

Maximum Mass, Moment of Inertia and compactness of Relativistic Stars

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- How heavy can a neutron star become before it collapses to a black hole?
- How does that limit change with rotation?
- Neutron star properties sensitively dependent on the modeling EOS
- Large uncertainties in EOS above nuclear density
- Approximately universal relations between certain quantities
- Constraints on EOS and quantities which are not directly observable
- Validity in rapid rotation?



Metric of a stationary, axisymmetric system:

$$ds^2 = -e^{\gamma+\rho} dt^2 + e^{\gamma-\rho} r^2 \sin^2 \theta (d\Phi - \omega dt)^2 + e^{2\mu} (dr^2 + r^2 d\theta^2)$$

Slow-Rotation Approximation:

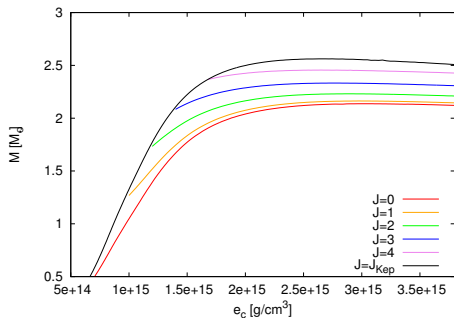
- Solve TOV-equations and scale-invariant differential equation for the angular velocity relative to the local inertial frame:
- $\frac{1}{r^4} \frac{d}{dr} (r^4 j \frac{\bar{\omega}}{dr}) + \frac{4}{r} \frac{dj}{dr} \bar{\omega} = 0$, $j(r) = \exp [(-\nu + \lambda)/2]$

Rapid Rotation:

- RNS-code
- KEH-approach, iteration until metric equations and first integral of the hydrostationary equilibrium equation are satisfied to a desired accuracy

Critical Mass

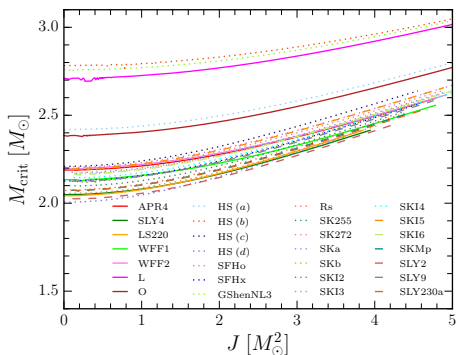
Critical Mass



- neutron star masses limited by instability to gravitational collapse and Kepler limit in case of rotation
- addition of small amount of rest mass through accretion leads to configuration with increased central density and mass

- process cannot continue indefinitely \rightarrow no new equilibrium with larger mass, finally star has to evolve dynamically towards a black hole
- turning point criterion: $\left. \frac{\partial M}{\partial \rho_c} \right|_J = 0$
- just sufficient but not necessary condition, actual neutral stability line at slightly smaller central densities (Takami, Rezzolla and Yoshida 2011)

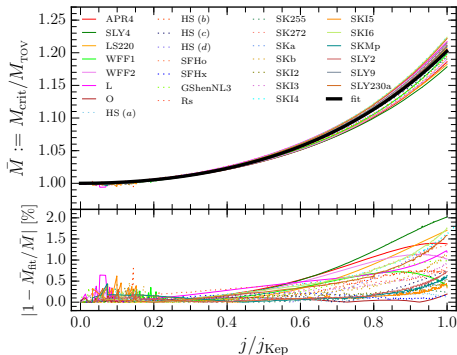
Critical Mass



- Lasota, Haensel, Abramowicz 1996: universal proportionality between masses and radii of static maximum-mass neutron stars and those of maximally rotating configurations

$$M_{\text{Kep}} \simeq b_1 M_{\text{TOV}},$$
$$R_{\text{Kep}} \simeq b_2 R_{\text{TOV}}$$

Critical Mass



- similarity in curves for critical mass as a function of j/j_{Kep}
- very small scatter if curves are properly normalised
- maximum error about two percent
- need j_{Kep}

Polynomial Fit:

$$M_{max} = 1 + a_2 \left(\frac{j}{j_{Kep}} \right)^2 + a_4 \left(\frac{j}{j_{Kep}} \right)^4$$

Critical Mass

- model with maximum mass close to models with maximum angular velocity Ω , angular momentum J and rest mass M_b (can be considered to coincide for practical purpose)

Mass-shedding velocity approximately expressed as (Haensel and Zdunik 1989):

$$\Omega_{Kep} \simeq \sqrt{M_{Kep}/R_{Kep}^3}$$

- Moment of inertia CMR^2 with C being a constant of order unity
- Express angular momentum as

$$J_{Kep} \simeq I_{Kep}\Omega_{Kep} \simeq CM_{Kep}R_{Kep}^2\Omega_{Kep} \simeq b_4\sqrt{M_{TOV}^3R_{TOV}}$$

- Dimensionless angular momentum:

$$j_{Kep} = J_{Kep}/M_{Kep}^2 \simeq b_4\sqrt{R_{TOV}/M_{TOV}} = b_4\sqrt{1/\mathcal{C}}$$

Fit only dependent on j and M_{TOV} :

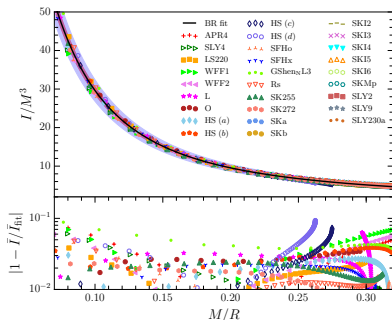
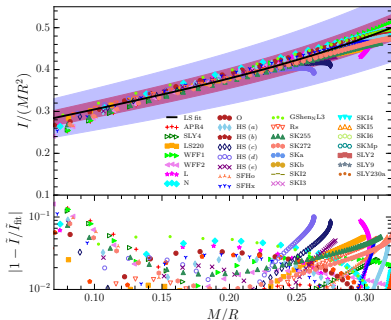
- $M_{crit} = (1 + c_2 \mathcal{C}_{TOV} j^2 + c_4 \mathcal{C}_{TOV}^2 j^4) M_{TOV}$
- now implicit function of angular momentum and properties of nonrotating configuration
- can be solved with simple root-finding algorithm

Maximum possible mass:

- $M_{max} := M_{crit}(j = j_{Kep}) = (1 + a_2 + a_4) M_{TOV}$
 $\simeq (1.203 \pm 0.022) M_{TOV}$

Universality in Moment of Inertia

I-ℓ-Relation

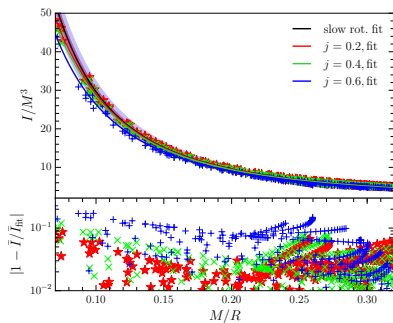
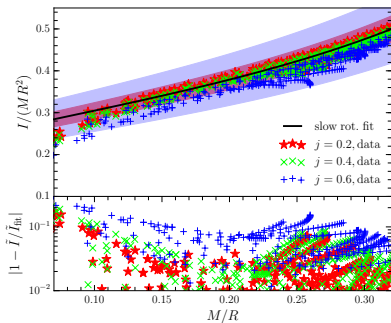


Lattimer and Schutz 2005

$$\frac{I}{MR^2} = \tilde{a}_0 + \tilde{a}_1 \ell + \tilde{a}_4 \ell^4$$

New Fit

$$\frac{I}{M^3} = a_1 \ell^{-1} + a_2 \ell^{-2} + a_3 \ell^{-3} + a_4 \ell^{-4}$$



- sequences with constant spin parameter j
- $I - \mathcal{C}$ relation depends on magnitude of rotation

Binding Energy

Difference between total rest mass and gravitational mass:

$$BE = M_b - M$$

Fit for dimensionless binding energy (Lattimer and Schutz 2005):

$$\frac{BE}{M} = \frac{c_1 \mathcal{C}}{1 - c_2 \mathcal{C}}$$

Polynomial fit:

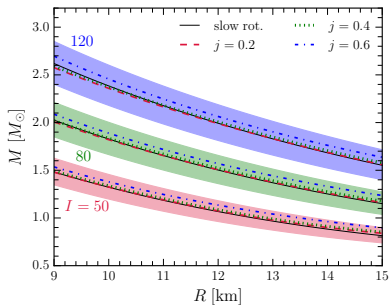
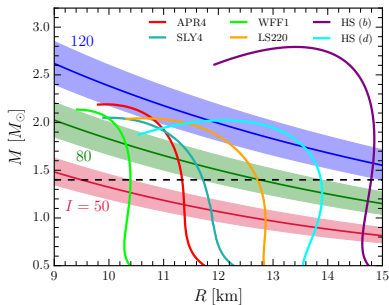
$$\frac{BE}{M} = d_1 \mathcal{C} + d_2 \mathcal{C}^2$$

Rotation: coefficients become j -dependent

$$\tilde{d}_1 = d_1(1 + \alpha j + \beta j^2)$$

$$\tilde{d}_2 = d_2(1 + \gamma j + \delta j^2)$$

Radius Measurement



- uncertainties in modeling of atmospheres/radiation processes
- goal: determination of I up to 10 % accuracy through periastron advance and geodetic precession
- simultaneous measurement of mass and moment of inertia of a radio binary pulsar would yield R
- estimate changes only slightly with rotation

Summary and Outlook

- universal relation also exhibited by unstable equilibrium solutions
- determination of critical mass for arbitrary angular momentum from properties of the nonrotating star
- $I - \mathcal{E}$ universality weaker for rapid rotation; but systematic behaviour along $j = \text{const.}$ sequences
- coefficients of fits for moment of inertia and binding energy dependent on rotation

Outlook:

- Differential rotation?
- Strange stars?
- Hybrid stars

→ Twin star sequences with more than one stable region?

Thank you for your attention!