

Gould, arXiv:2101.05528

- Tree-level is 100% away, one-loop is 20% away in $\log \Gamma$, system should be well described by one-loop (in linear space 10^{17}% and 10^{6}%)
  - Do we understand the discrepancy between lattice and perturbative results?
- Latent heat on the lattice vs. one-loop agree to 1% (in linear space)
  - Semiclassical expansion breaks down? Something else?



DO WE FULLY UNDERSTAND BUBBLE NUCLEATION?

- Tree-level is 100% away, one-loop is 20% away in $\log \Gamma$, system should be well described by one-loop (in linear space $10^{17}\%$ and $10^{60}\%$)
  - Do we understand the discrepancy between lattice and perturbative results?
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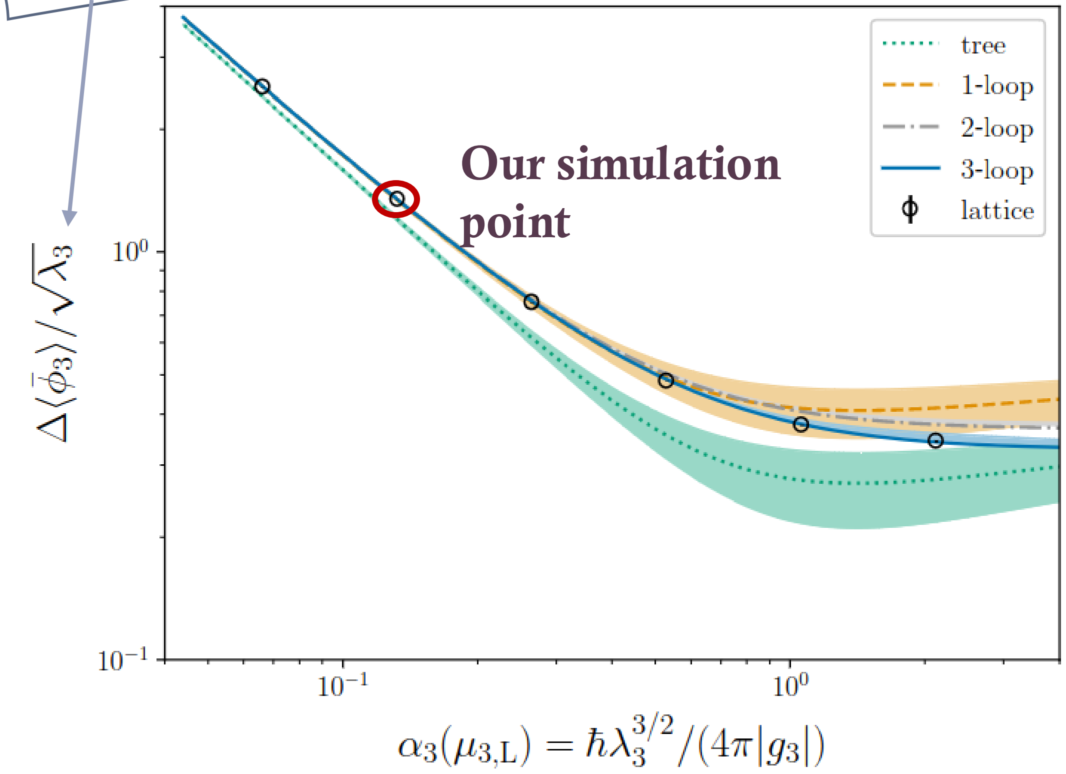
Proportional to latent heat

Gould, arXiv:2101.05528



Loop expansion parameter

CONCLUSIONS

- Allows us to calibrate the uncertainty in PT parameters when obtained from perturbative results
- Accurate computations of the nucleation rate are crucial for calculating e.g. the GW power spectrum
- Our simulations show us a suppression of the nucleation rate by a factor of 20 compared to the one loop estimate

One-bubble takeaway

Nucleation rate calculations are inaccurate in perturbation theory, lattice is significantly better

FAQ – DIMENSIONAL REDUCTION

- Fourier expansion

$$\int_\tau \int_x \left[\frac{1}{2}\psi(\tau,x)(-\nabla^2 - \partial_\tau^2 + m^2)\psi(\tau,x)\right] = \frac{1}{T}\sum_n \int_x \left[\frac{2}{2}\phi_n(x)(-\nabla^2 + (n\pi T)^2 + m^2)\phi_n(x)\right]$$

- Masses of the Fourier (Matsubara) modes are now

$$m_n^2 = (n\pi T)^2 + m^2$$

FAQ – DIMENSIONAL REDUCTION

Hard: $E \sim \pi T$

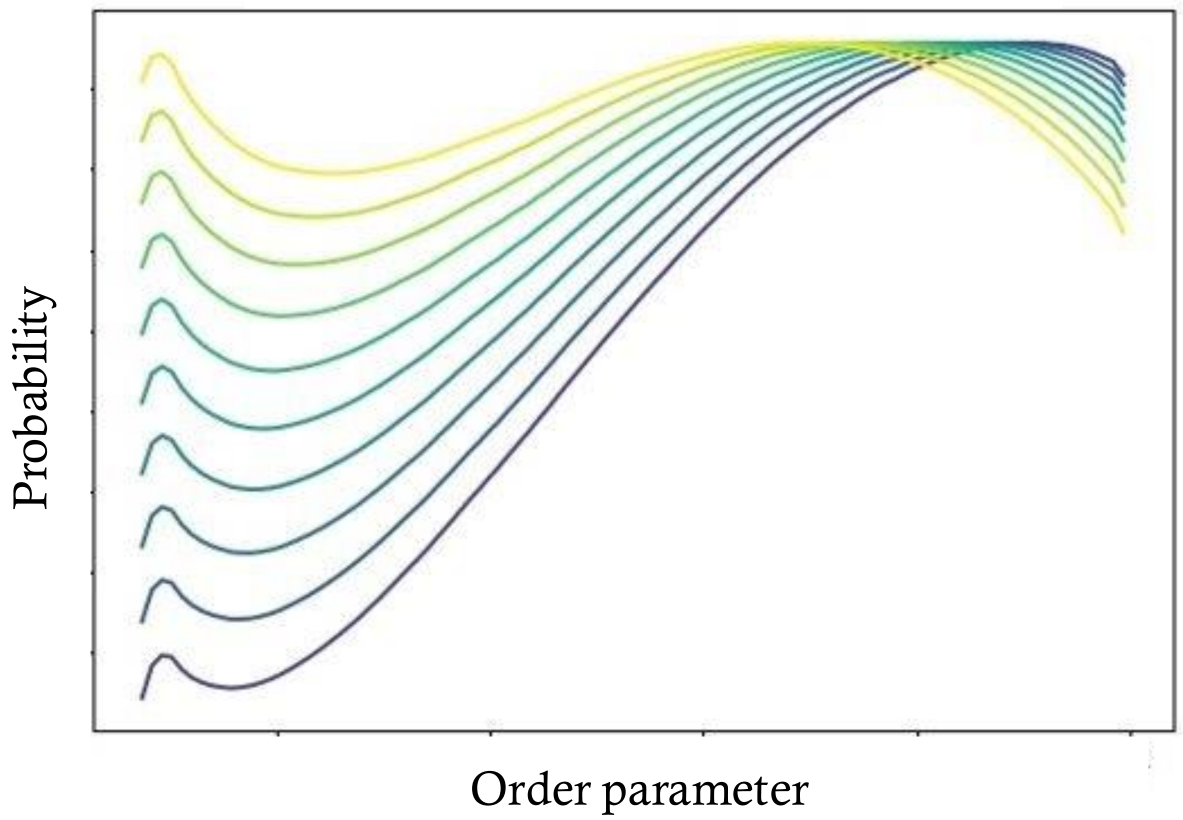
Soft: $E \sim gT$

Supersoft: $E \sim g^{3/2}T$

Ultrasoft: $E \sim g^2T$

FAQ - REWEIGHTING

- Simulations are computationally expensive → use reweighting the order parameter histogram at different parameter points
- In our case we reweight in two parameters



FAQ – HOW DO YOU KNOW LATTICE IS CORRECT?

- Compared to perturbation theory lattice is "exact"
- Method performs well in equilibrium physics (at critical temp) and perturbation theory and lattcice agree
- Less approximations than in perturbation theory
- The real time evolution matches the 4d time (no gauge field so even better)
- It is only when moving to bubble nucleation that we see a difference
  - Ok but what if your dynamical part calculation is wrong?
  - The statistical part x flux gives an upper limit to the bubble nucleation rate