Thermal false vacuum decay is not what it seems

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based on

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Tunnelling in QFT Online Seminar

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First order phase transitions: overview

Theory of decay of metastable state (“false vacuum”) covers a broad range of phenomena.

from boiling water to “boiling” vacuum of the Standard Model of particle physics

It plays an important role in various branches of physics.

from quantum matter to cosmology

Studies of decay of metastable state (“false vacuum”) have more than a century long history.

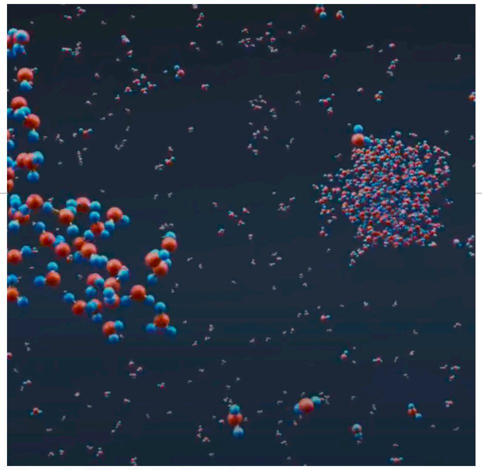
I will focus on the developments in the context of high-energy physics

We will consider **thermal** first order phase transitions:
the initial state is local thermal equilibrium — one can assign a temperature T.

Key observables: rate of decay, size of the “critical droplet” of new phase

Boiling water

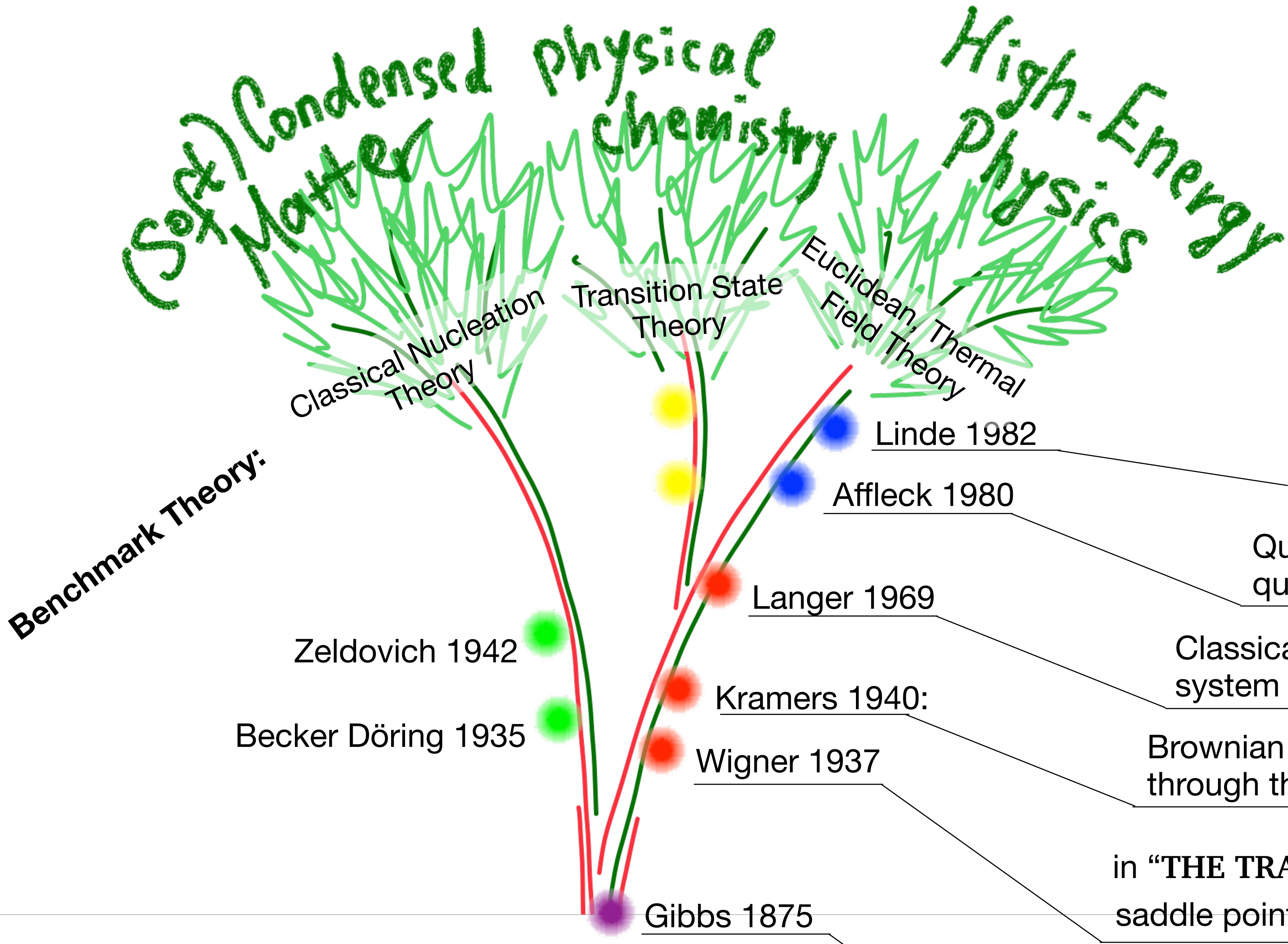
Higgs vacuum decay: Midjourney



water nucleation: MD simulation

K. K. Tanaka, A. Kawano & H.Tanaka, J. Chem. Phys. 2014

Thermal first order phase transitions: milestones



Classical Nucleation Theory

Transition State Theory

Euclidean, Thermal Field Theory

Benchmark Theory:

Zeldovich 1942

Becker Döring 1935

Linde 1982

Decay of false vacuum at finite temperature:
tunnelling and thermal activation in field theory

Affleck 1980

Quantum-statistical metastability:
quantum particle in a metastable well

Langer 1969

Classical-statistical theory of metastability:
system of many d.o.f. coupled to external heat bath

Kramers 1940:

Brownian motion of a classical particle
through the barrier at different viscosity

Wigner 1937

in “**THE TRANSITION STATE METHOD**”:
saddle point, negative mode, zero modes

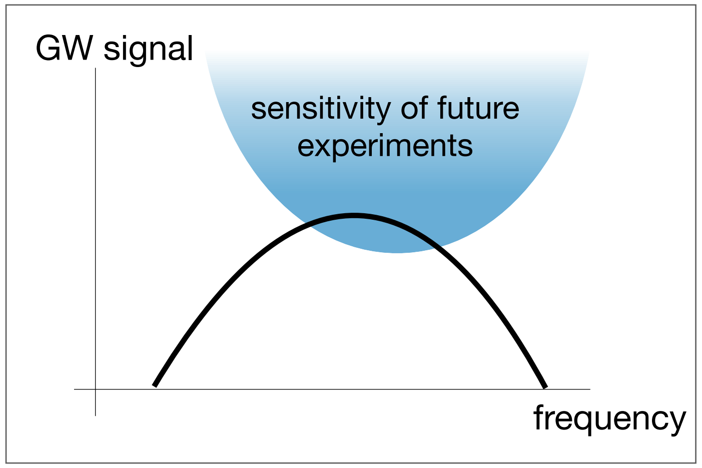
Gibbs 1875

in **“ON THE EQUILIBRIUM OF HETEROGENEOUS SUBSTANCES”:**
first discussion of the critical bubble, its size and free energy

Motivation in particle physics and cosmology

- **First order phase transitions in the early universe**

  Nucleating, propagating and colliding bubbles generate gravitational waves (GWs). This is a motivation to improve the existing and build new GW detectors.



- **Generation of baryon asymmetry of the universe**

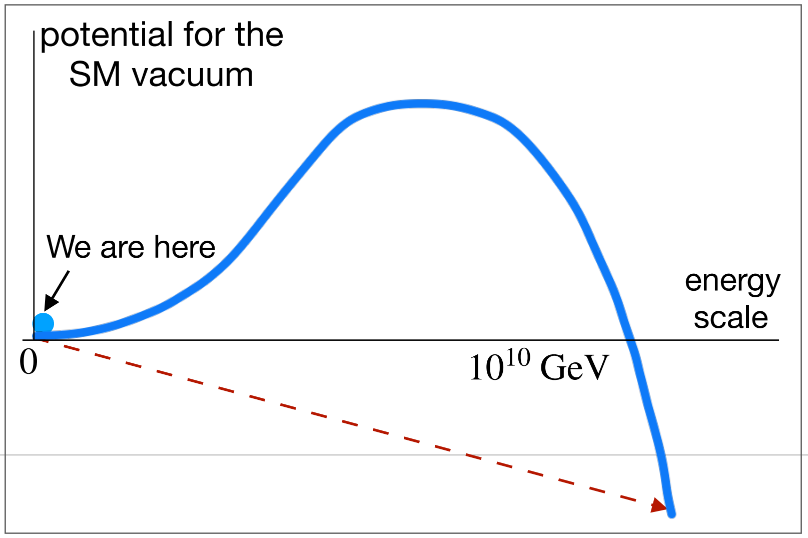
  Expanding bubbles moving through cosmic plasma can generate asymmetry between particles and anti-particles.

- **Metastability of the Standard Model vacuum**

  According to the measured values of the parameters of the Standard Model (SM), our "fundamental" vacuum may itself be metastable.

  In habitable parts of the present-day universe the decay probability is very small.

  This may not be so in extreme environments (e.g. black holes) or earlier epochs (e.g. inflation).



- **Experimental tests of nucleation theory**

  Zenesini et al, Nature Physics 20, 558–563 (2024) — first experimental result using a cold atom system

Thermal decay rate

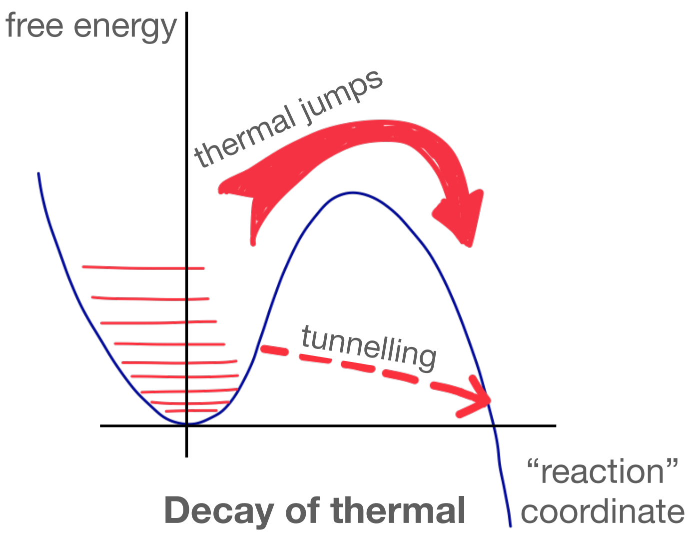
Decay probability per unit time and volume is $\Gamma = A(T)e^{-B(T)}$

exponential suppression — LO of WKB

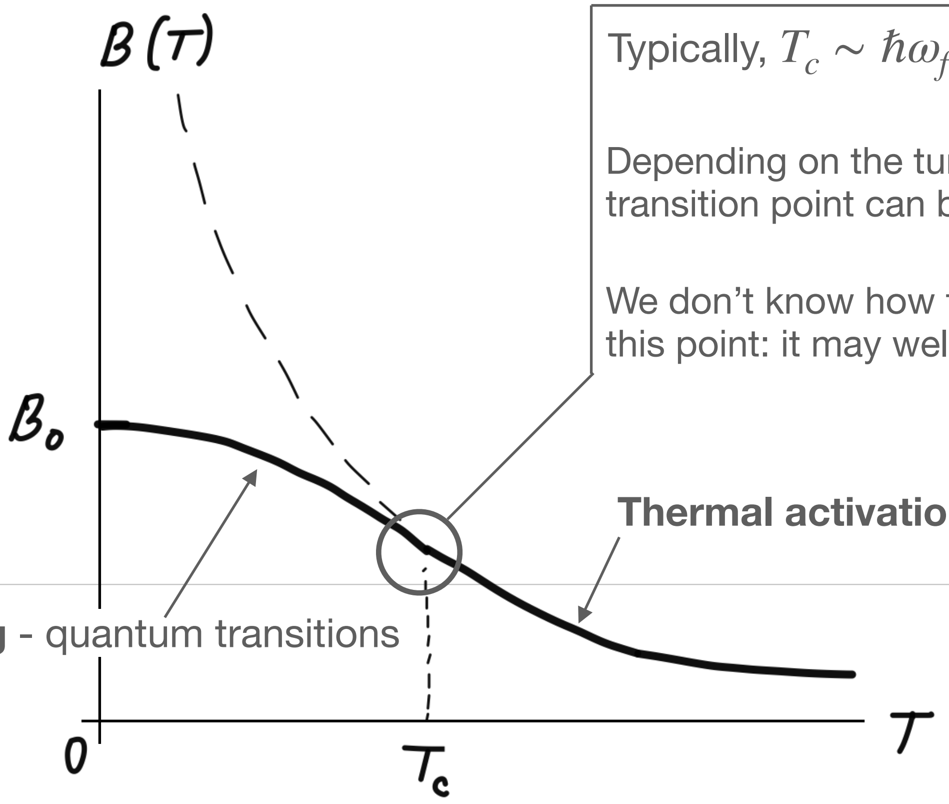
prefactor — NLO of WKB

$B(T)$ is computed using the stationary point of free energy (*bounce*)

$A(T)$ is found by evaluating the determinant of small fluctuations around the bounce



Decay of thermal metastable state



vacuum suppression →

Typically, $T_c \sim \hbar\omega_{fv}$

Depending on the tunnelling potential, the transition point can be smooth or only continuous.

We don't know how the **prefactor** behaves around this point: it may well be discontinuous.

Thermal activation - classical transitions

Tunneling - quantum transitions

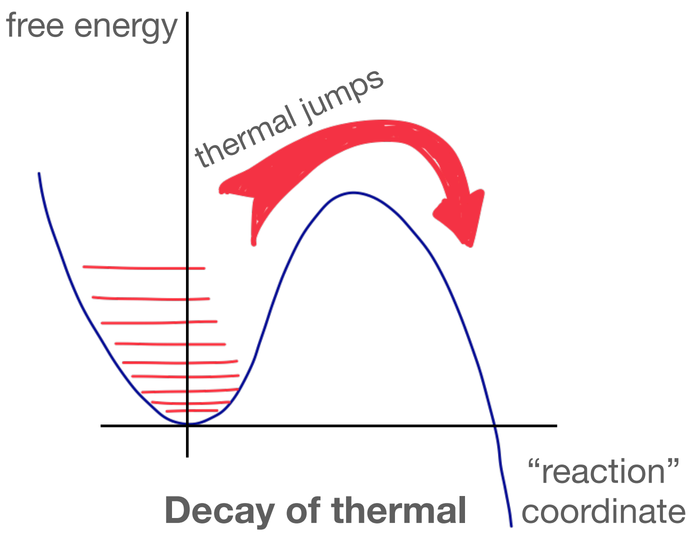
Exponential suppression of vacuum decay as a function of temperature

Affleck 80

Chudnovsky 92

Classical thermal decay rate

At T high (classical regime) but not too high (exponential — Boltzmann — suppression) the decay happens via the special thermodynamic fluctuation: **critical bubble**.



Decay of thermal metastable state

- **Standard thermal (Euclidean, equilibrium) theory** predicts:

Growth rate of the critical bubble's unstable mode

Free energy around the false vacuum

$$\Gamma_E = \frac{\omega_-}{\pi T}\frac{ImF(T)}{\mathscr{V}}$$

Volume

To test the predictions of the Euclidean theory and to study **dynamics** of the phase transition, one can run

- **Real-time, classical, lattice simulations**

They are applicable if occupation numbers of all relevant for the decay modes are big.

Such simulations have been employed for different purposes:

- Gould, Moore, Rummukainen — Vacuum decay, "multi-canonical sampling" + real-time evolution
- Alford, Feldman, Gleiser — Vacuum decay, **Langevin** dynamics
- Grigoriev, Rubakov, Shaposhnikov — Sphaleron transitions, soliton pair production, **Hamiltonian** dynamics

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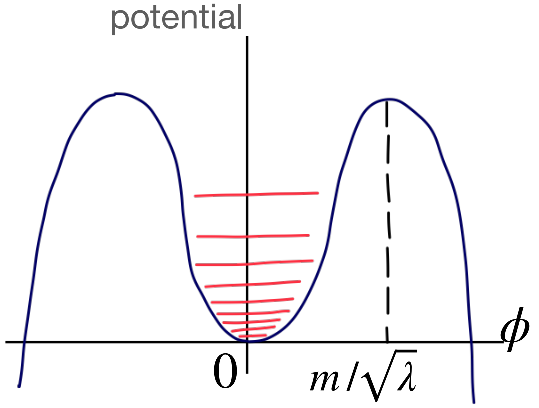
Setup

- We take the scalar field theory in 1+1 dimensions, with quartic self-interaction, and discretize it on the periodic lattice of size L with N sites and spacing a. This gives the following system:



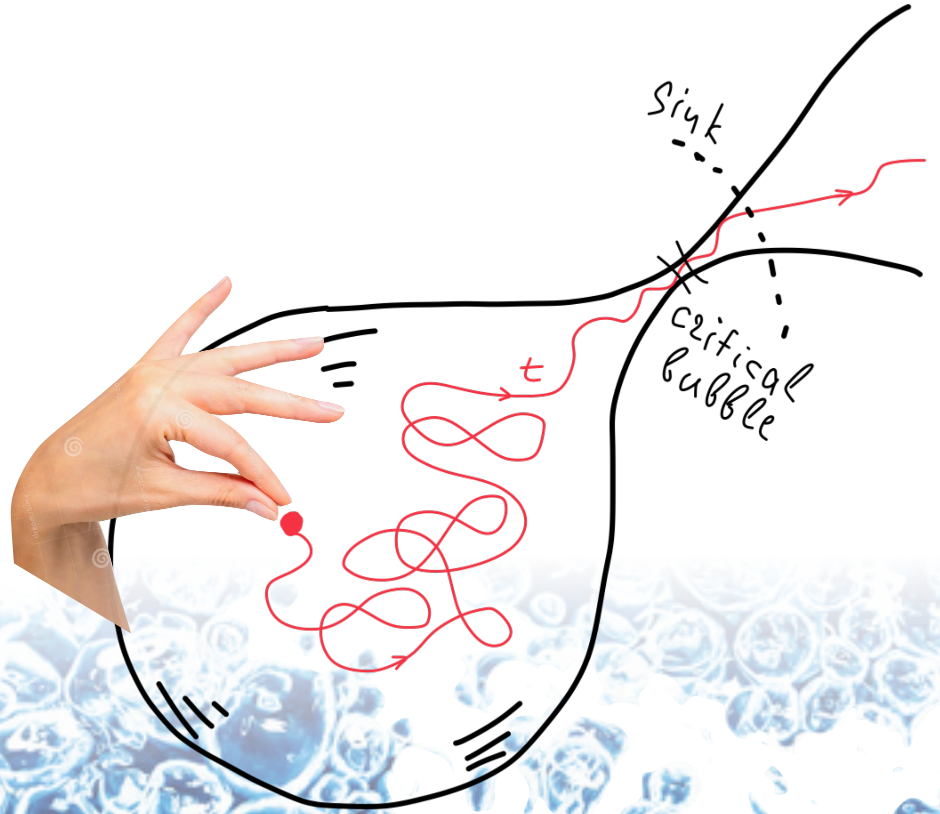
$$H = a\sum_{i=0}^{N-1}\left[\frac{\pi_i^2}{2} - \frac{1}{2}\phi_i(\Delta\phi)_i + \frac{m^2\phi_i^2}{2} - \frac{\lambda\phi_i^4}{4}\right]\,, \quad (\Delta\phi)_i = a^{-2}(\phi_{i+1} - 2\phi_i + \phi_{i-1})$$

lattice Laplacian

The system is evolved according to equations:

$$\begin{cases} \dot{\phi}_i = \pi_i \\ \dot{\pi}_i = (\Delta\phi)_i - m^2\phi_i + \lambda\phi_i^3 \end{cases}$$

- We prepare a suite of simulations with the initial state in thermal equilibrium around $\phi = 0$.



— It has the (almost) Rayleigh-Jeans spectrum, one can set it up explicitly.

The leading effect of self-interaction is the thermal correction to the mass:

$$m_{th}^2 = m^2 - \frac{3\lambda T}{2m}\,, \quad \underbrace{\frac{\lambda T}{m^3} \ll 1}_{\text{weak coupling condition}}$$

— It can also be set up implicitly, using Hamiltonian Monte-Carlo or Langevin evolution.

- We run the simulations until the decay happens (or time runs out)

Measuring decay rate

- **Euclidean theory predicts:**

  in the continuum limit

$$\Gamma_E = \frac{6m^2}{\pi}\sqrt{\frac{E_b}{2\pi T}}\,\mathrm{e}^{-E_b/T}$$

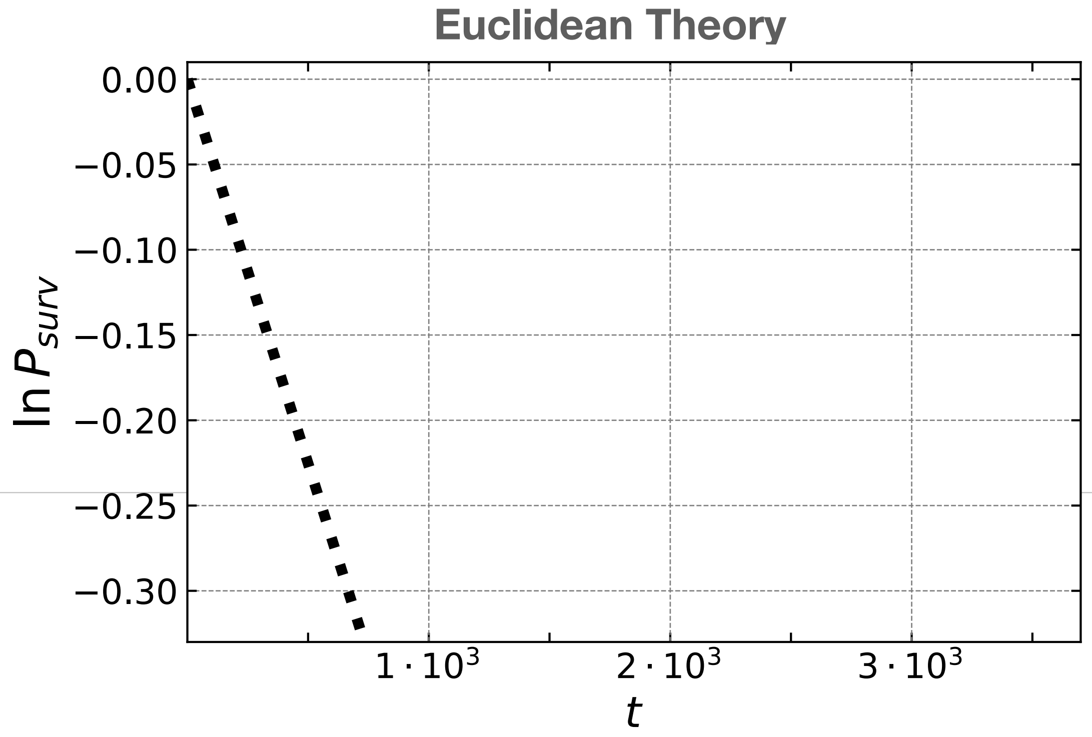
$$E_b = \frac{4m^3}{3\lambda}$$

— barrier (critical bubble) energy

- We measure the **survival probability** $P_{surv}(t)$

  For decays obeying the exponential distribution, it follows the law: $\ln P_{surv}(t) = \mathrm{const} - \Gamma L \cdot t$

  (we exclude early-time transients)



First surprise

- **Euclidean theory predicts:**

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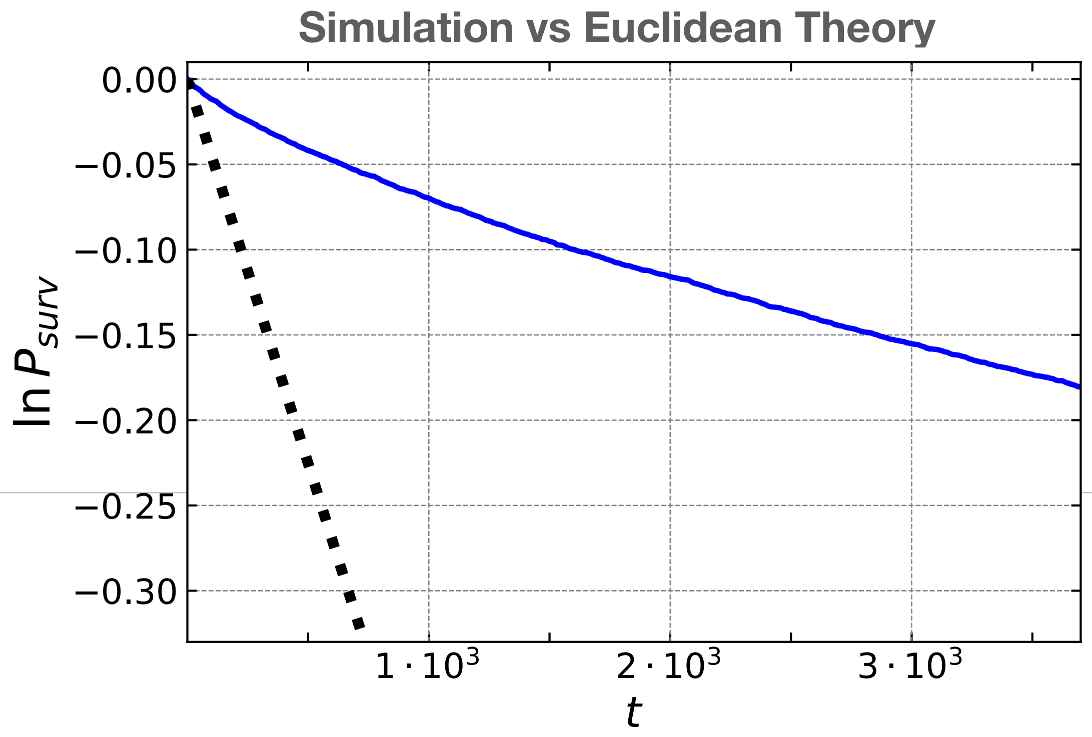
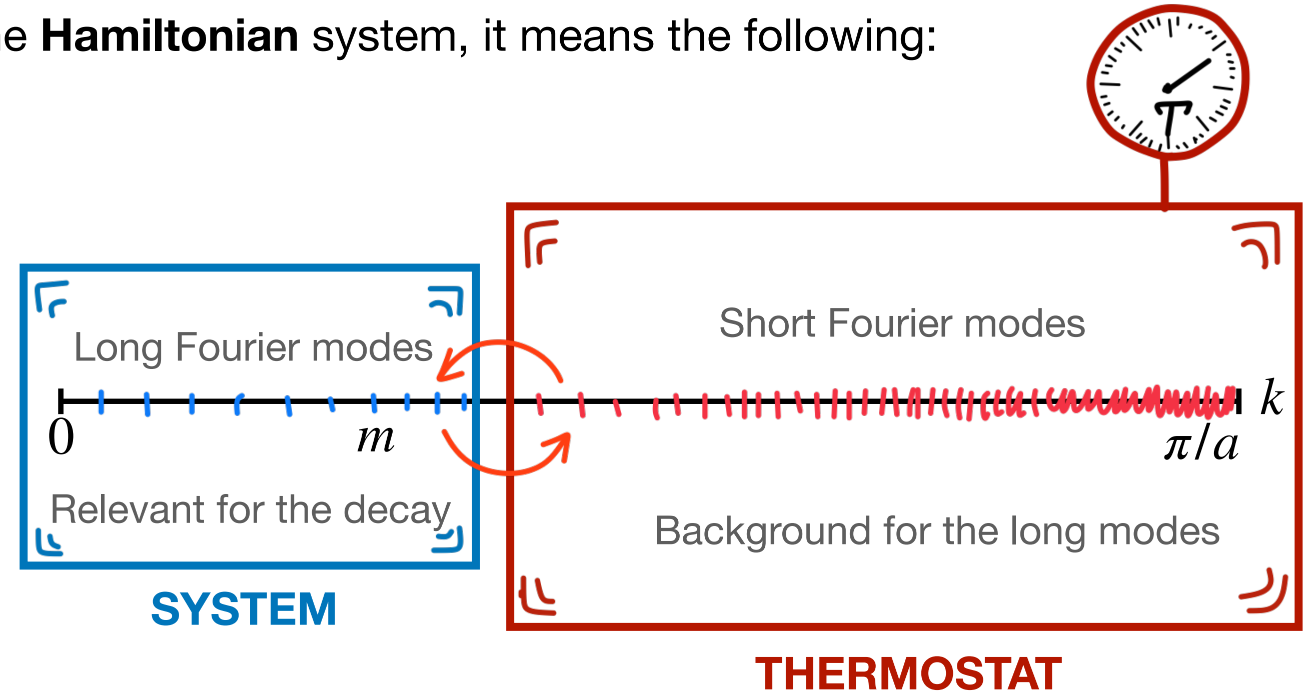


- Decay rate found in simulations is smaller than the Euclidean prediction
- It is, moreover, time-dependent, getting even smaller with time

What does it mean “thermal”?

For the **Hamiltonian** system, it means the following:



But **thermalisation** here is very **inefficient**: for modes with $\omega \sim m \sim$ (bubble size)$^{-1}$,

its time scale is $t_{th} \simeq \dfrac{(2\pi)^3}{m}\left(\dfrac{m^3}{\lambda T}\right)^4 \ggg m^{-1}$

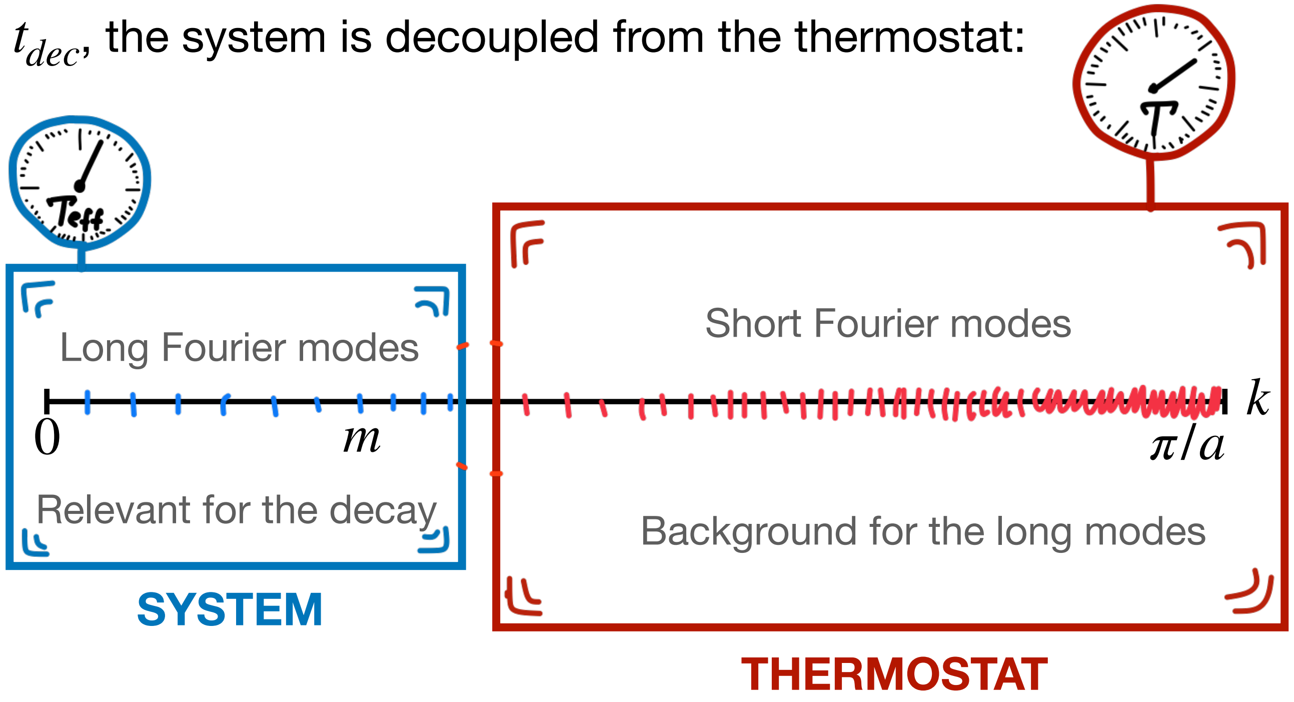
(due to $2 \to 4$ and $3 \to 3$ scattering processes)

Compare this with the decay time $t_{dec} \sim (\Gamma L)^{-1}$

In our simulations it happens that $t_{th} > t_{dec}$! This leads to the interesting effect…

Classical Zeno effect

At $t \lesssim t_{dec}$, the system is decoupled from the thermostat:

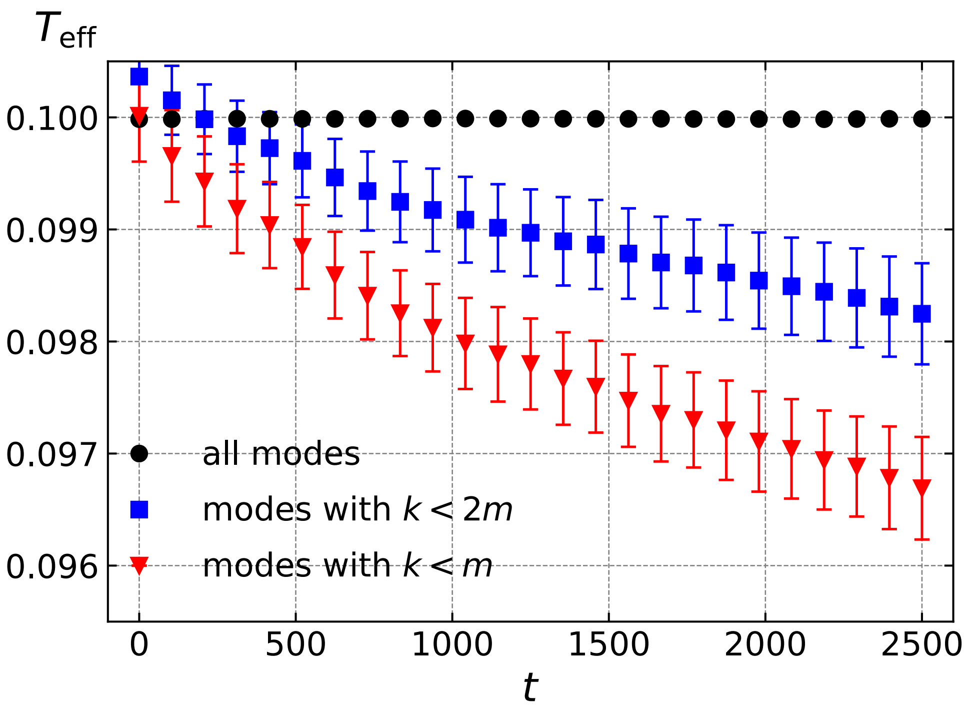


T_{eff} is preserved during the simulation.

Simulations with higher initial T_{eff} (due to statistical fluctuations) decay faster (on average).

→ Statistical properties of the ensemble change with time: long modes cool down.

Classical Zeno effect



Effective temperature of long modes
for simulations whose lifetime is longer than *t*

😱 Decay is a non-Markovian process (in this regime).

🤔 The longer we observe the system, the less chance it has to decay in the future: classical Zeno effect.

To find the unbiased rate, we extrapolate the slope of the survival probability curve to zero.

Second surprise

Recall that $\Gamma(T) = A(T)\exp(-E_b/T)$.

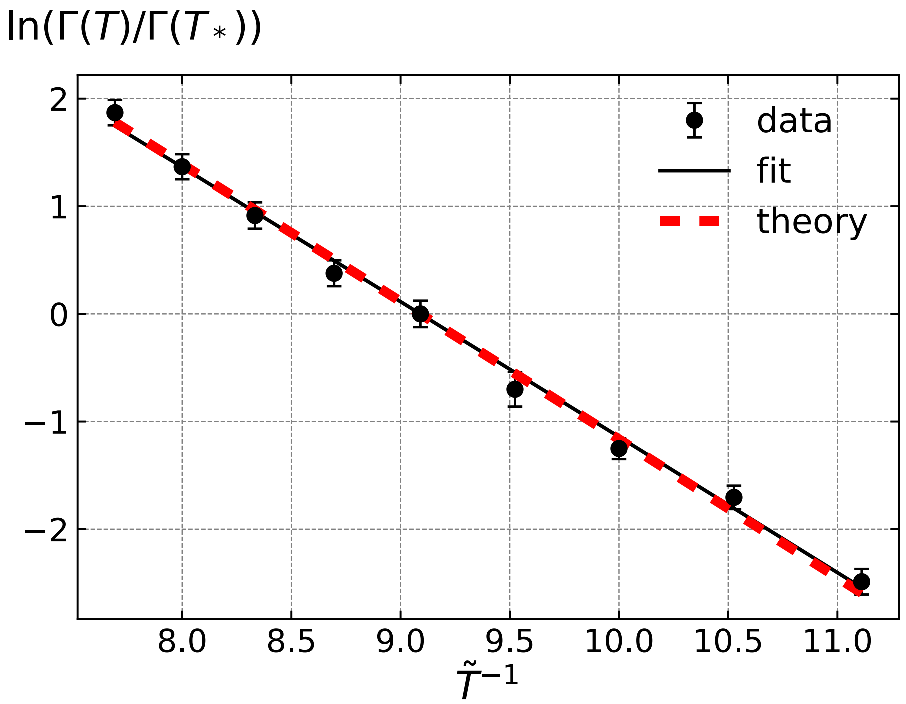
We measure the (unbiased) decay rate at different T and fit it with the formula

$$\ln\Gamma(T) = -\frac{1}{2}\ln T + \ln A - \frac{B}{T}$$

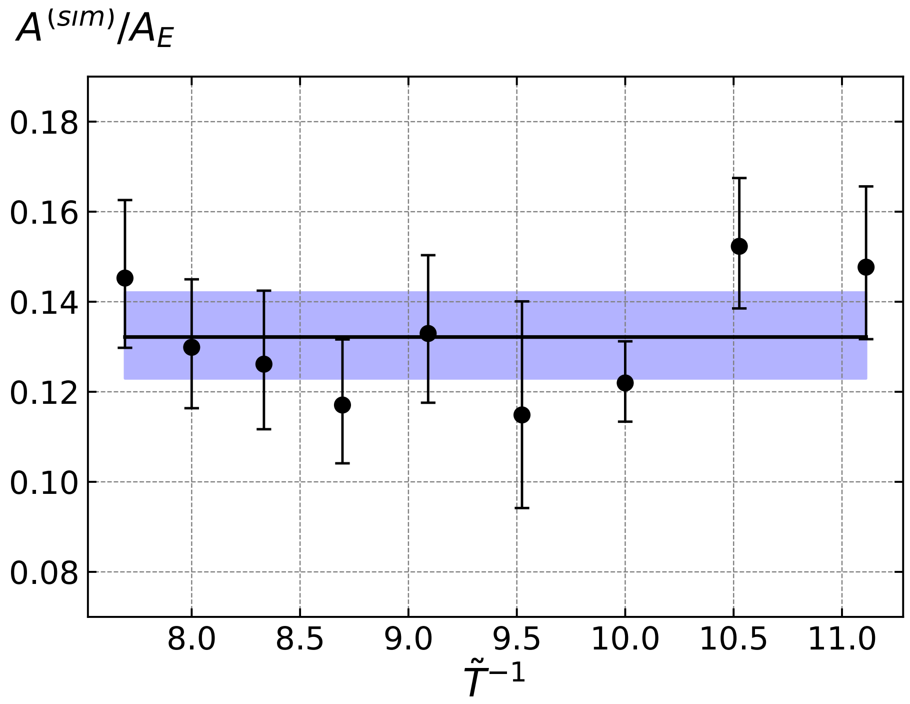
← critical bubble energy

prefactor (with the zero mode excluded)

from the zero mode in the prefactor



Critical bubble energy agrees perfectly with the Euclidean theory (<2% error bar)

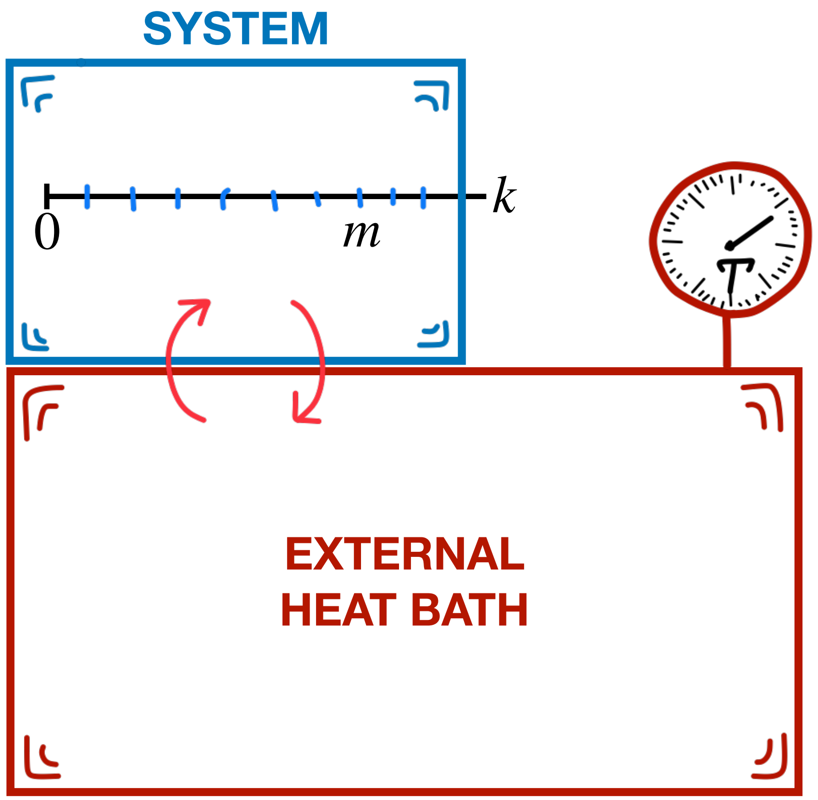


The measured prefactor is **smaller** by almost one order of magnitude.

Thermalization is still too slow?

Langevin evolution

We would like to couple the theory to an external heat bath with the controlled coupling strength:



This is done by promoting the Hamiltonian equations of motion to the **Langevin** equation:

$$\begin{cases} \dot{\phi}_i = \pi_i \\ \dot{\pi}_i = (\Delta\phi)_i - m^2\phi_i + \lambda\phi_i^3 - \eta\pi_i + \sigma\xi_i \end{cases}$$

linear damping

white noise

$$\langle\xi_i(t)\rangle = 0\ , \quad \langle\xi_i(t)\xi_j(t')\rangle = \delta_{ij}\delta(t-t')$$

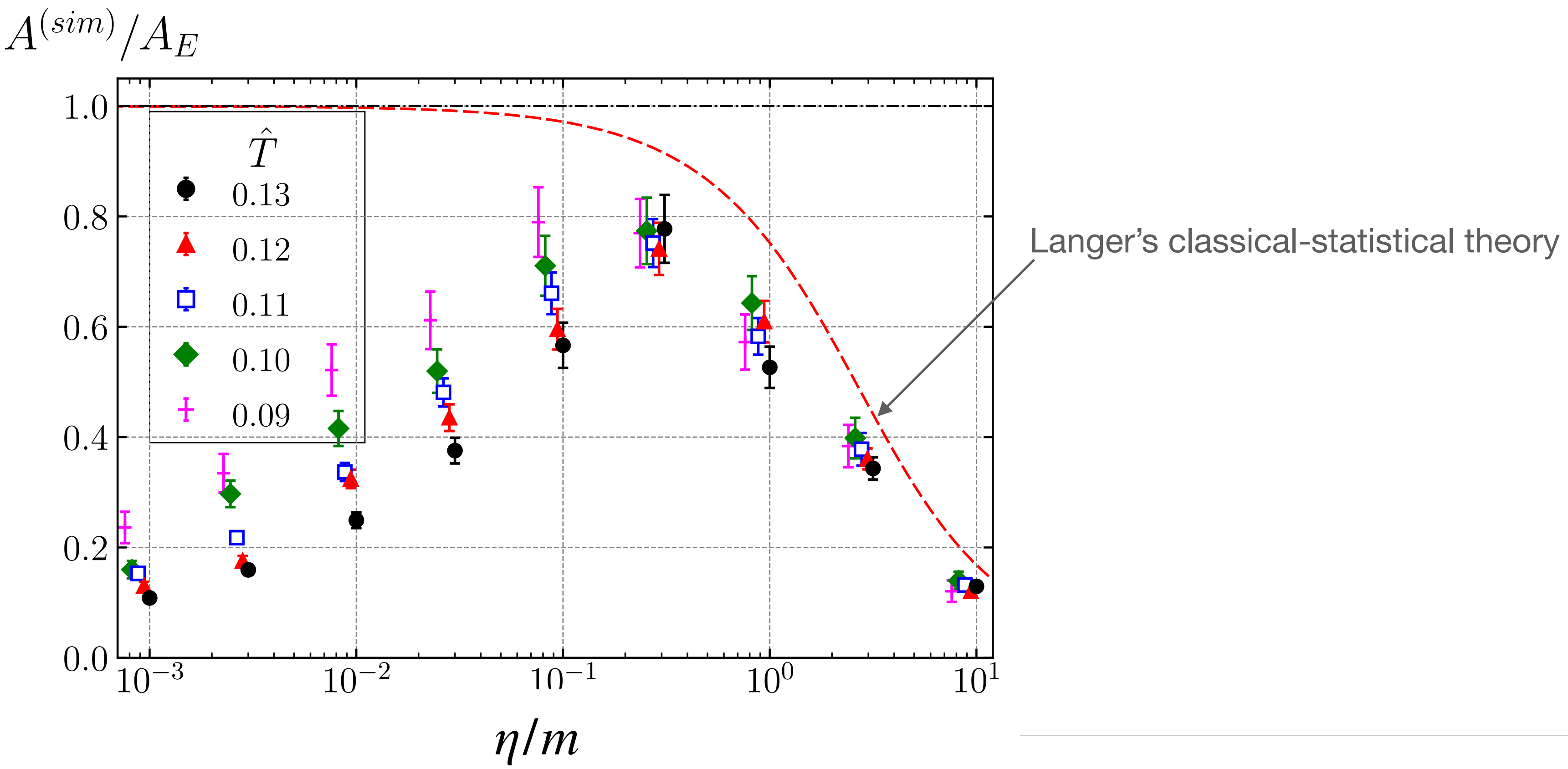
$$\sigma^2 = 2\eta T/a$$

by fluctuation-dissipation theorem

$$\rightarrow \quad t_{th} \sim \eta^{-1}$$

Main result

- No Zeno effect as long as $\eta \gtrsim \Gamma L$.
- As dissipation increases, Γ increases as well. It reaches maximum at $\eta/m \simeq 3 \cdot 10^{-1}$, then starts decreasing due to the over-damping of long modes.



Decay rate at various dissipation and temperature

Thermalization condition: conjecture

It looks like for the Euclidean theory to work, one must require:

In general, this is the effective free energy of the critical bubble

$$t_{th} \lesssim \frac{1}{\omega_-} \frac{E_b}{T}$$

Nucleation time

Big factor

— All our results are consistent with it.

— The analog of this condition is known in physical chemistry studying systems with a few d.o.f. coupled to the heat bath.

Hanggi, Talkner, Borkovec, Rev.Mod.Phys. 62 (1990)

— It is **unavoidably violated** in weakly-coupled theories with one coupling (one field)

In 3+1 dimensions as well

— In theories with many species and couplings, it **must be examined** on a case-by-case basis.

Experience shows that it is not easy to satisfy it even in theories with many d.o.f.

Discussion

- We need a theoretical derivation of the thermal rate without relying on the Euclidean theory.

  Work in progress…

- Is the thermalization condition satisfied in real-life systems such as

  — Standard Model thermal plasma in the early universe?
  — liquid droplet nucleation in supersaturated vapours?

  This is not obvious…

  Work in progress…

- How important are these results e.g. for cosmology?

  If one needs an accurate prediction of the decay rate or for the effects pertaining to the dynamics of bubble nucleation — these results are important.

  Besides, they might be relevant for other non-perturbative processes such as sphaleron transitions or production and collision of solitons.

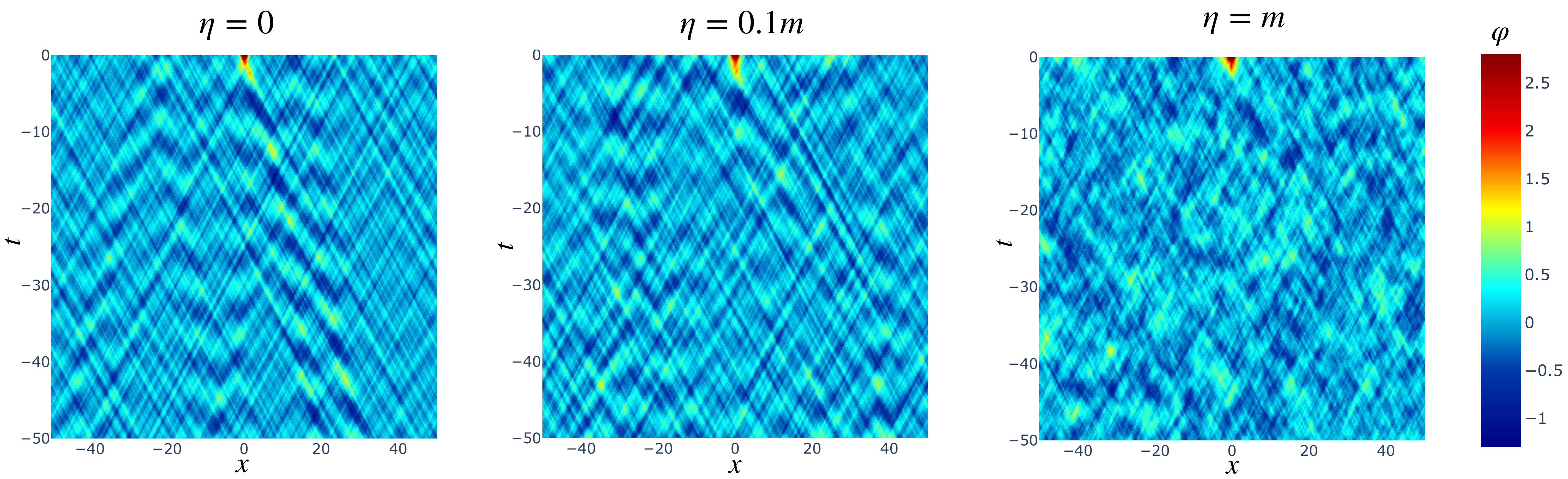
  Work in progress…

- Can we measure the deviation from thermality in experiment?

backup slides

Non-equilibrium dynamics of vacuum decay

When equilibrium is violated, interesting features appear in the field evolution prior to the decay.

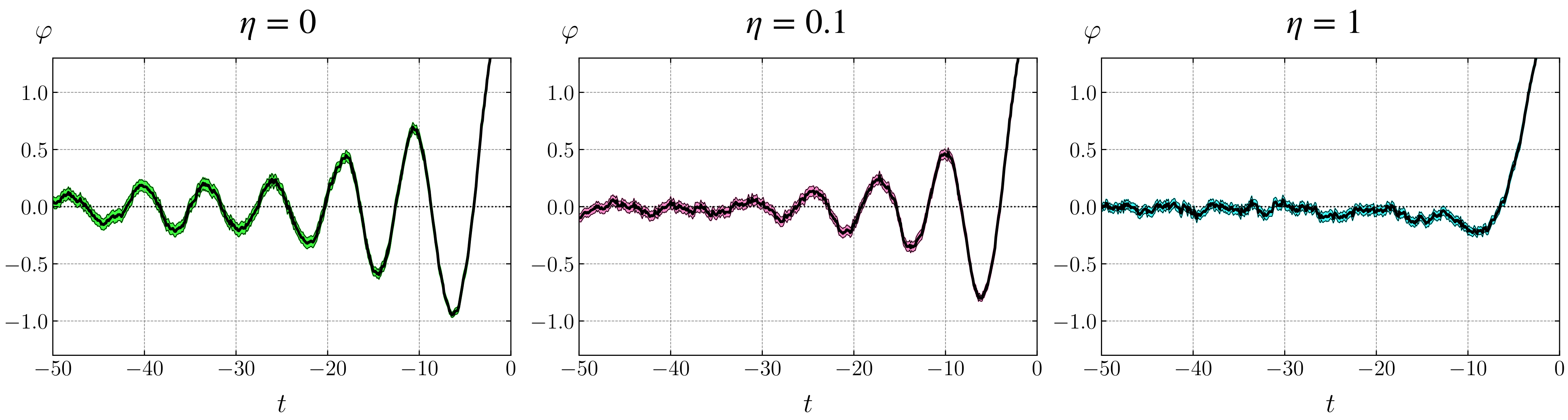


At small dissipation, we observe a population of nonlinear waves with $\omega < m$ — **oscillons**.

They disappear when $\eta > 0.1m$ and the system evolves due to the stochastic terms.

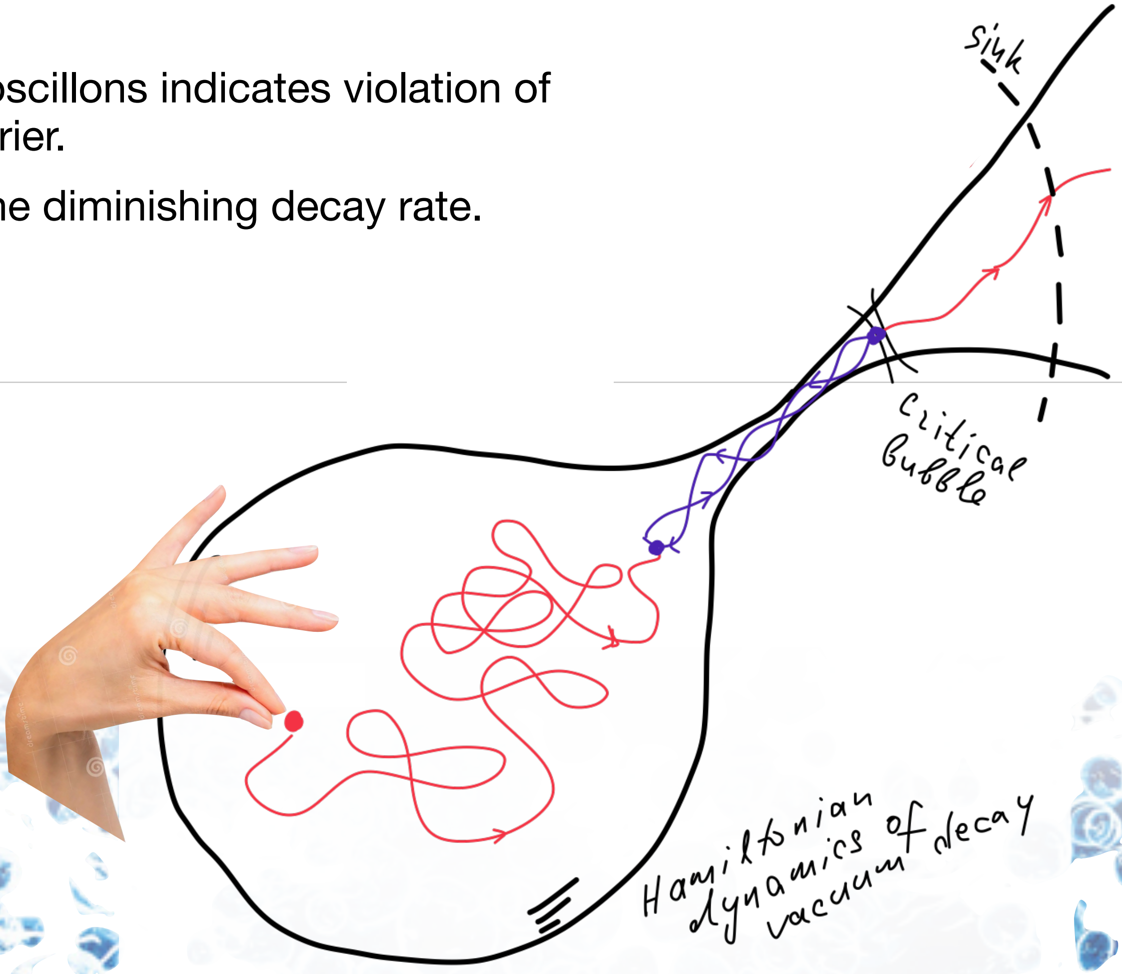
Non-equilibrium dynamics of vacuum decay

Stacking many oscillons together, we get the average **oscillonic precursor** to the critical bubble:



In our system, the presence of oscillons indicates violation of thermal equilibrium near the barrier.

Thus, they are correlated with the diminishing decay rate.



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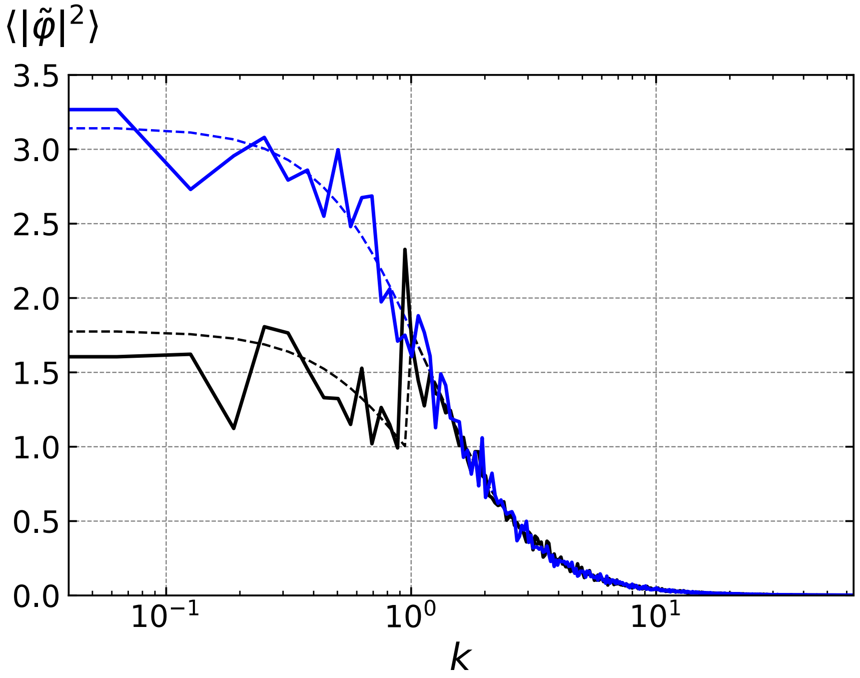
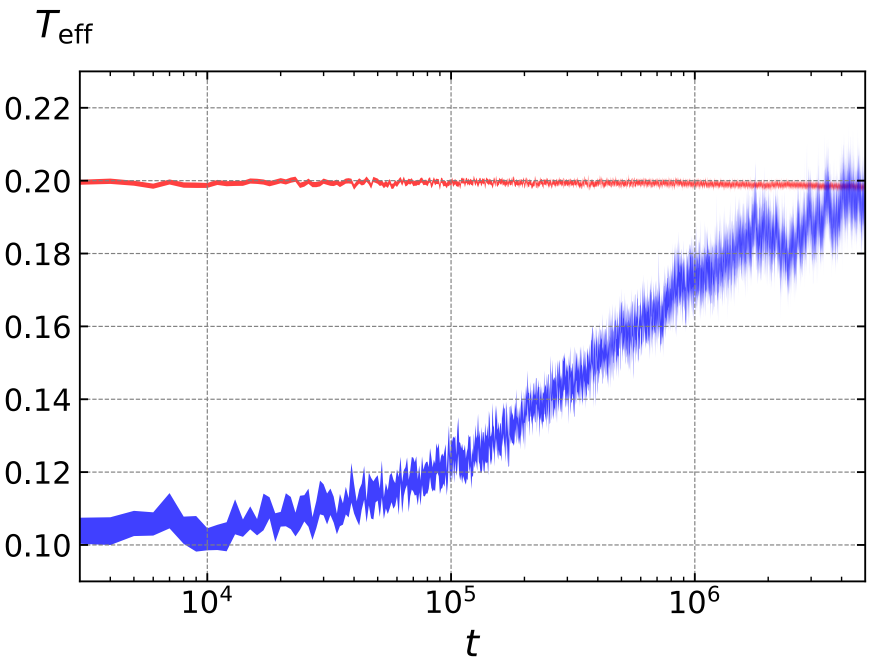
Thermalisation time

We perform the numerical experiment estimating the thermalisation time of long modes in the Hamiltonian system.



Initial and final spectra

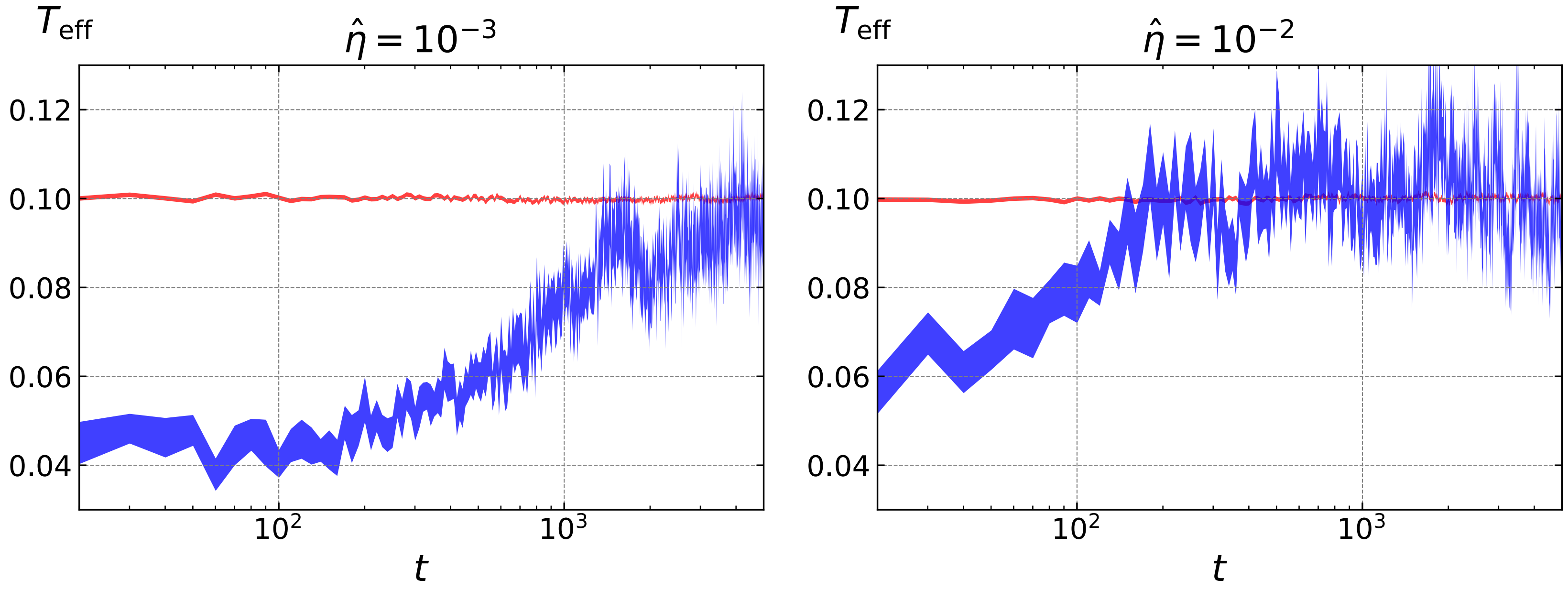


Effective temp. of long modes ($k < m$, blue) vs temp. of all modes (red)

The result agrees with the theoretical estimate $t_{th} \sim \dfrac{(2\pi)^3}{\tilde{T}^4}$

Thermalisation with external heat bath

We perform the numerical experiment estimating the thermalisation time of long modes with the Langevin evolution.



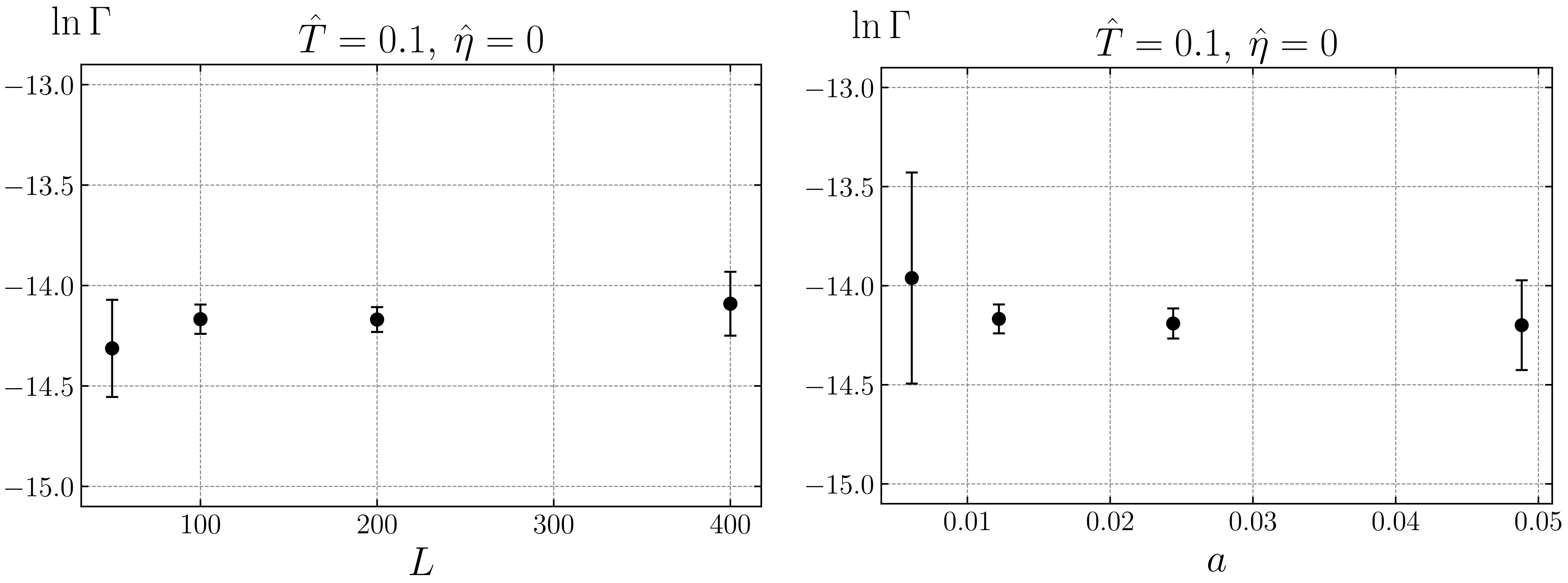
Effective temperature of long modes ($k < m$, blue) and the temperature of the ensemble ($k > m$, red)

The result agrees with the estimate $t_{th} \sim \eta^{-1}$.

Box size and lattice spacing

In simulations, we take $L = 100$ and $a \simeq 0.01$ (in units of mass).

The plots below demonstrate insensitivity of the decay rate to L and a.



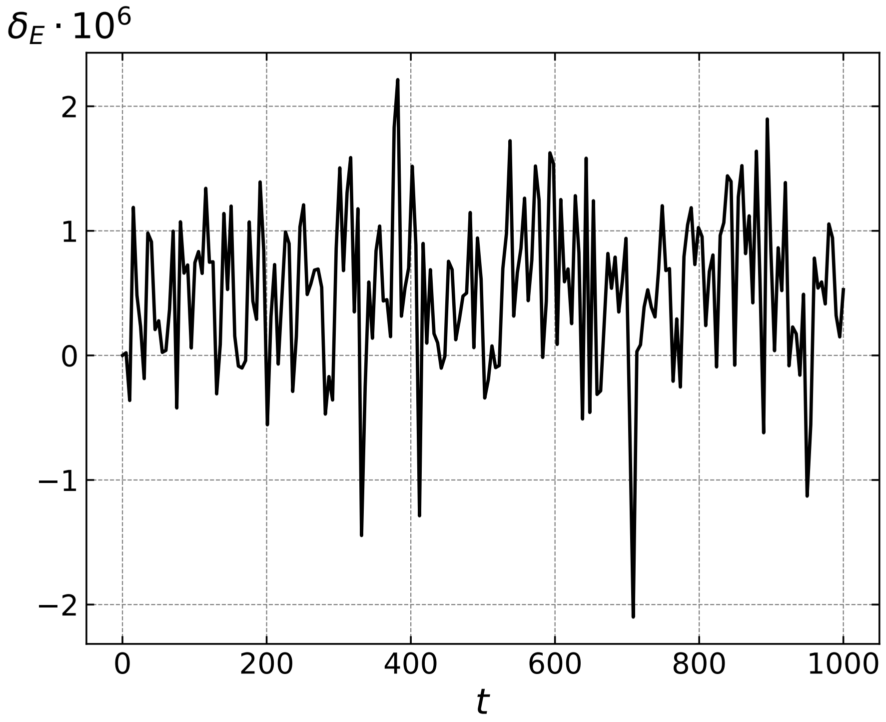
We use this to put the upper bound on the systematic error of the decay rate measurement.

Accuracy of numerical scheme

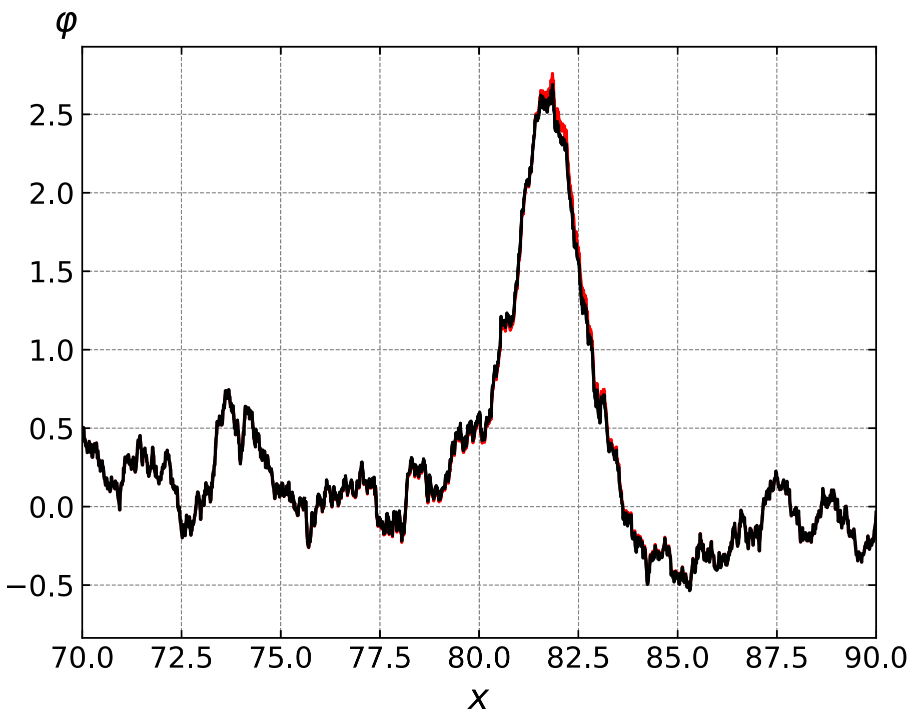
Hamiltonian dynamics

We use the 4th order pseudo-spectral, operator-splitting scheme.

The plots below show that it is enough to take $h/a \simeq 0.8$ to achieve the relative energy non-conservation $\lesssim 10^{-6}$.



Relative energy variation



Two decaying configurations evolved from the same initial state, with $h/a = 0.4, 0.8$.

Accuracy of numerical scheme

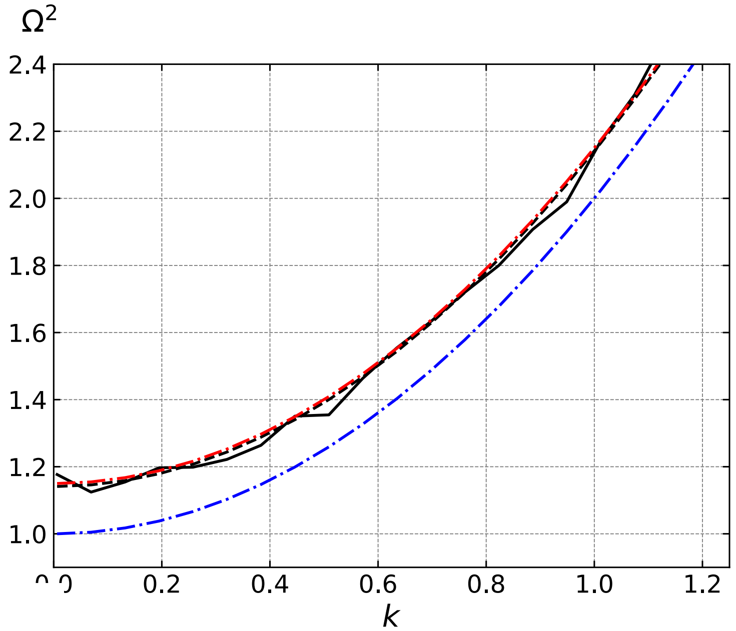
Langevin dynamics

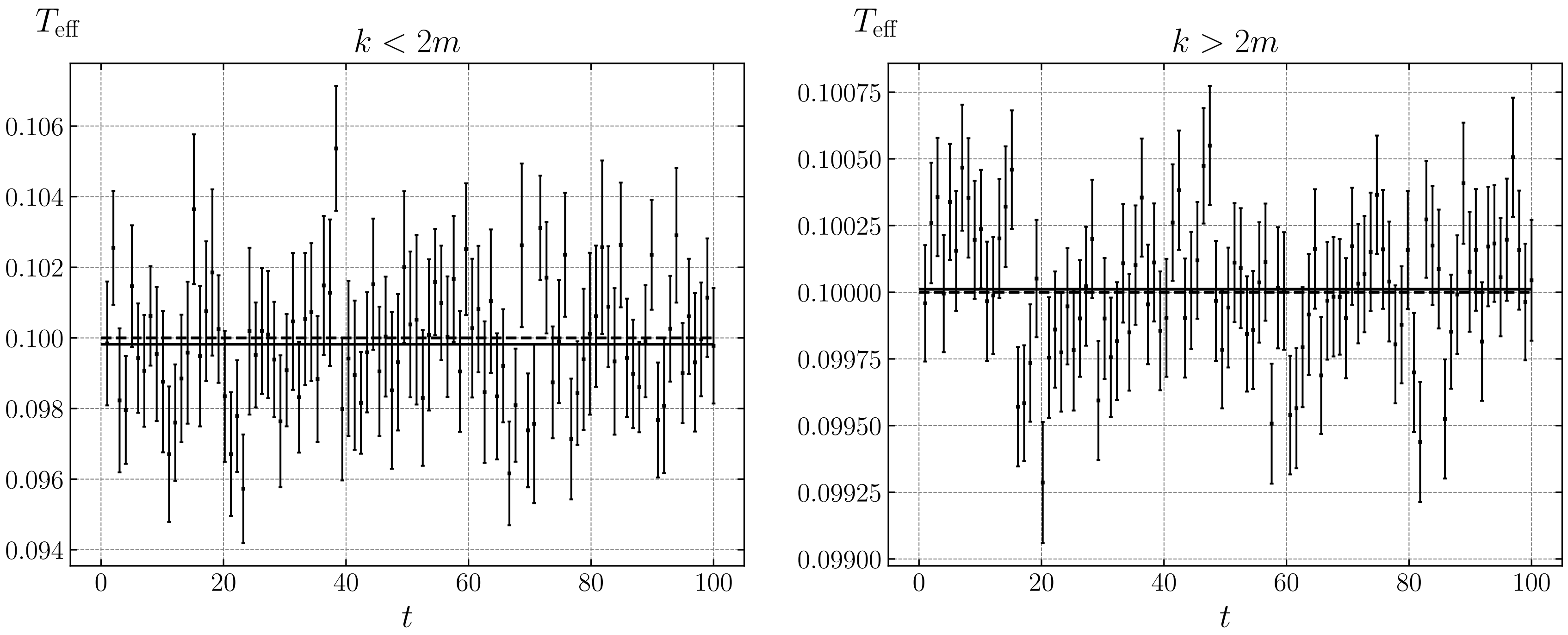
We use the 3rd strong order pseudo-spectral, operator-splitting scheme.

We took it from [Telatovich, Li, 1706.04237] but corrected their mistake.

The timestep is $h/a \simeq 0.25$ at $\eta \lesssim 1$ and $h/a \simeq 0.1$ at $\eta > 1$.

Dispersion relation measured in simulations (black), compared with the free (blue) and thermally-corrected (red) ones.





Effective temp. of long and short modes measured during the simulation.