Spin polarized neutron star matter and core-crust transition in the neutron stars

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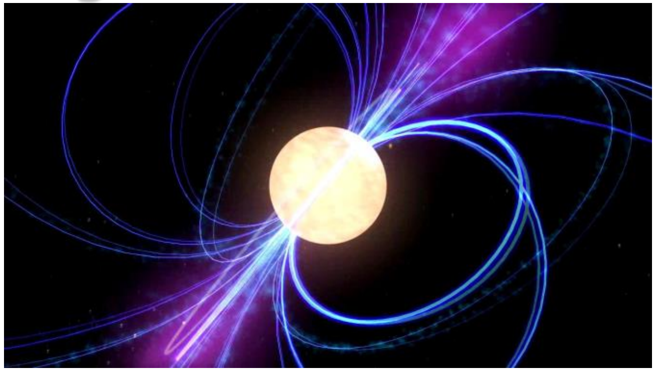
Pulsars & Magnetars

- Rotating neutron star with strong surface magnetic fields,

$$B = 10^{13} - 10^{15}\,G$$

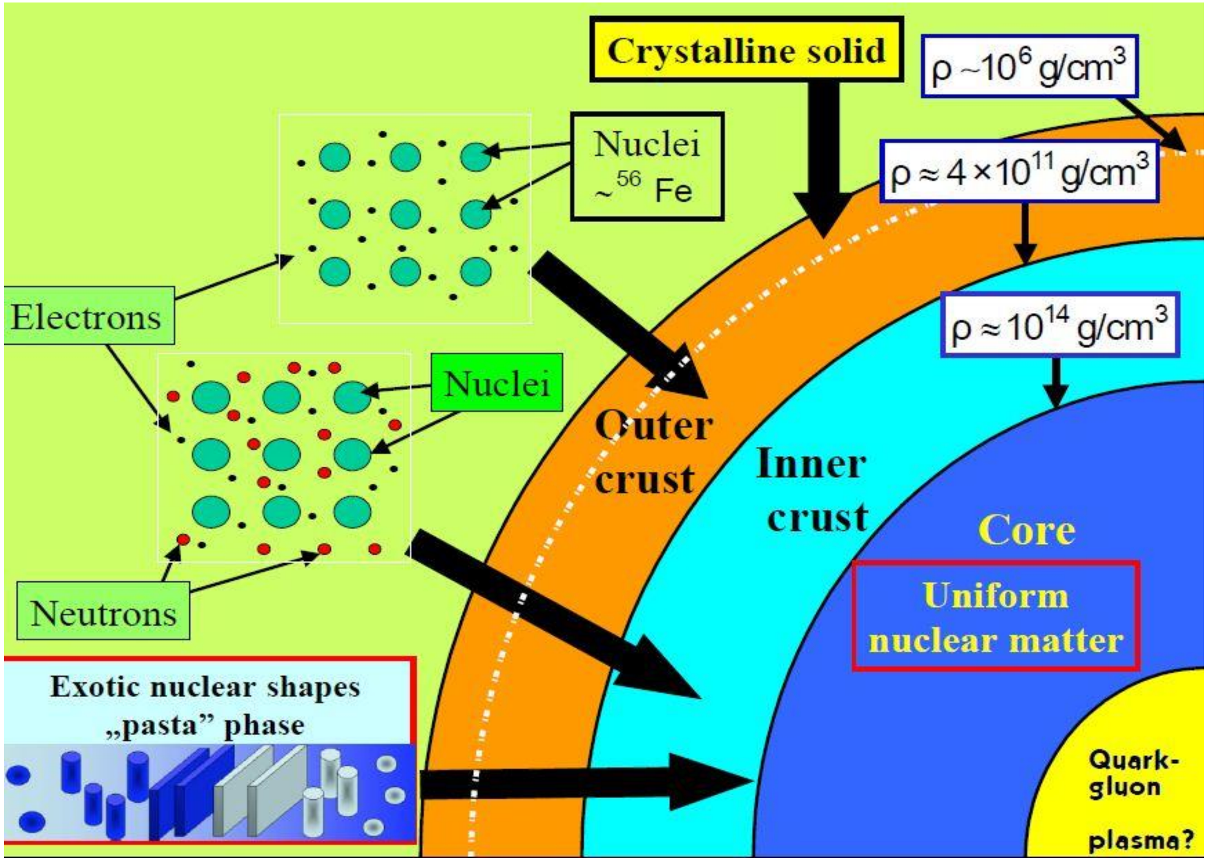


- According to virial theorem the strange of magnetic field at the core of neutron star may is

$$B \approx 10^{18}\,G$$

- Actually the exact origin of this magnetic field is not well known
- Spin polarized state may be occur or induce by magnetic field

Internal Structure of neutron stars



N. Chamel, *Vela pulsar glitches and nuclear superfluidity,indico.cern.ch*,(2014),

Spin polarized Matter

- Beta stable matter
  - We consider the homogeneous matter in the phase transition region as an infinite system of electrons and interacting nucleons, which are in equilibrium. In this system, the energy per baryon consists of two parts of contributions,

$$E = E_e + E_{\mathrm{nucl}}$$

  - electrons

E_e is the contribution of electrons is given by

$$E_e = \sum_{k \le k_e^F} \sqrt{m_e^2 + k^2} \cong \frac{\mu^4}{4\pi^2 \rho}$$

Where $\mu = \sqrt{m_e^2 + {k_e^F}^2} \cong k_e^F$ is electron chemical potential

Spin polarized Matter

- Beta stable matter
  - nucleons

The nucleonic part of neutron star matter is composed of neutrons and protons,

$$\rho = \left(\rho_n^{\uparrow} + \rho_n^{\downarrow}\right) + \left(\rho_p^{\uparrow} + \rho_p^{\downarrow}\right)$$

$$\delta_n = \frac{\rho_n^{\uparrow} - \rho_n^{\downarrow}}{\rho_n^{\uparrow} + \rho_n^{\downarrow}} \qquad \delta_p = \frac{\rho_p^{\uparrow} - \rho_p^{\downarrow}}{\rho_p^{\uparrow} + \rho_p^{\downarrow}} \qquad \beta = \frac{\rho_n - \rho_p}{\rho_n + \rho_p} = 1 - 2x$$

Now, we consider the cluster expansion of the energy functional up to the two-body term

$$E(\rho, x, \delta_n, \delta_p) = \frac{1}{A}\frac{\langle\psi|H|\psi\rangle}{\langle\psi|\psi\rangle} = E_1 + E_2$$

Spin polarized Matter

- Beta stable matter
  - nucleons

The one-body term E_1 for an asymmetrical nuclear matter is

$$E_1 = \sum_{\tau=n,p} \sum_{\sigma=\uparrow\downarrow} \sum_{k \le k_{F_\tau}^{\sigma}} \frac{k^2}{2m_\tau} \qquad k_{F_\tau}^{\sigma} = (6\pi^2 \rho_\tau^\sigma)^{1/3}$$

The two-body energy E_2 is,

$$E_2 = \frac{1}{2A} \sum_{ij} \langle ij | v(12) | ij - ji \rangle$$

two-body correlation function

two-body potential

$$v(12) = -\frac{\hbar^2}{2m} \left[f(12), \left[\nabla_{12}^2, f(12) \right] \right] + f(12) V(12) f(12)$$

Spin polarized Matter

- Beta stable matter
  - URCA processes

The ground state of the system can be determined by minimizing the total energy with respect to the proton fraction (beta-equilibrium condition). This minimization, together with the charge neutrality, imposes the following coupled constraints on our calculations:

$$\mu \approx 4(1-2x)S(\rho,\delta_n,\delta_p) \qquad \frac{\mu^3}{3\pi^2} - \rho x = 0$$

$$E_{\text{nucl}}(\rho,x,\delta_n,\delta_p) = E_{\text{snucl}} + S(1-2x)$$

Core-crust transition parameters

- The convexity of the energy per particle in the single phase leads to the following constraints for pressure and chemical potential

$$-\left(\frac{\partial P}{\partial v}\right)_{\mu} \succ 0, \qquad -\left(\frac{\partial \mu}{\partial q}\right)_{v} \succ 0$$

$$P = P_e + P_{\text{nucl}}, \quad v = 1/\rho, \quad q = x - x_e$$

H.B. Callen, Thermodinamics (Wiley-New York, 1985).

Core-crust transition parameters

- After doing some algebra and simplification, the above constraint leads to the following inequality,

$$V = \rho^2 \frac{\partial^2 E_{\text{nucl}}}{\partial \rho^2} + 2\rho \frac{\partial E_{\text{nucl}}}{\partial \rho}$$

$$-\left(\rho \frac{\partial^2 E_{\text{nucl}}}{\partial \rho \partial x} \right)^2 \left(\frac{\partial^2 E_{\text{nucl}}}{\partial x^2} \right) \succ 0$$

- The transition density, ρ_t , and the corresponding proton fraction, x_t , can been obtained by solving the coupled equations V = 0, eqs. beta-equilibrium condition and charge neutrality.

Results

Unpolarized matter

Method and model	ρ_s (fm^{-3})	ρ_t (fm^{-3})	P_t (MeV fm^{-3})	ε_t (MeV fm^{-3})	Ref.
LOCV, UV_{14} + TNI	0.167	0.106	0.543	100.16	
LOCV, AV_{18}	0.31	0.171	1.013	162.3	
BHF, AV_{18}	0.24	0.131	0.648	125.2	
BHF, AV_{18} + TBFa	0.187	0.099	0.541	93.78	
BHF, AV_{18} + TBFb	0.176	0.089	0.474	85.43	
BHF, AV_{18} + *micro* TBF	0.2	0.106	0.766		
DBHF, Bonn A	0.18	0.094	0.511		
IU-FSU	0.155	0.087	0.289	82.655	[12]
FSUmax	0.148	0.073	0.425	68.810	[12]
NL3max	0.148	0.083	0.550	78.290	[12]
TFcmax	0.148	0.079	0.692	75.454	[12]
MDI ($x = 0$)	0.16	0.091	0.645		[14]
MDI ($x = -1$)	0.16	0.093	0.982		[14]
APR, AV_{18} + UVIX	0.16	0.087	0.513		[14]
DBHF, Bonn B	0.185	0.1	0.393		[14]

M. Bigdeli and S. Elyasi, Eur. Phys. J. A **51**, 38 (2015).

[12] J. Piekarewicz, et al, Phys. Rev. C 90, 015803 (2014).

[14] A.Worley, P.G. Krastev, B.A. Li, Astrophys. J. 685, 390 (2008).

Results

- Now, we are in the position to discuss the results of our calculation for the core-crust transition parameters of the neutron stars. We have summarized the value of the core -crust transition parameters in table.

polarization	ρ_t	x_t
$\delta_n = \delta_p = 0$	0.171	0.0306
$\delta_n = \delta_p = 0.25$	0.1581	0.0319
$\delta_n = \delta_p = 0.5$	0.0907	0.0310
$\delta_n = \delta_p = 0.525$	0.078	0.0289
$\delta_n = \delta_p = 0.5425$	0.061	0.0249
$\delta_n = \delta_p = 0.6$	~ 0.0	

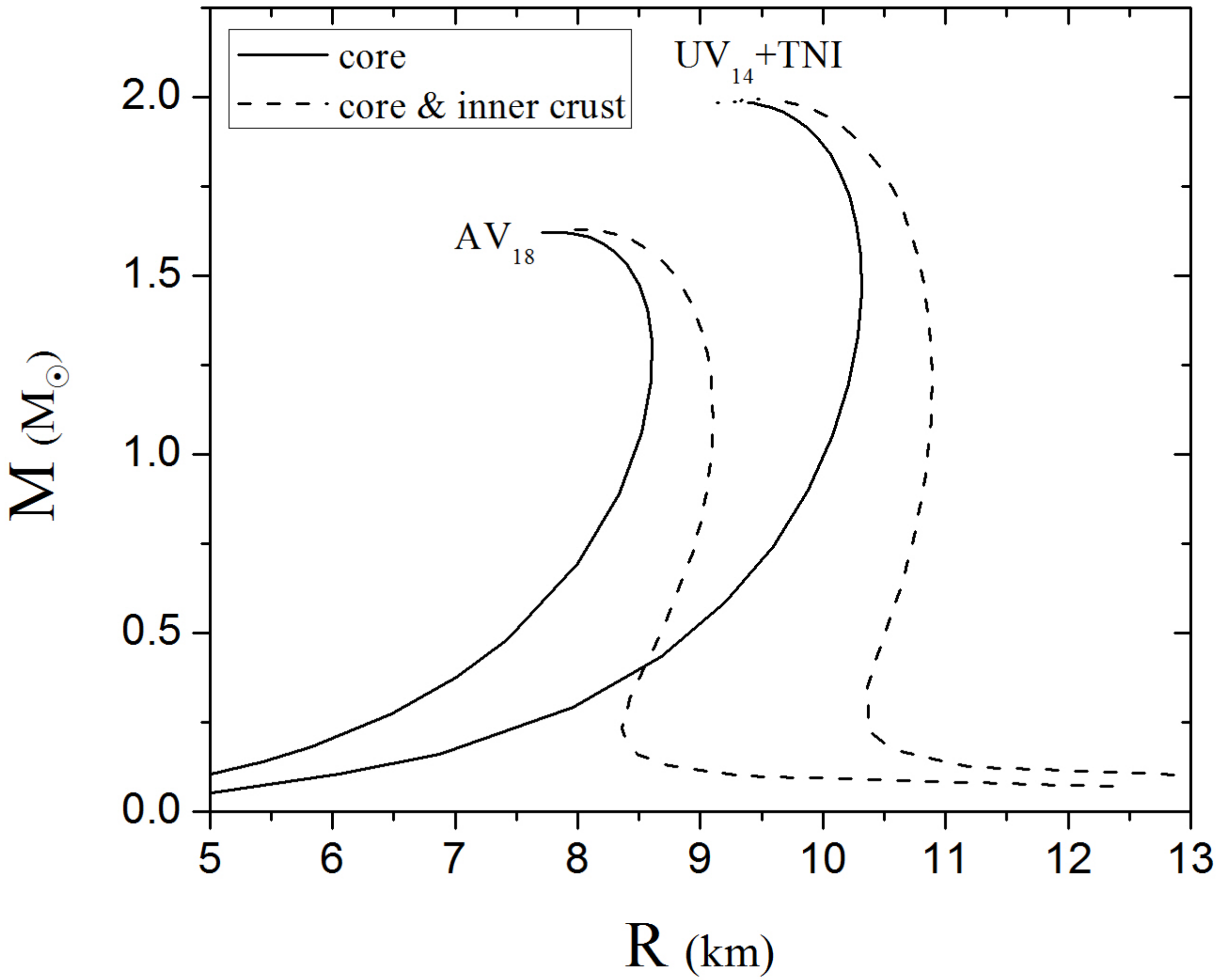
Results

Anti-ferromagnetic spin state

polarization	ρ_t	x_t
$\delta_n = \delta_p = 0$	0.171	0.0306
$\delta_n = -\delta_p = 0.25$	0.153	0.0298
$\delta_n = -\delta_p = 0.4$	0.1079	0.0272
$\delta_n = -\delta_p = 0.45$	0.0831	0.02381
$\delta_n = -\delta_p = 0.475$	0.0592	0.0195
$\delta_n = -\delta_p = 0.5$	~ 0.0	

Mass-Radius of neutron star

- Core and crust, for unpolarized case



Thank You!