

In the search for a new hybrid supernova EOS: A systematic analysis of hybrid stars and on the problem of reconfinement

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Examples of hadron-quark SN EOS

$B^{1/4} = 162 \text{ MeV}$	Sagert et al. 2009	$1.56 M_{\odot}$	explosion
$B^{1/4} = 165 \text{ MeV}$	Sagert et al. 2009	$1.50 M_{\odot}$	explosion
$B^{1/4} = 155 \text{ MeV}, a_s = 0.3$	Sagert et al. 2011	$1.67 M_{\odot}$	explosion
$B^{1/4} = 139 \text{ MeV}, a_s = 0.7$	Sagert et al. 2011	$2.04 M_{\odot}$	
$B^{1/4} = 145 \text{ MeV}, a_s = 0.7$	Sagert et al. 2011	$1.97 M_{\odot}$	
$B^{1/4} = 184 \text{ MeV}$	Nakazato et al. 2013	$1.36 M_{\odot}$	
$B^{1/4} = 209 \text{ MeV}$	Nakazato et al. 2008	$1.80 M_{\odot}$	

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Key Questions

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- Which kind of hybrid stars are still possible? → classification
- Different formulations of quark EOS?
- Which phase transition parameters of the quark EOS might be interesting for a new SN EOS?
- Where do we have to expect reconfinement and how big is its influence on the maximum mass?

Definition

Hybrid stars are neutron stars that consist of both, hadronic and quark matter.

Overview of the model used:

- Scenario introduced by Alford et al. (2013)
- Hadronic phase: HS(DD2) (new)
- Quark phase: Constant Speed of Sound approach (CSS) (Alford 2013)
- Phase transition: 1st order phase transition, parametrized (Alford 2013)

In this work: Generic quark EOS proposed by Alford et al.

Constant Speed of Sound EOS

$$\epsilon_{QM}(p) = c_{QM}^{-2}(p - p_0) + \epsilon_0$$

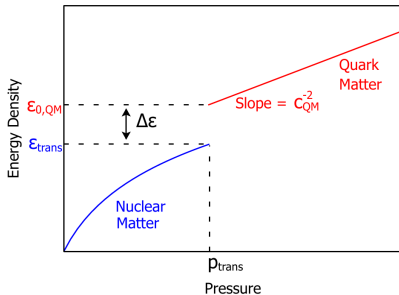
Properties:

- density-independent speed of sound c_{QM}
- $c_{QM}^2 = 1/3$ corresponds to weakly interacting massless quarks.
- $c_{QM}^2 = 1$ corresponds to interaction dominated quark matter.
Maximal value still consistent with SRT.

Isn't it too simple?

CSS shows good agreement for case $c_{QM}^2 = 1/3$ to more sophisticated models, as e.g. Nambu-Jona-Lasinio (NJL) (e.g. Benić 2014), Field-Correlator-Method (FCM) (Zappala 2014), bag model (Farhi and Jaffe 1984)

The Hybrid EOS



Phase Transition

1st order phase transition with a density jump at constant pressure from hadron to quark matter, based on local charge neutrality.

Figure : Schematic representation of the hybrid star EOS used Source: Alford, 2013

$$\epsilon(p) = \begin{cases} \epsilon_{HS(DD2)}(p) & p < p_{trans} \\ \epsilon_{HS(DD2)}(p_{trans}) + \Delta\epsilon + c_{QM}^{-2}(p - p_{trans}) & p > p_{trans} \end{cases}$$

Sequence of Calculations

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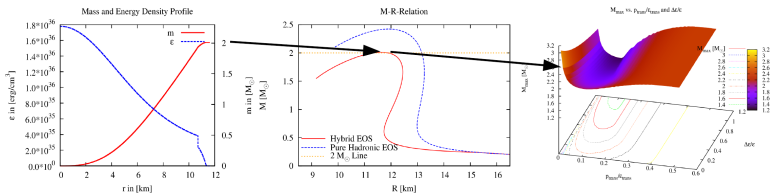
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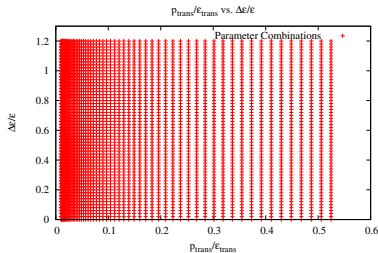
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- Determination of M-R relation by solving TOV equations.
- Variation of input parameters p_{trans} , $\Delta\epsilon$.
- 60 × 60 parameter combinations
- $p_{\text{trans}_{\min}} = 1 \cdot 10^{-4} \text{ MeV/fm}^3$
($n_B \sim 0.10 \text{ fm}^{-3}$)
- $p_{\text{trans}_{\max}} \approx 700 \text{ MeV/fm}^3$
($n_B \sim 0.96 \text{ fm}^{-3}$)
- $c_{\text{QM}}^2 = 1/3$
- $\Delta\epsilon/\epsilon = [0, 1.2]$



Alford's Classification of Hybrid Stars

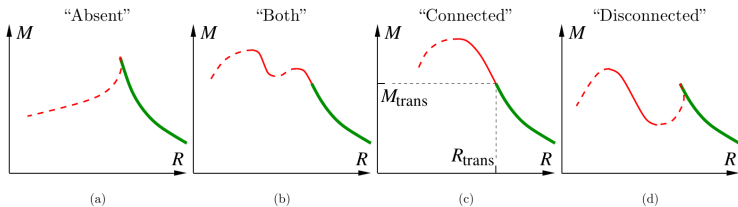
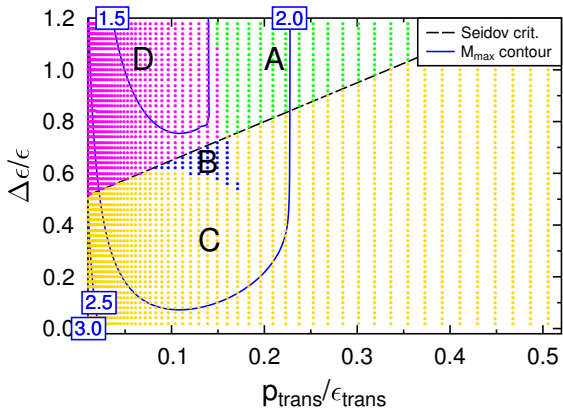


Figure : Four different possible M-R relation curves Source: Alford, 2013

- Two criteria for distinction:
 - Third family and continuous hybrid branch
- Third family: Hadronic phase building up \rightarrow set in phase transition \rightarrow phase of instability \rightarrow new stable branch
- Case b) and d) \rightarrow interesting for SN

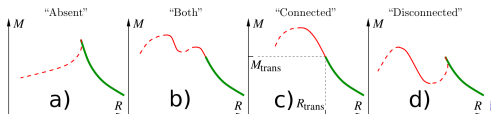
Results: Alford's Cases and Mass Contour Lines



Stable Hybrid Star Line

Analytic criterion derived by Seidov 1971

$$\frac{\Delta\epsilon_{\text{crit}}}{\epsilon_{\text{trans}}} = \frac{1}{2} + \frac{3}{2} \frac{p_{\text{trans}}}{\epsilon_{\text{trans}}}$$



A different Quark Model

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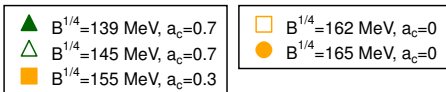
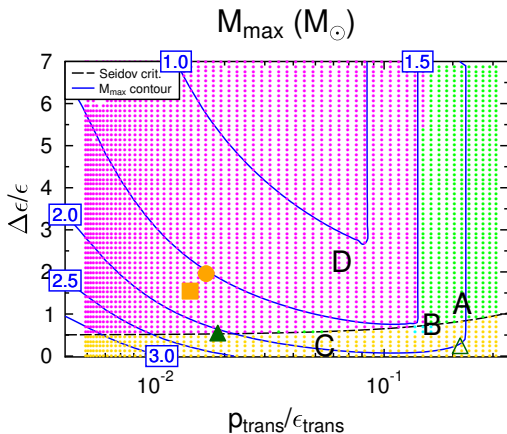
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- CSS is not a common parameterization for quark models!
- CSS is not extendible to finite temperatures.
- Often bag model is used!
 - For $T = 0$ and $m_s = 0$ and a non-vanishing interaction term a_4 , CSS model is transferable into bag model.
 - Extendible to finite temperatures

Restricting the parameter space



Reconfinement

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Classical parameter scan

Phase transition is modeled by using $p^{QM} = p^H$
 \rightarrow one fixed phase transition!

Thermodynamic consistent parameter scan

Phase transition is modeled by Maxwell construction:

- $p^{QM} = p^H$
- $\mu_B^{QM} = \mu_B^H$
- $T^{QM} = T^H$

Reformulating the CSS EOS leads to

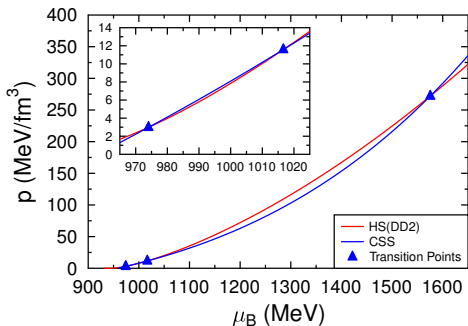
$$p(\mu_B)^{CSS} = \frac{c_{QM}^2}{1 + c_{QM}^2} (\epsilon_0 + p_0)$$

$$* \left[\left(\frac{\mu_B}{\mu_{B,0}} \right)^{\frac{1+c_{QM}^2}{c_{QM}^2}} + \frac{1}{c_{QM}^2} \right] - \epsilon_0.$$

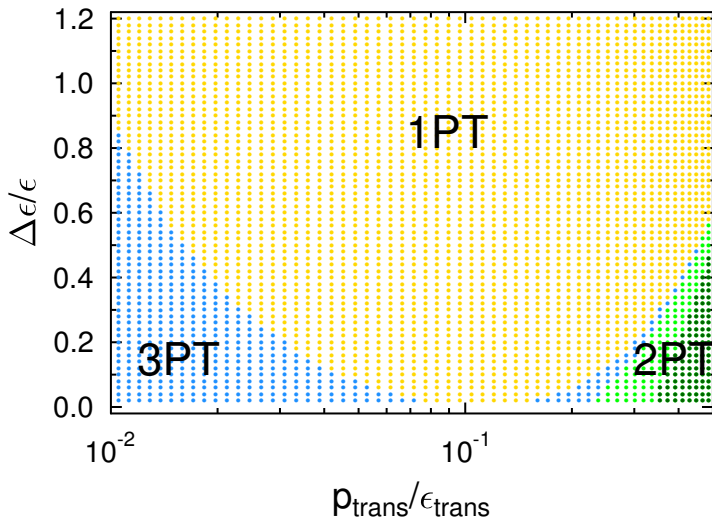
\rightarrow several phase transitions possible!

Reconfinement

- Deconfinement = hadron \rightarrow quark
- **Reconfinement** = quark \rightarrow hadron
- 3 Possibilities:
 - 1PT (HQ)
 - 3PT (HQHQ)
 - 2PT (QHQ) (absolutely stable quark matter)



Thermodynamical consistent phase transition



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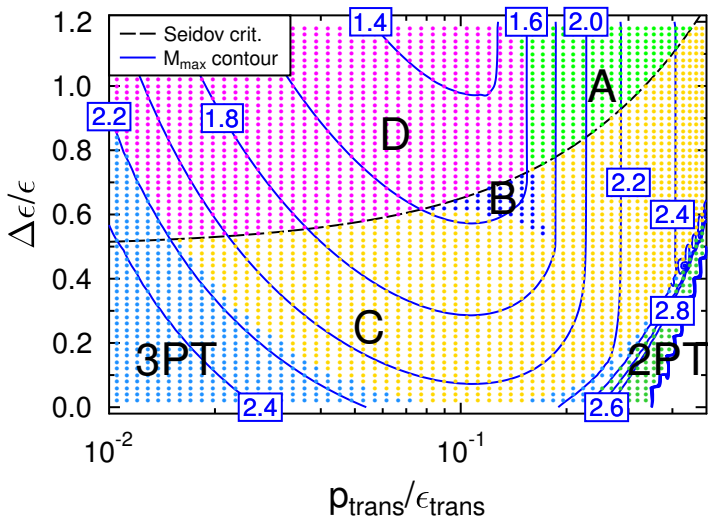
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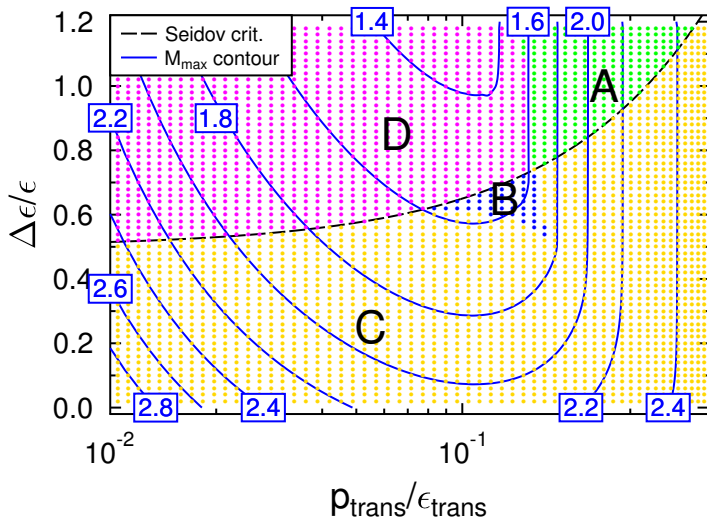
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Thermodynamical consistent phase transition



Original parameter scan



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Classical Parameter Scan

- Hybrid stars with third family branches and masses over $2 M_{\odot}$ are found.
- 1:1 correspondence between Constant Speed of Sound model and bag model exists.
- Parameter space for hybrid SN EOS candidates is very restricted.

Reconfinement Parameter Scan

- Taking into account thermodynamic consistency
- At $c_{QM}^2 = 1/3$, 3 PT's possible: (HQ), (QHQ), (HQHQ)
- Lowering of maximum mass due to reconfinement
- (QHQ) corresponds to absolutely stable QM

Thank you for your attention

Hadronic EOS: HS(DD2)

- Supernova EOS table at finite temperature and variable proton fraction available (Hempel & Schaffner-Bielich 2010, Fischer et al. 2014).
- Density-dependent relativistic mean field theory (DD2, Typel et al. 2010)
- Matter consists of n , p , e , A
- Nuclear matter properties are in good agreement with many different nuclear experiments.
- Maximum mass: $2.42 M_{\odot}$

Important

HS(DD2) EOS describes neutron star from crust to the outer core self-consistently. In this work: HS(DD2) at $T = 0.1$ MeV and β -equilibrium.

Backup HS(DD2)

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- $n_0 = 0.1491 \text{ fm}^{-3}$
- Binding energy $E_0 = 16.02 \text{ MeV}$
- Incompressibility $K = 243 \text{ MeV}$
- Symmetry energy $S = 31.67 \text{ MeV}$
- Slope of symmetry energy $L = 55.04 \text{ MeV}$
- See: EPJA Fischer et al. (2014)

Interacting model of Alford respectively Weissenborn 2005

Idea: Introduce phenomenological interaction term a_4 (and possibly a_2).

Alford (Weissenborn) model

$$\Omega_{\text{QM}} = \underbrace{\sum_{i=u,d,s,e} \Omega_i - \frac{3\mu^4}{4\pi^2}(1 - a_4) + B_{\text{eff}}}_{\text{Alford et al. (2005)}} + \left(\frac{3\mu^2}{4\pi^2} a_2 \right)$$

- a_4 term accounts for strong interaction QCD corrections
- a_2 can be interpreted as a term to take color superconductivity into account. In this case: $a_2 = m_s^2 - 4\Delta^2$ (Δ pairing gap).
- Here: Parameters are treated as generic interaction terms, which are freely varied without respect to their physical meaning.

Direct identification of Weissenborn's BAG model with CSS EOS

- Assumptions: $m_s = 0$ and $c_s^2 = 1/3$, $a_2 = 0$, non-vanishing a_4 term.

$$a_4 = 2 - \frac{\pi^2 \epsilon_0 + P_0}{3 \mu_0^4}$$

$$B_{\text{eff}} = \frac{1}{4} \epsilon_0 - \frac{3}{4} P_0$$

Non-interacting model

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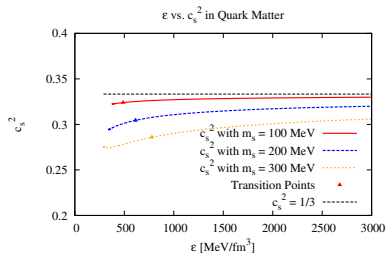
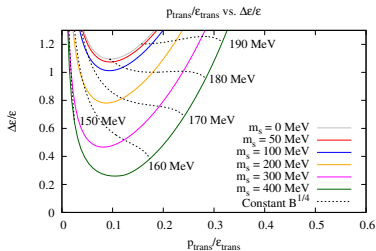
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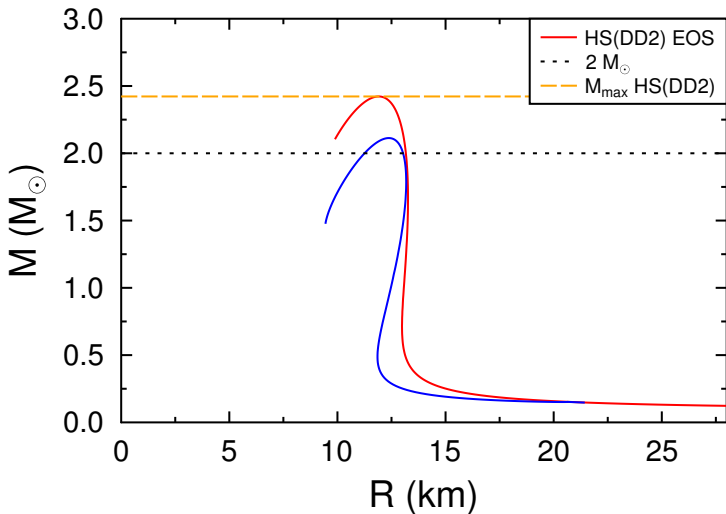
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- Non-interacting model: Only one curve. Variation of strange quark mass to cover parameter space.
- No $2 M_{\odot}$ cases possible.
- Low $\Delta\epsilon$ not realizable.
- With increasing strange quark mass \rightarrow speed of sound $c_s^2 \neq 1/3$ anymore.

Example



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$$c^2 = 1$$

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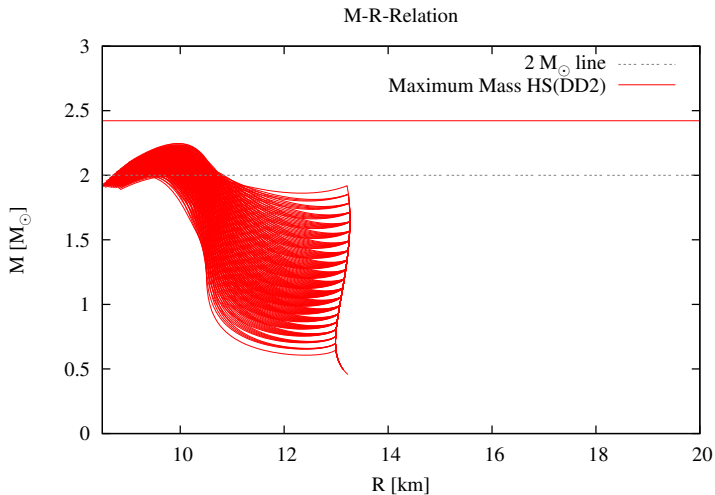
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Number density dependence of the parameter scan

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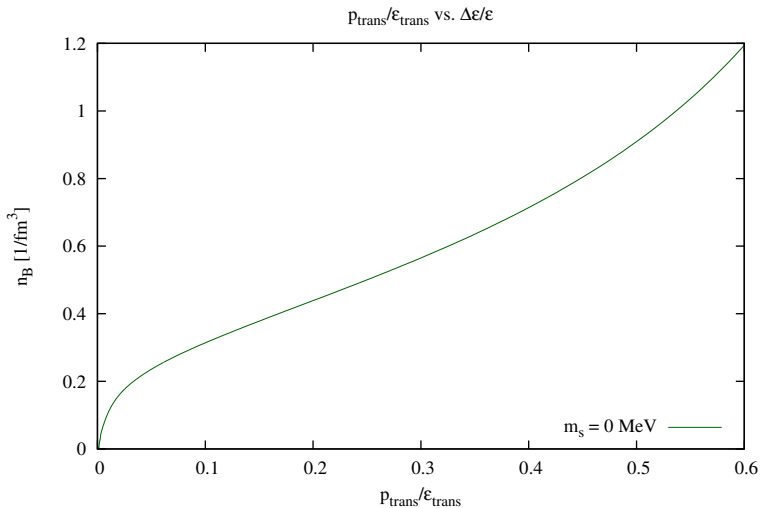
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Characteristics of Third Family Cases

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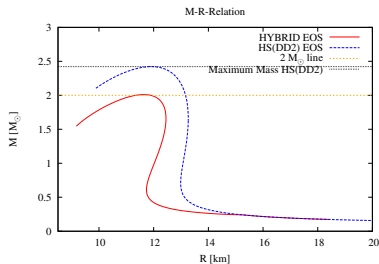
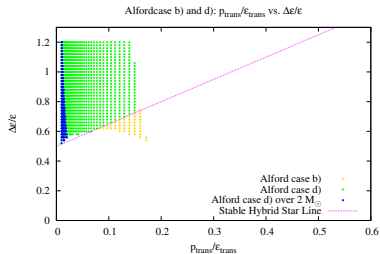
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- Third family cases with over $2 M_{\odot}$ found!
- Phase transition at low transition pressures
- Almost pure quark stars
- Third family, but not as commonly expected \rightarrow kink in M-R relation is almost invisible.

$p(\mu)$ -relation of CSS EOS

Start with

$$\epsilon = \epsilon_0 + \frac{1}{c_s^2}(P - P_0)$$

then use thermodynamical consistency and finally get

$$P(\mu) = \frac{c_s^2}{1 + c_s^2} \mu_0 n_0 \left[\left(\frac{\mu}{\mu_0} \right)^{\frac{1+c_s^2}{c_s^2}} + \frac{1}{c_s^2} \right] - \epsilon_0$$

- ϵ_0 and p_0 are fixed at the phase transition point.
- n_0 is fixed using thermodynamical consistency.
- We fix μ_0 by using chemical equilibrium at the phase transition point.

pQCD model Fraga et al. (2014)

- Based on state-of-the-art perturbative QCD EOS from Kurkela et al. (2010)
- Considered terms up to α_c^2 (strong-interaction coupling) as well as non-zero m_s
- Fraga et al. applied original EOS to electro-neutral and β -stable quark matter and fitted an easy to handle function

pQCD EOS of Fraga et al.

$$P_{\text{QCD}}(\mu_B, X) = P_{\text{SB}} \left(c_1 - \frac{a(X)}{(\mu_B/\text{GeV}) - b(X)} \right)$$

$$a(X) = d_1 X^{-\nu_1}, \quad b(X) = d_2 X^{-\nu_2}$$

$$P_{\text{SB}}(\mu_B) = \frac{3}{4\pi^2} (\mu_B/3)^4$$

pQCD model

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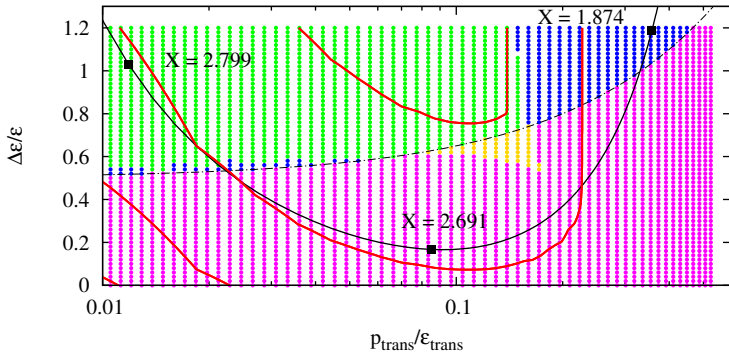
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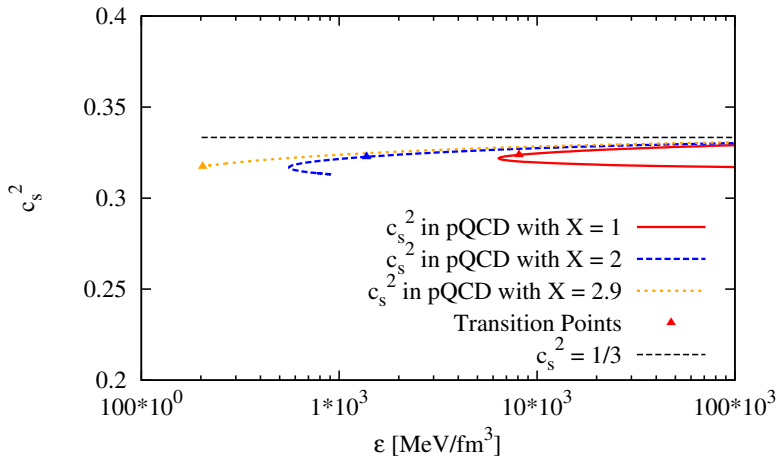
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All Four Alfordcases: $\rho_{\text{trans}}/\epsilon_{\text{trans}}$ vs. $\Delta\epsilon/\epsilon$



pQCD model - speed of sound dependence

ϵ vs. c_s^2 in Quark Matter



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pQCD vs. BAG model

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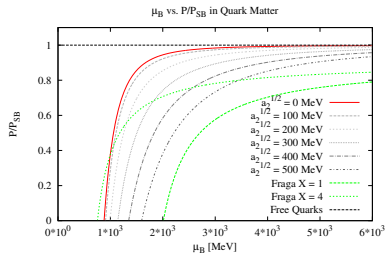
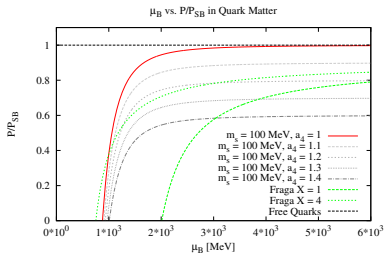
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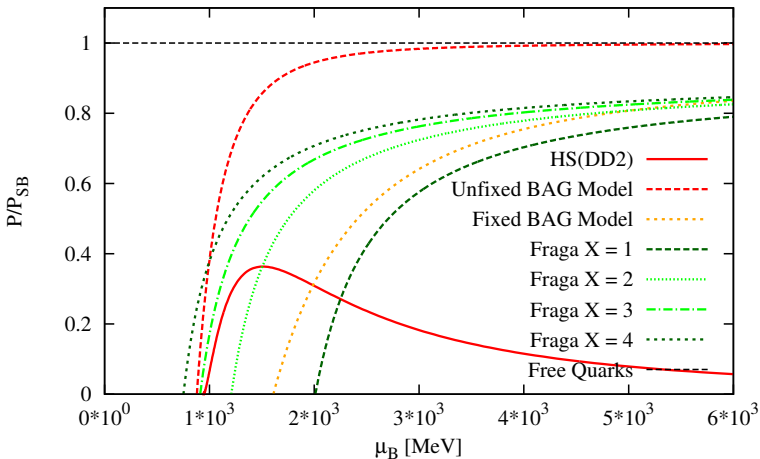
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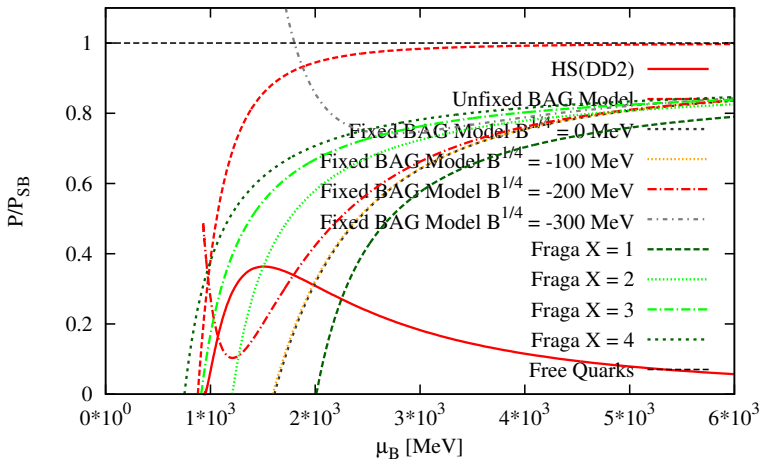
pQCD vs. BAG model

μ_B vs. P/P_{SB} in Quark Matter



pQCD vs. BAG model

μ_B vs. P/P_{SB} in Quark Matter



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