Microscopic Vortex Velocity for Neutron Stars

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Outline

- ✓ Description of vortex velocity for neutron stars
- ✓ The Bernoulli force and microscopic vortex velocity in the inner crust and outer core
- ✓ Implications for pulsar glitches
- ✓ Implications for magnetic field evolution from neutron star core

Vortex Line Velocity for Neutron Stars

- For drag forces (per unit vortex length) linear in velocity difference, equation of motion of a vortex line

$$\rho_s\vec{\kappa}\times(\vec{v}_{\rm L}-\vec{v}_{\rm s})-\eta(\vec{v}_{\rm c}-\vec{v}_{\rm L})=0$$

Magnus Force — Drag Force

- Vortex velocity in terms of dissipation angle (Bildsten & Epstein 1989)

$$\vec{v_L}=\omega R\left(\frac{1}{2}\sin 2\theta_d\,\hat{r}+\cos^2\theta_d\,\hat{\phi}\right)\qquad \tan\theta_d\equiv\frac{\eta}{\rho_s\kappa}$$

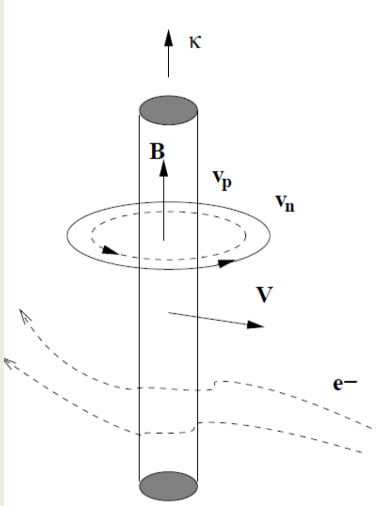
- $\Rightarrow \vec{v}_{\rm v}=\vec{v}_{\rm p}+(1-\beta')\,\vec{v}_{\rm np}+\beta\hat{\kappa}\times\vec{v}_{\rm np}$ $\quad \beta=\dfrac{\eta/\rho_s\kappa}{1+(\eta/\rho_s\kappa)^2},\ \beta'=\dfrac{(\eta/\rho_s\kappa)^2}{1+(\eta/\rho_s\kappa)^2}$

-Alpar, Langer & Sauls (1984)→Electron-magnetized vortex scattering (Andersson et al. 2006):

$$\beta\approx 4\times10^{-4}\left(\frac{\delta m_{\rm p}^*}{m_{\rm p}}\right)^2\left(\frac{m_{\rm p}}{m_{\rm p}^*}\right)^{1/2}\left(\frac{x_{\rm p}}{0.05}\right)^{7/6}\left(\frac{\rho}{10^{14}\,{\rm g\,cm^{-3}}}\right)$$

-Steady state superfluid spin-down (Link 2012):

$$\beta\cong\frac{1}{\omega_\infty t_{age}}\approx 1.6\times10^{-11}\left(\frac{\omega_\infty}{5\times10^{-2}\ \ {\rm rad\ s^{-1}}}\right)\left(\frac{t_{age}}{10^4{\rm yrs}}\right)$$



Vortex Velocity in Creep Model

- Vortex creep model (Alpar et al. 1984a, 1989) is a statistical model describing the motion of vortex lines in the inner crust superfluid through their interactions with the nuclei at lattice spacing $b << l_{\rm v} \approx 3 \times 10^{-3} \left(\frac{\Omega}{100 \text{ rad s}^{-1}} \right)^{-1/2} \text{cm}$

- Vortex velocity (creep rate) is a microscopic vortex velocity v_0 times the jump rate in radially outward diection as dictated by the spin-down torque:

$$v_{\rm r} = 2 v_0 e^{-E_{\rm p}/kT} \sinh \left(\frac{E_{\rm p}}{kT} \frac{\omega}{\omega_{\rm cr}} \right) \qquad v_{\rm r} = -\frac{r\dot{\Omega}}{2\Omega}$$

Microscopic Vortex Velocity

- Taking into account the superfluid density difference inside and outside of an inhomogeneity (lattice nuclei in the inner crust, flux tubes in the outer core) superfluid current continuity and vorticity equations for the superfluid velocity around a vortex become

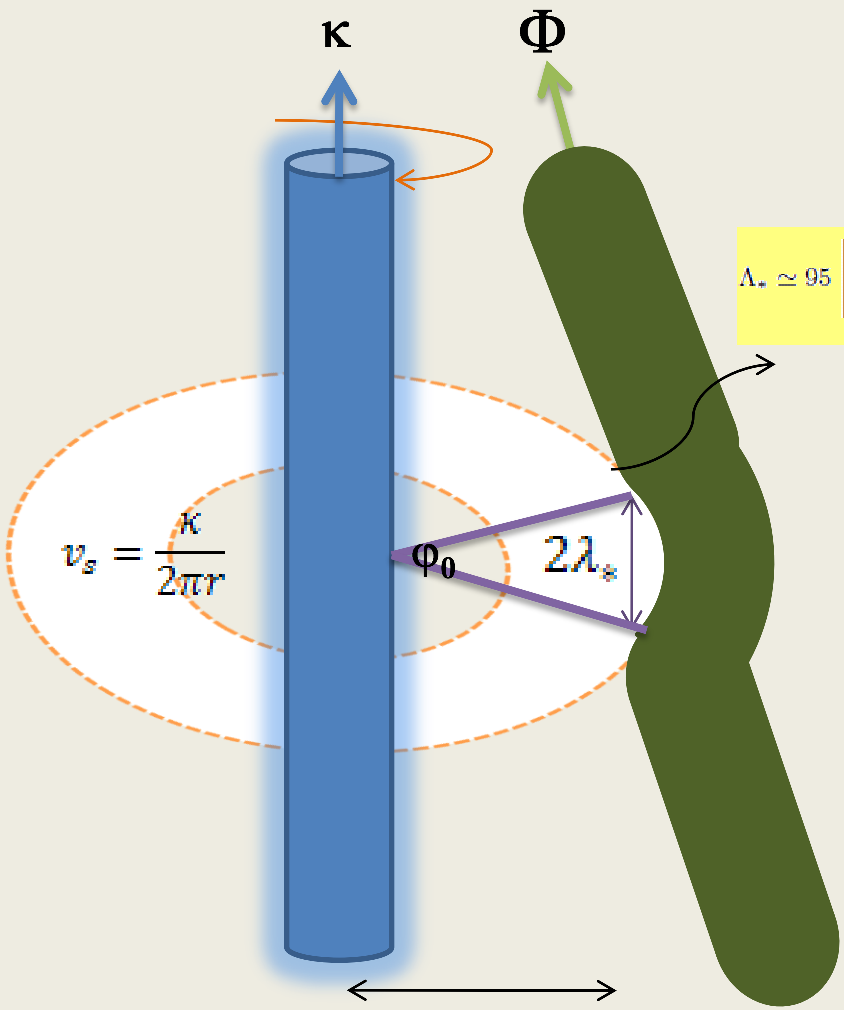
$$\rho_{\rm in} v_{\rm in}(r) = \rho_{\rm out} v_{\rm out}(r) \qquad r\left[\phi_0 v_{\rm in}(r) + (2\pi - \phi_0) v_{\rm out}(r)\right] = \kappa$$

- Vortex velocity field is different inside and outside of an inhomogeneity

$$v_{\rm in}(r) = \frac{\rho_{\rm out}}{\phi_0 \rho_{\rm out} + (2\pi - \phi_0)\rho_{\rm in}} \frac{\kappa}{r} \qquad v_{\rm out}(r) = \frac{\rho_{\rm in}}{\phi_0 \rho_{\rm out} + (2\pi - \phi_0)\rho_{\rm in}} \frac{\kappa}{r}$$

- Due to the presence of inhomogeneity there exsits an kinetic energy increment compared to the homogeneous superfluid:

$$\begin{aligned}
\Delta E &= 2R_{\rm L} \int_{R-R_{\rm L}}^{R+R_{\rm L}} r dr \left(\int_0^{\phi_0} \frac{1}{2}\rho_{\rm in} v_{\rm in}^2(r) d\phi + \int_{\phi_0}^{2\pi} \frac{1}{2}\rho_{\rm out} v_{\rm out}^2(r) d\phi - \int_0^{2\pi} \frac{1}{2}\rho_{\rm out} \left[\frac{\kappa}{2\pi r}\right]^2 d\phi \right) \\
&= R_{\rm L}\rho_{\rm out}\kappa^2 \left(\ln \frac{R+R_{\rm L}}{R-R_{\rm L}} \right) \left(\frac{\rho_{\rm in} R}{2R_{\rm L}(\rho_{\rm out} - \rho_{\rm in}) + 2\pi\rho_{\rm in} R} - \frac{1}{2\pi} \right).
\end{aligned}$$



$$\Lambda_* \simeq 95 \left[\left(\frac{m_p^*/m_p}{0.5} \right) \left(\frac{x_p}{0.05} \right)^{-1} \left(\frac{\rho}{10^{14}\ \mathrm{g\ cm^{-3}}} \right)^{-1} \right]^{1/2} \mathrm{fm}$$

$$l_\Phi = \left(\frac{B_\phi}{\Phi_C} \right)^{-1/2} \simeq 450 \left(\frac{B_\phi}{10^{14}\mathrm{G}} \right)^{-1/2} \mathrm{fm}$$

Microscopic Vortex Velocity Around Nuclear Cluster and Flux Tube Tangle

- Excess kinetic energy gradient gives rise to the Bernoulli force:

$$F_{\rm B} = -\frac{d\Delta E}{dr} = -R_{\rm L}\rho_{\rm out}\kappa^2\left[\left(\ln\frac{R+R_{\rm L}}{R-R_{\rm L}}\right)\left(\frac{2\rho_{\rm in}\Delta\rho R_{\rm L}}{(2R_{\rm L}\Delta\rho+2\pi\rho_{\rm in}R)^2}\right)-\frac{2R_{\rm L}}{R^2-R_{\rm L}^2}\left(\frac{\rho_{\rm in}R}{2R_{\rm L}\Delta\rho+2\pi\rho_{\rm in}R}-\frac{1}{2\pi}\right)\right] \quad \Delta\rho=\rho_{\rm out}-\rho_{\rm in}$$

- For the inner crust nuclear cluster $\rho_{in}>\rho_{out}$ as first shown by the seminal work Negele & Vautherin (1973).
- Inside a flux tube magnetic pressure < matter pressure $\Rightarrow$ buoyancy force ensues (Muslimov & Tsygan 1985):

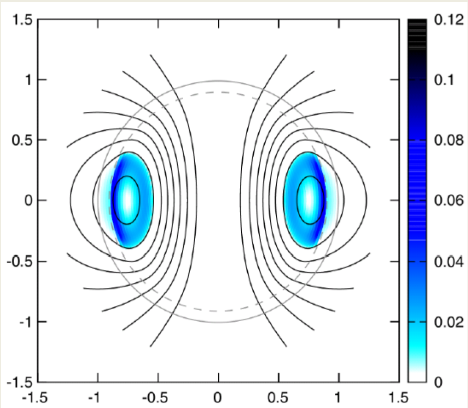
$$\Delta P(r) = \frac{H^2(r)}{8\pi}+\frac{1}{2}\rho_{\rm p}v_{\rm p}^2(r) \simeq \frac{1}{8\pi}\left[\frac{\Phi_0}{2\pi\Lambda_*^2}\ln\left(\frac{\Lambda_*}{r}\right)\right]^2+\frac{1}{2}\rho_{\rm p}v_{\rm p}^2(r) \quad \Delta\rho(r)\simeq\frac{d\rho}{dP}=\frac{\rho}{\Gamma P}\Delta P(r)$$

- Equating the Bernoulli force per unit length to the Magnus force yields the microscopic vortex velocity:

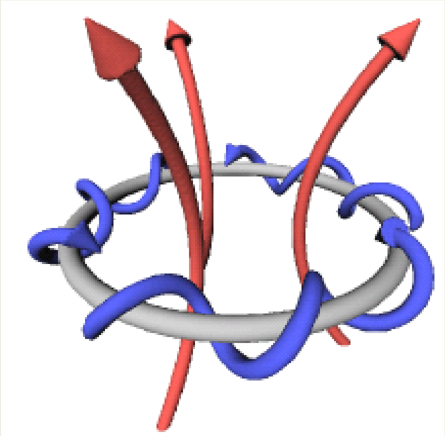
$$\frac{F_{\rm B}}{R}=\rho\kappa v_0 \longrightarrow v_0^{\rm crust}\sim 10^7\ {\rm cm\ s^{-1}} \gg v_0^{\rm core}\sim 1\ {\rm cm\ s^{-1}}$$

Properties of Magnetic Fields Inside Neutron Stars

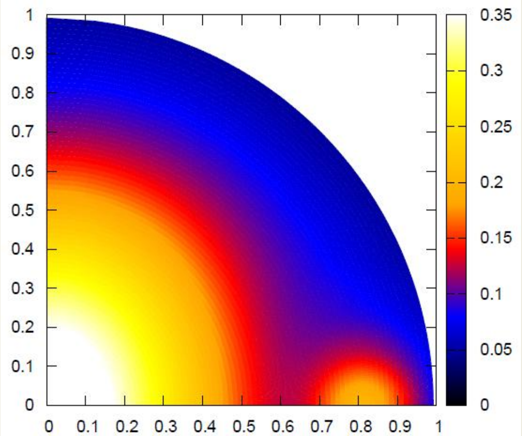
- Toroidal field component is confined within a closed field region where poloidal field vanishes.
- When toroidal field strength increases size of the closed field region decreases.
- When the toroidal field strength decreases closed field region shifts towards to the crust.
- In all cases E_{tor}/E_{mag} cannot be smaller than 10 percent.



Lander (2014)



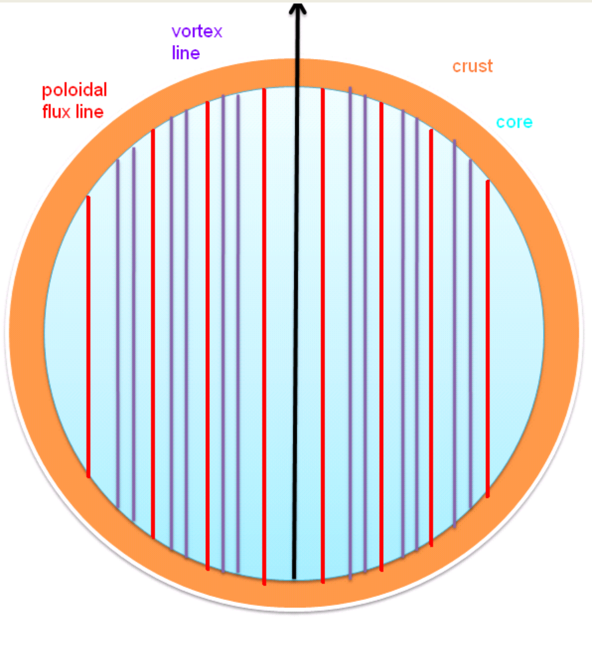
Braithwaite (2009)

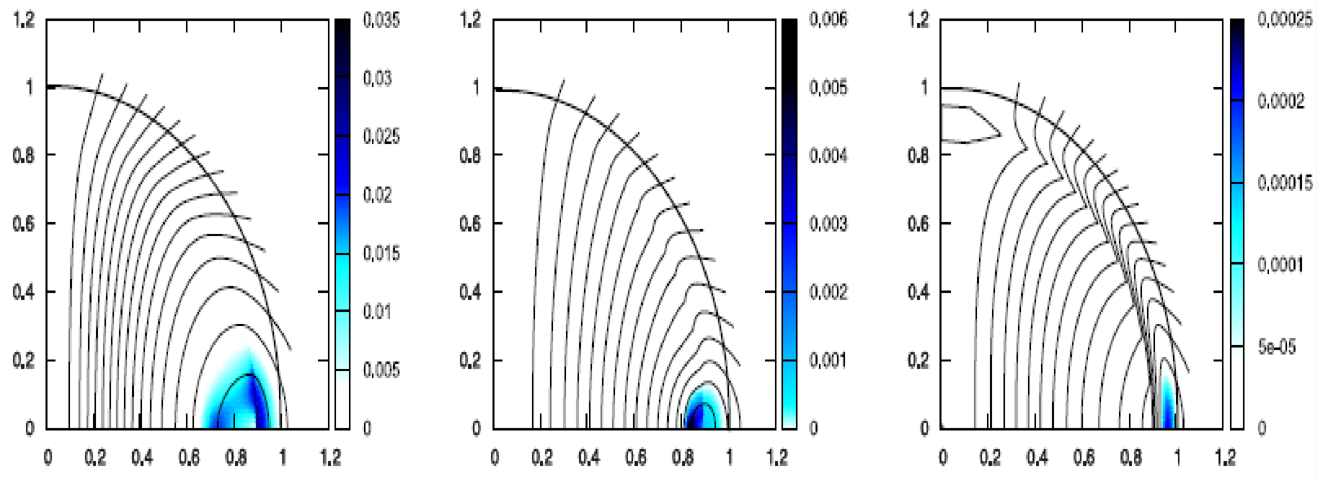


Lander et al. (2012)

Vortex Pinning and Creep Against Flux Tubes

- Flux tubes provide pinning/creep sites for vortex lines (Sidery & Alpar 2009, Gügercinoğlu & Alpar2014)
- If flux tubes have poloidal configuration, pinning and creep in the core will depend on the angle between the rotation and magnetic axes.
- By contrast, toroidal arrangement of flux tubes inevitably constrain the motion of the vortices.





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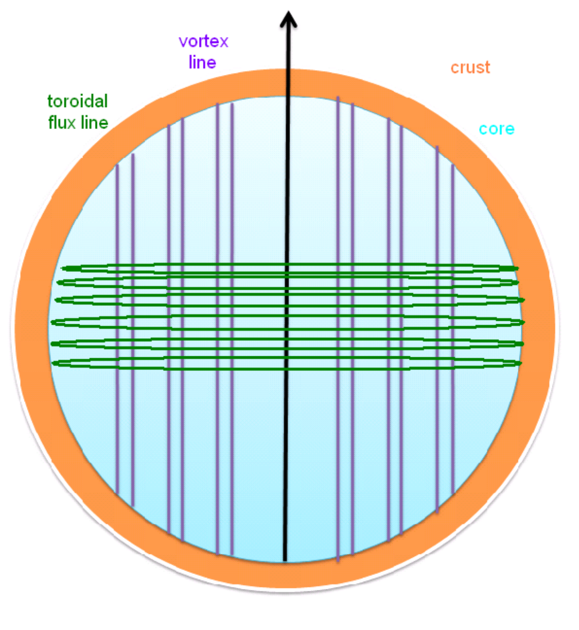


Vortex Line-Flux Tube Pinning

- In neutron star cores vortex line-flux tube pinning can occur via minimizing their condensation ve magnetic energies.

1) Proton density fluctuation inside flux tube center gives (Muslimov & Tsygan 1985, Sauls 1989):

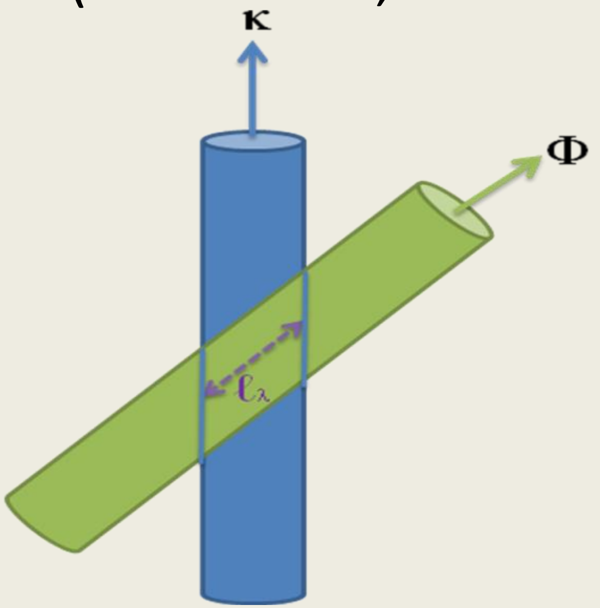
$$E_{\rm p} = \frac{3}{8} n_{\rm n} \frac{\Delta_{\rm p}^2}{E_{\rm F_p}^2} \frac{\Delta_{\rm n}^2}{E_{\rm F_n}} (\xi_{\rm n}^2 \xi_{\rm p}) = \frac{1}{\pi^5} \frac{\Delta_{\rm p}}{x_{\rm p}} \left(\frac{m_{\rm n}^*}{m_{\rm n}}\right)^{-2} \left(\frac{m_{\rm p}^*}{m_{\rm p}}\right)^{-1} \simeq 0.13{\rm MeV} \left(\frac{\Delta_{\rm p}}{1{\rm MeV}}\right) \left(\frac{x_{\rm p}}{0.05}\right)^{-1} \left(\frac{m_{\rm n}^*/m_{\rm n}}{1}\right)^{-2} \left(\frac{m_{\rm p}^*/m_{\rm p}}{0.5}\right)^{-1}$$

2) Vortex-flux tube magnetic interaction gives (Jones 1991, Chau et al. 1992)

$$E_{\rm p} = \frac{2\vec{B}_{\rm V} \cdot \vec{B}_{\Phi}}{8\pi} (\pi \Lambda_*^2 \ell_\Lambda) = \frac{\Phi_0^2}{16\pi^2} \frac{\ell_\Lambda}{\Lambda_*^2} \frac{\delta m_{\rm p}^*}{m_{\rm p}^*} \ln\left(\frac{\Lambda_*}{\xi_{\rm p}}\right) \cos\theta$$

- But both the vortex line and flux tube assumed straight during the interaction.

$$E_{\rm p} \approx 40{\rm MeV} \left(\frac{\delta m_{\rm p}^*/m_{\rm p}}{0.5}\right) \left(\frac{m_{\rm p}^*/m_{\rm p}}{0.5}\right)^{-1/2} \left(\frac{x_{\rm p}}{0.05}\right)^{1/2} \left(\frac{\rho_{\rm s}}{2\times 10^{14}\ {\rm g\ cm^{-3}}}\right)^{1/2}$$



Pinning Energy Calculation in the Core

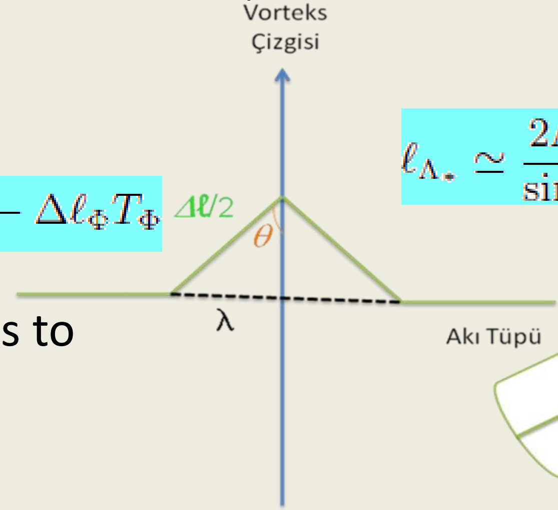
- However, neither the flux tube nor the vortex line have infinite rigidity.

$$T_{\rm v} \simeq 10^9 \left(\frac{\rho_{\rm s}}{2 \times 10^{14}\ {\rm g\ cm^{-3}}} \right)\ {\rm erg\ cm^{-1}}$$

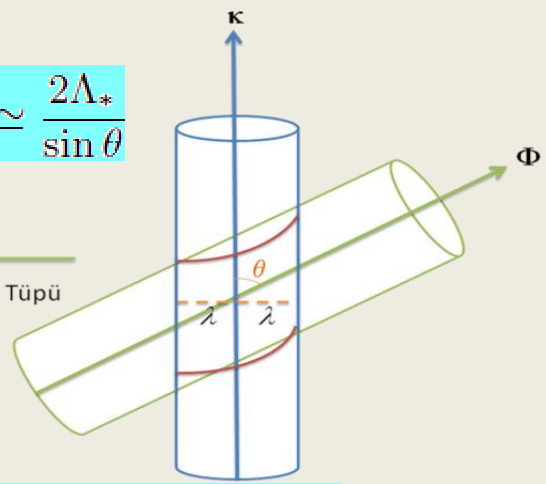
$$T_\Phi = \left(\frac{\Phi_0}{4\pi\Lambda_*} \right)^2 \ln\left(\frac{\Lambda_*}{\xi_{\rm p}} \right) \sim 10^7 \left(\frac{m_{\rm p}^*/m_{\rm p}}{0.5} \right)^{-1} \left(\frac{x_{\rm p}}{0.05} \right) \left(\frac{\rho_{\rm s}}{2 \times 10^{14}\ {\rm g\ cm^{-3}}} \right)\ {\rm erg\ cm^{-1}}$$

- When a flux tube comes closer to a vortex line, it bends and undergoes lengthening:

$$\ell_\Phi \approx 2\Lambda_* \left(\frac{1}{\sin\theta} - 1 \right)$$



$$\ell_{\Lambda_*} \simeq \frac{2\Lambda_*}{\sin\theta}$$



- The Pinning Energy Gain: $E_+ = E_{\rm p} - \Delta\ell_\Phi T_\Phi$

- Maximum energy gain corresponds to

$$\theta = \arccos\left(\frac{1}{2} \frac{\delta m_{\rm p}^*}{m_{\rm p}} \right)$$

- $\Rightarrow$

$$E_+ = \frac{\Phi_0^2}{8\pi^2\Lambda_*} \ln\left(\frac{\Lambda_*}{\xi_{\rm p}} \right) \left[\left(\frac{\delta m_{\rm p}^*}{m_{\rm p}} \right) \cot\left(\arccos\left(\frac{1}{2} \frac{\delta m_{\rm p}^*}{m_{\rm p}} \right) \right) - \frac{2}{\sin\left(\arccos\left(\frac{1}{2} \frac{\delta m_{\rm p}^*}{m_{\rm p}} \right) \right)} + 2 \right] \approx 4.8 {\rm MeV} \left(\frac{m_{\rm p}^*/m_{\rm p}}{0.5} \right)^{-1/2} \left(\frac{x_{\rm p}}{0.05} \right)^{1/2}$$
$$\times \left(\frac{\rho_{\rm s}}{2\times 10^{14}\ {\rm g\ cm^{-3}}} \right)^{1/2} \left[\left(\frac{\delta m_{\rm p}^*/m_{\rm p}}{0.5} \right) \cot\left(\arccos\left(\frac{1}{2} \frac{\delta m_{\rm p}^*/m_{\rm p}}{0.5} \right) \right) - \frac{2}{\sin\left(\arccos\left(\frac{1}{2} \frac{\delta m_{\rm p}^*/m_{\rm p}}{0.5} \right) \right)} + 2 \right].$$

- Thus, the pinning energy in the core reduces by a factor ~ 8 when the flux tube lengthening is taken into account.

Application to the Vortex Creep Model

- Vortex line-flux tube pinning maintains a maximum angular velocity lag found from equation→ $E_{+}/(\Lambda_{*} l_{\Phi}) = \rho_{s} \kappa R \omega_{cr}$

$$\omega_{cr} \cong 6.3 \times 10^{-2} \text{ rad s}^{-1} \left(\frac{\delta m_{p}^{*}/m_{p}}{0.5}\right)\left(\frac{m_{p}^{*}/m_{p}}{0.5}\right)^{-1}\left(\frac{x_{p}}{0.05}\right)\left(\frac{B_{\phi}}{10^{14}\text{G}}\right)^{1/2}$$

- Steady state angular velocity lag for vortex creep across toroidal arrangement of flux tubes in non-linear creep regime:

$$\omega_{\infty} = \omega_{cr}\left[1 - \frac{kT}{E_{+}} \ln\left(\frac{2\Omega_{s} v_{0}}{|\dot{\Omega}| R}\right)\right]$$

- $\Rightarrow E_{+} \approx 0.5$ MeV, $\omega_{\infty} \cong 3.3 \times 10^{-3}$ rads/s, $\omega_{cr} \cong 4.48 \times 10^{-3}$ rad/s
- Linear to non-linear creep regime transition occurs at $\left(\frac{E_{p}}{kT}\right)_{tr} = \ln\left(\frac{4\Omega v_{0}}{|\dot{\Omega}| r}\right)$
- Since $E_{+} \geq E_{p}|_{tr} \cong 0.12$ MeV→ vortex creep against toroidal flux tubes is always in non-linear regime as predicted by Gügercinoğlu & Alpar (2014).
- Other predictions of Gügercinoğlu & Alpar (2014) for pulsar glitches:

❖ e-folding timescale for post-glitch relxation ranges from a few days to months,

❖ Glitches in older pulsars end up with low recoveries.

Forces on the Flux Tubes

- Vortex acting force (Ding et al. 1993):

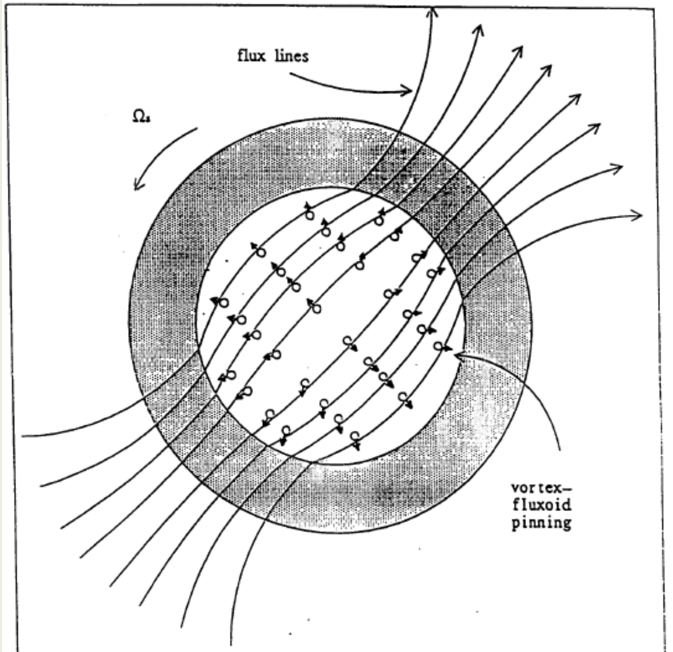
$$\vec{F}_{\rm n} = \frac{n_{\rm v}}{n_\Phi}\vec{F}_{\rm M} = \frac{2\Phi_0\rho R\Omega_{\rm s}(t)\omega(t)}{B_{\rm core}(t)}\hat{e}_{\rm r}$$

- Pinning force (Jahan-Miri 2000):

$$\vec{F}_{\rm p} = -\frac{\Lambda_*}{l_\Phi}\vec{F}_{\rm M}$$

- Buoyancy force (Muslimov & Tsygan 1985):

$$\vec{F}_{\rm b} = \left(\frac{\Phi_0}{4\pi\Lambda_*}\right)^2\frac{1}{R}\ln\left(\frac{\Lambda_*}{\xi_{\rm p}}\right)\hat{e}_{\rm r}$$



Ding et al. (1993)

- Viscous drag force (Harvey et al. 1986): Tension (Harvey et al. 1986):

$$\vec{F}_{\rm v} = -\frac{3\pi}{64}\frac{n_{\rm e}e^2\Phi_0^2}{E_{\rm F_e}}\frac{\vec{v}_{\rm p}}{c}$$

$$\vec{F}_{\rm t} = -\frac{R}{s_{\rm c}}\vec{F}_{\rm b}$$

Summary of Flux Expulsion from Neutron Star Core

- Electron orbital radius $r_L = E_{F_e}/eB \approx 3.3 \times 10^{-7} B_{12}^{-1} \mathrm{cm} >> l_\Phi \approx 4.55 \times 10^{-10} B_{12}^{-1/2} \mathrm{cm}$
- Then, electron scattering from individual flux tubes is irrelevant for neutron stars (Jones 1991, Harrison 1991).
- Chemical potential inhomogeneity arising from a change in electron distribution function is compansated by the Lorentz force.
- Buoyancy is the main driving force for flux expulsion out of the core (Jahan-Miri 2000).
- Flux tube velocity is very large $v_{\mathrm{p}} \approx 4 \times 10^{-7}$ cm/s (Jones 2006).

  $\tau_{\mathrm{decay}} \approx R / v_{\mathrm{p}} \sim 10^5$ yrs
- Vortex acting force and the pinning force affect the field decay in different ways for forward creep, comoving and reverse creep phases.

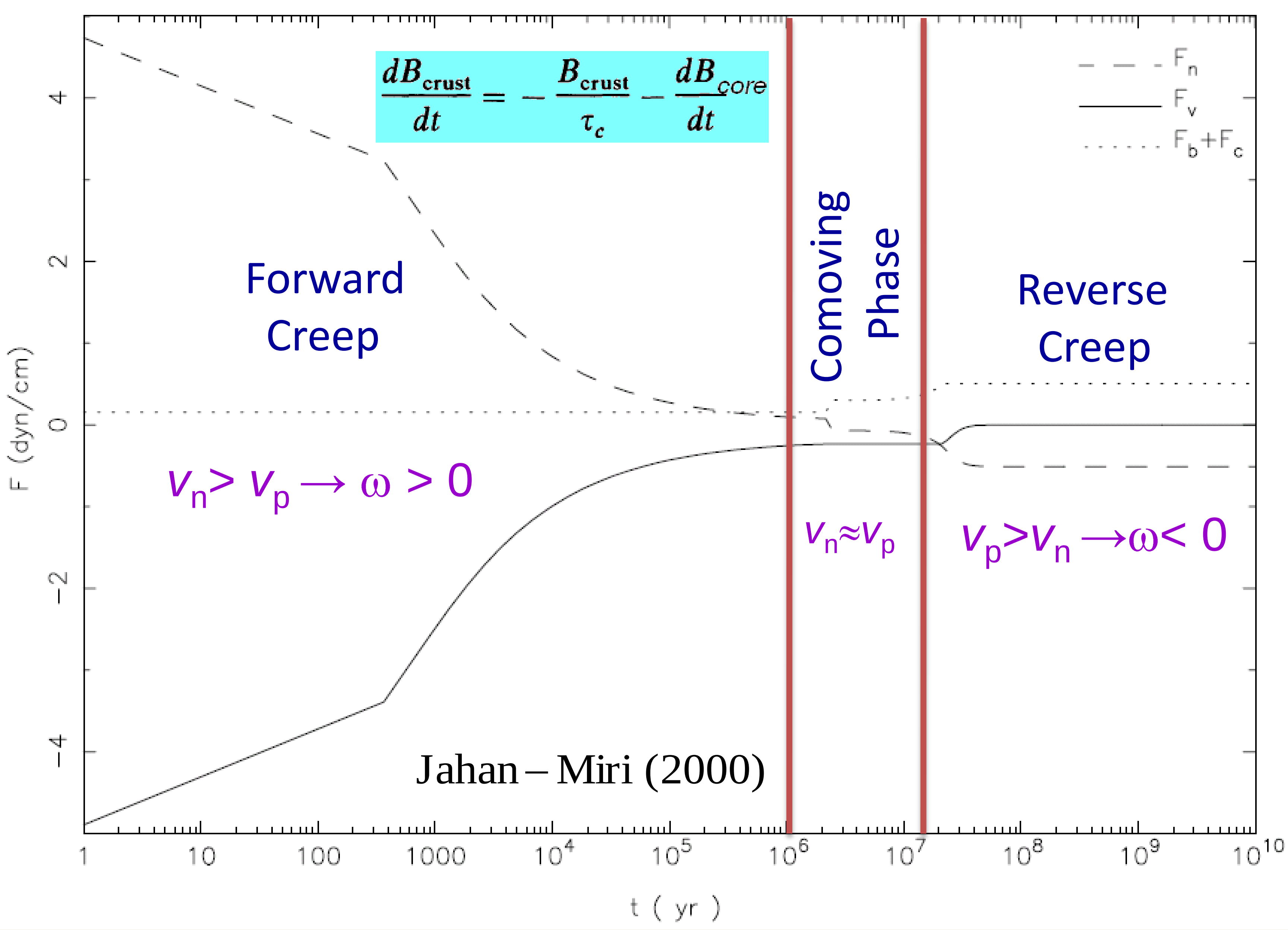
Srinivasan et al. (1990) Flux Tube Expulsion Model

Model Assumptions and Predictions

- Absolute pinning of vortex-flux line.
- Power law field decay:
  $$\mu(t) = \mu_0 \frac{\Omega(t)}{\Omega_0}$$
- No substantial decay is expected for isolated pulsars.
- In binary systems with wind braking significant field decay.
- Braking index→5

Caveats

- Very low crustal conductivity is assumed.
- Does perfect pinning realize in neutron stars?
- Other forces on flux tubes are ignored.
- High magnetic field pulsars in binary systems.



$$\frac{dB_{crust}}{dt} = -\frac{B_{crust}}{\tau_c} - \frac{dB_{core}}{dt}$$
Forward Creep
$v_n > v_p \rightarrow \omega > 0$
Comoving Phase
$v_n \approx v_p$
Reverse Creep
$v_p > v_n \rightarrow \omega < 0$
Jahan – Miri (2000)
F_n
F_v
$F_b + F_c$
F (dyn/cm)
t (yr)

Implications for Magnetic Field Evolution

- Flux tubes will be pushed by vortex through a force in terms of a microscopic velocity:

$$F_{\mathrm{v}-\Phi} \approx \frac{2\Phi_0 \rho R \Omega_{\mathrm{s}}(t) v_0}{B\mathrm{core}(t)} \sim 6 \times 10^{-2} \left(\frac{\Omega_{\mathrm{s}}}{100 \text{ rad s}^{-1}}\right) \left(\frac{\rho_{\mathrm{s}}}{2 \times 10^{14} \text{ g cm}^{-3}}\right) \left(\frac{B}{10^{12}\mathrm{G}}\right)^{-1} \left(\frac{v_0}{1 \text{ cm s}^{-1}}\right) \text{ dyne cm}^{-1}$$

- Vortex acting force is reduced by $\frac{v_0}{R\omega_\infty} \sim 2 \times 10^{-4}$
- Vortex line-flux tube pinning force $F_{\mathrm{p}} = \frac{E_+}{l_\Phi^2} \sim 4 \times 10^{14} \text{ dyne cm}^{-1}$
- But the timescale on which vortex line-flux tube remained pinned is $2\Lambda_*/v_0 \sim 10^{-11}$ s.
- The presence of vortex lines resists the buoyancy force on flux tubes and thus prevents further field decay.

Conclusions

- Vortex velocity around the nuclei and the flux tubes evaluated by incorporating the Bernoulli and the Magnus forces.
- $v_0^{\rm crust} \sim 10^7 \ {\rm cm\ s^{-1}} \gg v_0^{\rm core} \sim 1 \ {\rm cm\ s^{-1}}$
- In the inner crust both linear and non-linear creep regimes exist while in the outer core non-linear creep prevails.
- When the flux tube bending is taken into account $E_{\rm p}$ reduces by ~8.
- Due to the very low velocity vortex lines cannot push flux tubes tangle enough but resist field decay.
- There is no perfect pinning inside neutron stars.
- The details can be found in Gügercinoğlu & Alpar 2016, submitted to MNRAS.

Thank You for Attention…