Neutron stars in scalar-tensor theories with a massive scalar field

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Plan of the talk:

- Scalar-tensor theories with a massive scalar field
- Static and slowly rotating neutron stars
- Rapidly rotating neutron stars

Physical (Jordan) frame action:

$$S = \frac{1}{16\pi G_*}\int d^4x\sqrt{-\tilde{g}}\left[F(\Phi)\tilde{R} - Z(\Phi)\tilde{g}^{\mu\nu}\partial_\mu\Phi\partial_\nu\Phi - 2U(\Phi)\right] + S_m\left[\Psi_m;\tilde{g}_{\mu\nu}\right]$$

Conformal transformation $g_{\mu\nu} = F(\Phi)\tilde{g}_{\mu\nu}$ + redefining the scalar field terms

$$\left(\frac{d\varphi}{d\Phi}\right)^2 = \frac{3}{4}\left(\frac{d\ln(F(\Phi))}{d\Phi}\right)^2 + \frac{Z(\Phi)}{2F(\Phi)} \qquad \mathcal{A}(\varphi) = F^{-1/2}(\Phi),\ 2V(\varphi) = U(\Phi)F^{-2}(\Phi)$$

Einstein frame action (much simpler):

$$S = \frac{1}{16\pi G_*}\int d^4x\sqrt{-g}\left(R - 2g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi - 4V(\varphi)\right) + S_m[\Psi_m;\mathcal{A}^2(\varphi)g_{\mu\nu}]$$

Scalar-tensor theories with a massive scalar field

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi G_* T_{\mu\nu} + 2\partial_\mu \varphi \partial_\nu \varphi - g_{\mu\nu} g^{\alpha\beta} \partial_\alpha \varphi \partial_\beta \varphi - 2V(\varphi) g_{\mu\nu}$$

$$\nabla^\mu \nabla_\mu \varphi = -4\pi G_* k(\varphi) T + \frac{dV(\varphi)}{d\varphi}$$

✓ **Coupling function** $\boldsymbol{k(\varphi) = \frac{d \ln A(\varphi)}{d\varphi}}$

We shall consider two coupling functions

1) Brans-Dicke coupling $k(\varphi) = \alpha_0 \Leftrightarrow A(\varphi) = \exp(\alpha_0 \varphi)$

2) Theory with spontaneous scalarization $k(\varphi) = \beta\varphi \Leftrightarrow A(\varphi) = \exp\left(\frac{\beta}{2}\varphi^2\right)$, where $\beta < 0$

✓ **Massive scalar field with a potential** $V(\varphi) = \frac{1}{2} m_\varphi^2 \varphi^2$

L. Perivolaropoulos, PRD **81**, 047501 (2010); J. Alsing, E. Berti, C. M. Will, and H. Zaglauer, PRD **85**, 064041 (2012); M. Hohmann, L. J¨arv, P. Kuusk, and E. Randla, PRD **88**, 084054 (2013); A. Scharer, R. Ang´elil, R. Bondarescu, P. Jetzer, and A. Lundgren, PRD **90**, 123005 (2014); L. Jarv, P. Kuusk, M. Saal, and O. Vilson, PRD **91**, 024041 (2015)

- The recent astrophysical and cosmological observations have severely constrained the basic parameters of the scalar-tensor theories with a massless scalar field leaving a narrow window for new physics beyond general relativity.

The situation changes drastically if we consider a massive scalar field.

- The scalar field mass m_φ leads to a finite range of the scalar field of the order of its Compton wavelength $\lambda_\varphi = 2\pi/m_\varphi$. In other words the presence of the scalar field will be suppressed outside the compact objects at distances $D > \lambda_\varphi$. This means in turn that all observations of compact objects involving distances greater than λ_φ not put constraints, or at least stringent constraints, on the scalar tensor theories.

Massive Brans-Dicke theory

- In the case of massive Brans-Dicke theory with $m_{\varphi} \geq 2 \times 10^{-25} GeV$ $(or\ \lambda_{\varphi} \leq 10^{11} m)$ the Solar System observations can not put constraints on the Brans-Dicke parameter α_0 and all values of α_0 $(\omega_{BD} > -3/2)$ are observationally allowed.

- The massive gravitational scalar suppresses also the dipole radiation and the compact binaries can not constrain severely the Brans-Dicke parameter if their orbit radius is significantly greater than λ_{φ}.

Scalar-tensor theory with $\boldsymbol{k(\varphi) = \beta\varphi}$

- The mass of the scalar field can effectively suppress the scalar gravitational waves and reconcile the scalar-tensor theories with the binary neutron star observations for a much larger range of β. In more rigorous terms, if the Compton wave-length of the scalar field λ_{φ} is much smaller than the separation of the two stars in the binary system the emitted scalar gravitational radiation will be negligible.

Theoretical and observational bounds on the parameters

- The orbital separation for all these systems is roughly of the same order $10^9 m$, or equivalently $m_\varphi \geq 10^{-16}\, GeV$. For such values of m_φ, β is practically unconstrained.
- An upper limit on m_φ can be imposed based on the requirement that the mass term does not prevent the scalarization of the star. Namely, the characteristic length scale of the star should be smaller than the Compton wavelength which leads to $\mathrm{m}_\varphi \leq 10^{-9}\, eV$.

The allowed range for m_φ is $10^{-16}\, eV \leq m_\varphi \leq 10^{-9}\, eV$ $(10^2\, m \leq \lambda_\varphi \leq 10^9\, m)$.

F. M. Ramazanoglu , F. Pretorius, PRD 93, 064005 (2016); S. Yazadjiev, D. Doneva, D. Popcev, PRD D **93**, 084038 (2016)

- The coupling constants (and the coupling functions in general) of the scalar-tensor theories with a massive scalar field, which are observationally allowed, can differ significantly from those in the massless case. This fact naturally leads us to the conclusion that the compact objects in general, and the neutron stars in particular, with a massive scalar field could in principle have rather different structure and properties in comparison with their counterparts in the massless case.

Scalar-tensor theories of gravity – neutron stars

Spacetime metric

$$ds^2 = -e^{2\phi(r)}dt^2 + e^{2\Lambda(r)}dr^2 + r^2(d\theta^2 + \sin^2\theta d\vartheta^2) - 2\omega(r,\theta)r^2 sin^2\theta d\vartheta dt$$

Dimensionally reduced equations

$$\frac{1}{r^2}\frac{d}{dr}\left[r(1-e^{-2\Lambda})\right] = 8\pi G A^4(\varphi)\tilde{\rho} + e^{-2\Lambda}\left(\frac{d\varphi}{dr}\right)^2 + \frac{1}{2}V(\varphi),$$

$$\frac{2}{r}e^{-2\Lambda}\frac{d\phi}{dr} - \frac{1}{r^2}(1-e^{-2\Lambda}) = 8\pi G A^4(\varphi)\tilde{p} + e^{-2\Lambda}\left(\frac{d\varphi}{dr}\right)^2 - \frac{1}{2}V(\varphi),$$

$$\frac{d^2\varphi}{dr^2} + \left(\frac{d\phi}{dr} - \frac{d\Lambda}{dr} + \frac{2}{r}\right)\frac{d\varphi}{dr} = 4\pi G\alpha(\varphi)A^4(\varphi)(\tilde{\rho} - 3\tilde{p})e^{2\Lambda} + \frac{1}{4}\frac{dV(\varphi)}{d\varphi}e^{2\Lambda},$$

$$\frac{d\tilde{p}}{dr} = -(\tilde{\rho} + \tilde{p})\left(\frac{d\phi}{dr} + \alpha(\varphi)\frac{d\varphi}{dr}\right),$$

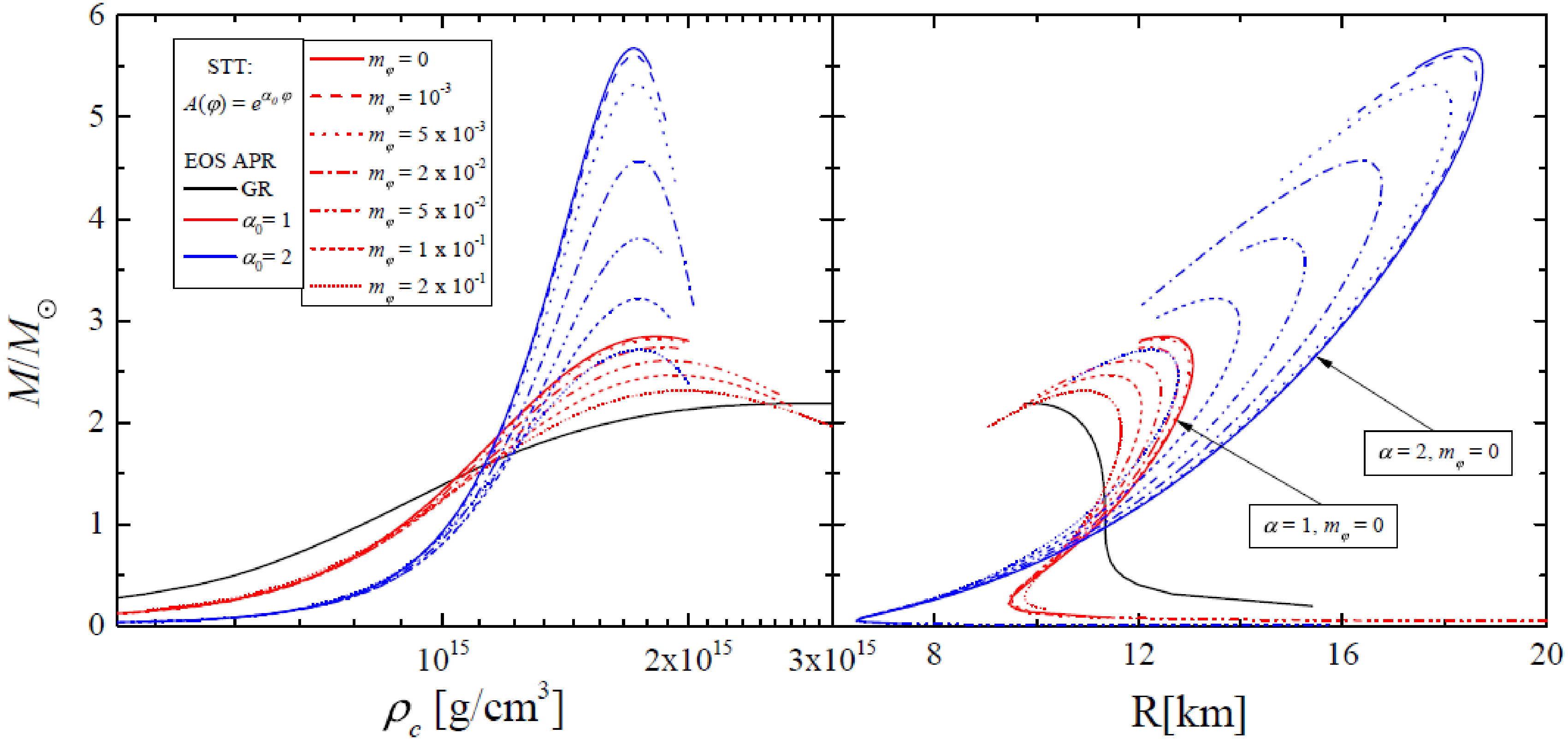
$$\frac{e^{\Phi-\Lambda}}{r^4}\partial_r\left[e^{-(\Phi+\Lambda)}r^4\partial_r\bar{\omega}\right] + \frac{1}{r^2\sin^3\theta}\partial_\theta\left[\sin^3\theta\partial_\theta\bar{\omega}\right] = 16\pi G A^4(\varphi)(\tilde{\rho} + \tilde{p})\bar{\omega},$$

$$\bar{\omega} = \Omega - \omega.$$

Neutron stars in massive Brans-Dicke theory

$$m_\varphi \rightarrow m_\varphi R_0 = 2\pi\, R_0/\lambda_\varphi \qquad R_0 = 1.47664\, km$$

Brans-Dicke coupling $k(\varphi) = \alpha_0 \Leftrightarrow A(\varphi) = \exp(\alpha_0\varphi)$



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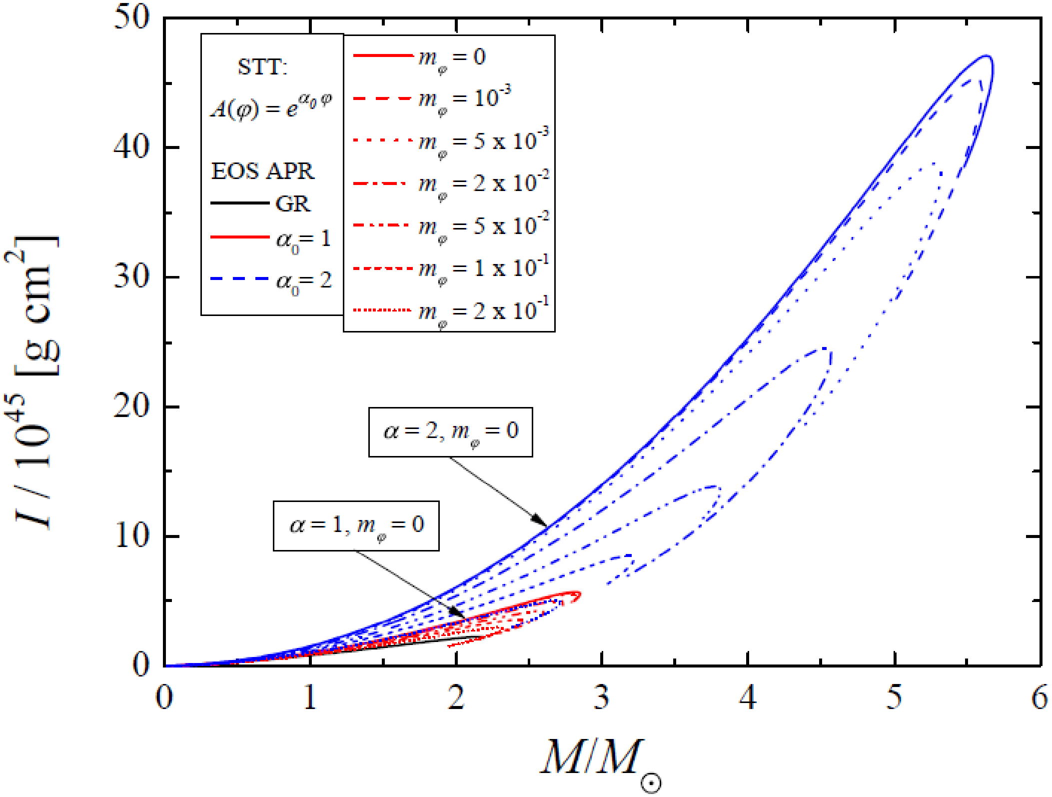
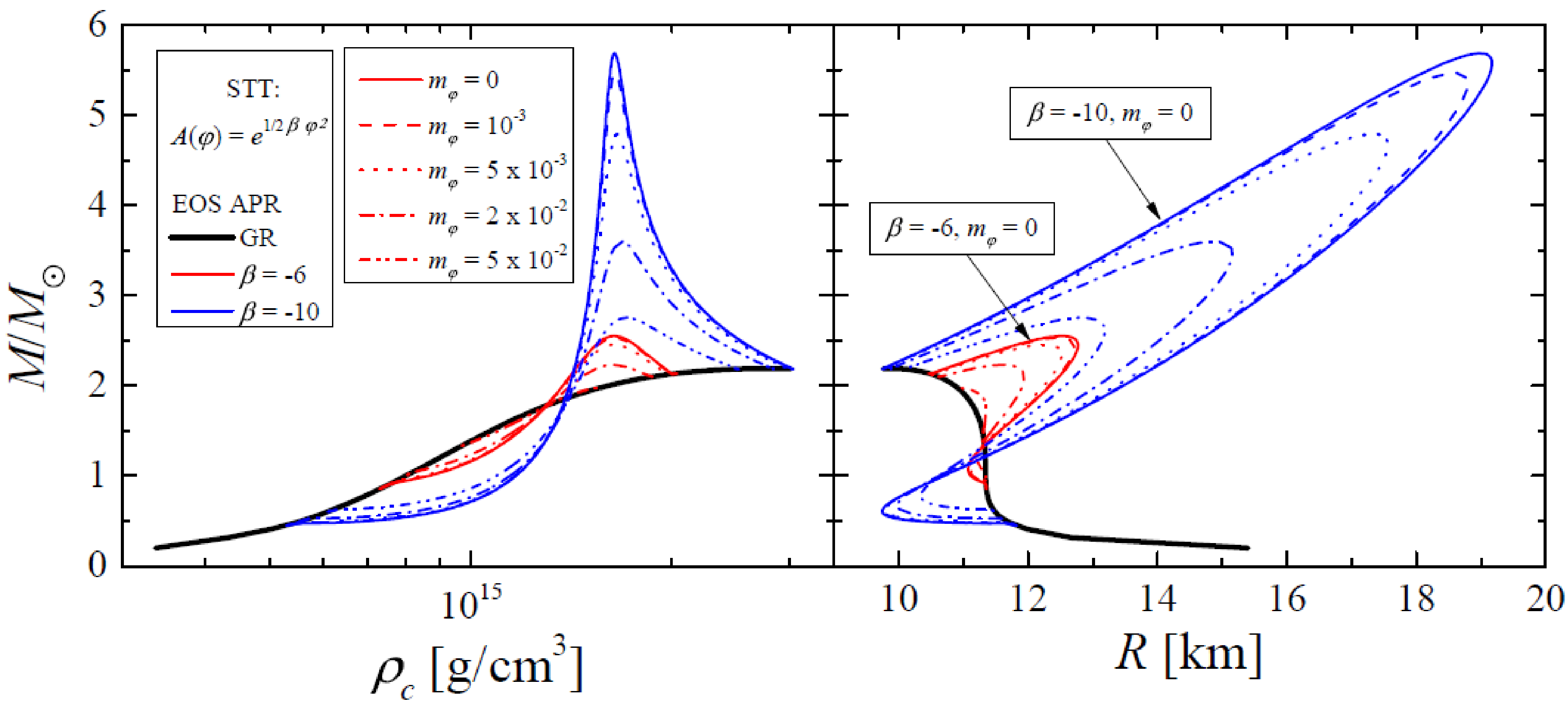


Spontaneous scalarization with a massive scalar

Theory with **spontaneous scalarization** $k(\varphi) = \beta\varphi \Leftrightarrow A(\varphi) = \exp\left(\frac{\beta}{2}\varphi^2\right), \beta < 0$



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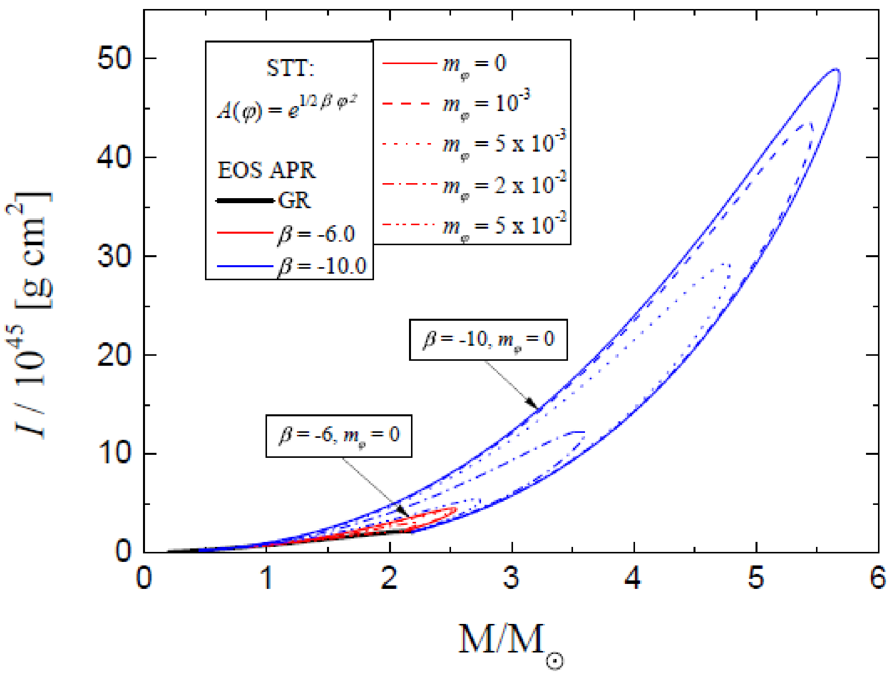
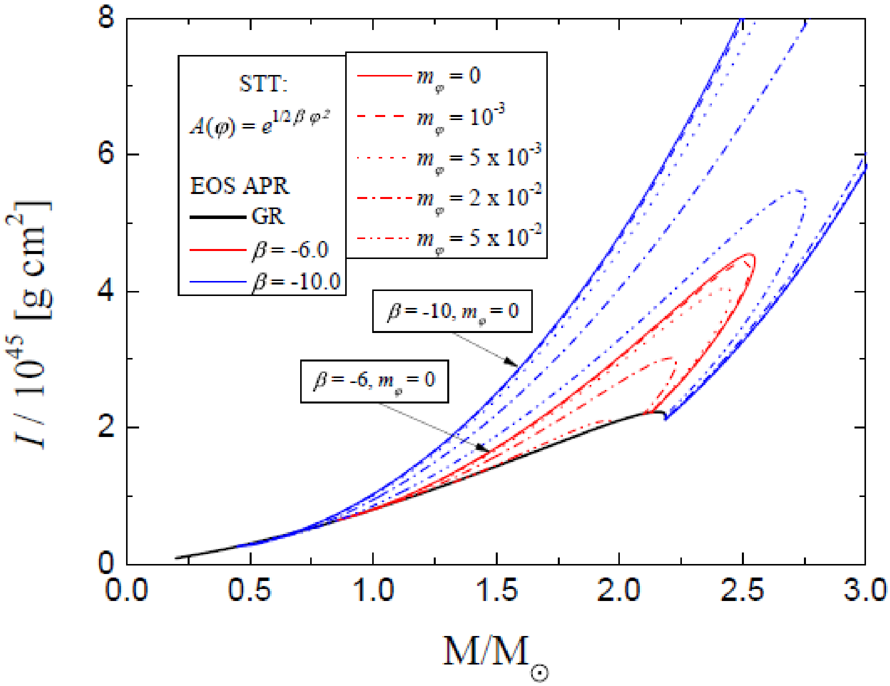




Dimensionally reduced field equations for rapidly rotating stars

A general ansatz for a stationary and axisymmetric metric:

$$ds^2 = -e^{\gamma+\sigma}dt^2 + e^{\gamma-\sigma}r^2\sin^2\theta(d\phi - \omega dt)^2 + e^{2\alpha}(dr^2 + r^2 d\theta^2)$$

The reduced field equations for the metric functions:

$$\left(\Delta + \frac{1}{r}\partial_r - \frac{\mu}{r^2}\partial_\mu\right)\left(\gamma e^{\gamma/2}\right) = e^{\gamma/2}\left\{(16\pi p - 4V(\varphi))e^{2\alpha} + \right.$$
$$\left. + \frac{\gamma}{2}\left[(16\pi p - 4V(\varphi))e^{2\alpha} - \frac{1}{2}(\partial_r\gamma)^2 - \frac{1}{2}\frac{1-\mu^2}{r^2}(\partial_\mu\gamma)^2\right]\right\},$$

$$\Delta(\sigma e^{\gamma/2}) = e^{\gamma/2}\left\{8\pi(\varepsilon + p)e^{2\alpha}\frac{1+v^2}{1-v^2} + r^2(1-\mu^2)e^{-2\sigma}\left[(\partial_r\omega)^2 + \frac{1-\mu^2}{r^2}(\partial_\mu\omega)^2\right] + \frac{1}{r}\partial_r\gamma - \frac{\mu}{r^2}\partial_\mu\gamma\right.$$
$$\left. + \frac{\sigma}{2}\left[(16\pi p - 4V(\varphi))e^{2\alpha} - \frac{1}{r}\partial_r\gamma + \frac{\mu}{r^2}\partial_\mu\gamma - \frac{1}{2}(\partial_r\gamma)^2 - \frac{1}{2}\frac{1-\mu^2}{r^2}(\partial_\mu\gamma)^2\right]\right\},$$

$$\left(\Delta + \frac{2}{r}\partial_r - \frac{2\mu}{r^2}\partial_\mu\right)\left(\omega e^{\gamma/2-\sigma}\right) = e^{\gamma/2-\sigma}\left\{-16\pi\frac{(\varepsilon+p)(\Omega-\omega)}{1-v^2}e^{2\alpha} + \right.$$
$$\omega\left[-\frac{1}{r}\partial_r(\frac{1}{2}\gamma + 2\sigma) + \frac{\mu}{r^2}\partial_\mu(\frac{1}{2}\gamma + 2\sigma) - \frac{1}{4}(\partial_r\gamma)^2 - \frac{1}{4}\frac{1-\mu^2}{r^2}(\partial_\mu\gamma)^2 + \right.$$
$$+(\partial_r\sigma)^2 + \frac{1-\mu^2}{r^2}(\partial_\mu\sigma)^2 - r^2(1-\mu^2)e^{-2\sigma}\left((\partial_r\omega)^2 + \frac{1-\mu^2}{r^2}(\partial_\mu\omega)^2\right)$$
$$\left.\left. -8\pi\frac{\varepsilon(1+v^2) + 2pv^2}{1-v^2}e^{2\alpha} - 2V(\varphi)e^{2\alpha}\right]\right\}.$$

Field equations

The reduced field equation for the metric function α:

$$\partial_\mu \alpha = -\frac{\partial_\mu \gamma + \partial_\mu \sigma}{2} - \left\{(1-\mu^2)(1+r\partial_r\gamma)^2 + [-\mu + (1-\mu^2)\partial_\mu\gamma]^2\right\}^{-1} \times$$

$$\left\{\frac{1}{2}\left[r\partial_r(r\partial_r\gamma) + r^2(\partial_r\gamma)^2 - (1-\mu^2)(\partial_\mu\gamma)^2 - \partial_\mu[(1-\mu^2)\partial_\mu\gamma] + \mu\partial_\mu\gamma\right] \times [-\mu + (1-\mu^2)\partial_\mu\gamma] + \right.$$

$$+\frac{1}{4}[-\mu + (1-\mu^2)\partial_\mu\gamma] \times \left[r^2(\partial_r\gamma + \partial_r\sigma)^2 - (1-\mu^2)(\partial_\mu\gamma + \partial_\mu\sigma)^2 + 4r^2(\partial\varphi)^2 - 4(1-\mu^2)(\partial_\mu\varphi)^2\right] +$$

$$+\mu r\partial_r\gamma[1 + r\partial_r\gamma] - (1-\mu^2)r(1+r\partial_r\gamma)\left[\partial_\mu\partial_r\gamma + \partial_\mu\gamma\partial_r\gamma + \frac{1}{2}(\partial_\mu\gamma + \partial_\mu\sigma)(\partial_r\gamma + \partial_r\sigma) + 2\partial_\mu\varphi\partial_r\varphi\right] +$$

$$+\frac{1}{4}(1-\mu^2)e^{-2\sigma}\left[-[-\mu + (1-\mu^2)\partial_\mu\gamma][r^4(\partial_r\omega)^2 - r^2(1-\mu^2)(\partial_\mu\omega)^2] + \right.$$

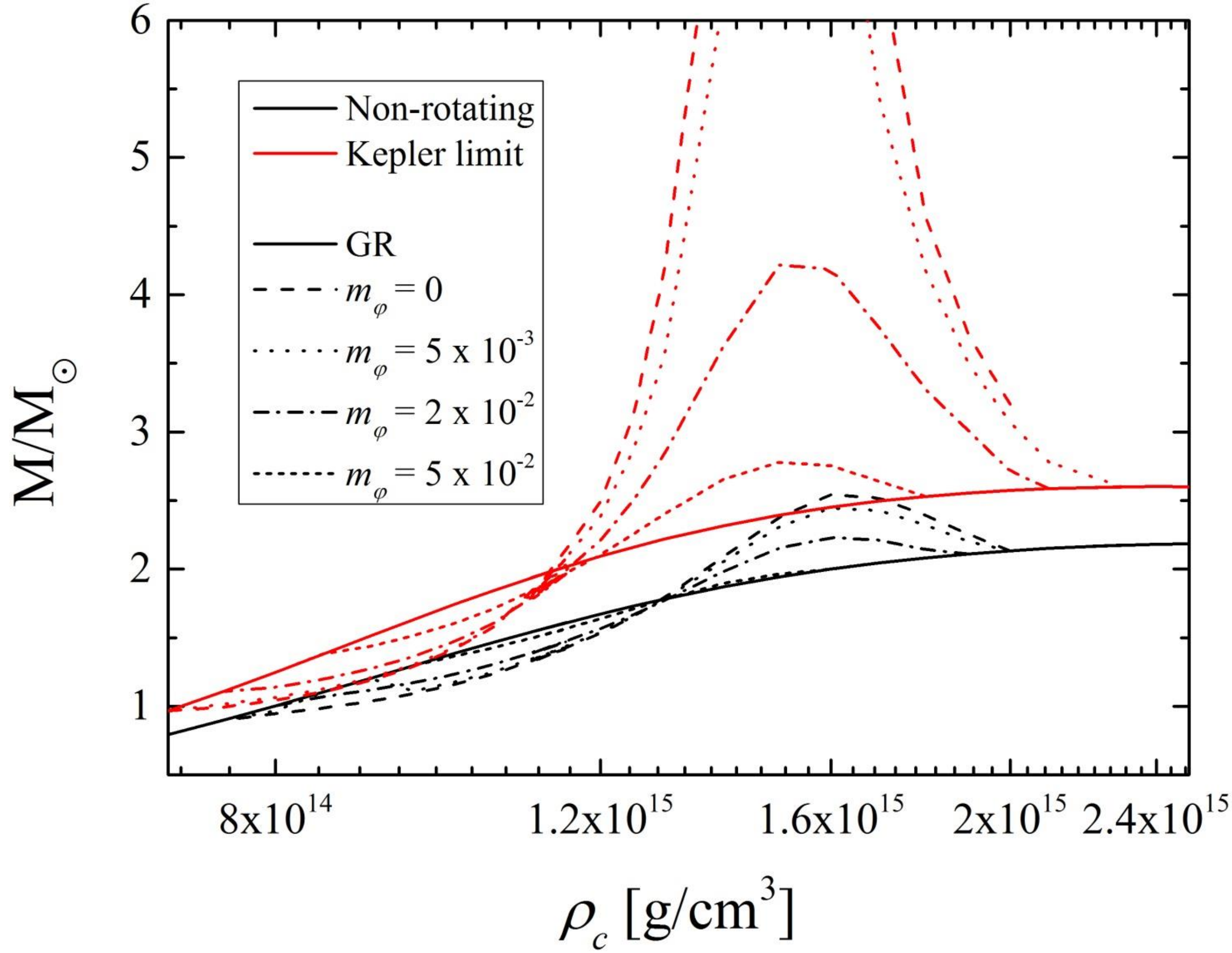
$$\left.\left. +2(1-\mu^2)r^3\partial_\mu\omega\partial_r\omega(1+r\partial_r\gamma)\right]\right\}.$$

Equations for the scalar field and for hydrostationary equilibrium:

$$\Delta\varphi = -\partial_r\gamma\partial_r\varphi - \frac{1-\mu^2}{r^2}\partial_\mu\gamma\partial_\mu\varphi + \left[4\pi k(\varphi)(\varepsilon - 3p) + \frac{dV(\varphi)}{d\varphi}\right]e^{2\alpha}$$

$$\frac{\partial_i\tilde{p}}{\tilde{\varepsilon} + \tilde{p}} - \left[\partial_i(\ln u^t) - u^t u_\phi \partial_i\Omega - k(\varphi)\partial_i\varphi\right] = 0.$$

Theory with spontaneous scalarization $\beta = -6$



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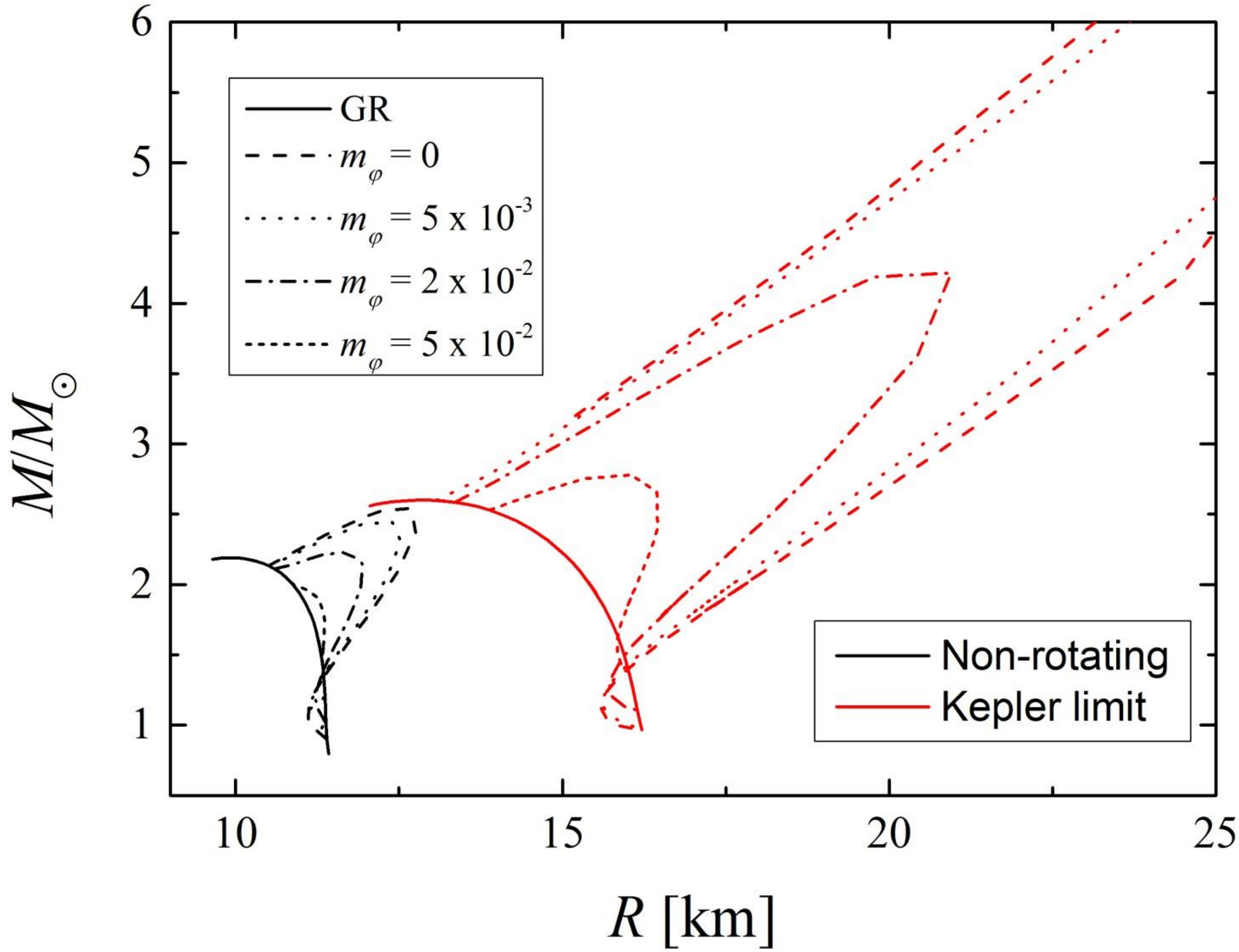

Conclusions

1. In scalar-tensor theories with a massless scalar field neutron stars differ almost marginally from GR if one considers coupling parameters that are in agreement with the present observations.

2. The inclusion of scalar field mass changes the picture dramatically. It suppresses the scalar field at length scale of the order of the Compton wavelength which helps us reconcile the theory with the observations for a much broader range of the coupling parameters.

3. The structure and the properties of the neutron stars in in massive STT can differ drastically from the pure GR solutions if sufficiently large masses of the scalar field are considered.

THANK YOU!