A SIMPLE MODEL FOR

PULSAR GLITCHES

ArXiv:

M. Antonelli, P. Pizzochero
"Axially symmetric equations for differential pulsar rotation with superfluid entrainment"

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Outline:

- How to construt a model for pulsar glitches that:

- Framework: two-components model
- Embeds the most important physical ingredients in a simple (but consistent) way
- Non uniform star, realistic EOS & layered structure: “micro(r)” → “hydro(x)”

- MAIN WORKING ASSUMPTION:

- Axial symmetry → at the end the dynamical equtions will depend only on “x”
- The charged component is a rigid body

- Results:

- General set of equations valid for any “cylindrical model”
- Dynamical generalization of the “snowplow model” with entrainment
- Estimate of superfluid angular mometum reservoir → upper limits on NS masses

Details of the cylindrical model are on ArXiv (submitted to MNRAS):

M. Antonelli, P. Pizzochero
“Axially symmetric equations for differential pulsar rotation with superfluid entrainment”

Mass estimates: in preparation!

M. Antonelli, B. Haskell, P. Pizzochero, S. Seveso

PULSAR GLITCHES (basic facts)

- **Lack of radiative/pulse profile changes**

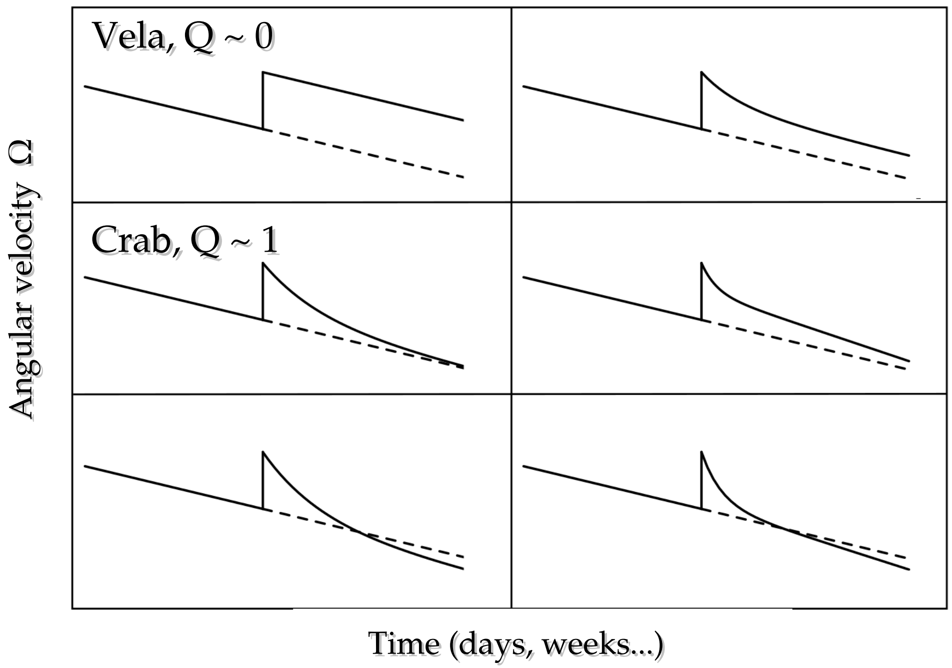
  → Evidence for internal origin

- **Long recoveries**

  → Thought to be due to superfluid component in the star

- **Diverse phenomenology**

  → different ages/mass/rotational parameters..?



Key point: to describe glitches we need that a NS is comprised of (at least) two components that exchange angular momentum.

Can we identify the (two?) components ?

Which part of the NS provides the angular momentum to spin-up the "observable component" ?

The **“minimal”** model

The inner crust & core contain a neutron superfluid (superfluid n-component).

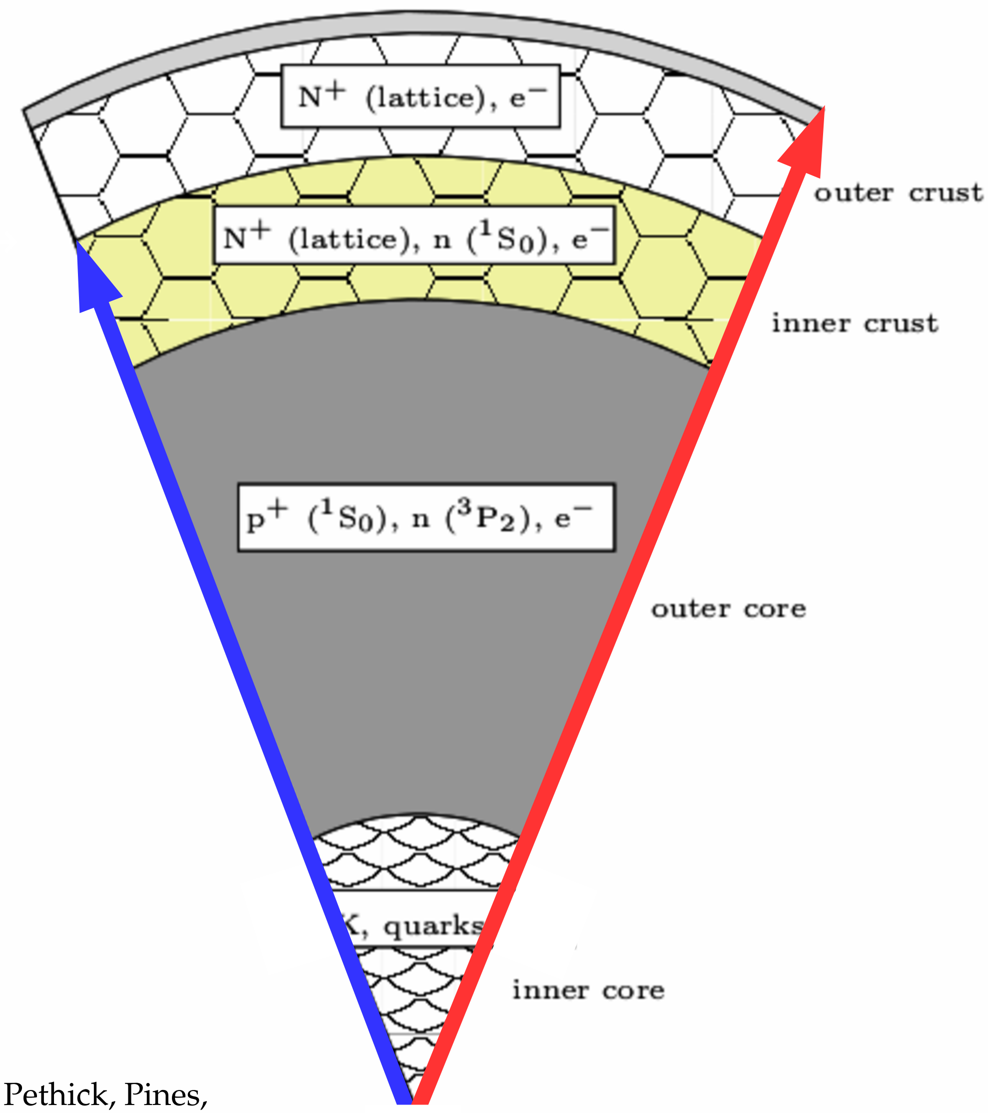
Everything else (proton superconductor, electron gas) is locked with the solid crust into the magnetic field (rigid p-component).

braking torque

“crust” $$I_c\dot{\Omega}_c = -\alpha - \frac{I_c}{\tau_c}(\Omega_c - \Omega_n)$$

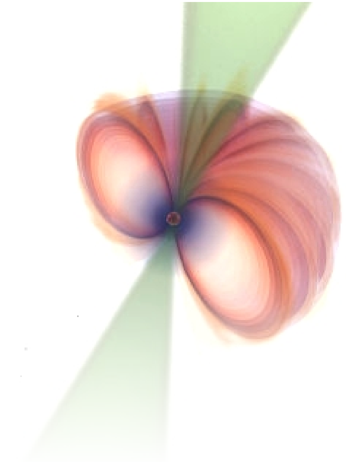
“neutrons” $$I_n\dot{\Omega}_n = \underbrace{\frac{I_c}{\tau_c}(\Omega_c - \Omega_n)}_{\text{Mutual friction}}$$

Mutual friction



Baym, Pethick, Pines, Ruderman (1969)

Glitch mechanism



- The p-component follows the observed spin down of the pulsar

- Where vortex lines are pinned, the n-component cannot follow p...

 ...a velocity lag builds up between n and p

 ...motion of a neutron vortex is affected by fluid flow past it

- Hydrodynamical effect: when the Magnus force $\simeq$ pinning force the vortex line unpins and is expelled $\rightarrow$ n looses angular momentum, p gains the same amount

$$\boldsymbol{f}_D = -\eta(\boldsymbol{x})(\boldsymbol{v}_L(\boldsymbol{x}) - \boldsymbol{v}_p(\boldsymbol{x}))$$ Drag force

$$\boldsymbol{f}_M = \rho_n(\boldsymbol{x})\boldsymbol{\kappa} \times (\boldsymbol{v}_L(\boldsymbol{x}) - \boldsymbol{v}_n(\boldsymbol{x}))$$ Magnus force

} Expulsion of vortex lines from bulk superfluid } Vortex mediated mutual friction between n and p

UNPINNING $\rightarrow$ (thermally activated) (soc, "external" trigger?)

Local: vortex creep **Global: avalanche**

Models of pulsar glitches

Nice review: Haskell & Melatos (2015)

- Exchange of angular momentum → 2 components
- Long timescales → one component is superfluid

$$\frac{p_i^{\rm n}}{m_{\rm n}} = v_i^{\rm n} + \varepsilon_{\rm n}(v_i^{\rm p} - v_i^{\rm n})$$
$$\frac{p_i^{\rm p}}{m_{\rm p}} = v_i^{\rm p} + \varepsilon_{\rm p}(v_i^{\rm n} - v_i^{\rm p})$$

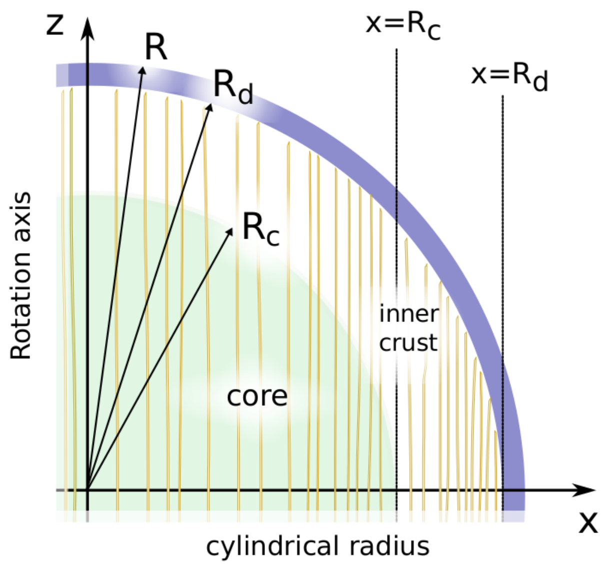
Multifluid hydro (with entrainment)

$$\partial_t \rho_{\rm x} + \nabla_i(\rho_{\rm x} v_{\rm x}^i) = 0$$

$$(\partial_t + v_{\rm x}^j \nabla_j)(v_i^{\rm x} + \varepsilon_{\rm x} w_i^{\rm yx}) + \nabla_i(\tilde{\mu}_{\rm x} + \Phi) + \varepsilon_{\rm x} w_{\rm yx}^j \nabla_i v_j^{\rm x} = f_i^{\rm x}/\rho_{\rm x} + \nabla_j D_i^j$$

$$f_i^{\rm x} = 2\rho_{\rm n}\mathcal{B}'\epsilon_{ijk}\Omega^j w_{\rm xy}^k + 2\rho_{\rm n}\mathcal{B}\epsilon_{ijk}\hat{\Omega}^j\epsilon^{klm}\Omega_l w_m^{\rm xy}$$

Mutual friction



"Cylindrical reduction"

By assuming columnar flow we can project the complicated 3D problem into a simpler one (1D radial) → this means that inside the star the vorticity of the superfluid is assumed to be parallel to the rotation axis of the star

1D **cylindrical** model:

(x, θ, z) usual cyl. coord.

Assumptions: $\boldsymbol{p}_n(\boldsymbol{x}) \cdot \hat{\boldsymbol{e}}_\theta = p_n(x)$ Azimuthal component of the superfluid momentum function of "x" only

$\boldsymbol{v}_p = x \Omega_p \hat{\boldsymbol{e}}_\theta$ Charged component rotates as a rigid body (due to short Alf. timescale & electron screening length)

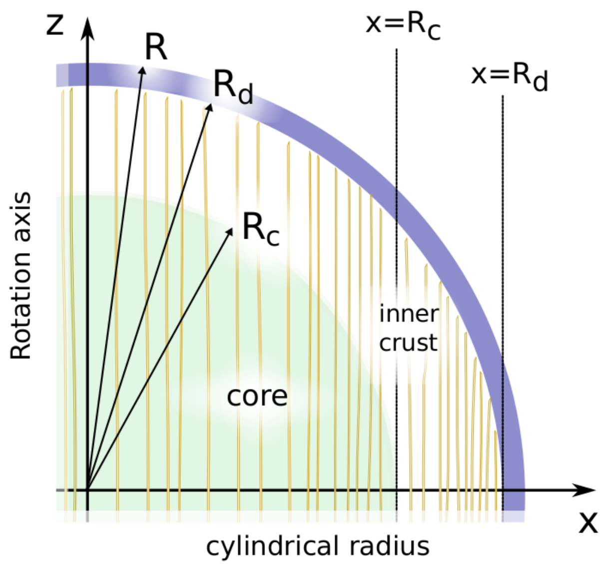
$$\Omega_v(x) = \frac{p_n(x)}{m_n \, x}$$

Is the quantity that obeys Fey. relation

$$\Omega_v(x) = \frac{\kappa \, N(x)}{2\pi \, x^2}$$

$$\boldsymbol{v}_n(x,z) = x \frac{\Omega_v(x) - \epsilon_n(r)\Omega_p}{1 - \epsilon_n(r)} \, \hat{\boldsymbol{e}}_\theta$$

Even in the simplest situation of straight vortices is non-columnar



To encode entrainment consistently into a cylindrical model we have to consider a "v" (fictitious) component and the usual (rigid) "p" component

$$I_v = \frac{8\pi}{3} \int_0^R dr \, r^4 \, \frac{\rho_n(r)}{1 - \epsilon_n(r)}$$

$$I_p = \frac{8\pi}{3} \int_0^R dr \, r^4 \, \rho_p(r) \frac{1 - \epsilon_n(r) - \epsilon_p(r)}{1 - \epsilon_n(r)}$$

$$I_p + I_v = I$$

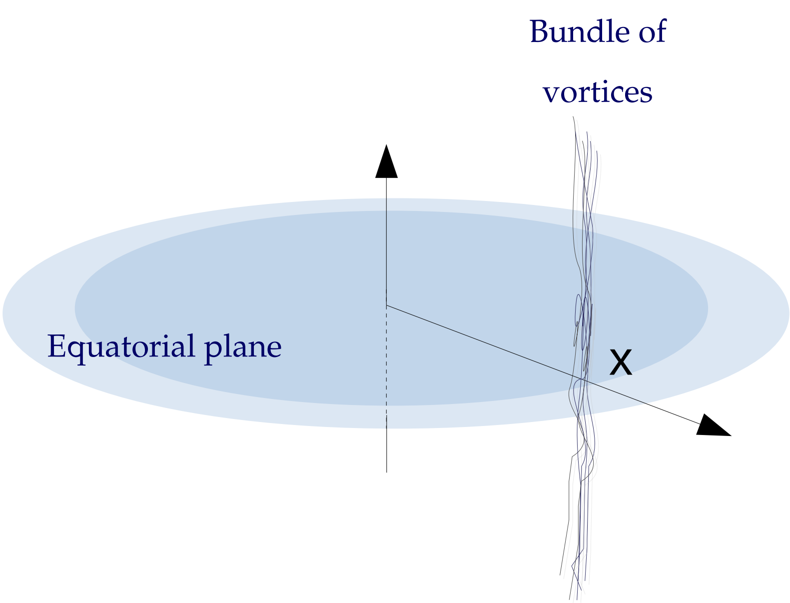
Macroscopic equations (similar eqs without entrainment → Alpar et al. 1984)

- The star spins-down under a constant external torque:

$$\int d^3 x \rho_p(\boldsymbol{r}) \left(\boldsymbol{x} \times \dot{\boldsymbol{v}}_p(x)\right) = -\boldsymbol{T}_{ext} - \int d^3 x \rho_n(\boldsymbol{r}) \left(\boldsymbol{x} \times \dot{\boldsymbol{v}}_n(\boldsymbol{x})\right)$$

$$\boldsymbol{p}_n(\boldsymbol{x}) = m_n \left[\left(1 - \epsilon_n(\boldsymbol{x})\right) \boldsymbol{v}_n(\boldsymbol{x}) + \epsilon_n(\boldsymbol{x}) \boldsymbol{v}_p(\boldsymbol{x}) \right]$$

- v-component: $\Omega_v(x) = \dfrac{p_n(x)}{m_n\, x}$ → Feynman relation + continuity



$$\partial_t n_v + \frac{1}{x} \partial_x \left(x\, n_v\, v_L^x\right) = 0$$

2D density of vortex lines

Radial vel. of vortices

$$n_v(x) = \frac{(\nabla \times \boldsymbol{p}_n) \cdot \hat{\boldsymbol{e}}_z}{\kappa\, m_n} = \frac{1}{\kappa}\left(2\Omega_v(x) + x \partial_x \Omega_v(x)\right)$$

$$v_L^x = x\, \mathcal{B}[\Omega_v, \Omega_p, x] \left(\Omega_v(x) - \Omega_p\right)$$

Dimensionless drag functional

Angular velocity lag v-p

Radial (avereage) velocity of vortices

Macroscopic equations

- Just carry out the integrations on the z axis, from 0 to: $z(x) = \sqrt{R^2 - x^2}$

The general form of the equations → formally the same for the case without entrainment

$$\partial_t \Omega_v(x,t) = -\mathcal{B}[\Omega_v, \Omega_p, x]\,(2\Omega_v + x\partial_x \Omega_v)(\Omega_v - \Omega_p)$$

$$\partial_t \Omega_p(t) = -(1+q)\dot{\Omega}_\infty - q\langle\, \partial_t \Omega_v(t)\,\rangle$$

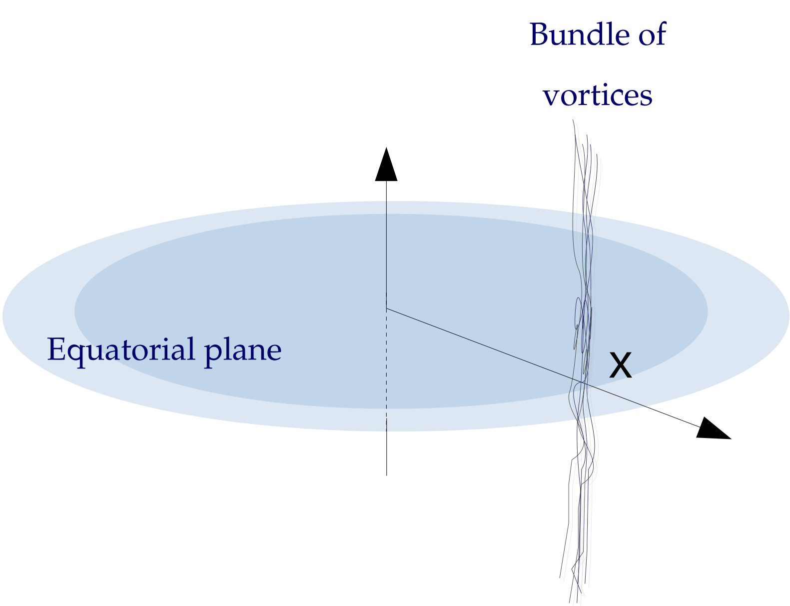
$$q = I_v / I_p$$

$$\dot{\Omega}_\infty = T_{ext}/I$$

$$dI_v(x)/I_v = g(x)dx$$

Equation of motion for (rigid) vortex lines:

$$\boldsymbol{F}_{tot} = \int_L dl(\boldsymbol{x})\,[\boldsymbol{f}_M(\boldsymbol{x}) + \boldsymbol{f}_D(\boldsymbol{x}) + \boldsymbol{f}_P(\boldsymbol{x})] = 0$$



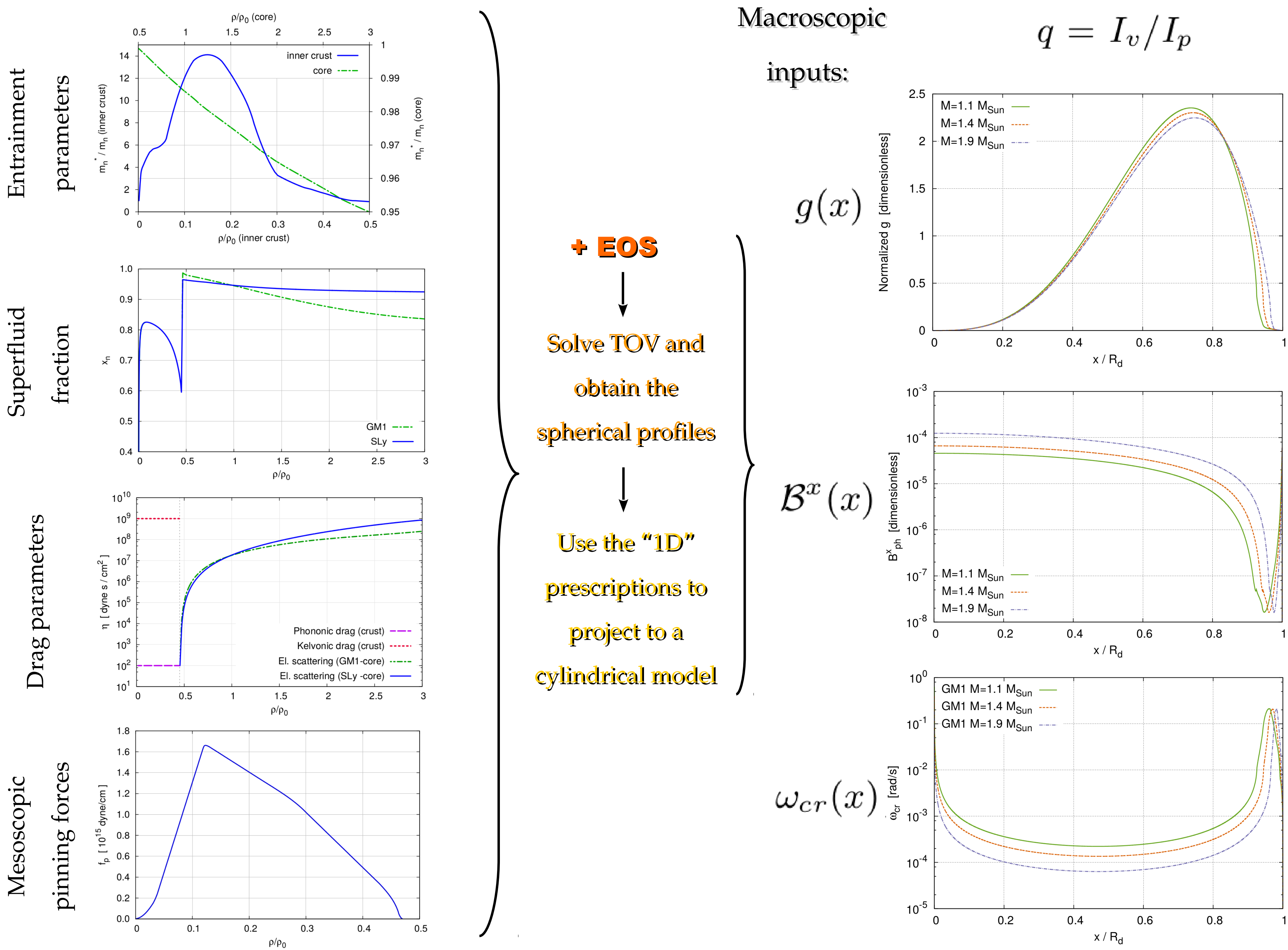
$$v_L^x = x\,\mathcal{B}^x(x)\,\omega(x,t)$$

Here: rigid vortices approximation → Snowplow like drag functional

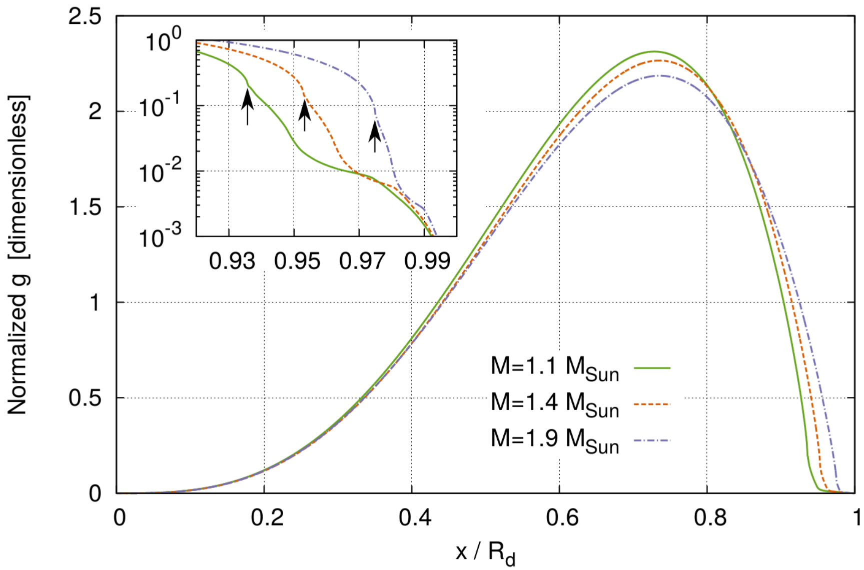
$$F_P(x) = |\boldsymbol{F}_M(x)|_{\boldsymbol{v}_L = \boldsymbol{v}_p}$$

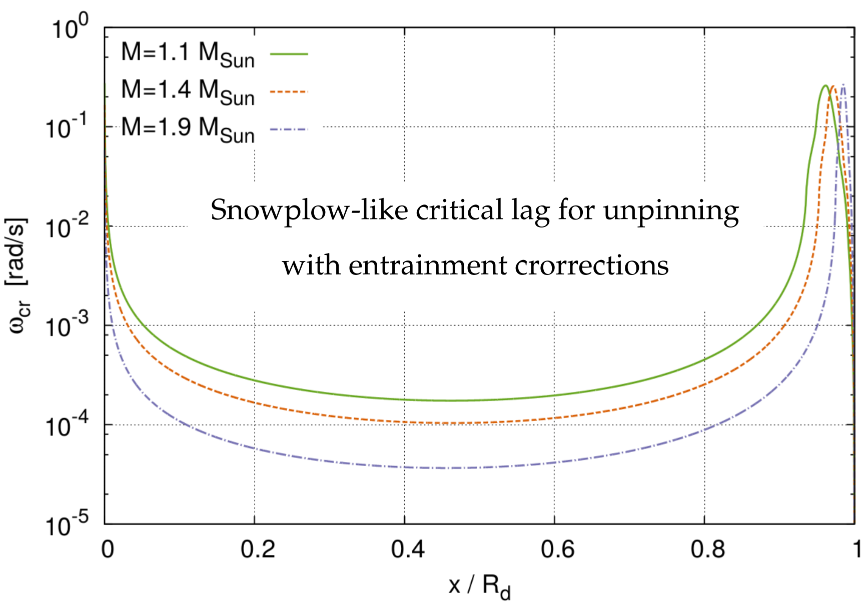
$$Y[\omega, x] = \theta(|\omega(x)| - \omega_{cr}(x))$$

$$\mathcal{B}[\Omega_v, \Omega_p, x] \approx Y[\omega, x]\,\mathcal{B}^x(x)$$

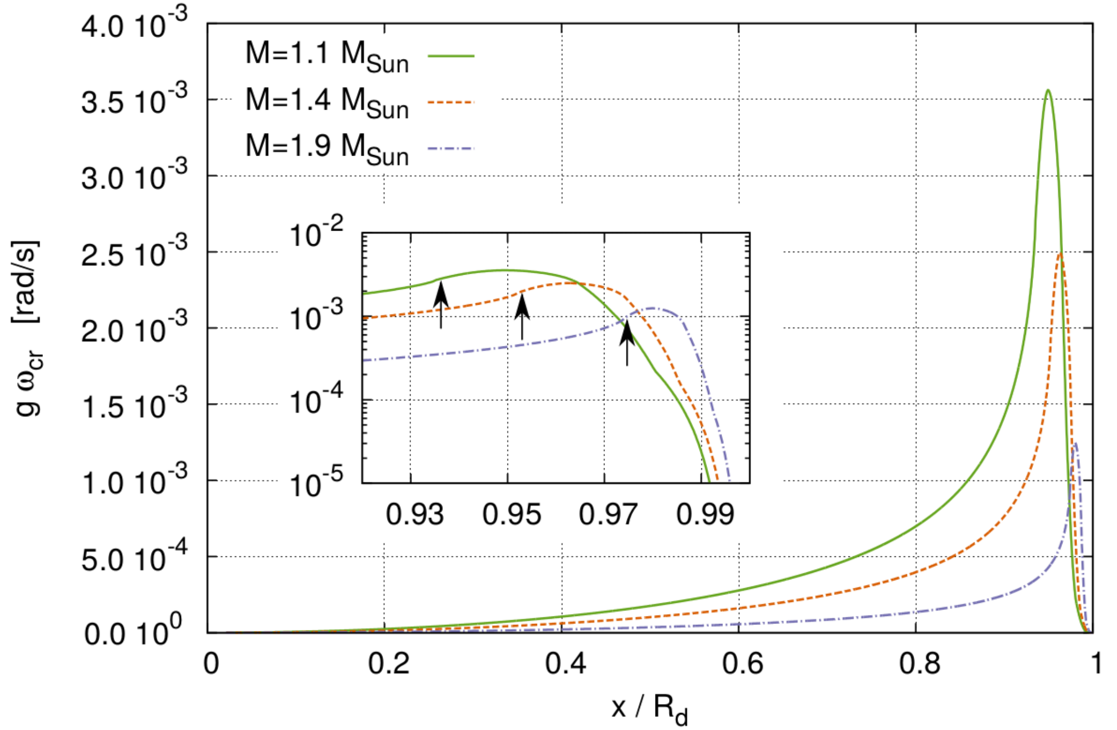

Entrainment parameters
Superfluid fraction
Drag parameters
Mesoscopic pinning forces
ρ/ρ0 (core)
inner crust
core
m*n / mn (inner crust)
m*n / mn (core)
ρ/ρ0 (inner crust)
GM1
SLy
xn
ρ/ρ0
Phononic drag (crust)
Kelvonic drag (crust)
El. scattering (GM1-core)
El. scattering (SLy -core)
η [dyne s / cm²]
fp [10¹⁵ dyne/cm]
+ EOS
Solve TOV and obtain the spherical profiles
Use the "1D" prescriptions to project to a cylindrical model
Macroscopic inputs:
$q = I_v / I_p$
$g(x)$
$\mathcal{B}^x(x)$
$\omega_{cr}(x)$
M=1.1 MSun
M=1.4 MSun
M=1.9 MSun
Normalized g [dimensionless]
Bxph [dimensionless]
GM1 M=1.1 MSun
GM1 M=1.4 MSun
GM1 M=1.9 MSun
ωcr [rad/s]
x / Rd

Angular momentum reservoir





Arrows indicate the radius of the cylindrical region that is completely immersed into the crust



Examples for Sly EOS.

The product g(x) ω_{cr}(x) is proportional to the contribution to angular momentum reservoir given by a cylindrical shell at radius x

(given that the lag is critical → **upper bound**)

...simulation of a **GIANT** glitch

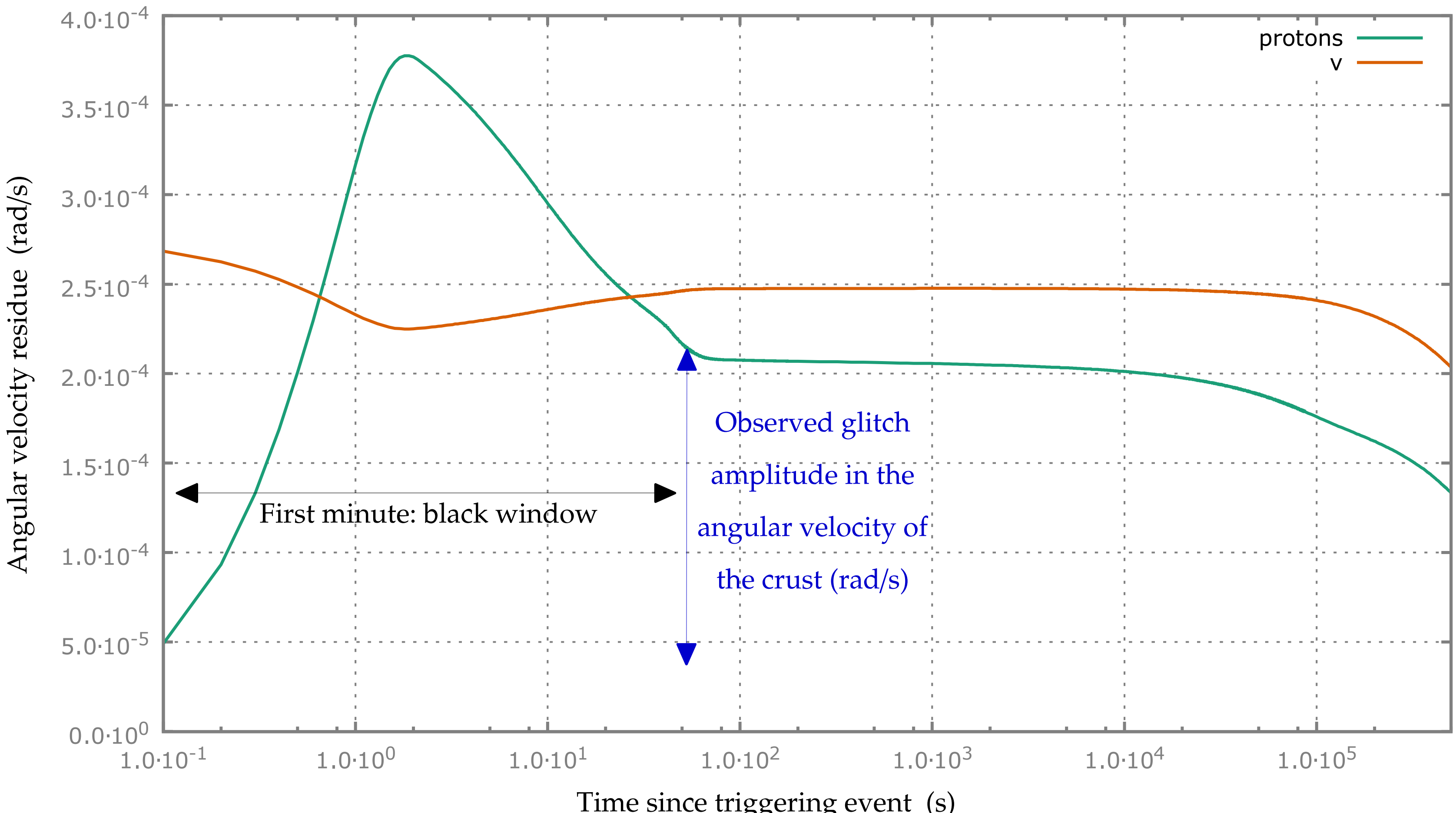
$$\begin{cases} \partial_t \Omega_v(x,t) = -\mathcal{B}[\Omega_v, \Omega_p, x]\,(2\Omega_v + x\partial_x\Omega_v)(\Omega_v - \Omega_p) \\ \partial_t \Omega_p(t) = -(1+q)\dot{\Omega}_\infty - q\langle\, \partial_t \Omega_v(t)\,\rangle \end{cases}$$

Here: rigid vortices approximation → Snowplow like drag functional

$$\mathcal{B}[\Omega_v, \Omega_p, x] \;\approx\; Y[\omega, x]\; \mathcal{B}^x(x)$$

Glitch amplitude (Vela): $\Delta\nu / \nu \sim 10^{-6} \rightarrow \Delta\nu \sim 10^{-5}$ Hz

Differential rotation of the superfluid + NON uniform structure → different timescales in the relaxation



Dynamical simulation: S. Seveso

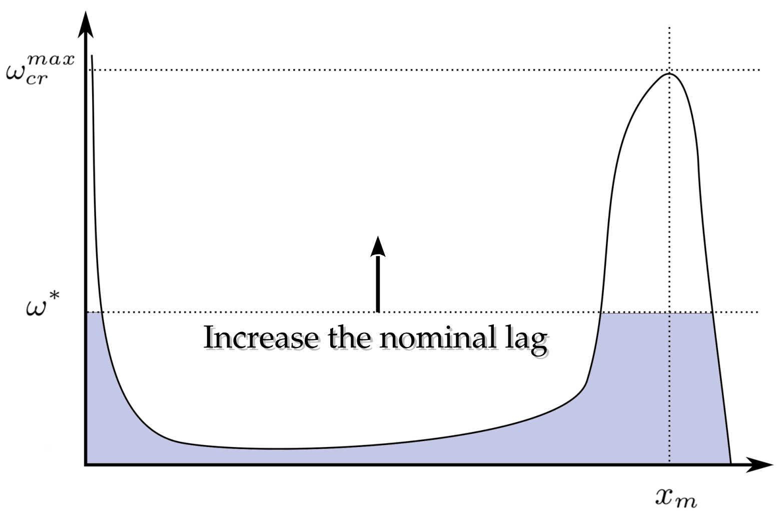
Mass upper bounds... **(just an example!)**

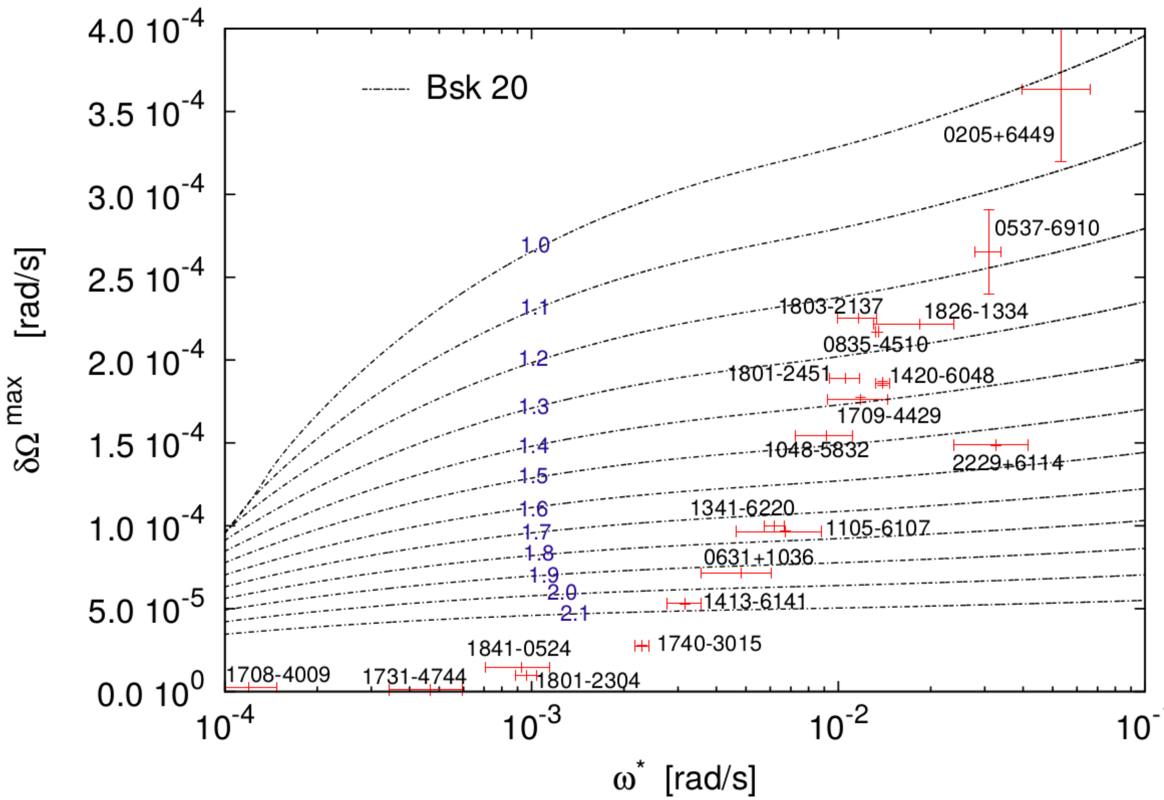
$$L_v(t) = I_v \langle \Omega_v(x,t) \rangle$$
$$\Delta L[\omega] = L_v - I_v \Omega_p = I_v \langle \omega(x) \rangle$$

$$I\Omega_p + I_v \langle \omega_{pre} \rangle = I(\Omega_p + \delta\Omega_p) + I_v \langle \omega_{post} \rangle$$
$$\langle \omega_{post} \rangle = 0,$$

$$\downarrow$$

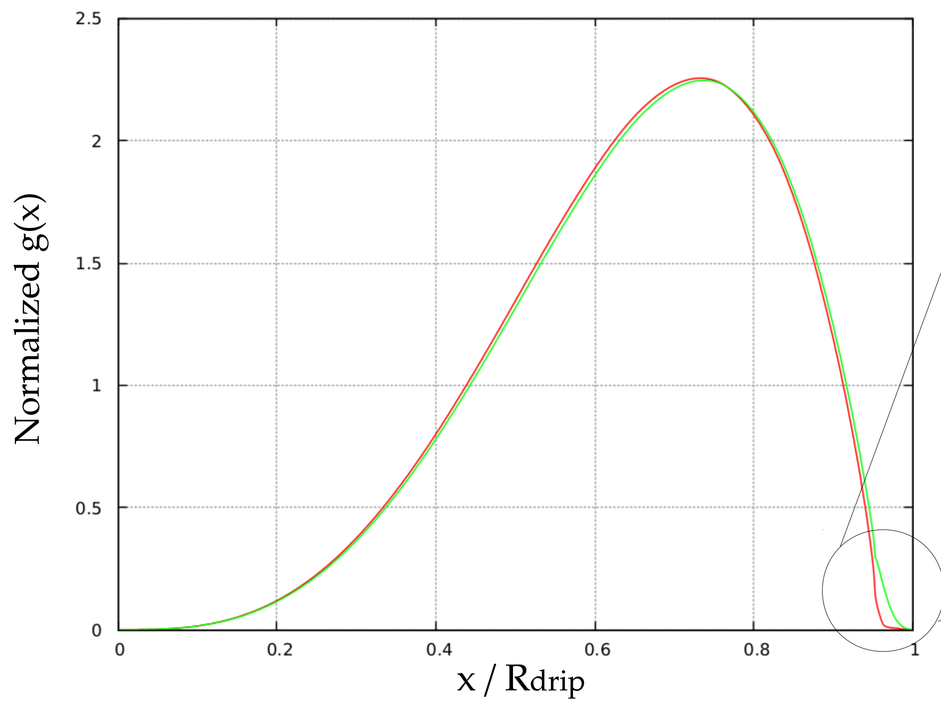
$$\delta\Omega_p^{max} = \frac{I_v}{I} \langle \omega_{cr}(x) \rangle$$

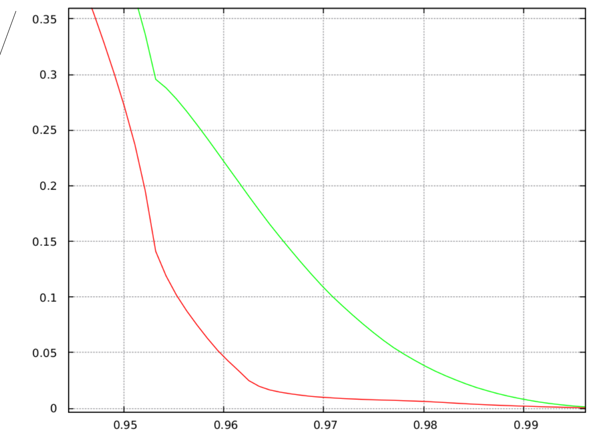




NOTE: it is possible to do the same job using the consistent dynamical equations instead of using this "snowplow-like" prescription

Entrainment corrections

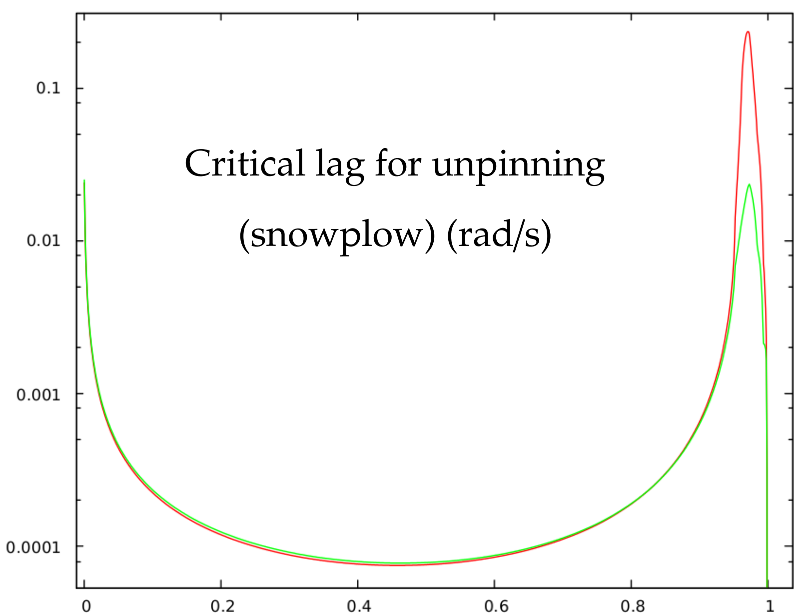


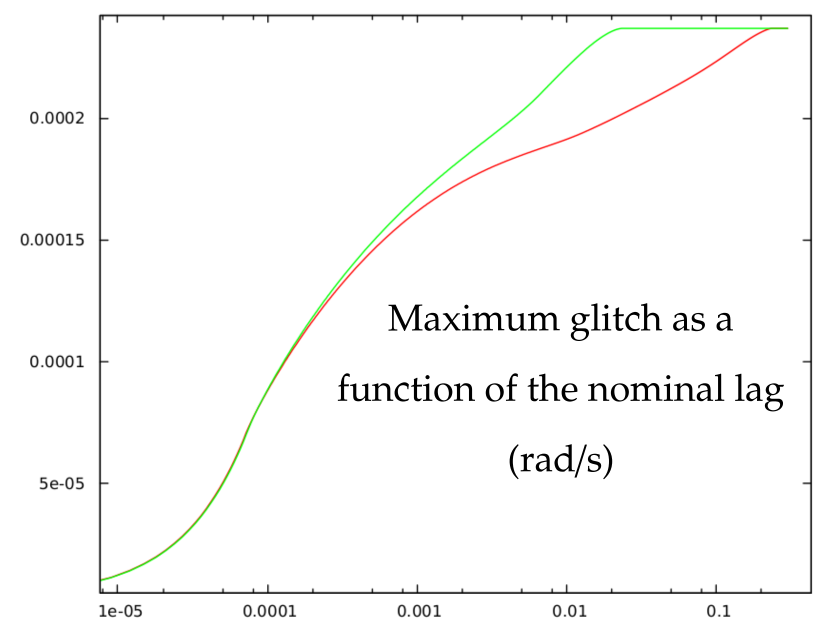


Eos: Sly, M = 1.4 Msun

Red→ with entrainment (Chamel)

Green → no entrainment





Turbulence?

(Andersson, Sidery, Comer - 2007)

Study of mutual friction (force per unit volume) based on superfluid He analogy

Mutual friction for straight vortices...

$$f_i^{\rm mf} = \mathcal{B}'\rho_{\rm n} n_v \epsilon_{ijk}\kappa^j w^k_{\rm np} + \mathcal{B}\rho_{\rm n} n_v \epsilon_{ijk}\epsilon^{klm}\hat\kappa^j \kappa_l w_m^{\rm np} - \rho_{\rm n}\tilde\nu n_v \left[\mathcal{B}'\hat\kappa^j\nabla_j\kappa_i + \mathcal{B}\epsilon_{ijk}\kappa^j\hat\kappa^l\nabla_l\hat\kappa^k\right]$$

...and with curvature

$$f_i^{\rm mf} = \frac{8\pi^2\rho_{\rm n}}{3\kappa}\left(\frac{\chi_1}{\chi_2}\right)^2 \mathcal{B}^3 w^2_{\rm pn} w_i^{\rm pn}$$

Gorter-Mellink

$$f_{\rm HV} \approx \mathcal{B}\rho_{\rm n}\kappa n_v w_{\rm pn}$$

Straight vortex array: “Hall-Vinen type” force

$$\chi_1/\chi_2 \sim 1 \qquad n_v\kappa \approx 2\Omega_{\rm n}, \quad \kappa = h/2m_n$$

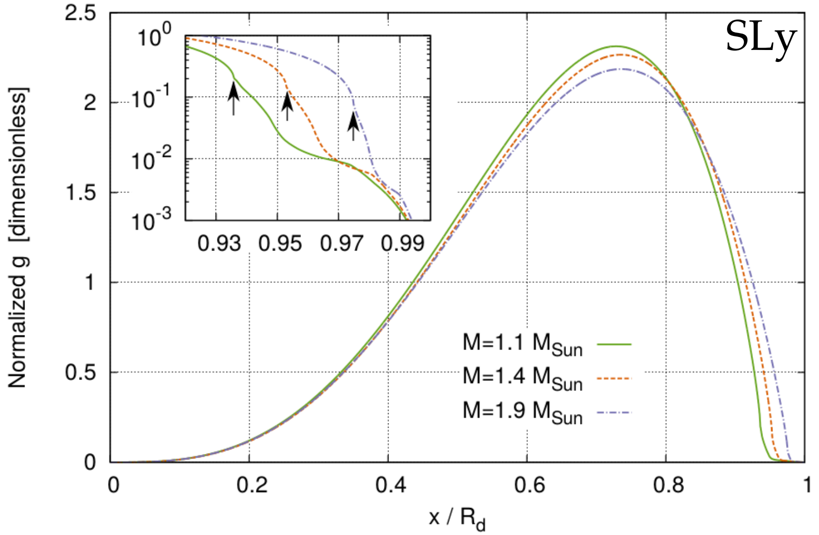
$$w_{\rm pn} = r\Delta\Omega = r(\Omega_{\rm p} - \Omega_{\rm n})$$

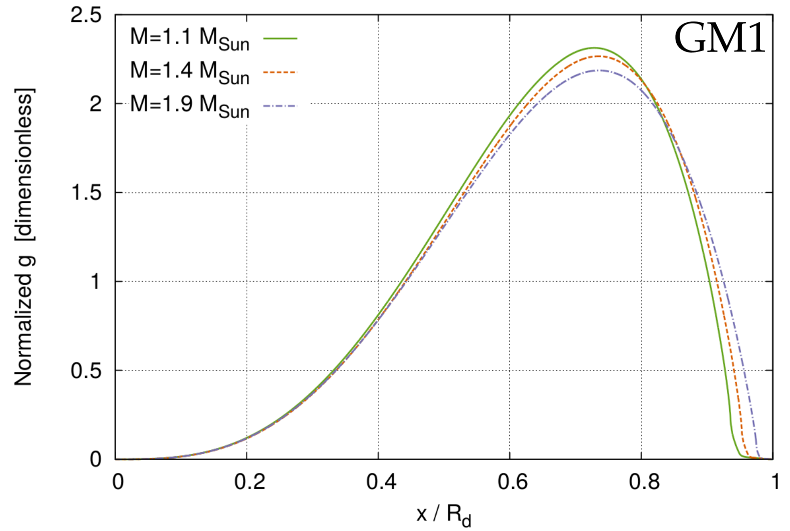
$$\frac{f_{\rm GM}}{f_{\rm HV}} \approx 250\left(\frac{r}{10^6{\rm cm}}\right)^2\left(\frac{\Delta\Omega/\Omega_{\rm n}}{5\times10^{-4}}\right)^2\left(\frac{P}{1{\rm s}}\right)^{-1}$$

Here r is the cylindrical radius, GM more important wrt HV at great r → where vortices are “more pinned”

Some results for Sly & GM1 equations of state

Moments of inertia weights look the same (also because of normalization on the unit interval)





$$\tau^{-1} = 2(1+q)\langle \mathcal{B}^{x}_{\text{eff}} \rangle \Omega_p$$

	M $(M_\odot)$	R_d (km)	R_c (R_d)	$I/10^{45}$ (g cm^2)	I_v (I)	q	τ_{ph} (P)	τ_{kv} (P)	$\langle\omega_{cr}\rangle$ $(10^{-4}$rad/s)	$\delta\Omega_p^{max}$ $(10^{-4}$rad/s)
SLy	1.1	11.41	0.936	0.86	0.928	12.9	62.1	13.2	4.17	3.87
	1.4	11.43	0.953	1.09	0.941	16.1	29.2	12.1	2.51	2.37
	1.9	10.92	0.974	1.35	0.958	22.9	9.0	7.2	0.92	0.89
GM1	1.1	13.08	0.926	1.14	0.901	9.2	445	13.9	6.99	6.30
	1.4	13.28	0.945	1.51	0.913	10.5	261	17.1	4.37	3.92
	1.9	13.24	0.966	2.04	0.916	10.9	136	26.3	2.04	1.87

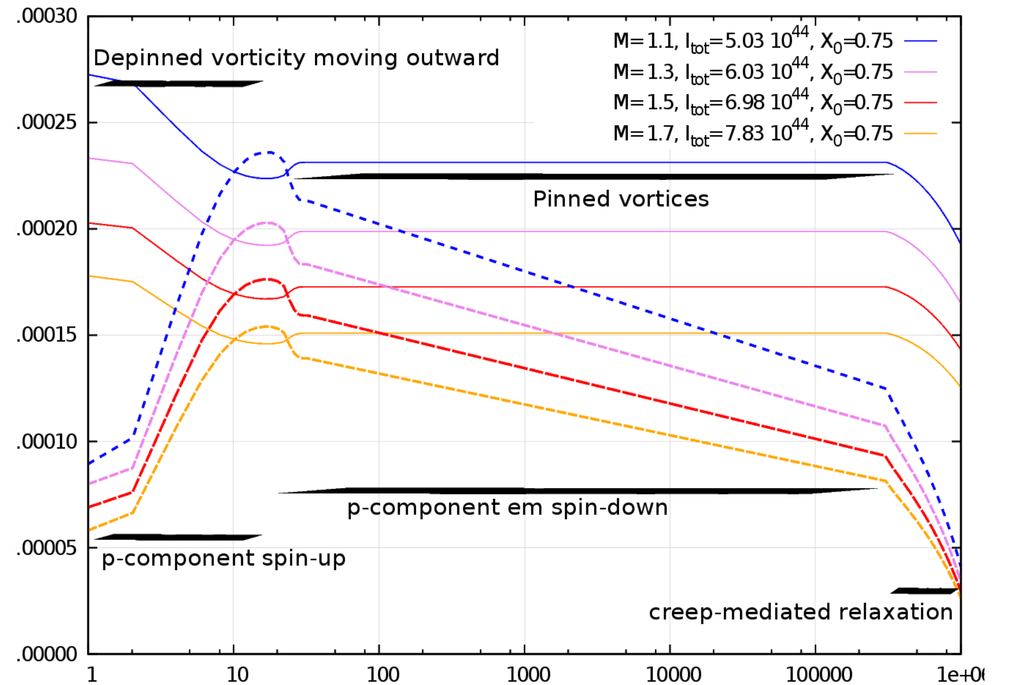
Estimates of the rise time

(all vortices unpinned)

Thanks for the attention,

QUESTIONS ?

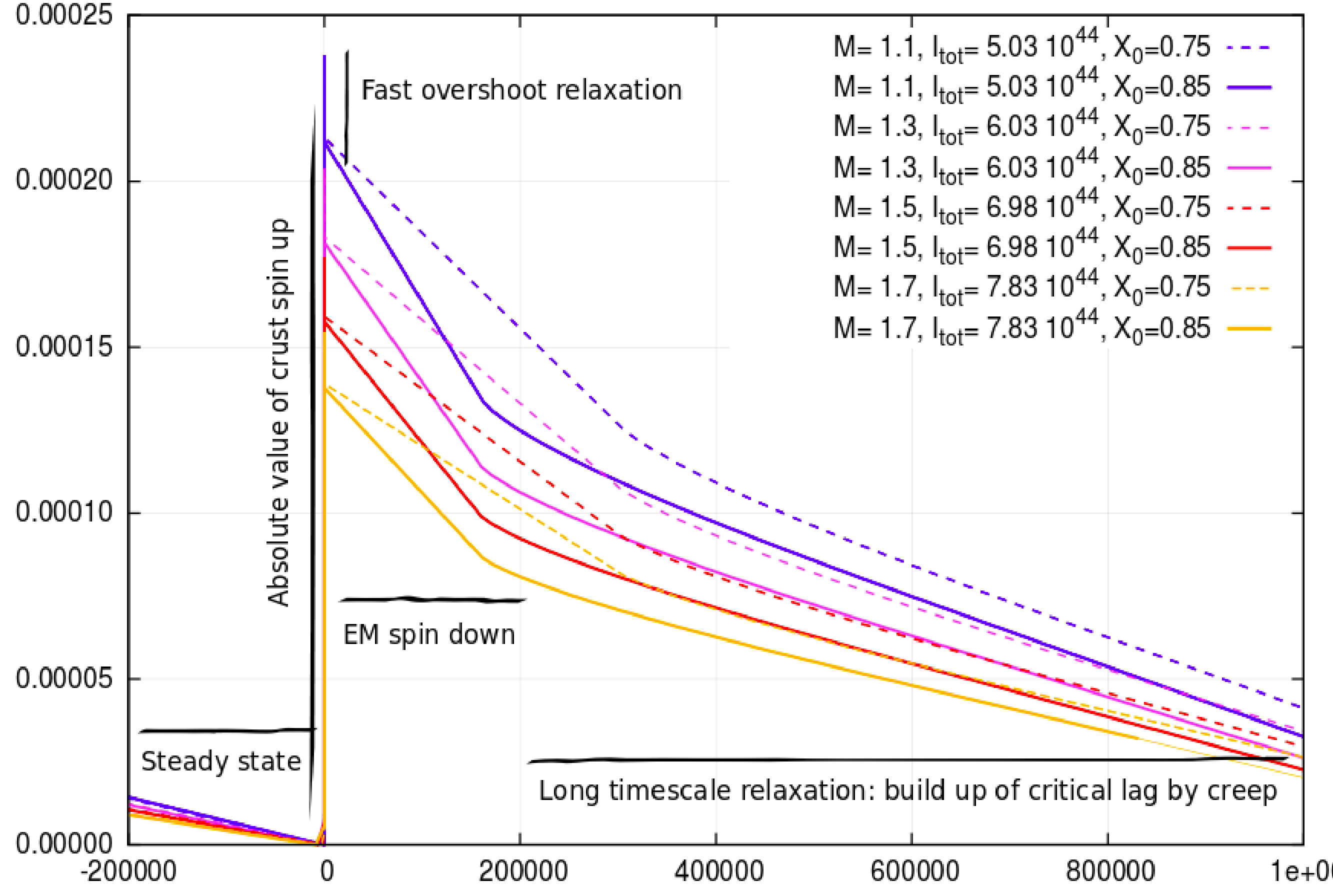
...why a **simple** but **consistent** model?



Dynamical simulation with:

- Polytropic EOS
- Proton fractions by hand
- Entrainment by hand
- Drag by hand (rise times are the same!)
- "Similar" pinning force profile
  (Pizzochero, "snowplow model")

Important:
Large glitch → high mass

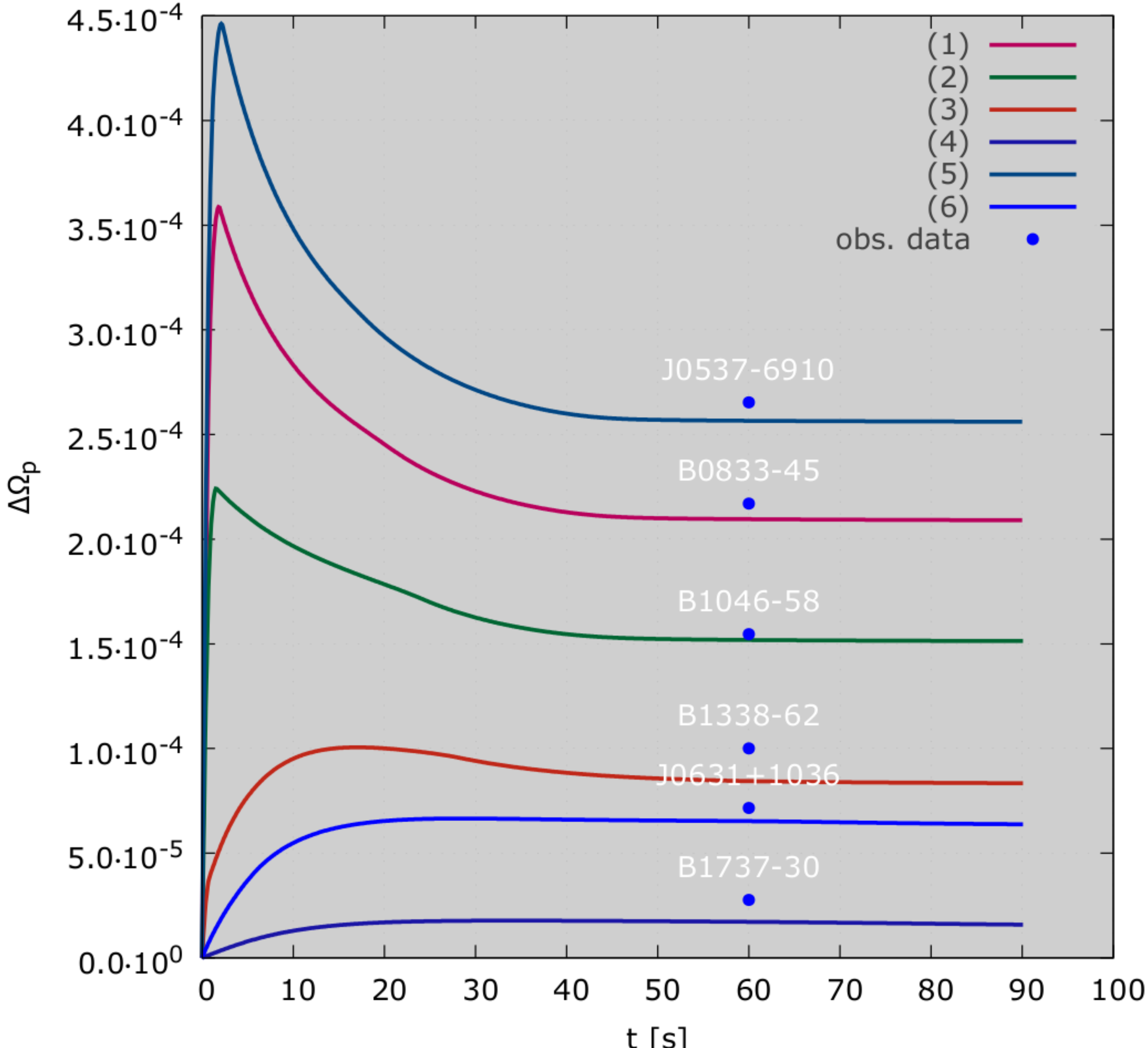


...why a **simple** but **consistent** model?

Dynamical simulations with:

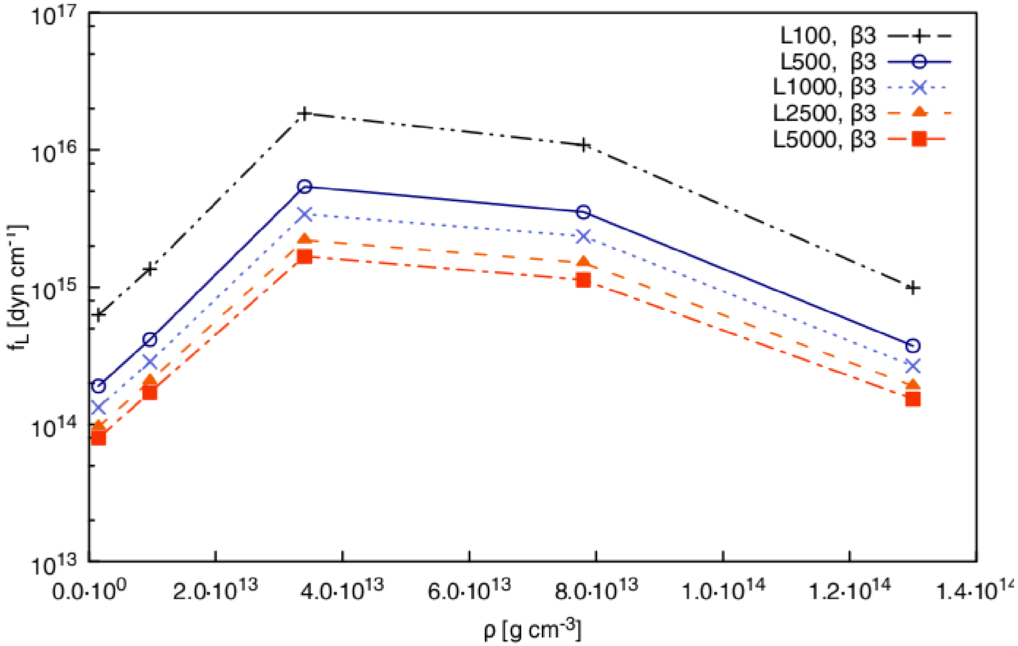
- GM1 EOS
- EOS consistent proton fractions
- Entrainment in the core & crust (Chame
- Consistent drag in the core
  (el. scattering, Alpar, Andersson...)
- Drag in the crust (phonons, Jones 1991)
- New MESOSCOPIC pinning forces
  (Seveso, Haskell, Pzzochero 2016)

Large glitch → low mass
(true for all the EOSs that we used)



Dynamical simulation: S. Seveso

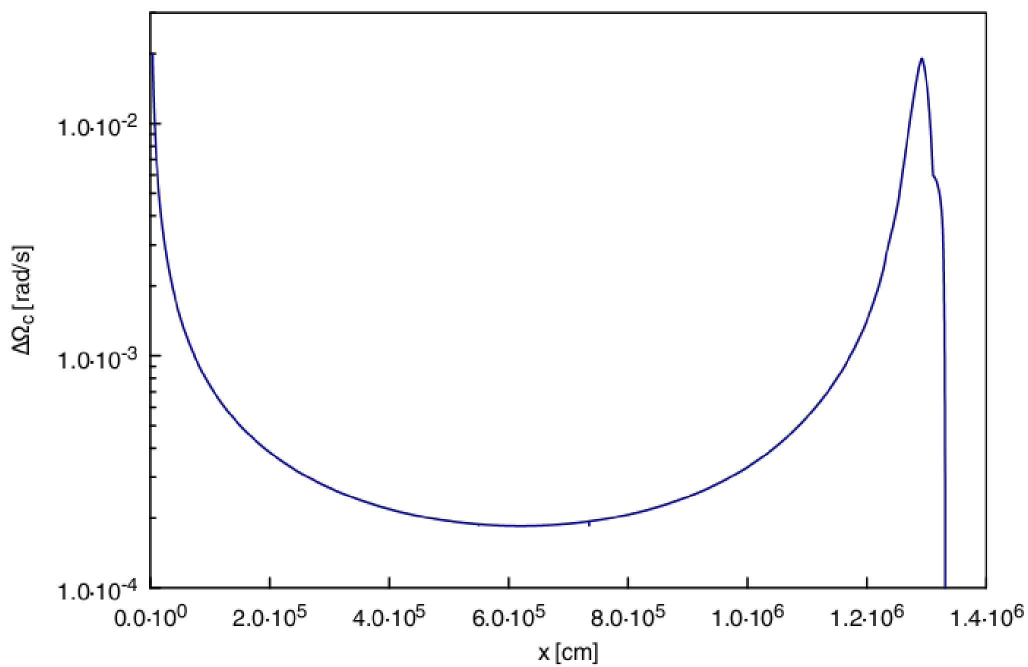
Mesoscopic pinning forces



Mesoscopic pinning forces

(vortex-lattice interaction per unit length of vortex line)

S. Seveso, F. Grill,
P. Pizzochero, B. Haskell



Resulting critical lag for unpinning (without entrainment)

$$Y[\omega, x] = \theta(|\omega(x)| - \omega_{cr}(x))$$