Excluded volume effects in the neutron star EoS

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The paradigm: Van der Waals Equation

$$\left[P + a\left(\frac{N}{V}\right)^2\right]\left[\frac{V}{N} - b\right] = kT$$

reproduces qualitative properties the isotherms of real gases with a liquid-gas transition and a critical isotherm of first order transitions.
The parameter b stands for the eigenvolume of the N molecules.
There is no complete theoretical justification.
$\rightarrow$ Virial expansion; $\rightarrow$ Beth-Uhlenbeck EoS

Equation of state (EoS)

Connects pressure, temperature, and particle density

But does not provide full information about the state.
The partition function is necessary!

Hadron resonance gas and excluded volume

Excluded volume concept

Hadrons are not point particles, and repulsive interactions can be implemented via an excluded-volume approximation whereby the volume available for the hadrons to move in is reduced by the volume they occupy.

Basic references

- R. Hagedorn and J. Rafelski, Phys. Lett. **B 97**, 136 (1980).
- R. Hagedorn, Z. Phys. C: Part. Fields **17**, 265 (1983).
- M. I. Gorenstein, V. K. Petrov, and G. M. Zinovjev, Phys. Lett. **B 106**, 327 (1981).

Microscopic foundation

"Hard core" repulsive interaction is inevitable for composite objects with a fermionic substructure because of the fundamental Pauli principle.
This holds in particular in statistical systems with cluster formation, e.g., for nuclei in nuclear matter and for hadrons in quark matter.

Excluded volume (EV) hadron resonance gas (HRG)

A hadron species α has volume v_α. The fraction of volume V available for hadrons to move in is

$$\Phi = V_{\rm av}/V = (V - \sum_\alpha v_\alpha N_\alpha)/V = 1 - \sum_\alpha v_\alpha n_\alpha$$

In the grand canonical ensemble, the substitution $V \to \Phi V$ leads to a transcendental equation for the pressure of the EV-HRG ($p_\alpha^{\rm pt}$ is the partial pressure for a pointlike hadron α)

$$p(T,\mu) = \sum_\alpha p_\alpha^{\rm pt}(T,\tilde{\mu}_\alpha)\ ; \quad \tilde{\mu}_\alpha = \mu_\alpha - v_\alpha p(T,\mu)$$

Since pressure is a thermodynamical potential, all other EoS can be derived from it, e.g., the baryon density ($\mu_\alpha = b_\alpha \mu_B + s_\alpha \mu_S + q_\alpha \mu_Q$, with baryon number $b_\alpha = 0, \pm 1$ for hadron α)

$$\rho_B = \frac{\partial p}{\partial \mu_B} = \frac{\sum_\alpha b_\alpha n_\alpha^{\rm pt}(T,\tilde{\mu}_\alpha)}{1 + \sum_\beta v_\beta n_\beta^{\rm pt}(T,\tilde{\mu}_\beta)}\ .$$

Volumes $v_\alpha = \frac{1}{2}\frac{4\pi}{3}\sum_\beta (r_\alpha + r_\beta)^3$ are related to hadron radii r_α.

For unique radii $r_\alpha = r_\beta = r$ this becomes the Van der Waals volume $v_\alpha = v = 4 \cdot (4\pi r^3/3)$.

Results - I

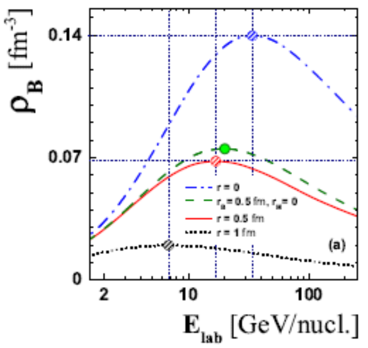
Baryon density at the chemical freeze-out in heavy-ion collisions

Chemical freeze-out line in the $T - \mu_B$ plane

$$\frac{T}{\mathrm{GeV}} = 0.166 - 0.139 \left(\frac{\mu_B}{\mathrm{GeV}}\right)^2 - 0.053 \left(\frac{\mu_B}{\mathrm{GeV}}\right)^4$$

$$\frac{\mu_B}{\mathrm{GeV}} = \frac{1.308}{1 + 0.273\sqrt{s_{NN}}/\mathrm{GeV}}$$

$$\sqrt{s_{NN}} = \sqrt{2m_N E_{\mathrm{lab}} + 2m_N^2}\ ; \ m_N = 0.939\ \mathrm{GeV}$$

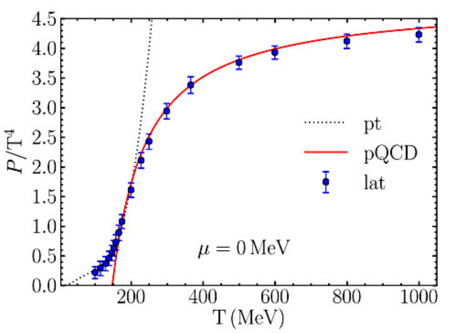


The excluded volume lowers the collision energy E_{lab} where the highest baryon density at freeze-out is reached!

V.V. Begun, M. Gazdzicki, M.I. Gorenstein, Phys. Rev. **C88** (2013) 024902

Hadron resonance gas and perturbative QCD

M. Albright, J. Kapusta, and C. Young - Phys. Rev. C **90**, 024915 (2014)



The dotted curve represents the parameter-free, point-particle hadron-resonance gas.

The solid curve represents perturbative QCD with two parameters adjusted to fit the lattice result

Find a means for switching from a hadron-resonance gas at low temperature, preferably treating the hadrons not as point particles but as extended objects, to a plasma of weakly interacting quarks and gluons at high temperature.

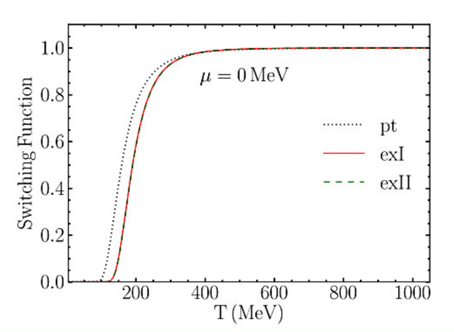
Switching from hadrons to quarks and gluons

Switching function $S(T,\mu)$

$$P(T,\mu) = S(T,\mu)P_{qg}(T,\mu) + [1 - S(T,\mu)]P_h(T,\mu)$$

$$S(T,\mu) = e^{-\theta(T,\mu)}; \quad \theta(T,\mu) = \left[\left(\frac{T}{T_0}\right)^r + \left(\frac{\mu}{\mu_0}\right)^r\right]$$

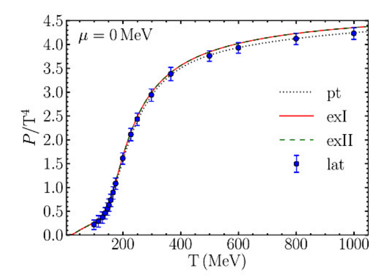
with integer $r = 5$ ($r = 4$ for pointlike hadrons).

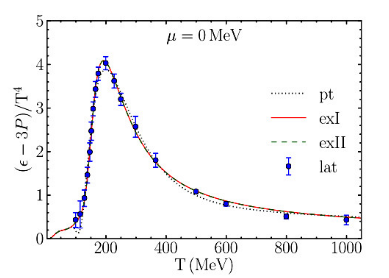


Results - II

Perfect fit of lattice QCD thermodynamics at $\boxed{\mu = 0}$

Only minor advantage of excluded volume model over pointlike particle model for interaction measure $(\varepsilon - 3p)/T^4$ at $T \sim 100 - 120$ MeV

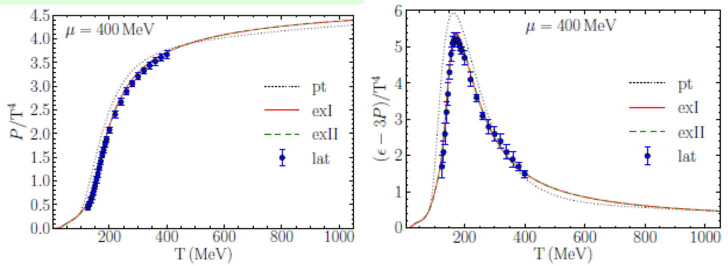




Results - III

Perfect fit of lattice QCD thermodynamics at $\boxed{\mu = 400 \text{ MeV}}$

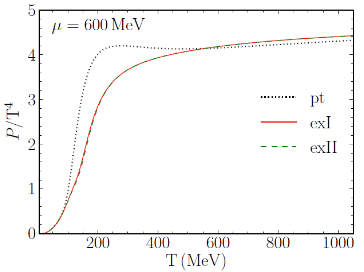
Clear advantage of excluded volume model over pointlike particle model for both, pressure p/T^4 and interaction measure $(\varepsilon - 3p)/T^4$ in transition region $T \sim 100 - 300$ MeV

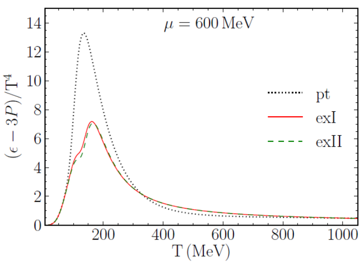


Results - IV

Prediction! No lattice QCD thermodynamics data at $\boxed{\mu = 600\text{ MeV}}$

Strong effect of excluded volume over pointlike particle model for both, pressure p/T^4 and interaction measure $(\varepsilon - 3p)/T^4$ in transition region $T \sim 100 - 300$ MeV





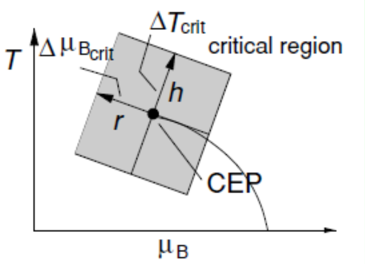
Question: Critical Endpoint (CEP) ?

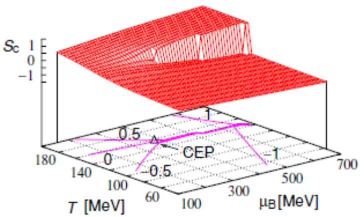
Idea: Change the switching function to one with a critical endpoint, e.g., inspired by the Ising model with critical exponents from that universality class *)

$$S_c(T,\mu_B) = D\sqrt{\Delta T_{\rm crit}^2 + \Delta\mu_{B,\rm crit}^2}\, s_c(T,\mu_B)$$

where $s_c(T,\mu_B)$ is the singular part of the entropy density and D a dimensionless constant. The entropy density of the hybrid EoS with CEP is

$$s(T,\mu_B) = \frac{1}{2}\{1-\tanh[S_c(T,\mu_B)]\}s_H(T,\mu_B) + \frac{1}{2}\{1+\tanh[S_c(T,\mu_B)]\}s_Q(T,\mu_B)$$

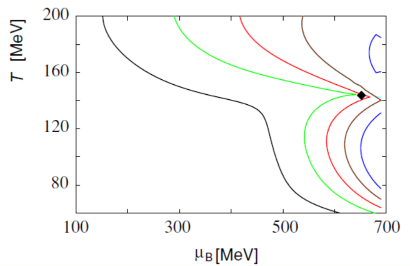


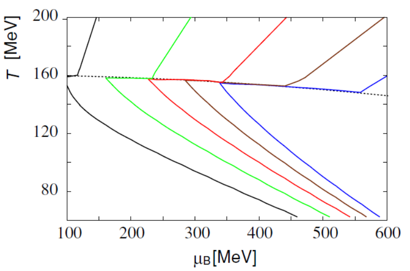


*) For details, see: C. Nonaka & M. Asakawa, Phys. Rev. **C71** (2005) 044904

Question: Critical Endpoint (CEP) ?

Hydrodynamic evolution of a heavy-ion collision shall follow isentropic trajectories in the QCD phase diagram with CEP (left figure) and with 1st order transition all over (right figure), e.g. by Maxwell construction:
Both the pressure and chemical potential should have the same value in both hadronic and quark matter phases.

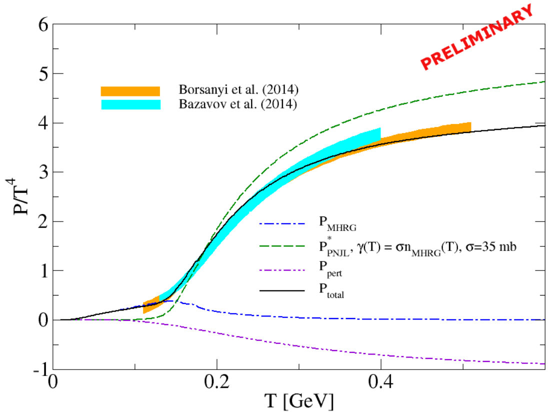




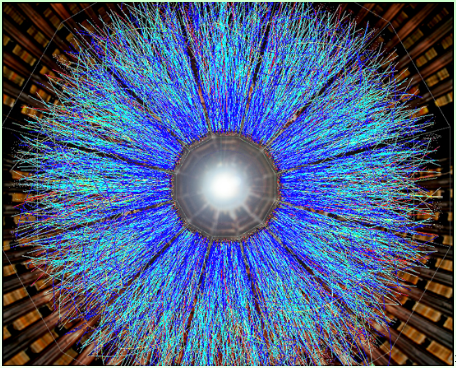
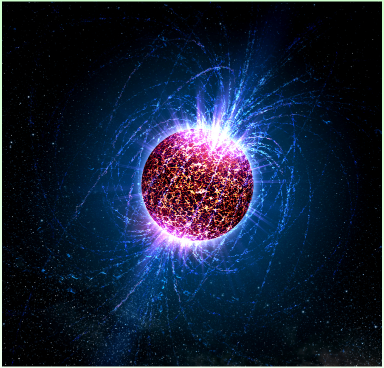
C. Nonaka & M. Asakawa, Phys. Rev. **C71** (2005) 044904

Results B-D-T

D. Blaschke, A. Dubinin, L. Turko
Mott-hadron resonance gas and lattice QCD thermodynamics (in prep.)



Heavy-Ion Collisions → Neutron Stars

 $\Longrightarrow$ 

Excluded volume in the hadronic phase and the higher-order repulsive interactions in the quark phase

Cold nuclear matter

Problem: Saturation of nuclear matter: Nuclear liquid-gas transition!
→ Relativistic density functional (DD2) - Fermi statistics
→ Generalized excluded volume approach *)

Ansätze for available volume fraction:

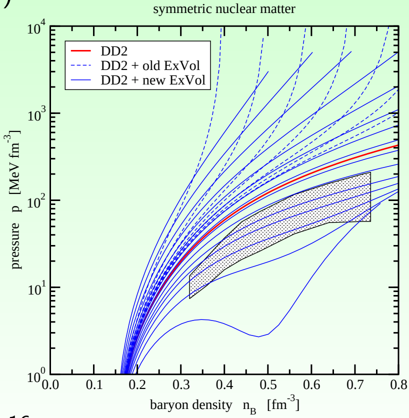
old ExVol $\Phi_n = \Phi_p = 1 - v n_b$

maximum possible density $n_{\rm max} = 1/v$

new ExVol

$$\Phi_n = \Phi_p = \begin{cases} 1 & \text{if } n_b \le n_{\rm sat} \\ \exp\left[-\frac{v|v|}{2}(n_b - n_{\rm sat})^2\right] & \text{if } n_b > n_{\rm sat} \end{cases}$$

Advantages:
→ nuclear matter at $n_b \le n_{\rm sat}$ not affected
→ no divergence of pressure
→ negative v for attraction → phase transition

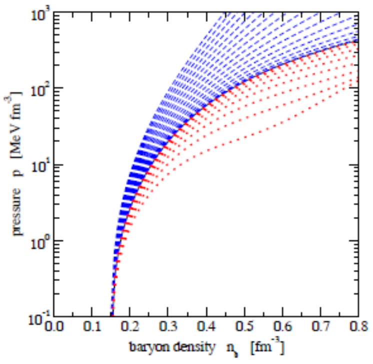


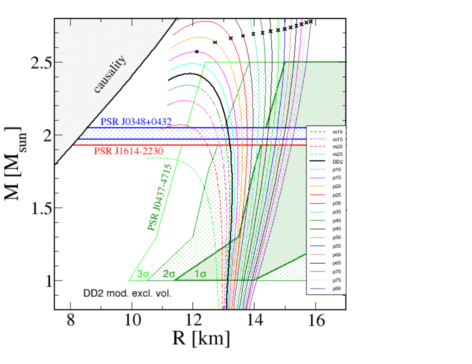
*) For details, see: S. Typel, Eur. Phys. J. **A 52** (2016) 16

Cold neutron star matter

→ isospin asymmetry: $\beta-$equilibrium; charge neutrality with electrons

→ DD2 ExVol (left panel): $v > 0$ (blue), $v < 0$ (red), $v = 0$ (black)





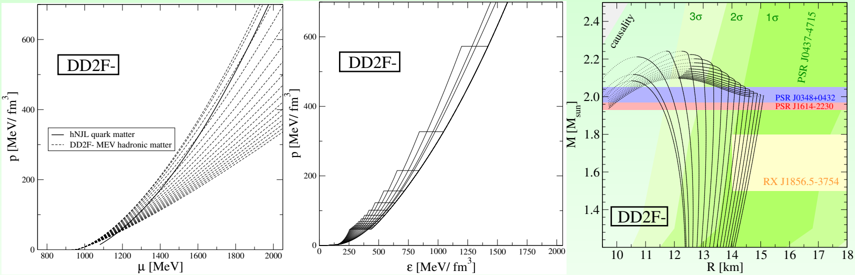
→ TOV equations → $M - R$ relation: positive v - larger $M_{\max}$ and R

Problem: causality violation (crosses)! Solution: deconfinement transition

Deconfinement in neutron star matter

→ finite hadron volume (quark substructure) entails deconfinement (overlapping bags)
→ Maxwell construction: DD2F- with varying v vs. fixed chiral quark matter model (hNJL):

Both the pressure and chemical potential should have the same value in both hadronic and quark matter phases.



Result: larger excluded volume entails earlier deconfinement with larger latent heat
Consequence: hybrid stars with quark core only for stiffer (large excluded volume) hadronic EoS!

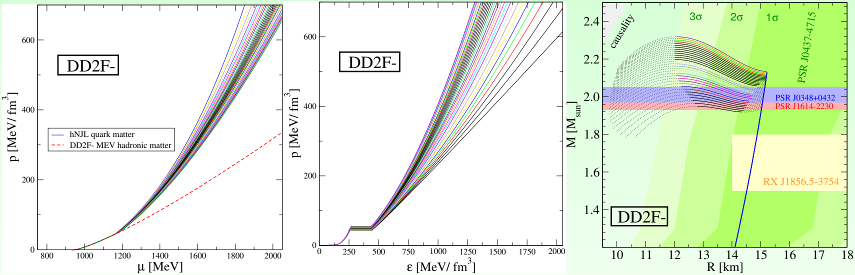
For Bayesian analysis, see: D. Alvarez-Castillo et al., Eur. Phys. J. **A 52** (2016) 69

For twin EoS, see: S. Benic et al., Astron. & Astrophys. **577** (2015) A40

Deconfinement in neutron star matter

→ finite hadron volume (quark substructure) entails deconfinement (overlapping bags)
→ Maxwell construction: DD2F- with fixed v vs. hNJL quark matter with varying stiffness :

Both the pressure and chemical potential should have the same value in both hadronic and quark matter phases.



Striking result: high-mass twin stars as observable signature for quark matter core!
Consequence: twin detection signals 1st order transition → CEP in QCD Phase diagram !

For Bayesian analysis, see: D. Alvarez-Castillo et al., Eur. Phys. J. **A 52** (2016) 69

For twin EoS, see: S. Benic et al., Astron. & Astrophys. **577** (2015) A40

Bayesian analysis of the most probable EoS

Problem: Find the most probable set of hybrid EoS parameters for

$\rightarrow$ for hadronic EoS (DD2F, excl. volume p) and quark matter EoS (hNJL, stiffness η_4)

$\rightarrow$ under ficticious radius constraints for known two high-mass pulsars

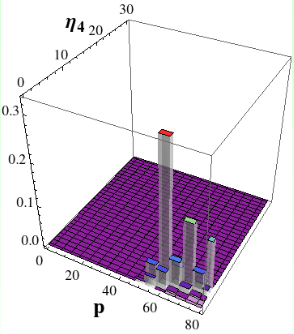
Result:

well identified (see figure) hybrid EoS with $v = 6$ fm^3, when the known high-mass pulsars:

$M_{\rm Demorest} = 1.97 \pm 0.04$ M$_\odot$
$M_{\rm Antoniadis} = 2.01 \pm 0.04$ M$_\odot$

would be twin stars with:

$R_{\rm Demorest} = 11.0 \pm 0.5$ km
$R_{\rm Antoniadis} = 15.0 \pm 0.5$ km



Goal:

Measure the radii of high-mass pulsars, e.g., with upcoming satellite mission NICER (Neutron star Interior Composition ExploreR)

*) For details, see: D. Alvarez-Castillo et al., Eur. Phys. J. **A 52** (2016) 69

Conclusions

- The excluded volume (EV) approach to hadronic and hybrid matter improves the agreement of model EoS with lattice QCD thermodynamics (where we have data for comparison)
- The EV is an inevitable consequence of the quark substructure of hadrons and approximately accounts for the Pauli blocking effect between hadrons. It is thus a precursor effect of deconfinement.
- The EV effect triggers a stiffening of the hadronic EoS and a lowering of the deconfinement density.
  It is a necessary condition for obtaining high-mass twin stars as a striking observable signal of a strong first-order deconfinement transition
- Observing high mass twin stars would be an indirect proof of the existence of a CEP in the QCD phase diagram, the goal of present and upcoming heavy-ion collision programmes.
- High-mass pulsars are already observed, it remains to measure their radii and to find that they are significantly different ! This may be done in near future with: NICER, SKA, LOFT, . . .