Cool quark matter

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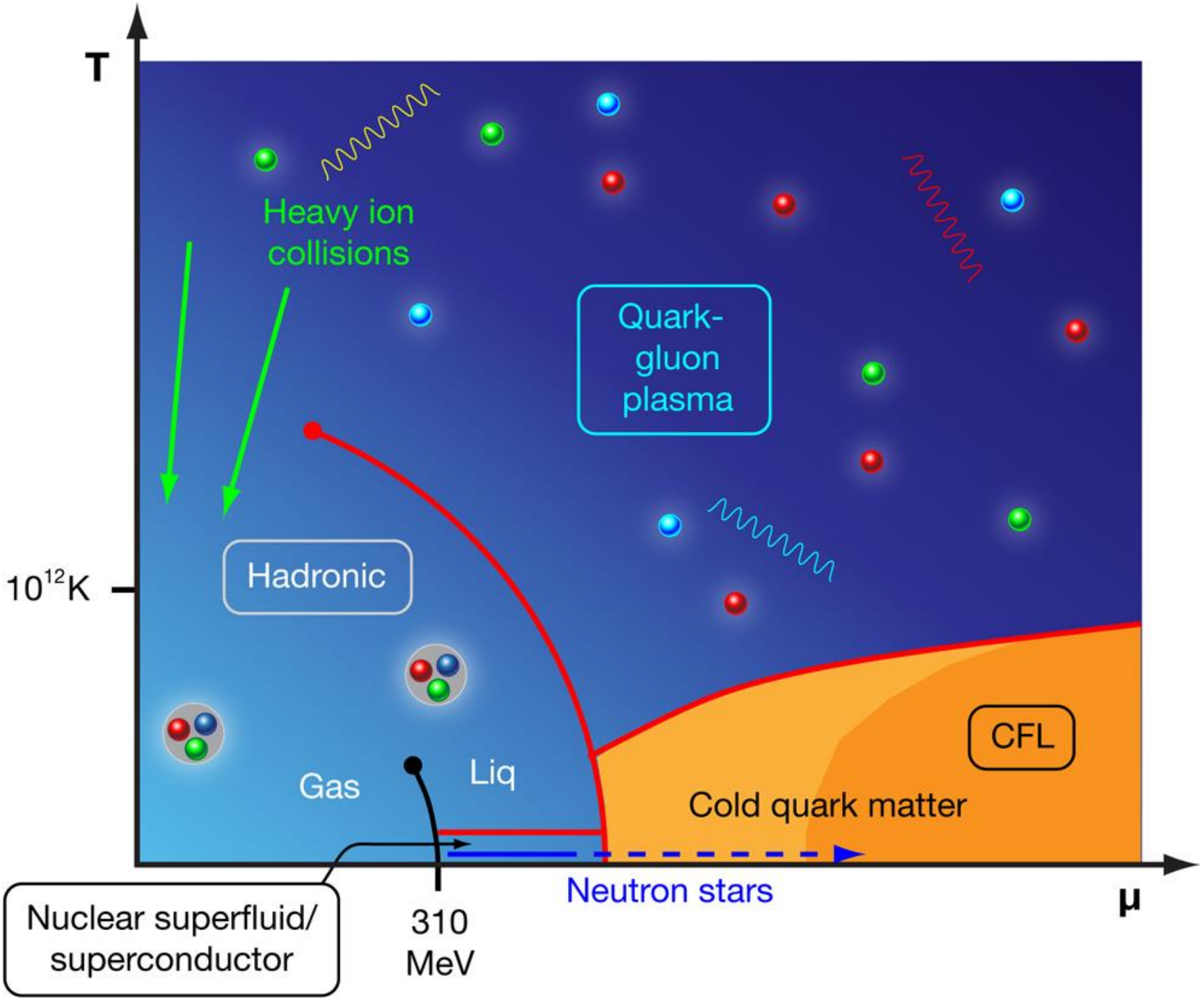
Based on: Aleksi Kurkela, AV, arXiv:1603.00750 [hep-ph]

Ultimate goal in nuclear theory: quantitative description of systems from single nuclei to nuclear and quark matter from first principles, i.e. QCD

Tools: Lattice QCD, Effective Field Theory, perturbation theory


T
Heavy ion collisions
Quark-gluon plasma
Hadronic
10^{12}K
Gas
Liq
CFL
Cold quark matter
Neutron stars
μ
Nuclear superfluid/ superconductor
310 MeV

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This talk: bulk thermodynamics at small but nonzero temperatures

I. Perturbation theory at finite density: from quark matter to neutron stars
II. Challenges at small but nonzero temperature
III. New result: perturbative EoS of cool quark matter

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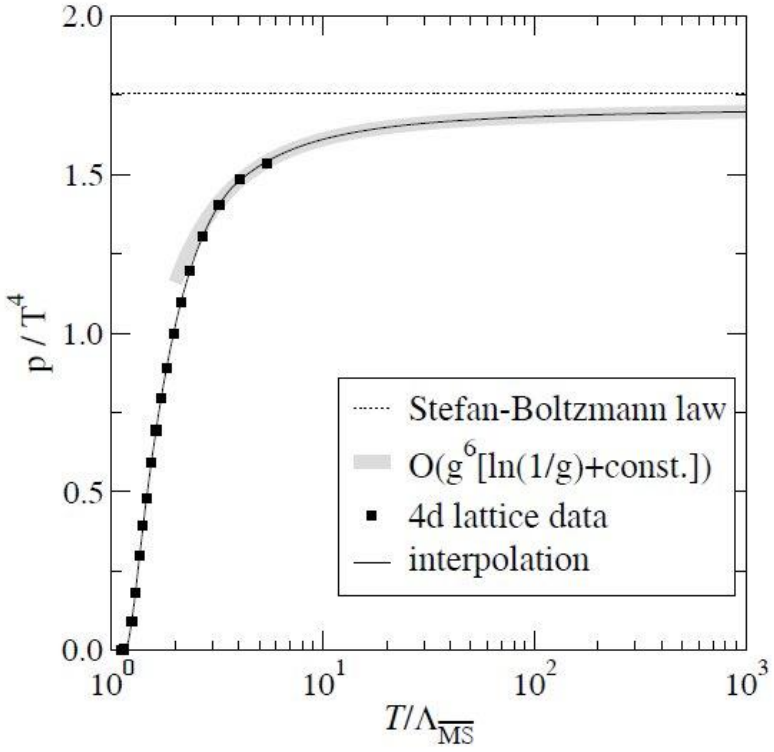
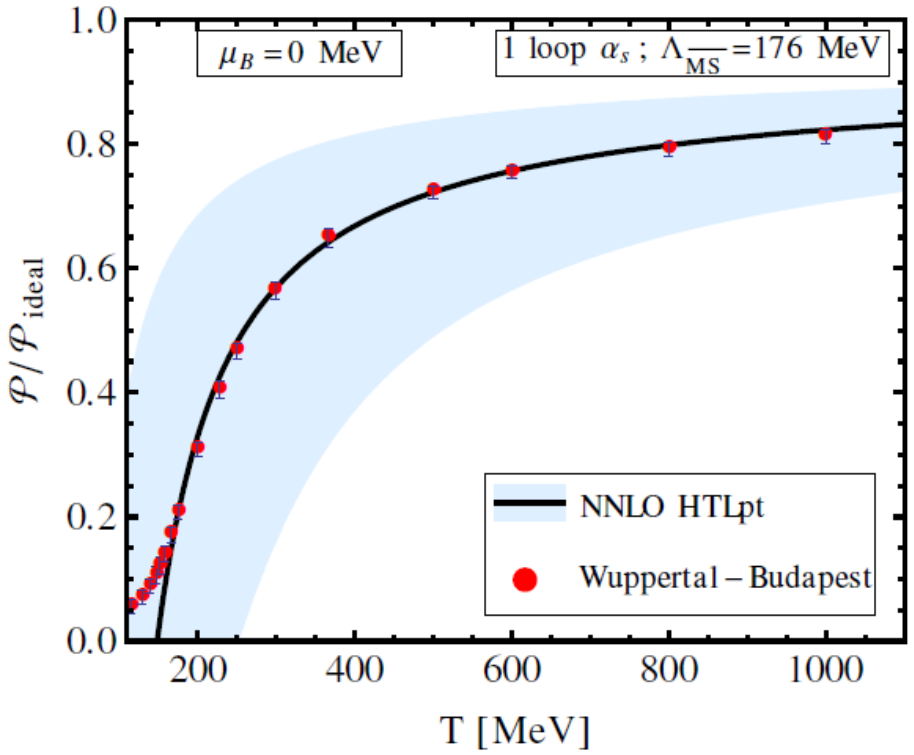
High temperatures: pQCD standard tool complementing lattice, with excellent agreement above ca. $2T_c$





Kajantie, Laine, Rummukainen, Schröder, PRD 67 (2003)
Laine, Schröder, PRD 73 (2006)
Andersen, Leganger, Strickland, Su, JHEP 1108 (2011)

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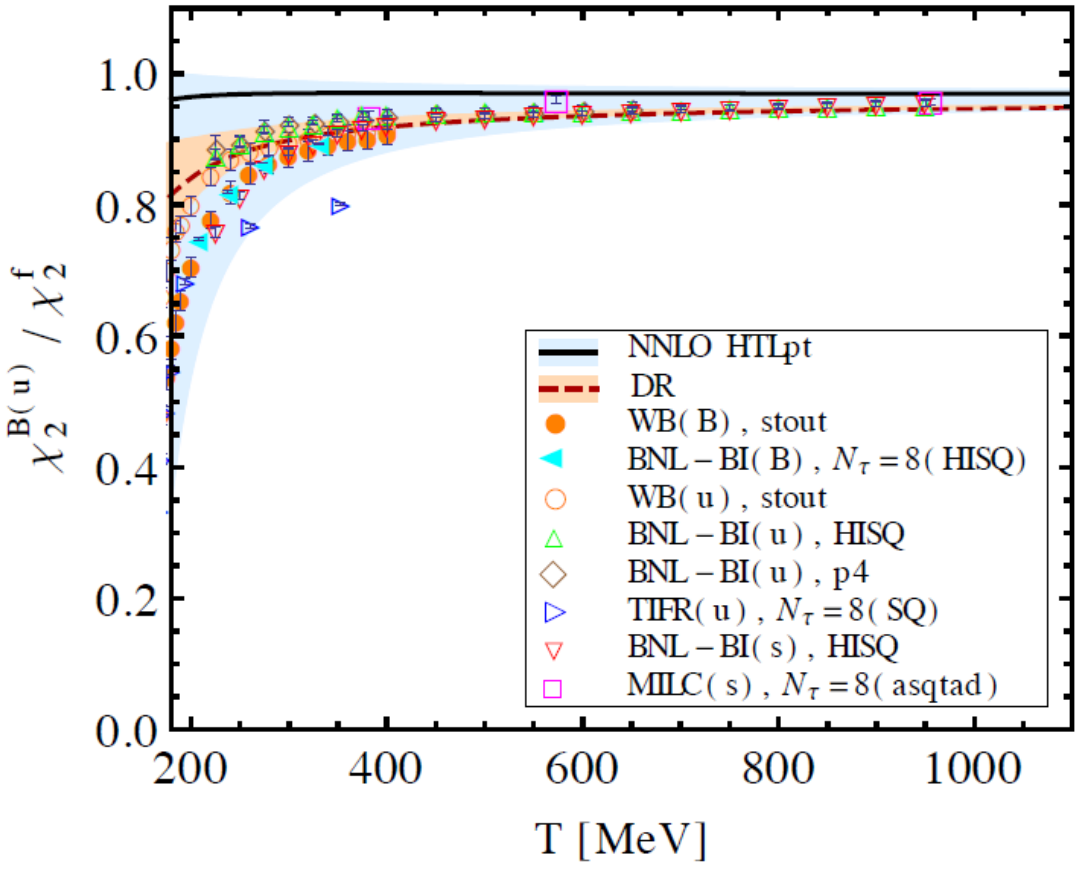
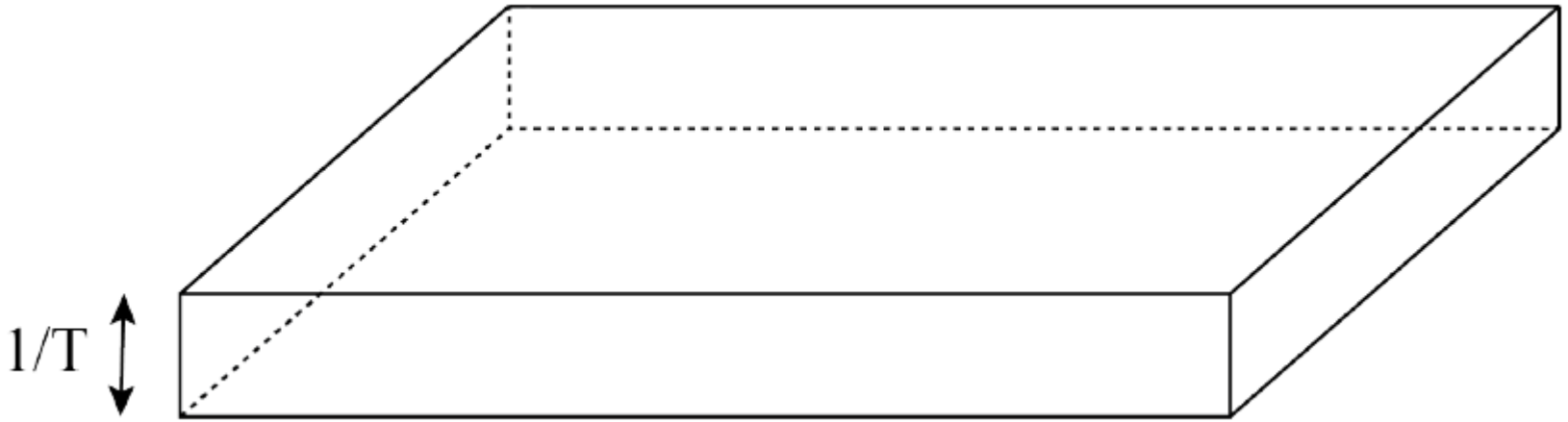


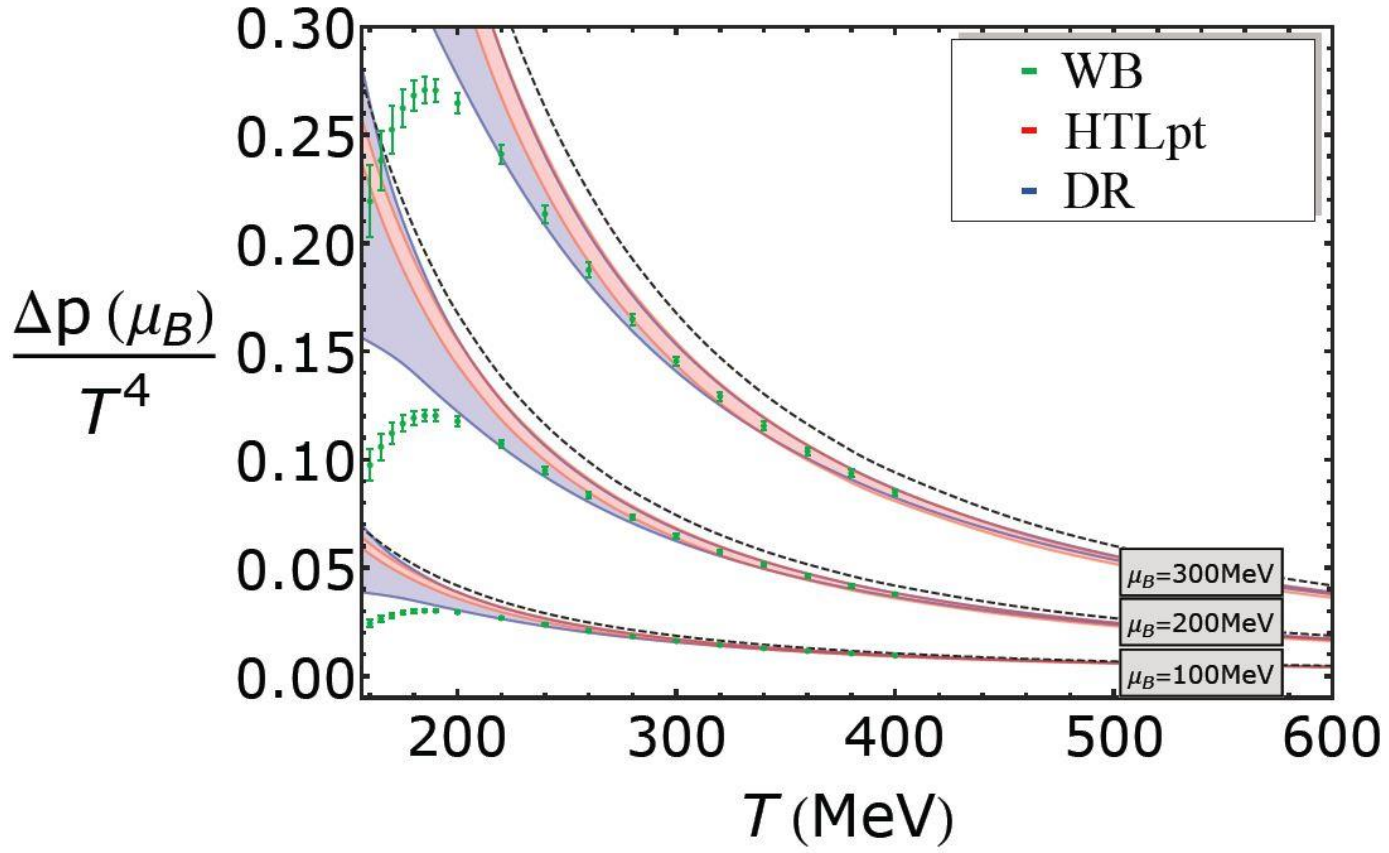
AV, PRD 67, PRD 68 (2003)
Mogliacci, Andersen, Strickland, Su, AV, JHEP 1312 (2013)
Haque, Andersen, Mustafa, Strickland, Su, PRD 89 (2014)

Key observation behind improvement:
Dimensional reduction and effective theories
HTL and EQCD → Resummation of soft modes



$$p_{\rm QCD}(T,\mu) \quad = \quad p_{\rm E}(T,\mu) + \frac{T}{V}\ln\int \mathcal{D}A_i^a\,\mathcal{D}A_0^a \exp\left\{-S_{\rm E}\right\}$$

No Sign Problem → Perturbation theory works even (better) at finite density!



AV, PRD 67, PRD 68 (2003)

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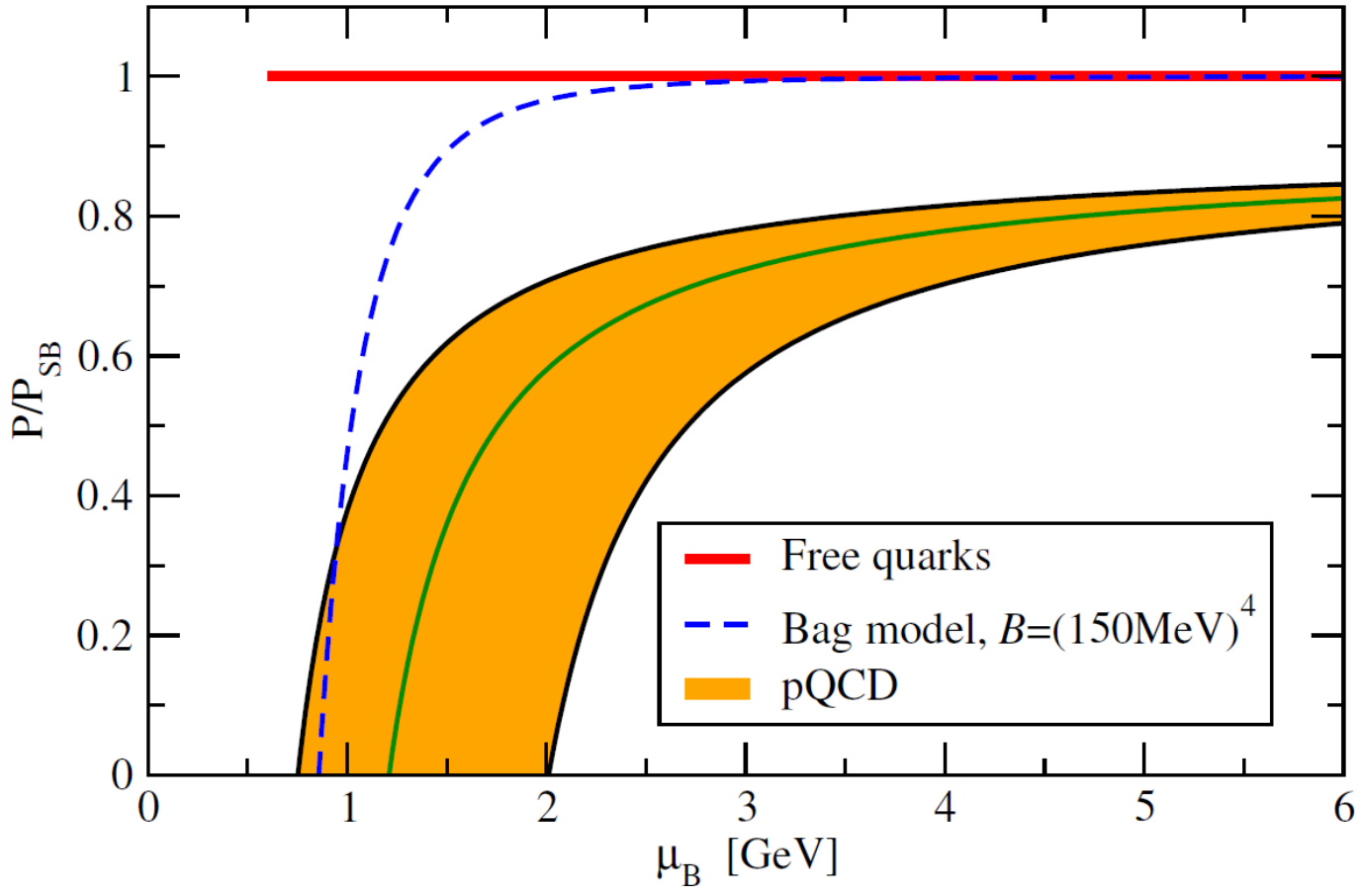
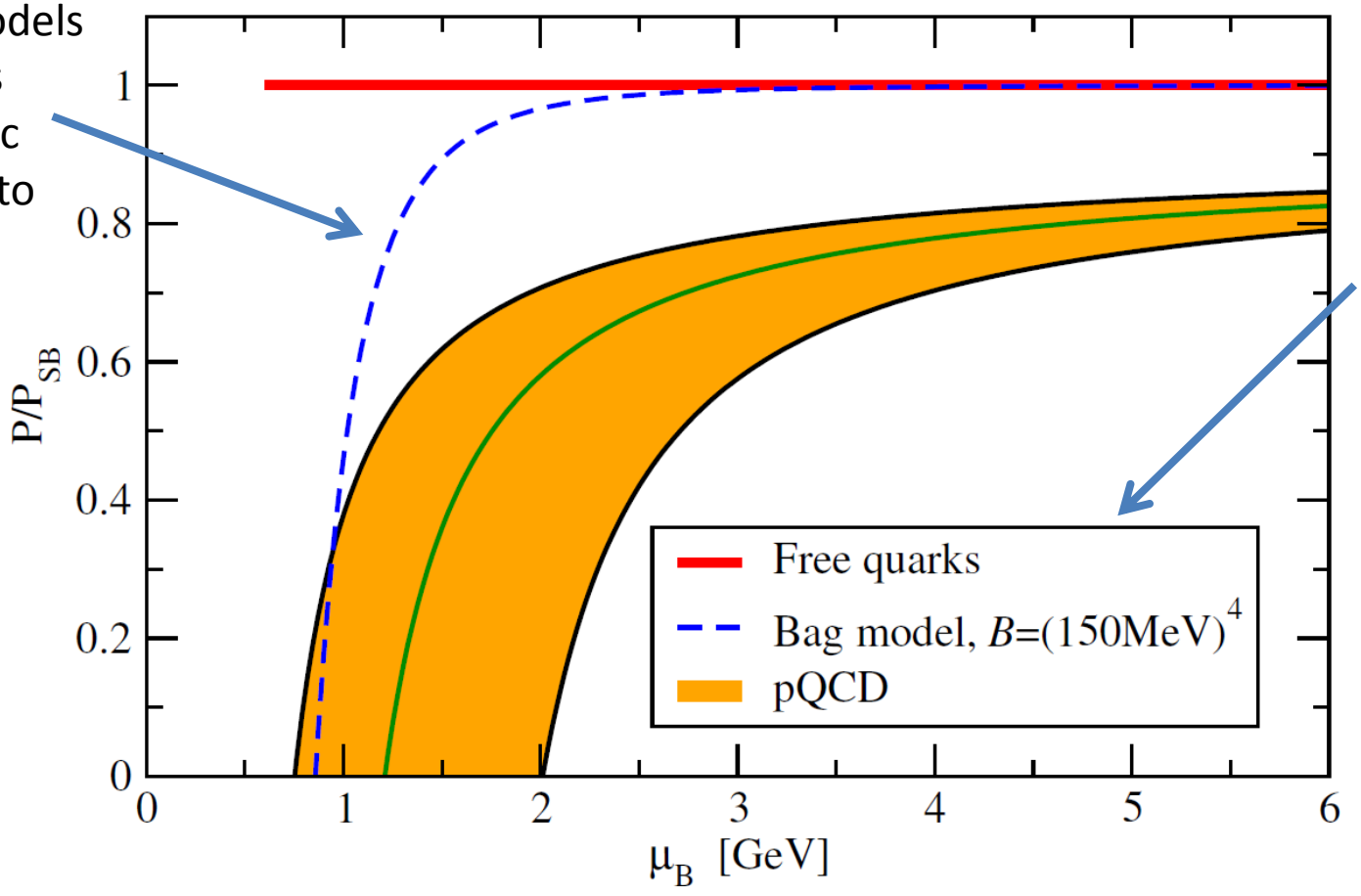
pQCD at $T = 0$: $O(g^4)$ with massive quarks



Kurkela, Romatschke, AV, PRD 81 (2010)

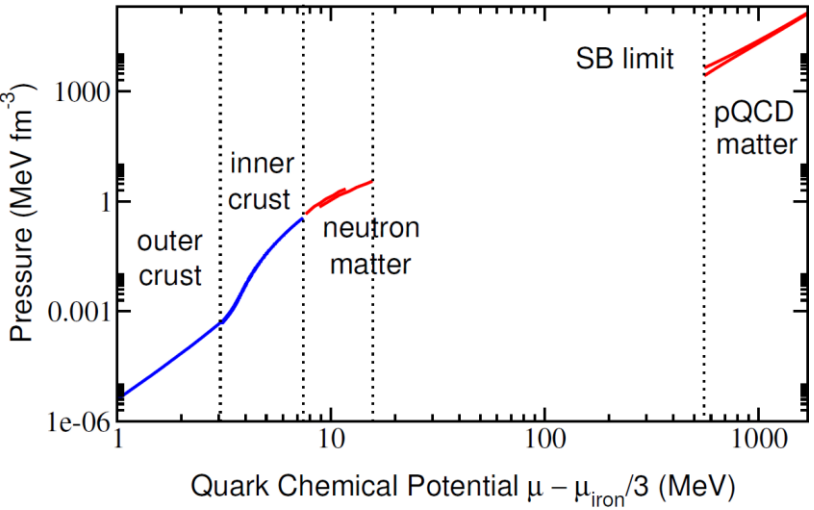
pQCD at $T = 0$: $O(g^4)$ with massive quarks

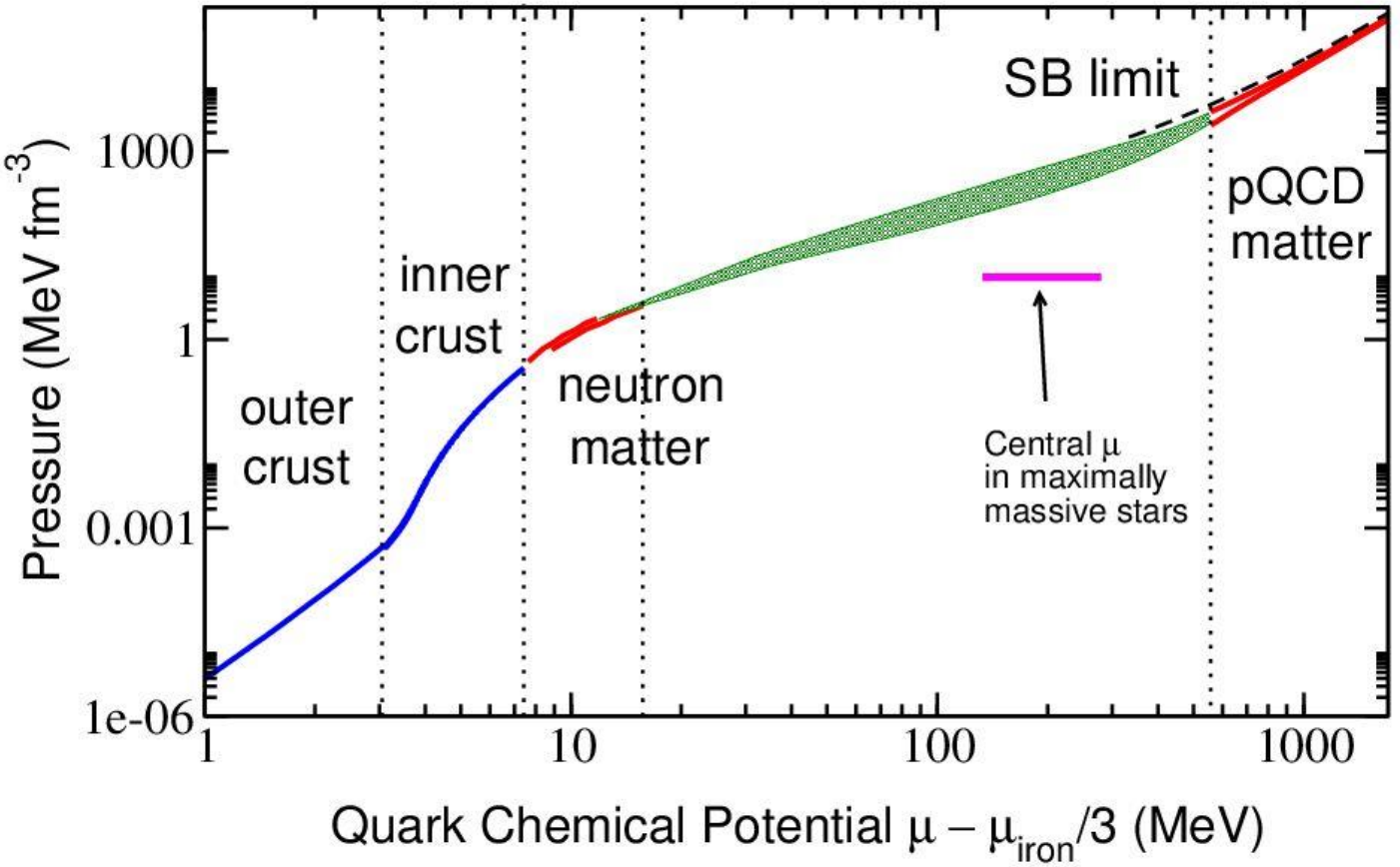
Simple models often miss logarithmic approach to free limit



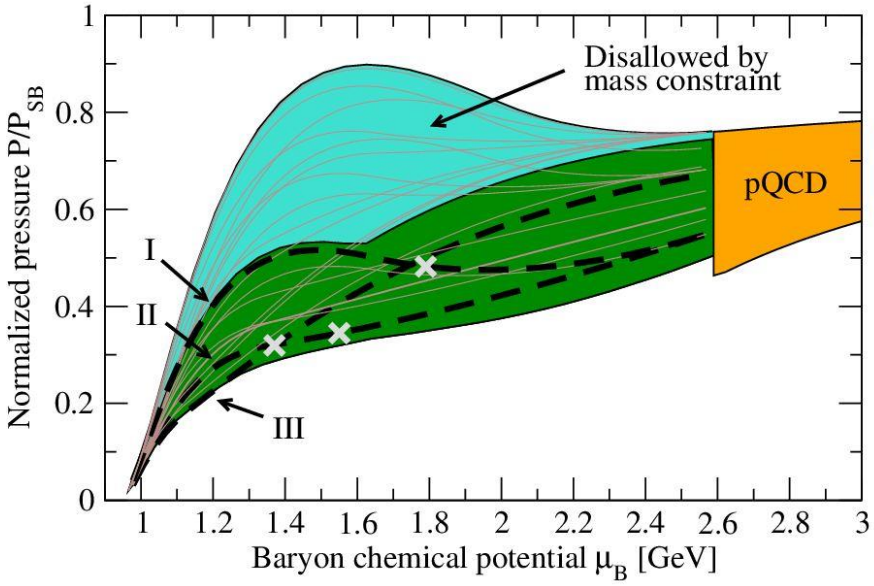
No lattice prediction!

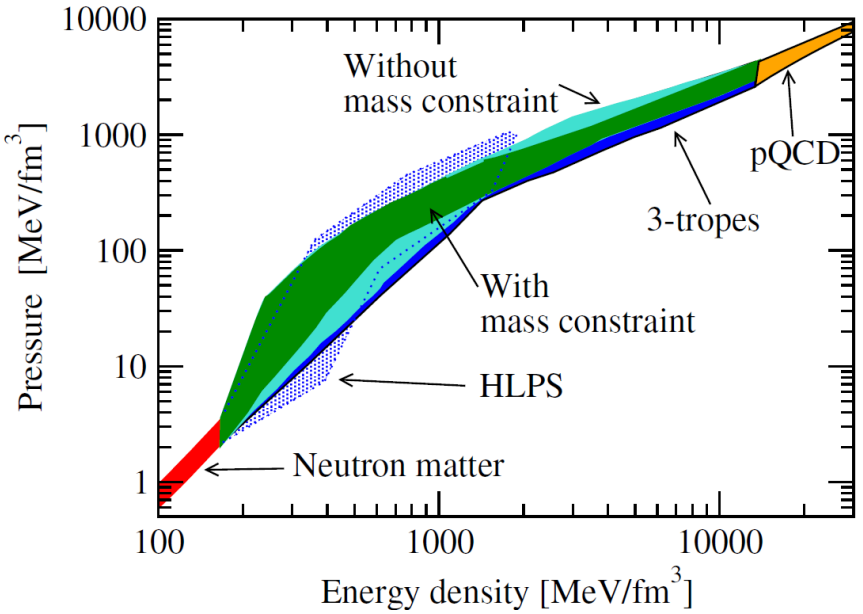
Kurkela, Romatschke, AV, PRD 81 (2010)





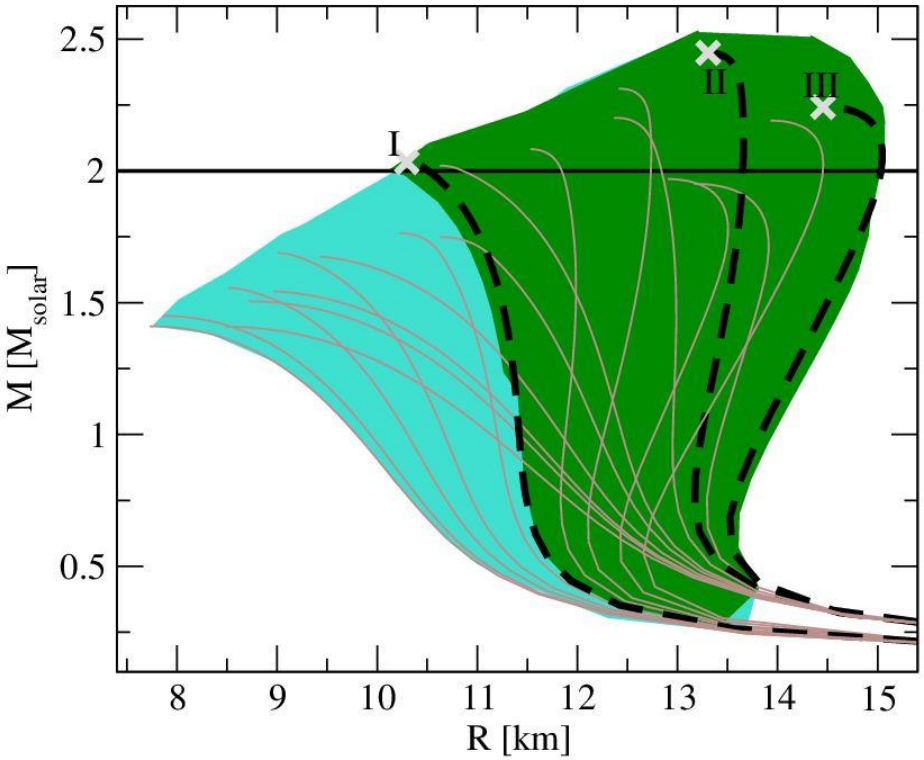
Kurkela, Fraga, Schaffner-Bielich, AV, APJ 789 (2014)





Main goal of future work:

- Extend EoS of $T = 0$ QM to full four-loop order → dramatically reduced uncert.
- Further constrain neutron star EoS at moderate density
- $O(g^6 \ln g)$ already *almost* complete



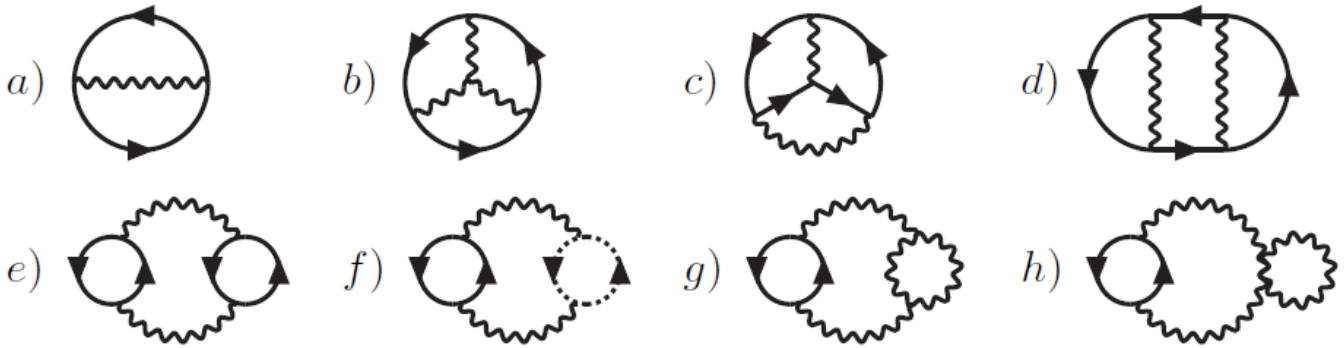
Ghisoiu, Gorda, Kurkela, Romatschke, AV,
In preparation

I. Perturbation theory at finite density: from quark matter to neutron stars
II. Challenges at small but nonzero temperature
III. New result: perturbative EoS of cool quark matter

$$\Omega(T,\mu_u,\mu_d,\mu_s,m_s) = -T\log\int \mathcal{D}\bar{\psi}\mathcal{D}\psi\mathcal{D}A_\mu e^{-\int d^3x\int_0^{1/T}d\tau\mathcal{L}_{\rm QCD}},$$

$$\mathcal{L}_{\rm QCD} = \frac{1}{4}F^a_{\mu\nu}F^a_{\mu\nu} + \bar{\psi}_i(\gamma_\mu D_\mu + m_i - \mu_i\gamma_0)\psi_i$$

Perturbation theory: expansion of partition function in powers of gauge coupling *g* → Vacuum or bubble diagrams



$$p_n^{\rm bos} = 2\pi nT, \qquad p_n^{\rm ferm} = (2n+1)\pi T - i\mu$$

$$\int \frac{d^{4-2\epsilon}p}{(2\pi)^{4-2\epsilon}} \rightarrow T\sum_{p_n}\int \frac{d^{3-2\epsilon}p}{(2\pi)^{3-2\epsilon}}$$

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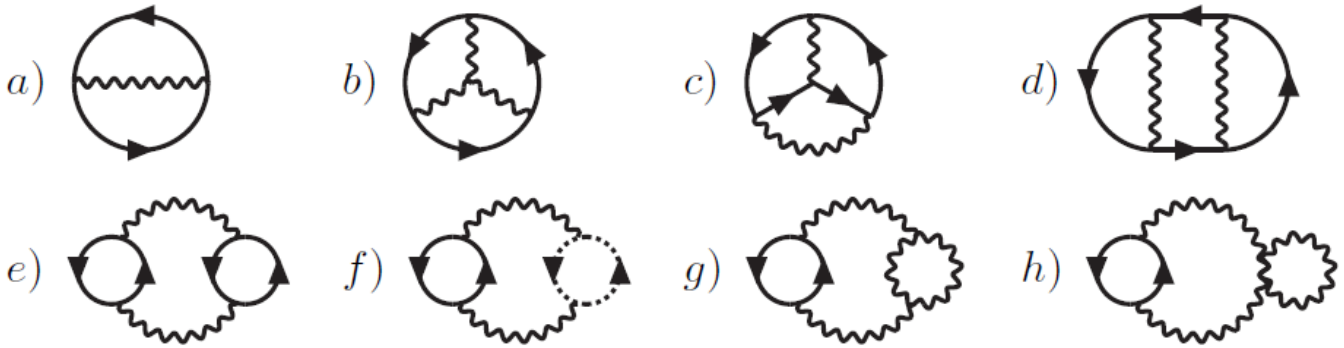
Problem in pQCD: infrared divergences at three-loop order from long-range (static) gauge fields



$$-\omega^2 + k^2 \to -\omega^2 + k^2 + \Pi(\omega,k)$$

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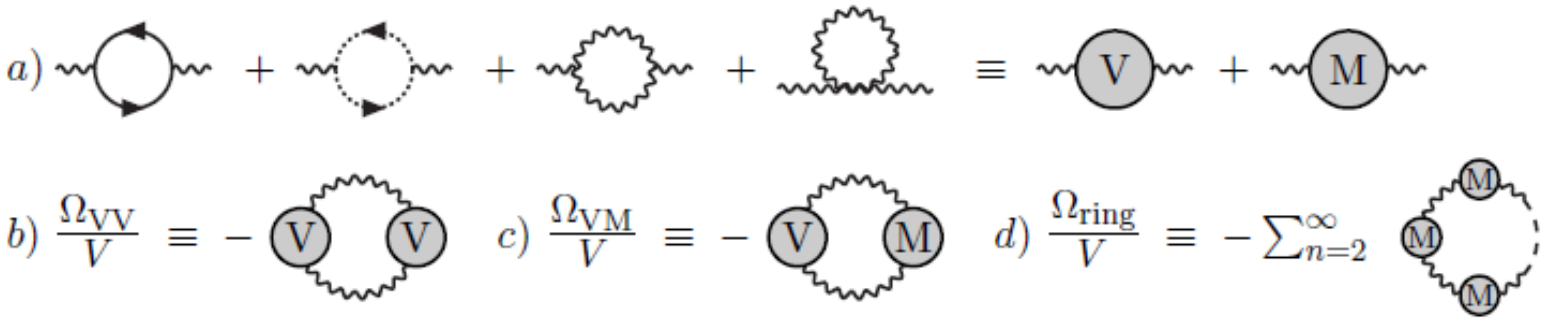
Solution: Resummation of IR sensitive contributions to the EoS: sum (certain) diagrams to infinite order or use EFT



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$$p_{\rm QCD} = p^{\rm naive}_{\rm QCD} + p^{\rm res}_{\rm DR} - p^{\rm naive}_{\rm DR} + p^{\rm res}_{\rm HTL} - p^{\rm naive}_{\rm HTL}$$

Effective theory for $n = 0$ Matsubara mode. Necessary at $T \neq 0$; vanishes when $T \to 0$.

Effective description for $n \neq 0$ Matsubara mode. Dominates in the $T = 0$ limit.

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Status of weak coupling EoS today:

- $\mu_B = 0$: $O(g^6 \ln g)$ Kajantie, Laine, Rummukainen, Schröder, PRD 67 (2003)
- $0 < \mu_B \leq T/g$: $O(g^6 \ln g)$ AV, PRD 68 (2003)
- $T = 0$: $O(g^4)$ /w finite masses Kurkela, Romatschke, AV, PRD 81 (2010)

Problematic region: small but nonzero temperatures

- High order expansion in $\tau \equiv T/(g\mu_B)$ around $T = 0$ – small radius of converge; "Hard Dense Loops" Rebhan et al. 2002-2005
- Very cumbersome diagrammatic resummation connecting $T = 0$ and $T \sim \mu_B$ Ipp, Kajantie, Rebhan, AV, PRD 74 (2006)

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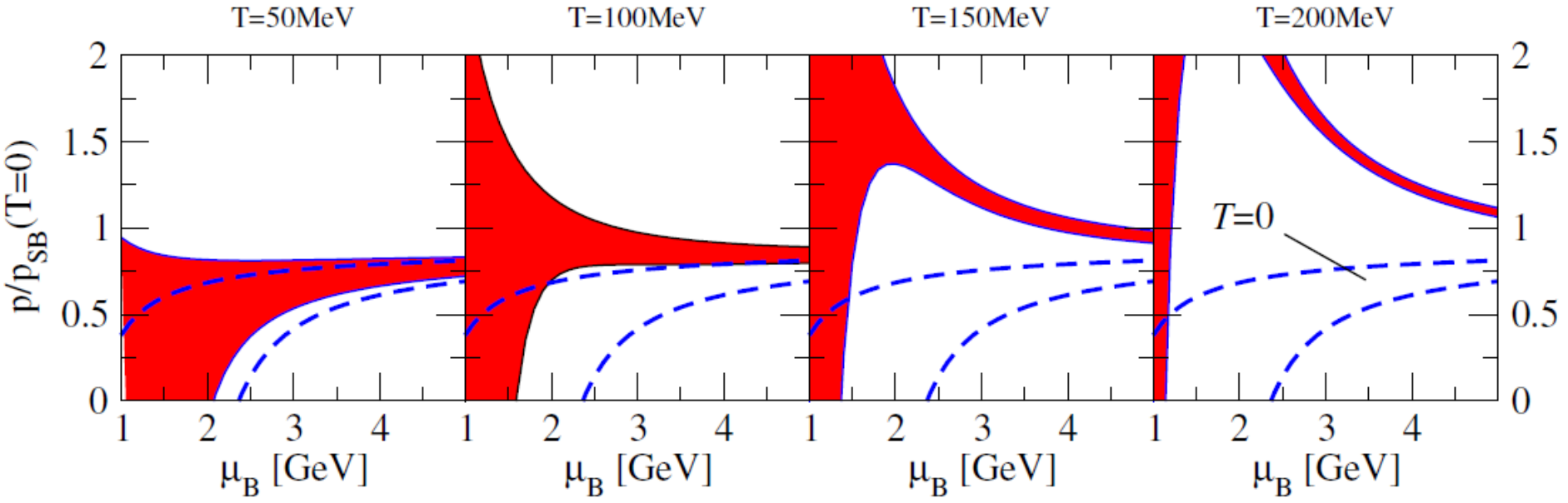
New: analytic result combining DR and HTL resummations →
Small temperatures under control Kurkela, AV, arXiv:1603.00750 [hep-ph]



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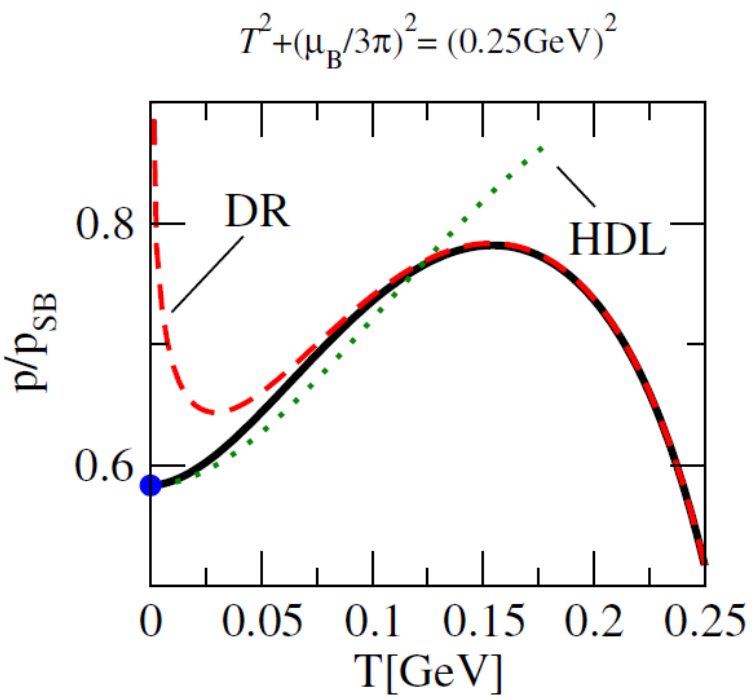

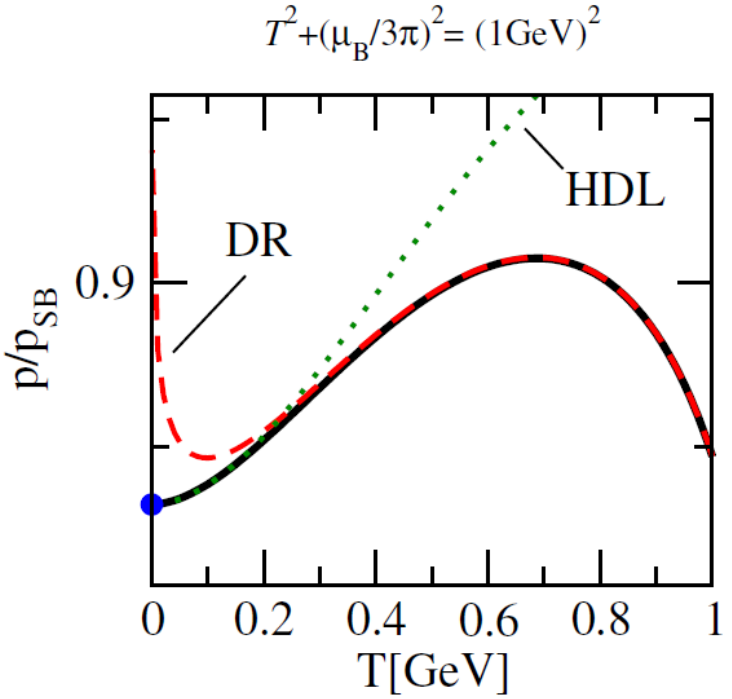

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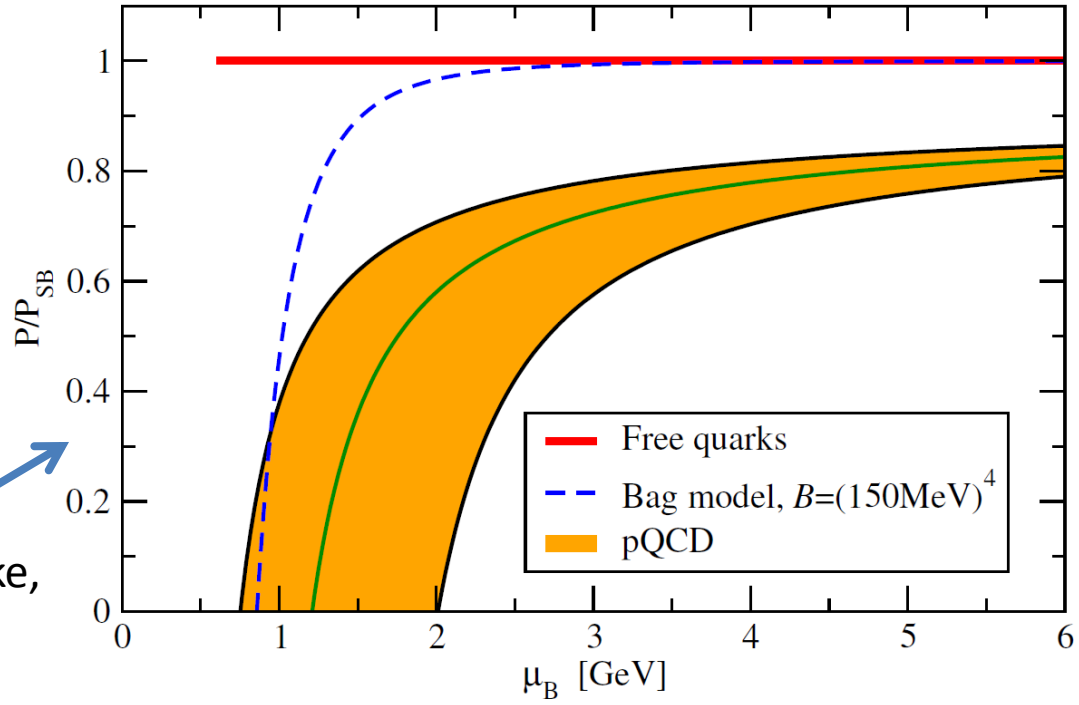
- Analyticity: result trivially extendible outside beta equilibrium and charge neutrality
  - Leading finite-*T* correction: $O\left(T^2\ln\frac{T}{\mu_B}\right)$
- Potential uses in neutron star merger calculations: need finite temperature corrections on a large density interval
  - Plan: Constrain low-density EoSs with new result

Conclusions



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