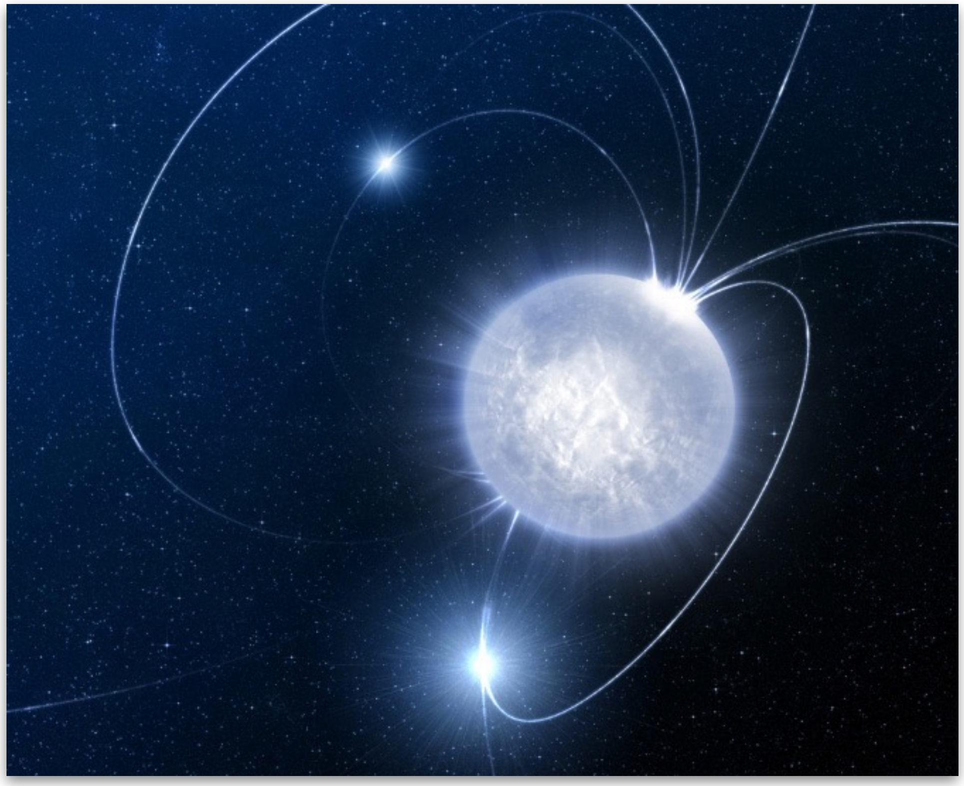
Unstable neutron star oscillations: dynamics & gravitational waves



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CompStar 2016, Istanbul

Neutron stars as GW sources

“Burst” emission
(not discussed here)

- Binary neutron star mergers (our safest bet for detection)
- Pulsar glitches
- Magnetar flares

Continuous emission
(next slide)

GWs from *rotating* neutron stars

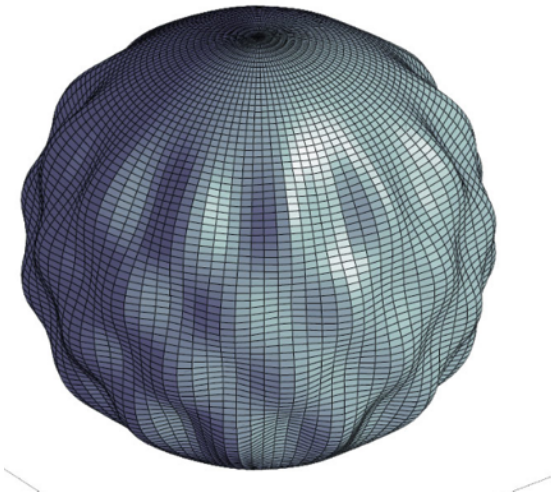
Continuous emission

Non-axisymmetric mass quadrupole ("mountains")

(talk by Ian Jones)

Fluid part (oscillations)

(this talk)



The CFS instability summarized

- The Chandrasekhar-Friedman-Schutz instability (1970s) is *secular*: the fluid must be coupled to some dissipative mechanism (GWs, EMs, fluid viscosity).

- Oscillation modes: energy rate of change due to GWs emission:

$$\dot{E}_{\rm mode} = -\omega_r \sum_{\ell \geq 2} \omega_i^{2\ell+1} N_\ell \left(\Big|\underset{\substack{\downarrow \\ \text{mass} \\ \text{multipoles}}}{D_{\ell m}}\Big|^2 + \Big|\underset{\substack{\downarrow \\ \text{current} \\ \text{multipoles}}}{J_{\ell m}}\Big|^2 \right)$$

$$\underset{\substack{\downarrow \\ \text{inertial frame} \\ \text{frequency}}}{\omega_i} = \underset{\substack{\searrow \\ \text{rotating frame} \\ \text{frequency}}}{\omega_r} - m\Omega$$

- CFS instability:

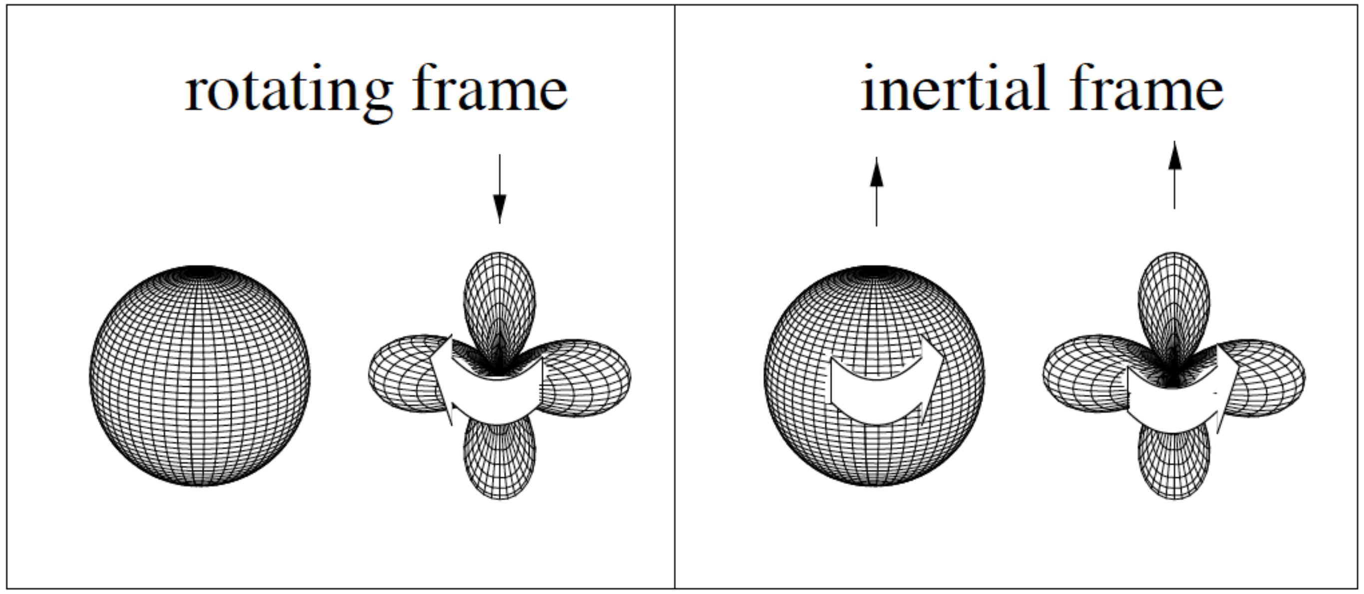
$$\dot{E}_{\rm mode} = -\frac{2E_{\rm mode}}{\tau_{\rm gw}} > 0 \;\Leftrightarrow\; \omega_i \omega_r < 0$$

For $\omega_r > 0 \Rightarrow \Omega > \dfrac{\omega_r}{m}$

mode is “frame-dragged”

mode
pattern speed

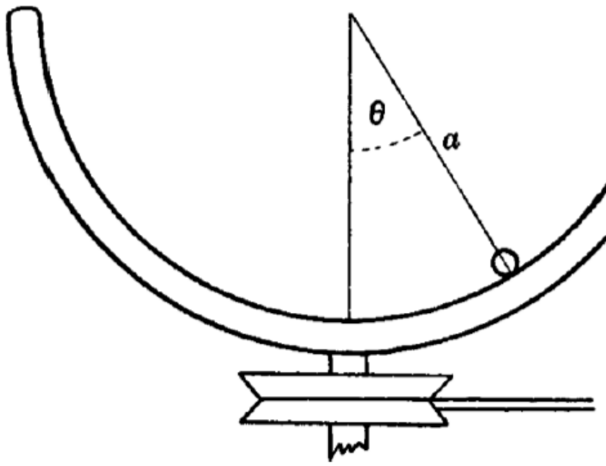
$$\dot{\varphi} = -\frac{\omega}{m}$$



[Andersson & Kokkotas 2001]

CFS mechanical analogue [Lamp 1908]

- Particle inside a rotating bowl, perturbed from equilibrium $\theta=0$, in the presence of friction.



equation of motion (rotating frame):

$$\zeta(t) = x(t) + iy(t)$$

$$\ddot{\zeta} + (\,2i\Omega + \underset{\substack{\downarrow \\ \text{friction}}}{\lambda}\,)\dot{\zeta} + \left(\frac{g}{a} - \Omega^2\right)\zeta = 0$$

general solution:

$$\zeta(t) = A\, e^{-i(\Omega \mp \sqrt{g/a})t} e^{-\frac{1}{2}\lambda(1 \mp \Omega\sqrt{a/g})t}$$

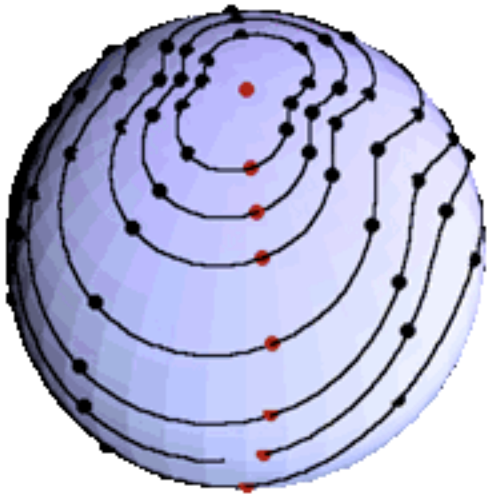
- Motion *unstable* for: $\lambda > 0$ and $\Omega > \sqrt{g/a}$

- This is an example of a viscosity-driven “CFS” instability.

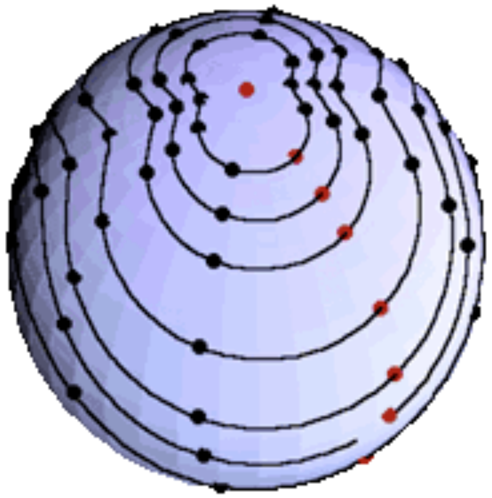
The r-mode instability

- The r-modes is a special class of *inertial waves*, characterised by nearly horizontal fluid motion.
- r-modes are CFS-unstable for *any* spin frequency Ω.
- Their GW radiation is special in the sense that it is dominated by the current multipoles.
- The $\ell = m = 2$ r-mode is the most unstable one, with a typical growth timescale of ~ 1 min.
- GW frequency: $f_{\rm gw} = f_{\rm mode} \approx \frac{4}{3} f_{\rm spin}$

The signal falls within the LIGO sensitivity band for fast-spinning stars.



corotating frame



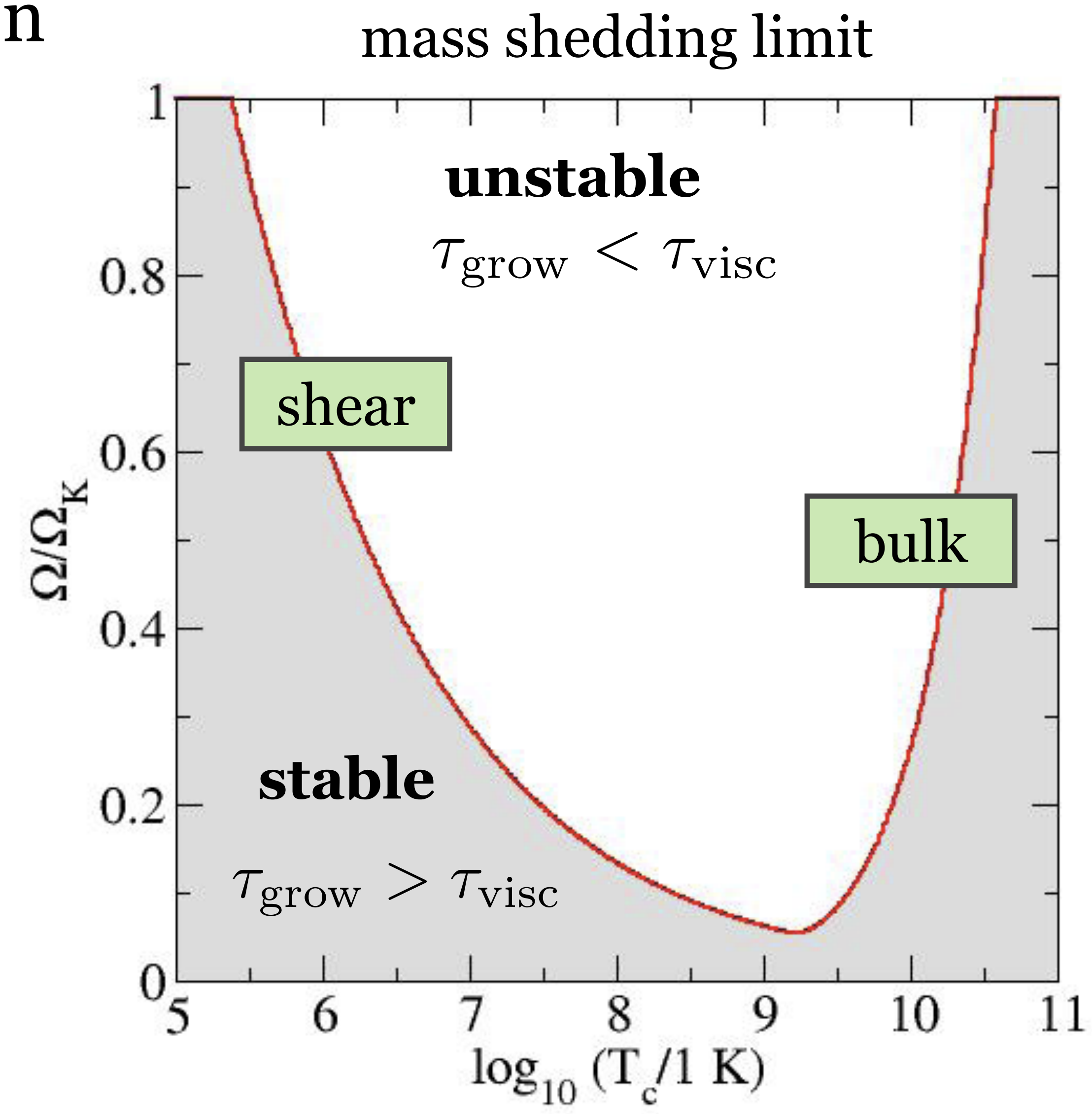
inertial frame

[Figure credit: Hanna & Owen]

The r-mode instability window

- The r-mode instability is active for any rotation but can be damped by viscous processes.
- The *spin-temperature instability window* is “large” but depends on uncertain core-physics.
- “Minimal” model: accounts for damping only due to shear and bulk viscosity.
- Once active, the instability’s GW signal is largely determined by the mode’s *velocity amplitude* α:

$$\delta v_{\mathrm{mode}} \sim \alpha \left(\frac{r}{R}\right)^2 \Omega R$$



r-mode saturation amplitude

- Several mechanisms could limit the r-mode's maximum amplitude:

  - Non-linear coupling with short-wavelength modes (mostly inertial):

$$\alpha_{\rm sat} \sim 10^{-4} - 10^{-3}$$

  - Dissipative "cutting" of proton flux tubes by neutron vortices:

$$\alpha_{\rm sat} \sim 10^{-6} - 10^{-5}$$

  - Winding up of magnetic field lines by the r-mode's differential flow ?

$$\alpha_{\rm sat} \sim 10^{-6} - 10^{-4}$$

r-modes: spin-down upper limits

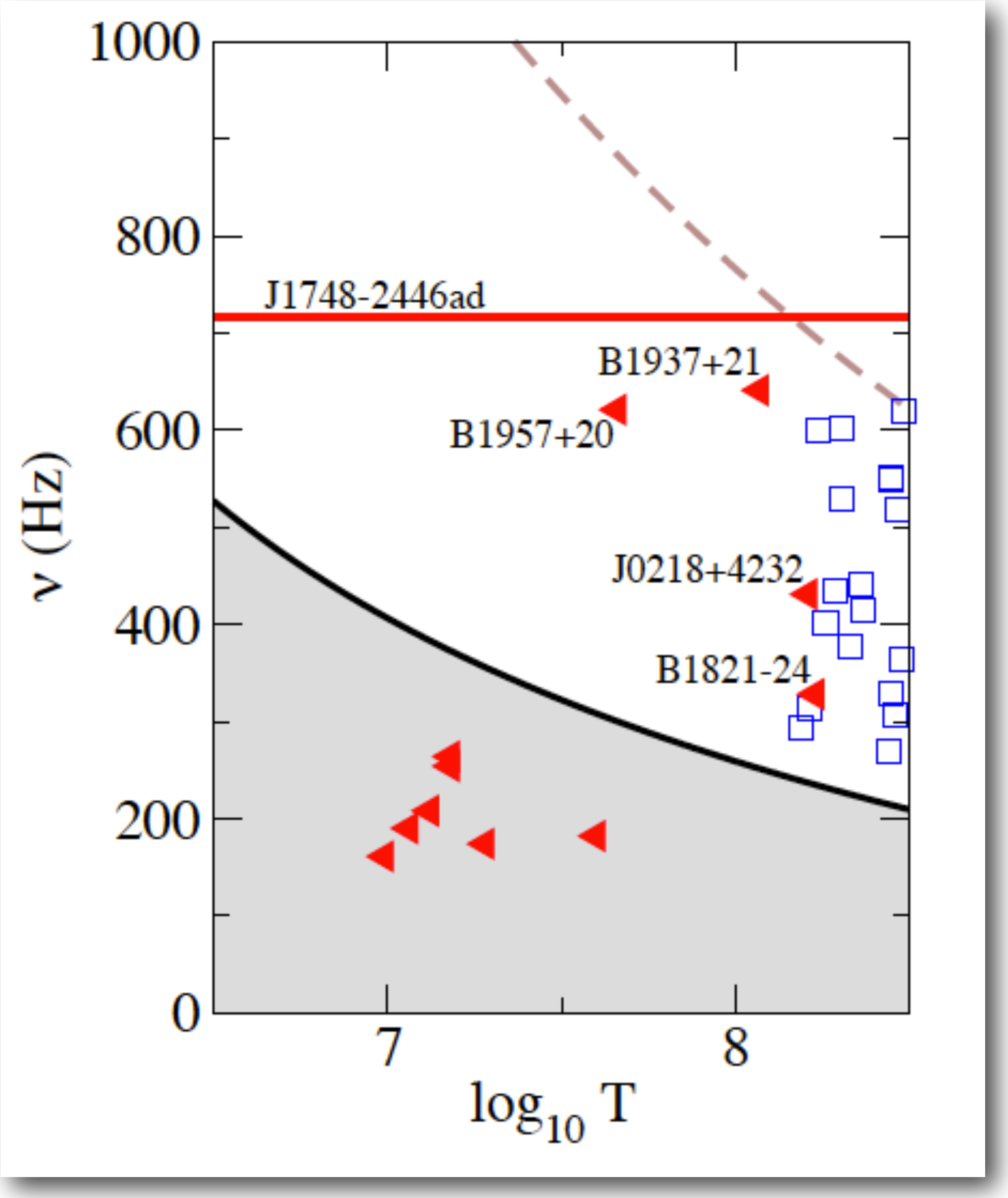
- Assume a “minimum-physics” instability window.

- Then, several LMXBs and MSPs with measured $f_{\rm spin}, \dot{f}_{\rm spin}$ are potentially r-mode unstable.

- Obtain upper limits for the amplitude by assuming spin down only via r-mode GW radiation. The outcome is tiny:

$$\alpha_{\rm sat} \lesssim 10^{-7}$$

- This of course assumes that the systems are r-mode unstable in the first place.



blue: LMXBs
red: MSPs (T data: upper limits)

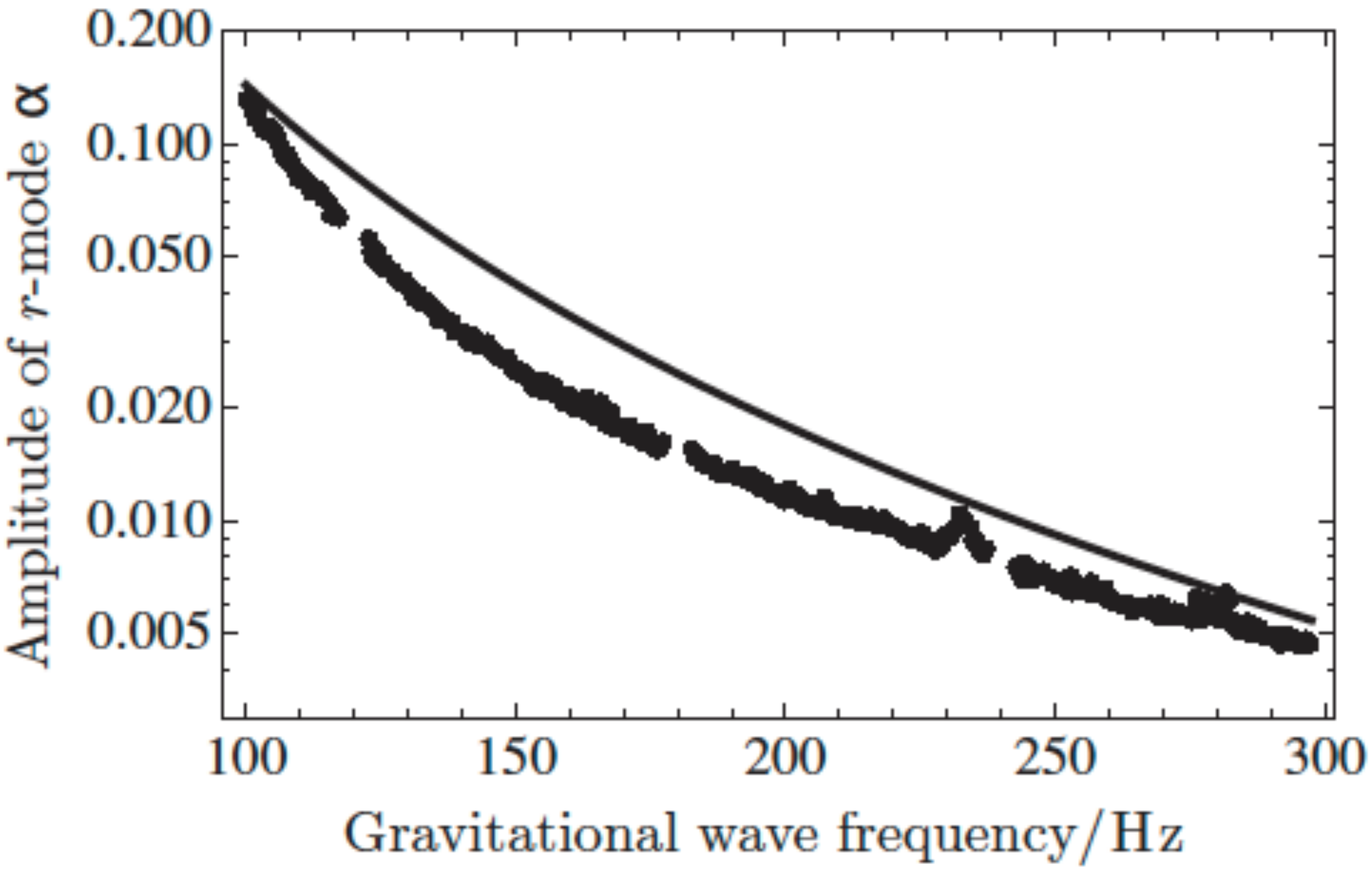
[Figure credit: N. Andersson]

Direct upper limit: Cassiopeia A

- The Cas A supernova remnant hosts the youngest known neutron star (~ 300 years old). The system is “famous” for its thermal evolution: it is seen cooling in “real” time.

- LIGO has set direct upper limits on GW strain assuming r-mode emission

- The spin frequency is not known, but is likely to be too low for ground-based detectors.



[Aasi et al. 2014]

LMXBs: spin equilibrium

- LMXB spin distribution:

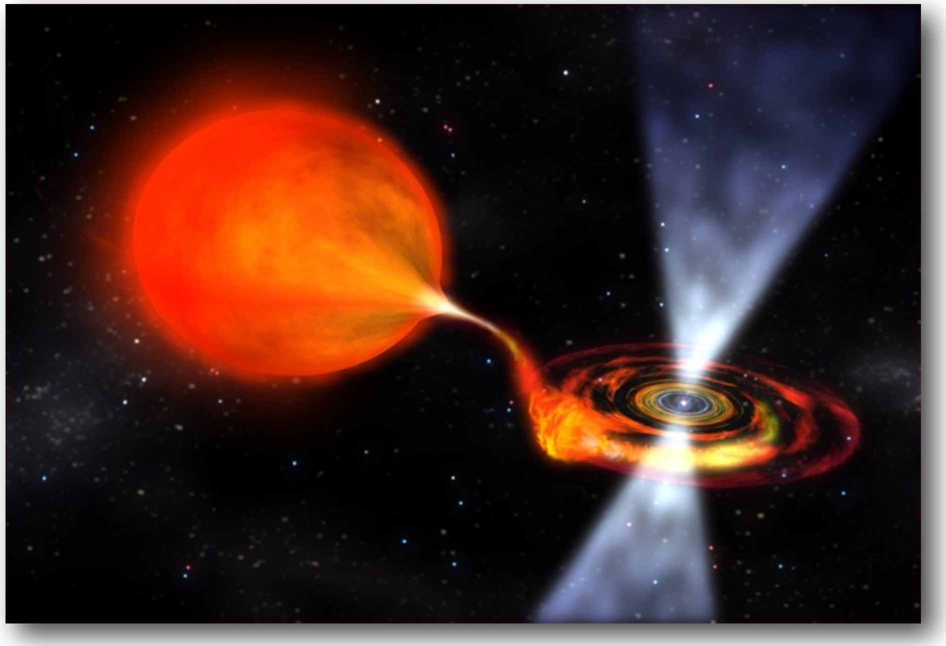
$$200\,\mathrm{Hz} \lesssim f_{\rm spin} \lesssim 600\,\mathrm{Hz}$$

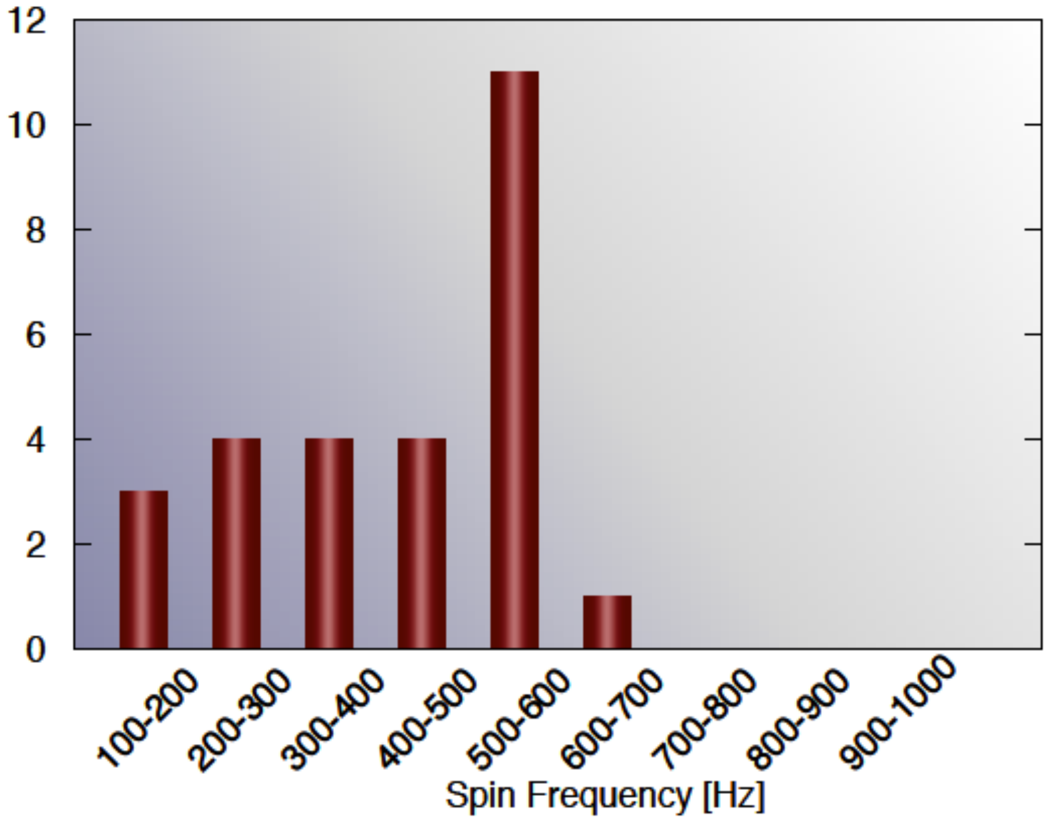
- This is well below the mass-shedding limit:

$$f_{\rm spin} \ll f_{\rm Kepler} \sim 1.5\,\mathrm{kHz}$$

- Accretion lasts $\sim 10^7 - 10^8\,\mathrm{yr}$, enough time for LMXBs to straddle the Kepler limit.

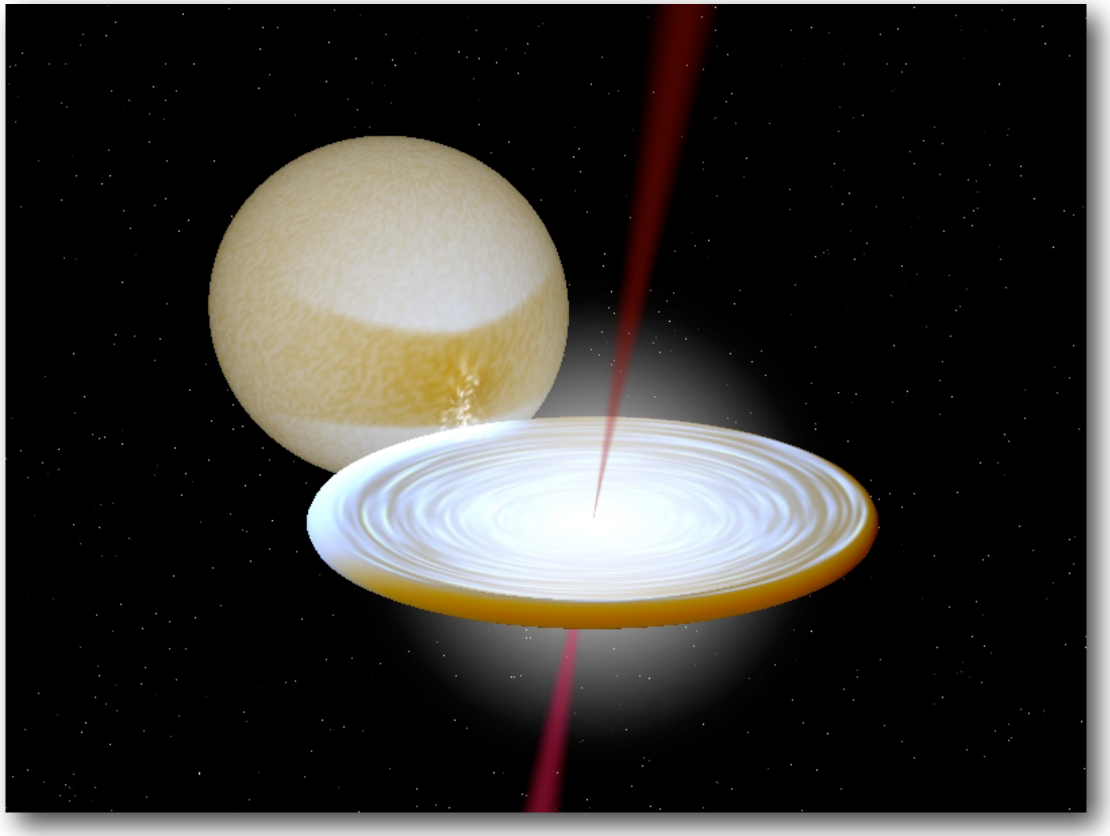
- Some process seems to halt the spin-up.





[Figure credit: A. Patruno]

LMXBs: halting accretion



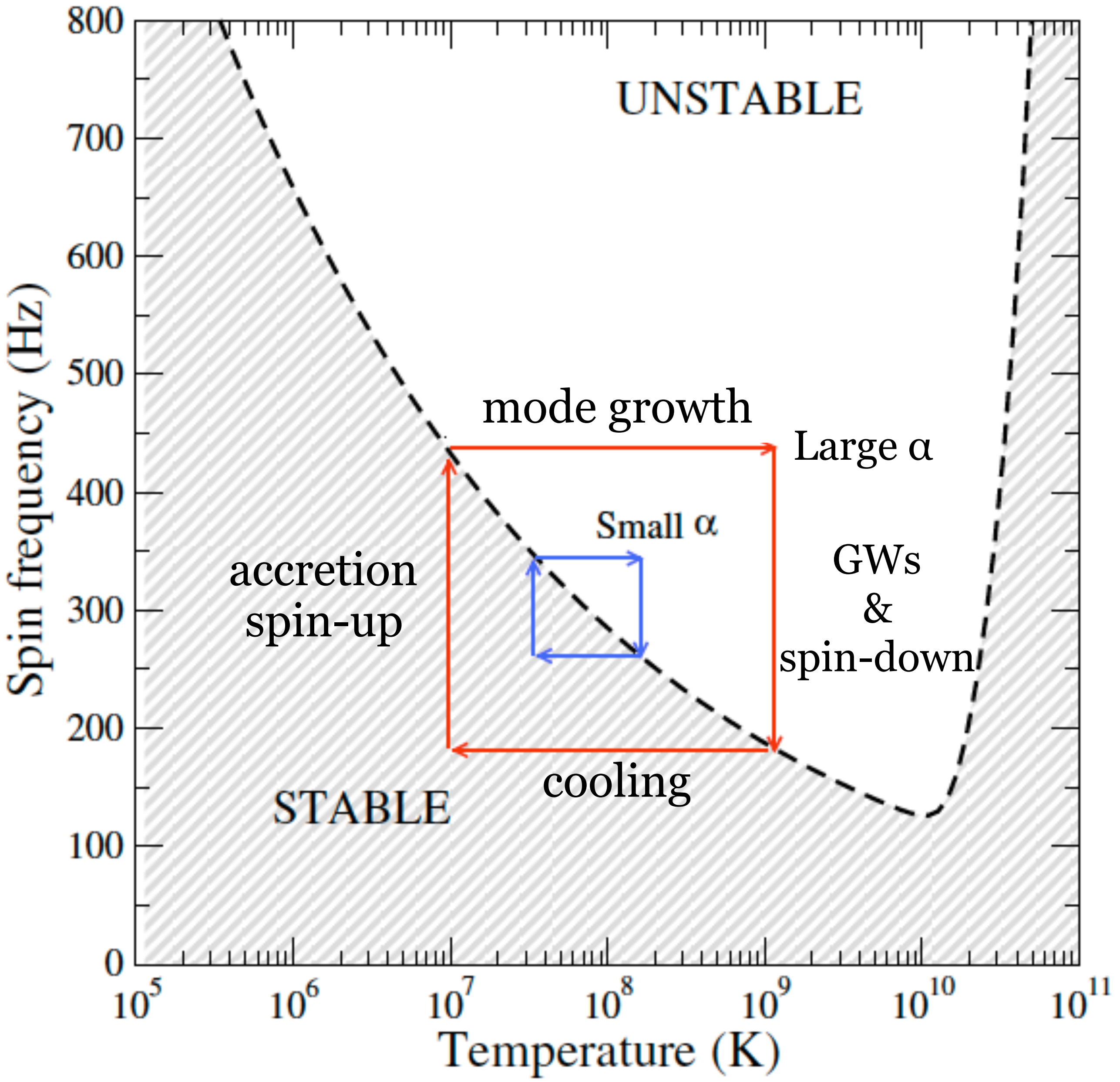
- *Three* mechanisms have been invoked :
  - Coupling between the stellar magnetic field and the accretion disc.
  - GW torque by unstable r-modes or a “mountain”.
- The r-mode amplitude required to *balance the accretion torque:*

$$\alpha_{\rm acc} \sim 10^{-6}$$

- Magnetic disk coupling can provide the necessary spin-down torque (although the underpinning accretion theory needs improvement)
- A hint: the measured spin down of two accreting systems in quiescence (SAX J1808 & XTE J1814) is consistent with the one caused by a “canonical” surface dipole field $B \sim 10^8$ G.

Spin-temperature evolution

- The r-mode-driven evolution depends on two main factors:
  - The T-slope of the curve at the point of entry.
  - The mode's saturation amplitude.
- LMXBs are likely to become unstable in the negative T-slope portion of the instability curve.
- The figure shows the resulting thermal runaway evolution ("Levin cycle").



[Haskell et al. 2014]

r-mode cycle: detectability

- The detectability of r-mode-“cycling” LMXBs is a subtle issue.
- The GW duty cycle (=fraction of the cycle spent in GW emission) is :

$$D \approx \frac{t_{\rm cycle}}{10^7\,{\rm yr}} \approx \frac{10^{-11}}{\alpha^2}$$

- If α is too big, D is too low and no system would be observed being unstable.
- Combine D with the LMXB birth rate ~ $10^{-5}/{\rm yr}/{\rm galaxy}$ and lifetime ~ 10^7 yr and estimate the amplitude for which a system is always “on” in our galaxy:

$$D \lesssim 10^{-2} \;\Rightarrow\; \alpha \lesssim 10^{-4}$$

- For the system to be detectable at (say) 10 kpc we need: $\alpha \gtrsim 10^{-6}$
- A small-ish r-mode amplitude is actually better for detecting LMXBs!

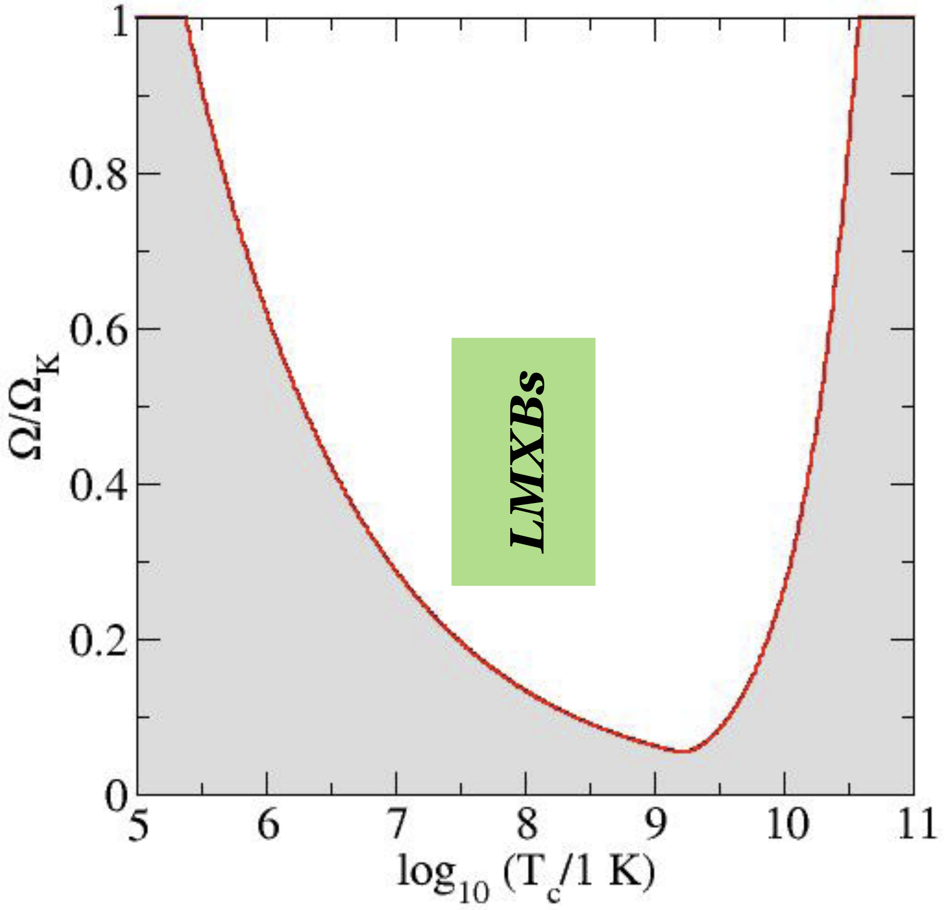
r-mode puzzle?

- Several LMXBs (and perhaps some MSPs) reside well inside the “minimal” instability window.

- These systems should experience r-mode-driven evolution and GW spin-down.

- This is *not* what observations suggest.

  Possible resolutions:



- *Additional damping*
  (e.g. friction at the crust-core boundary, exotica in the core, …).

- r-mode amplitude *much smaller* than current theoretical predictions.

r-modes: extra damping

- Several other mechanisms could dampen the r-mode instability:
  - An Ekman-type boundary layer at the crust-core interface.
  - Bulk viscosity due to exotica (hyperons/quark matter).
  - Superfluid mutual friction due to vortex-fluxtube interactions.
  - Coupling between the r-mode and superfluid modes.

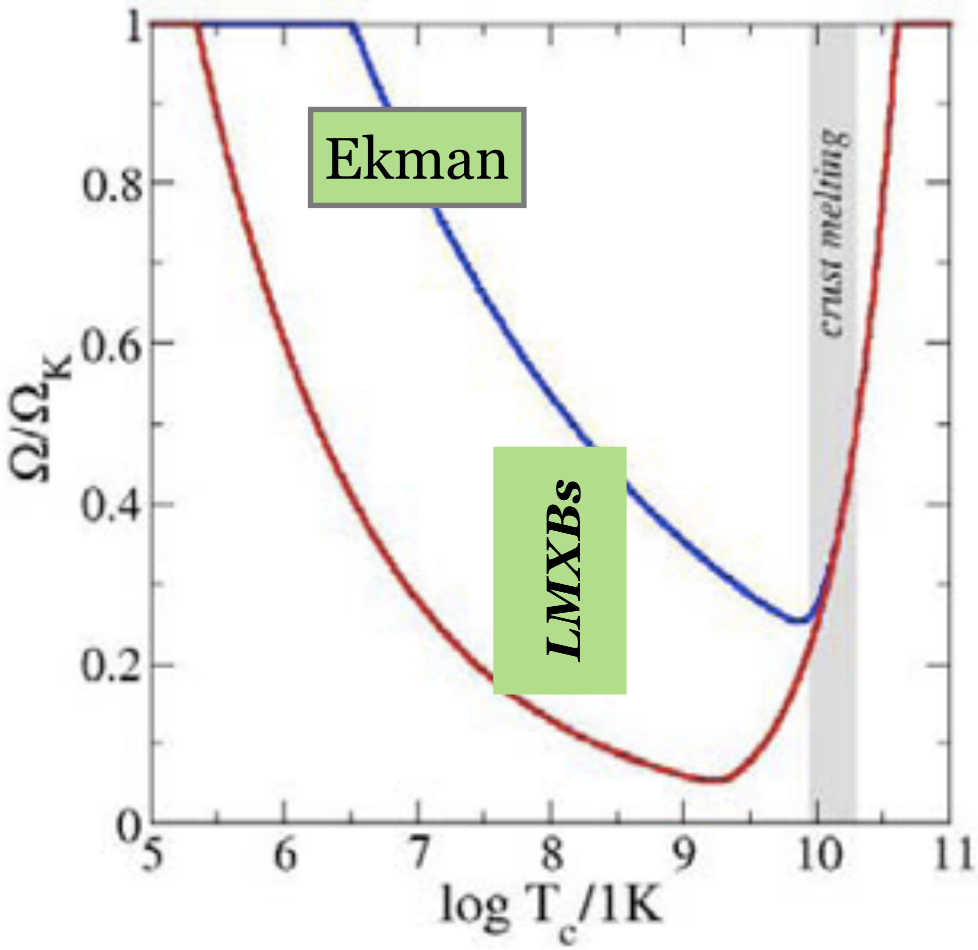
The role of the crust

- r-mode damping could be easily dominated by the viscous "rubbing" at the base of the crust

- The crust is more like a jelly than solid: the resulting crust-core "slippage" reduces damping

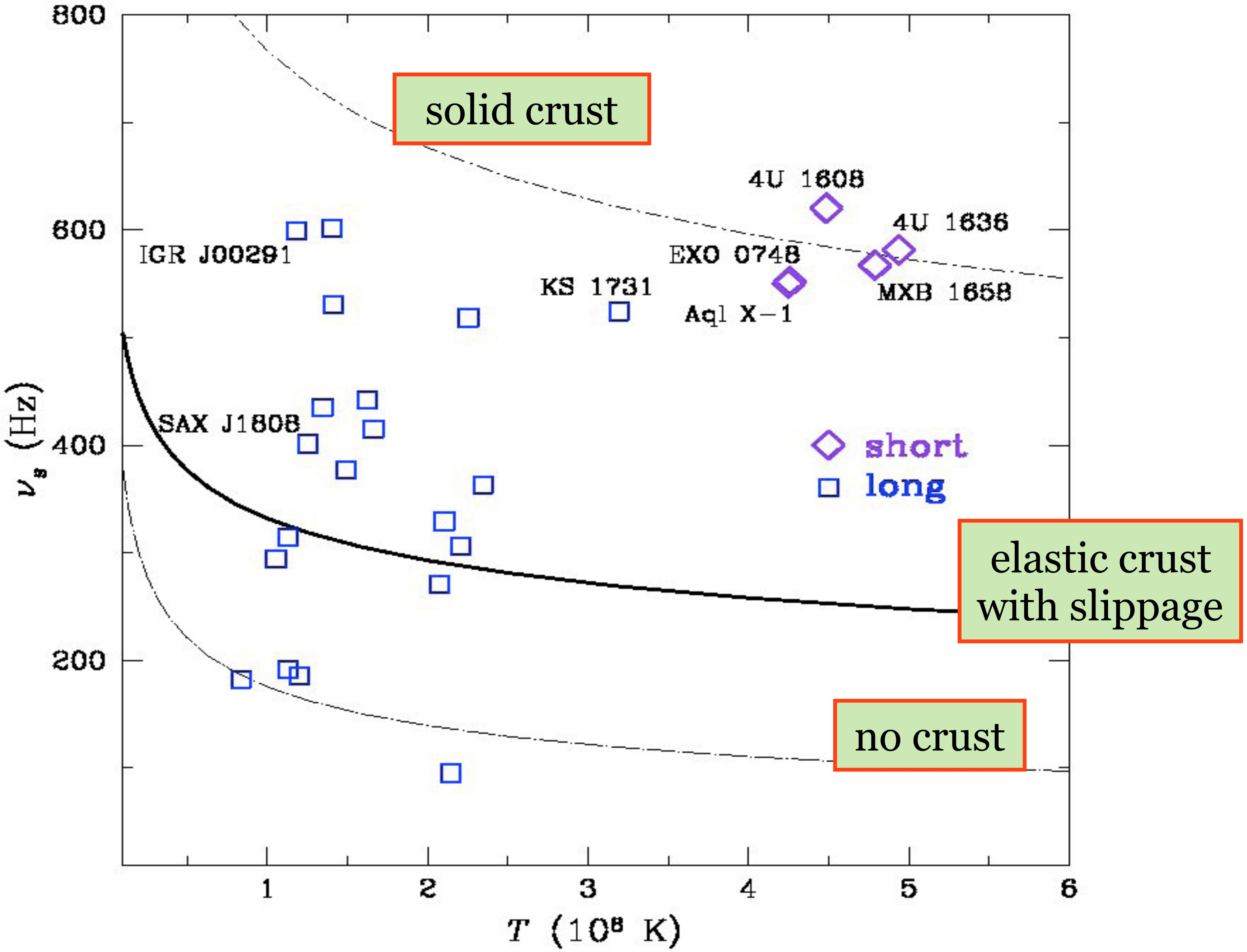
- Resonances between the r-mode and torsional crustal modes may also play a key role.

- Existing work assumes a "sharp" crust-core transition … but how safe is this assumption?



[KG & Andersson 2006]

r-mode window: “theory vs observations”



[Ho et al. 2011]

Magnetic boundary layer

- The Ekman layer physics is significantly modified by the local B-field:

– Crust-core *slippage is suppressed* (= damping amplified)

– Above a threshold, the B-field enhances the damping rate.

– The layer's thickness grows with B, so the B-field shouldn't be too strong.

- In LMXBs (and MSPs) the magnetic field ($B \sim 10^8$ G) can indeed lead to enhanced damping, provided the outer core is superconducting:

$$\frac{\dot{E}_B}{\dot{E}_{\rm visc}} \approx 30 \left(\frac{B}{10^8 \text{ G}} \right)^{1/2}$$

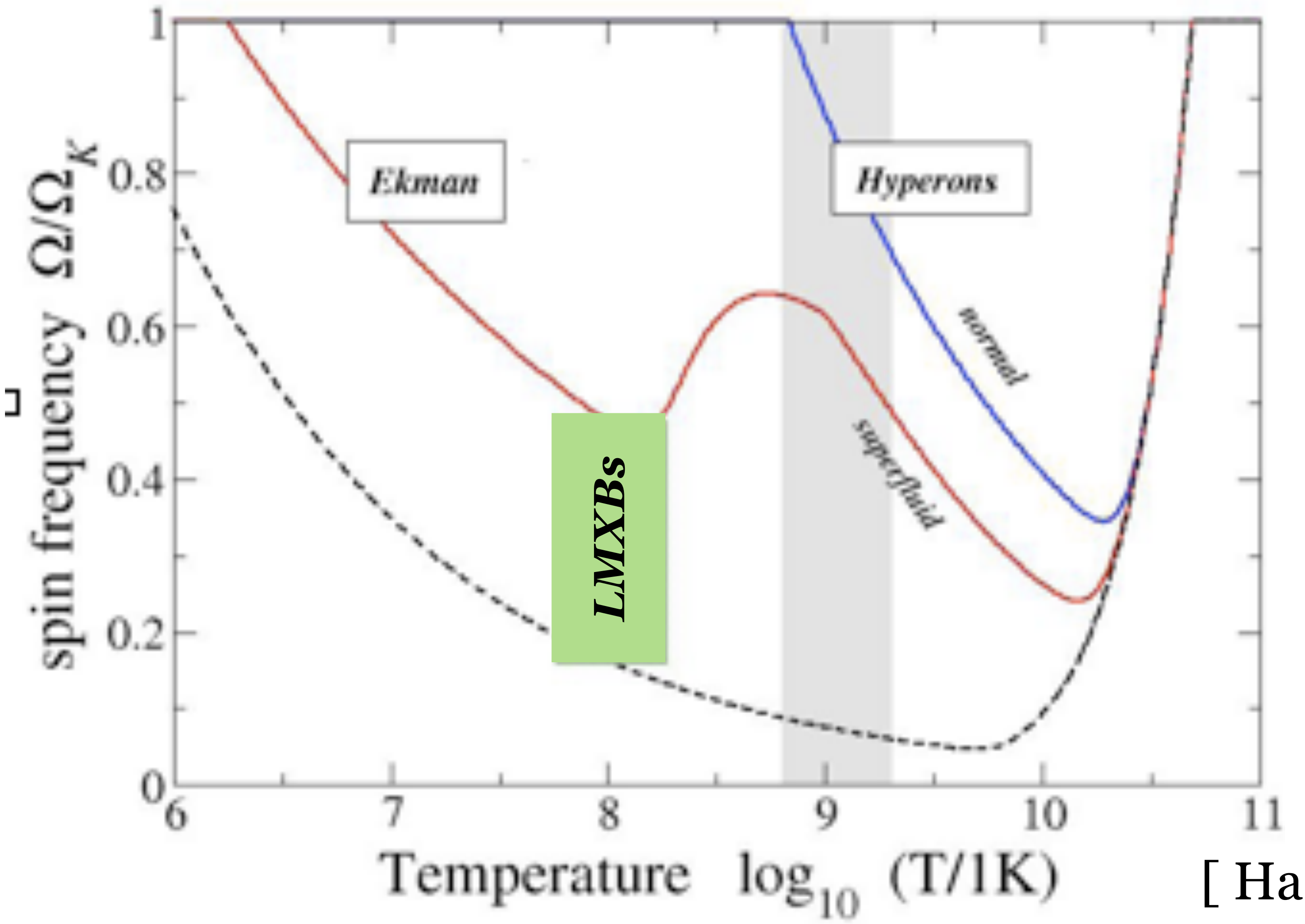
- This (approximate) result would render these systems r-mode-*stable*.

- But: we need more realistic crust-core interface modelling (with superfluidity, superconductivity, finite thickness transition etc.)

Exotic bulk viscosity

- A neutron star core populated by hyperons and/or quarks leads to strong bulk viscosity and a significantly modified r-mode instability window.

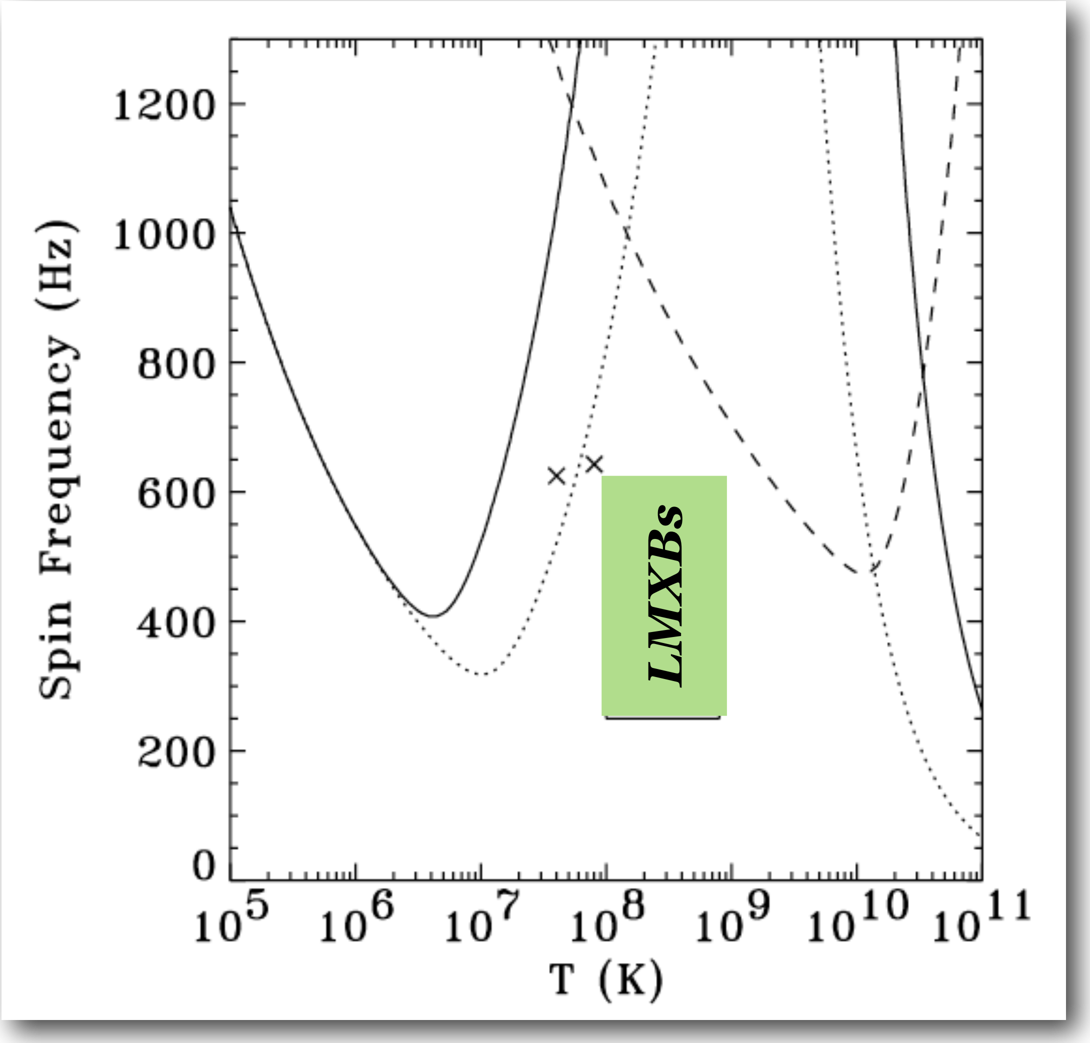
- Below we show examples of such windows but these can vary as a function of the poorly known properties of exotic matter.



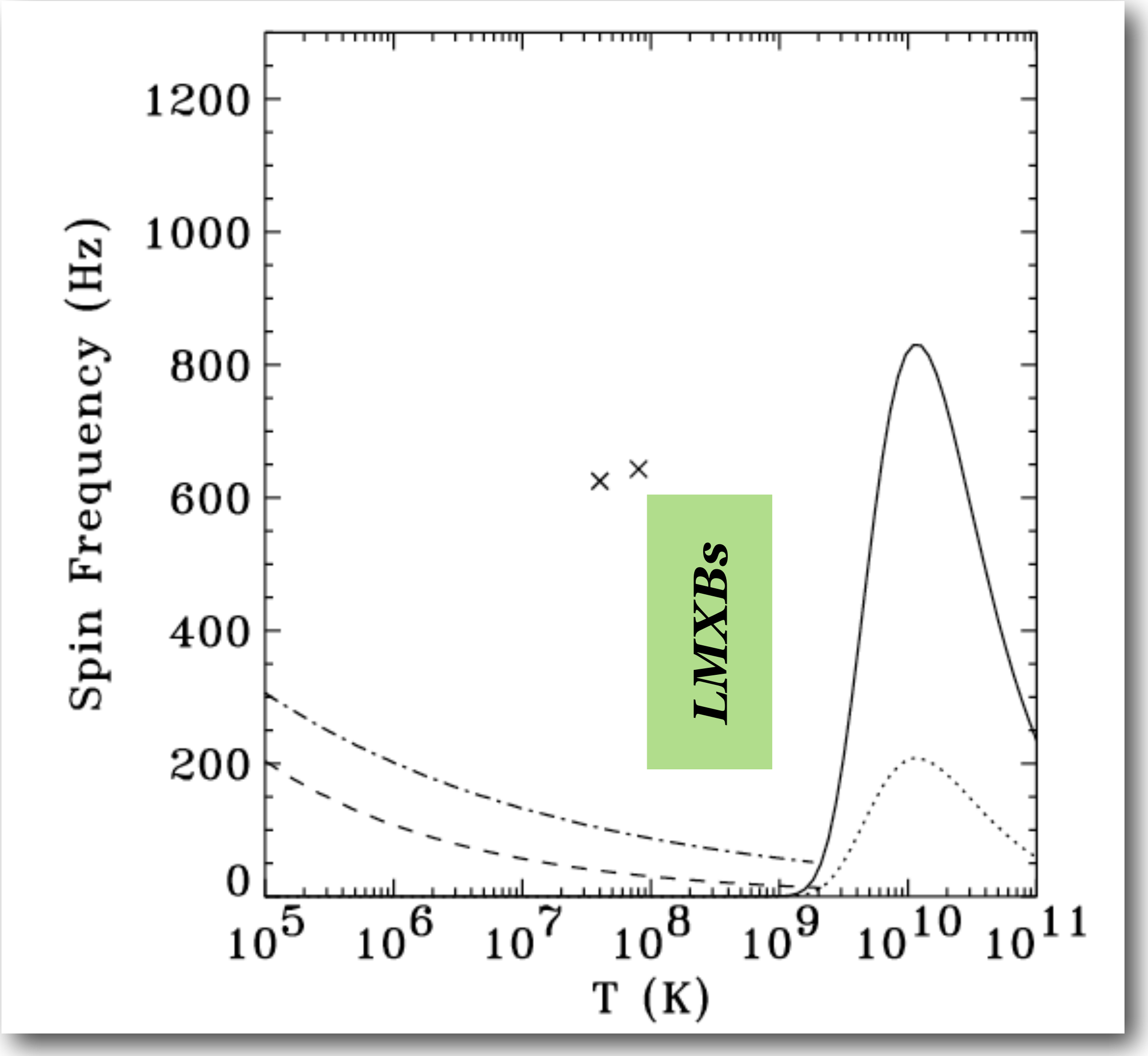
[Haskell & Andersson 2010]

Exotic bulk viscosity

Quarks (without pairing)



Quarks (with pairing)

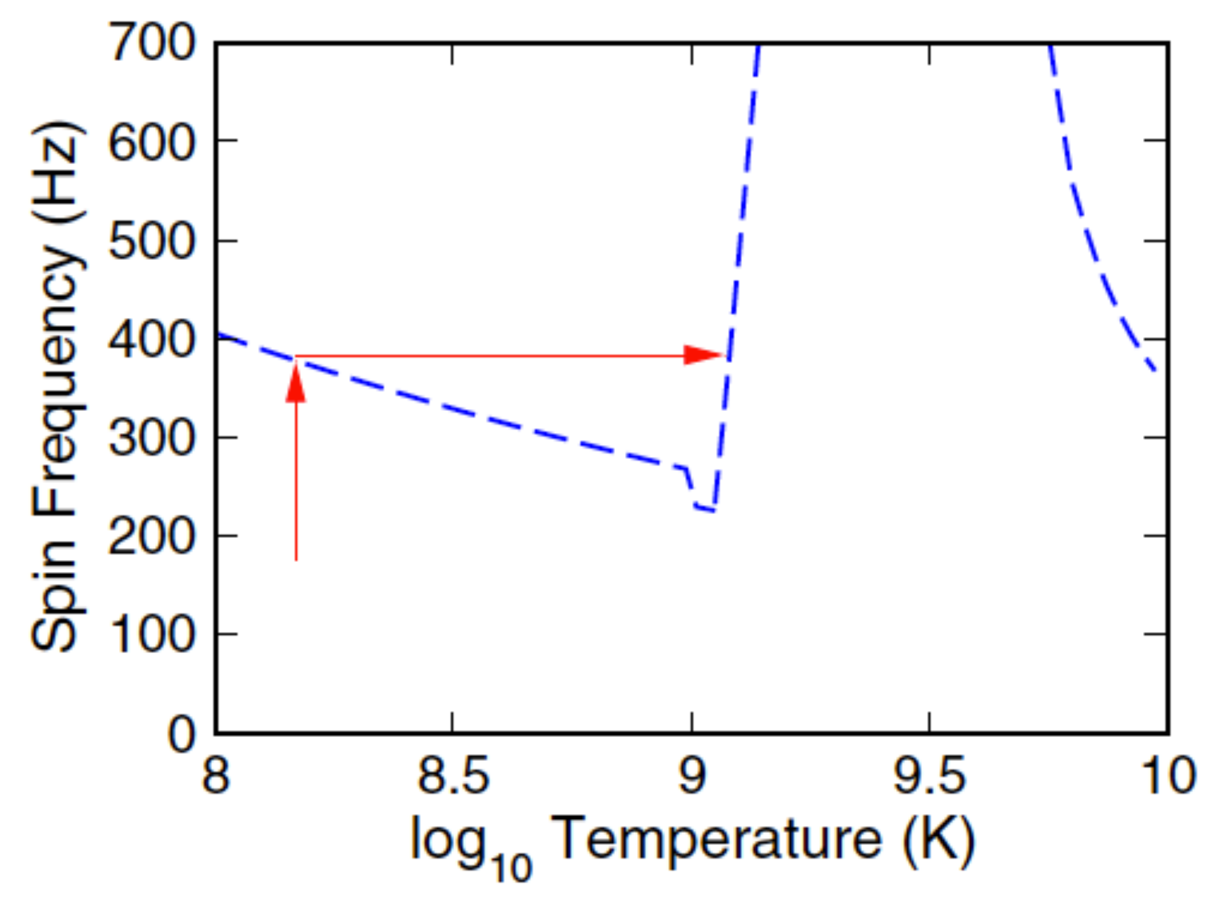


[Madsen 2000]

Persistent r-mode emission

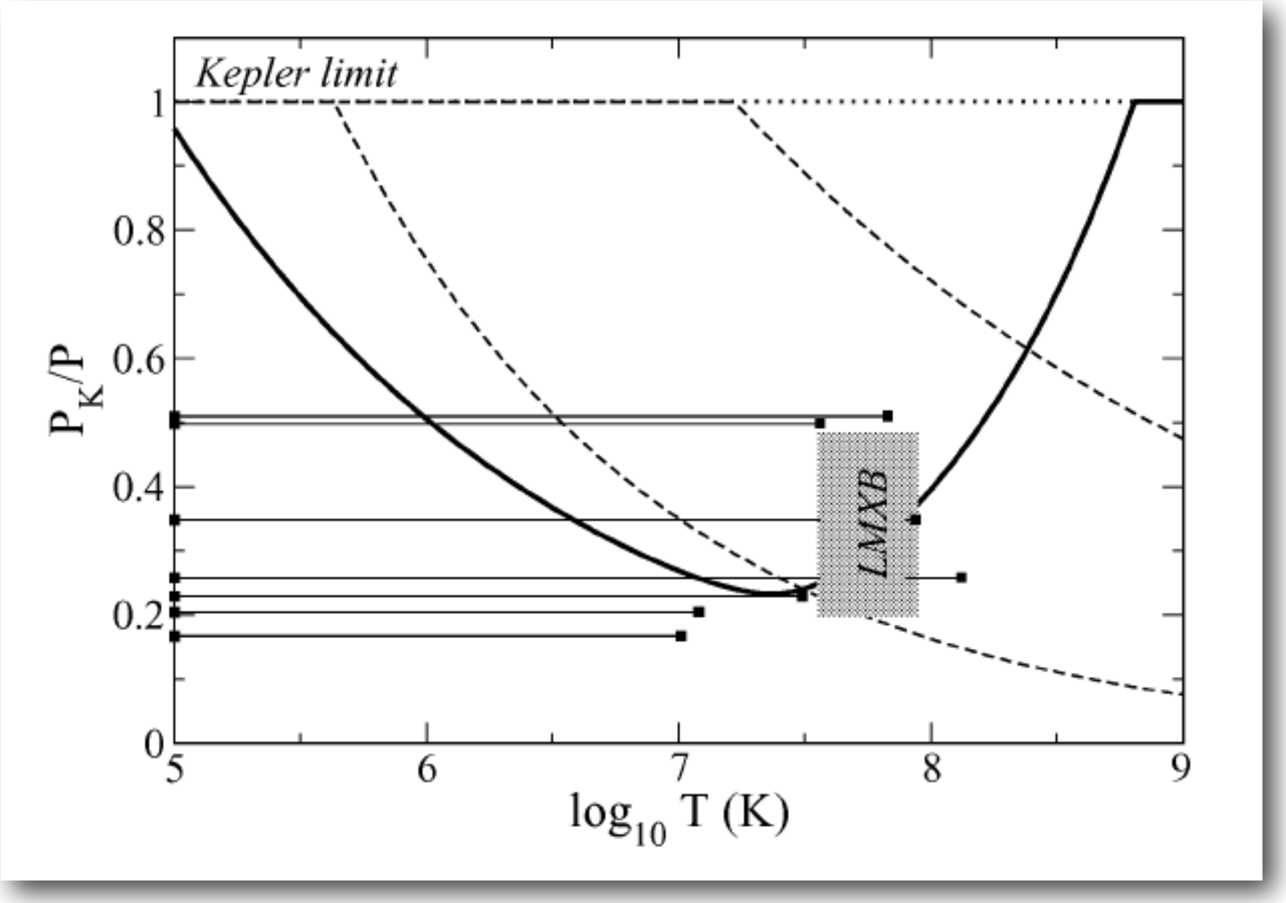
- The presence of exotica may drive LMXB-evolution near the positive T-slope instability curve, and the emission of persistent GWs. This could be potentially detectable by advanced detectors.

Hyperon core



[Nayyar & Owen 2006]

Quark core



[Andersson et al. 2002]

r-mode observed? XTE 1751

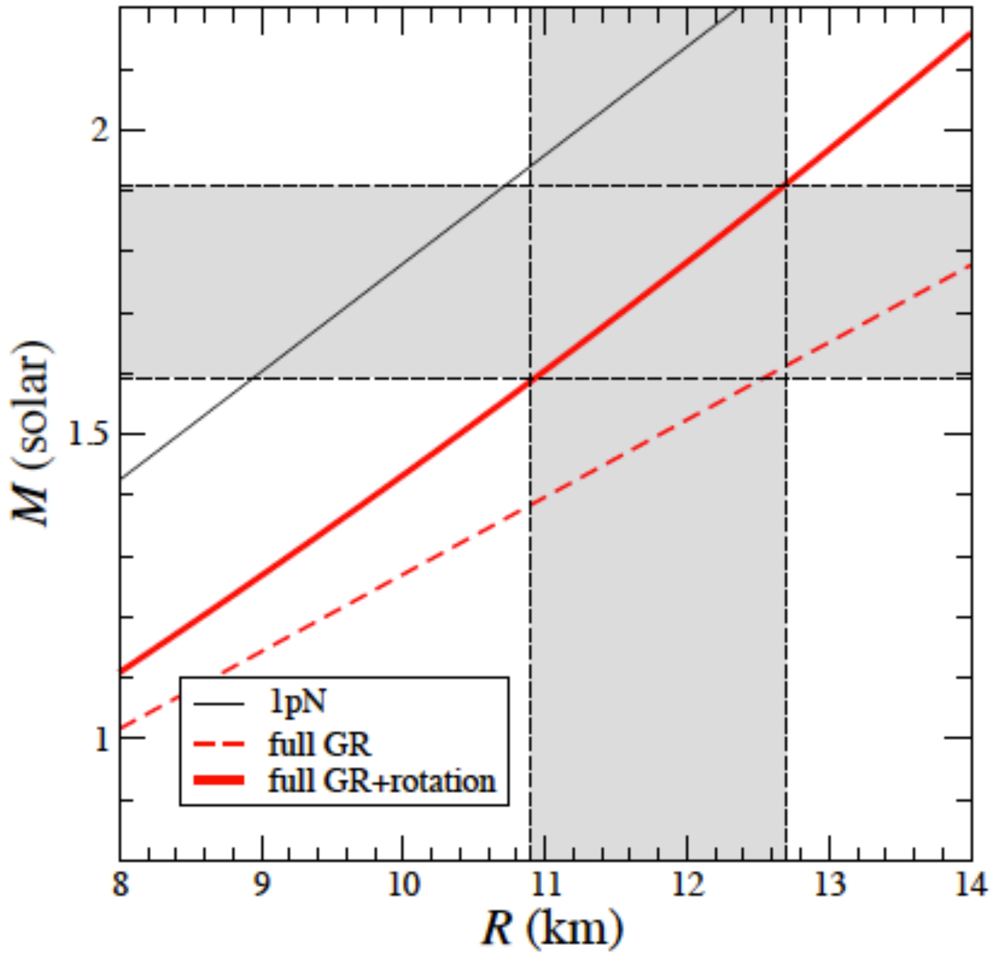
- XTE 1751-305 is an AMXP (accretion-powered X-ray pulsar)

- Recent result: a coherent oscillation was discovered in a burst light curve:

$$f_{\rm osc} = 0.572 f_{\rm spin}$$

- Provided the light curve is modulated by a global mode, the observed signal could be an r-mode. The numbers can match provided we account for *relativistic corrections* in the mode frequency.

- But: inferred r-mode amplitude is too large to be reconciled with the system's spin evolution.



[Andersson et al. 2014]

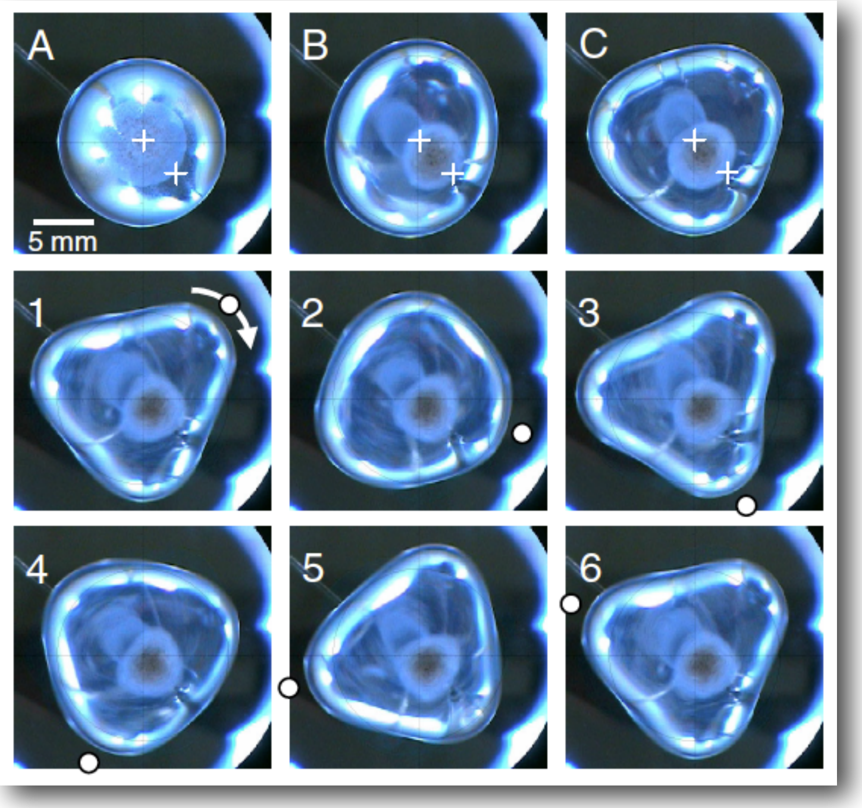
The *f*-mode instability

- The *f*-mode is a powerful emitter of GWs (via the mass multipoles) but only becomes unstable at high rotation:

$$\Omega > \Omega_{\rm CFS} \approx 0.9\,\Omega_{\rm K}$$

- The instability is active in the high-temperature regime, appropriate for newborn stars. At lower temperatures, it is suppressed by superfluid vortex mutual friction.

- The growth timescale strongly depends on the spin difference $\Omega - \Omega_{\rm CFS}$ and the stellar compactness M/R. *Need for GR calculations.*



"*f*-mode instability" in a rotating liquid drop

f-mode: instability window

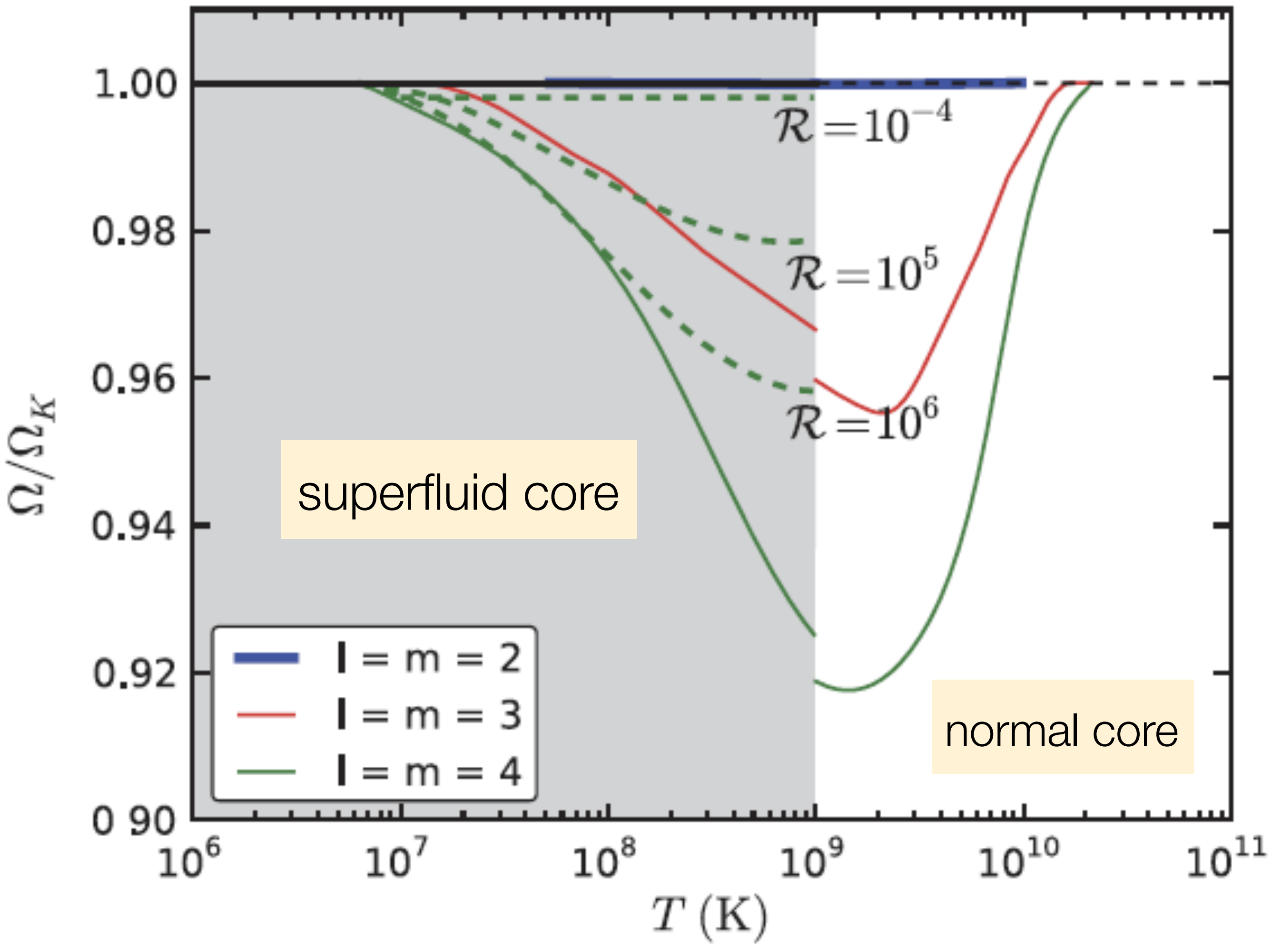
- GR calculations:

$$\tau_{\text{grow}} \sim 10^4 - 10^6 \text{ s}$$

(this is about a factor ~ 10 shorter than earlier Newtonian results).

- Typically, the $\ell = m = 4$ mode is the most unstable one.

- The growth timescale can become much shorter (and the instability window much bigger) for massive neutron stars.



[Gaertig et al. 2011]

Stellar model:

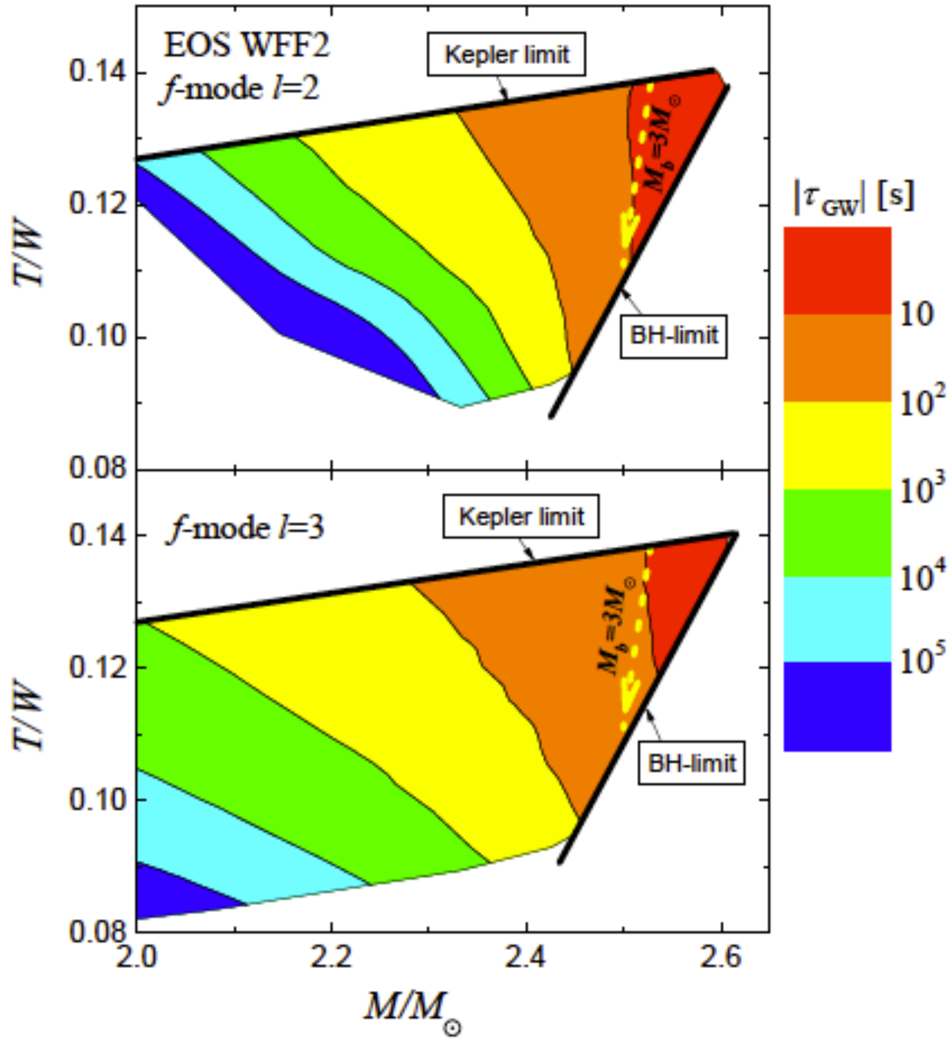
N = 0.73 polytrope

$M = 1.48 M_\odot, \quad R = 10.47 \,\text{km}$

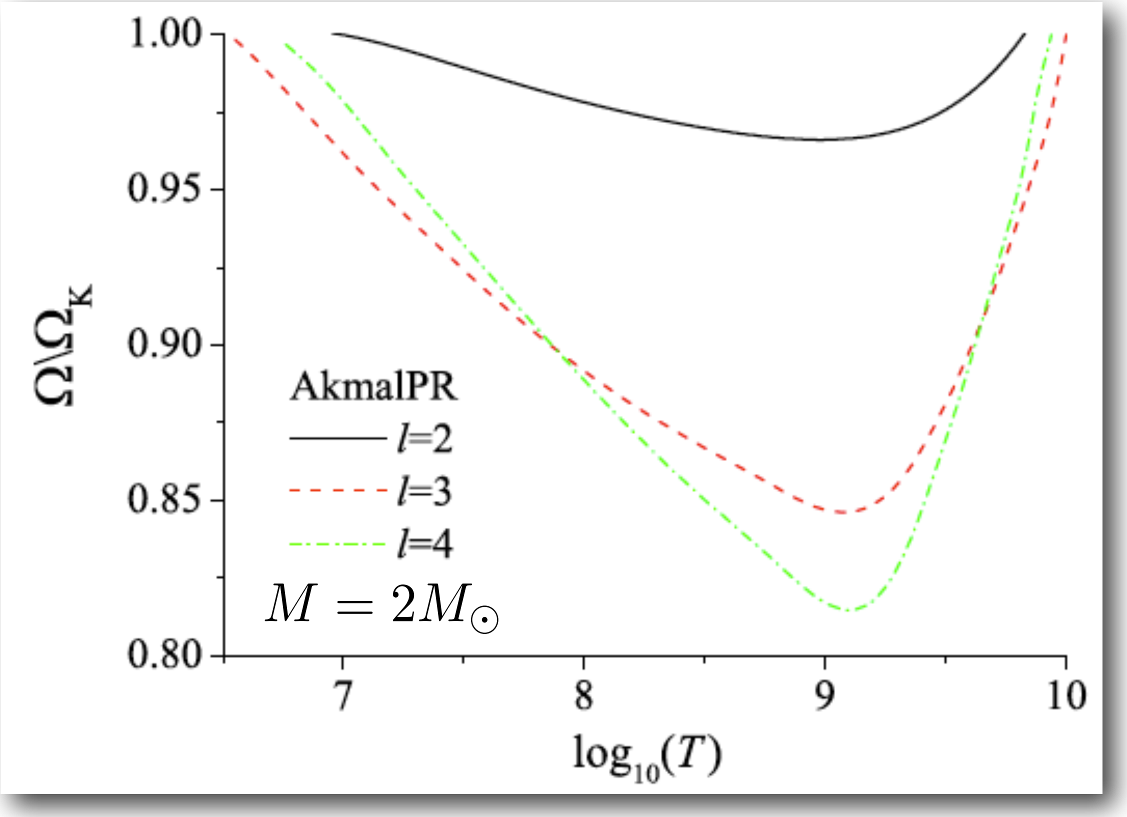
f-mode: (supra) massive neutron stars

- An "optimal" arena for the *f*-mode instability is a post-merger-formed massive neutron star ($M \gtrsim 2M_\odot$). Such a system is associated with short GRBs.

- For the expected mass range the growth timescale can be < 100 sec.



[Doneva et al. 2013, 2015]

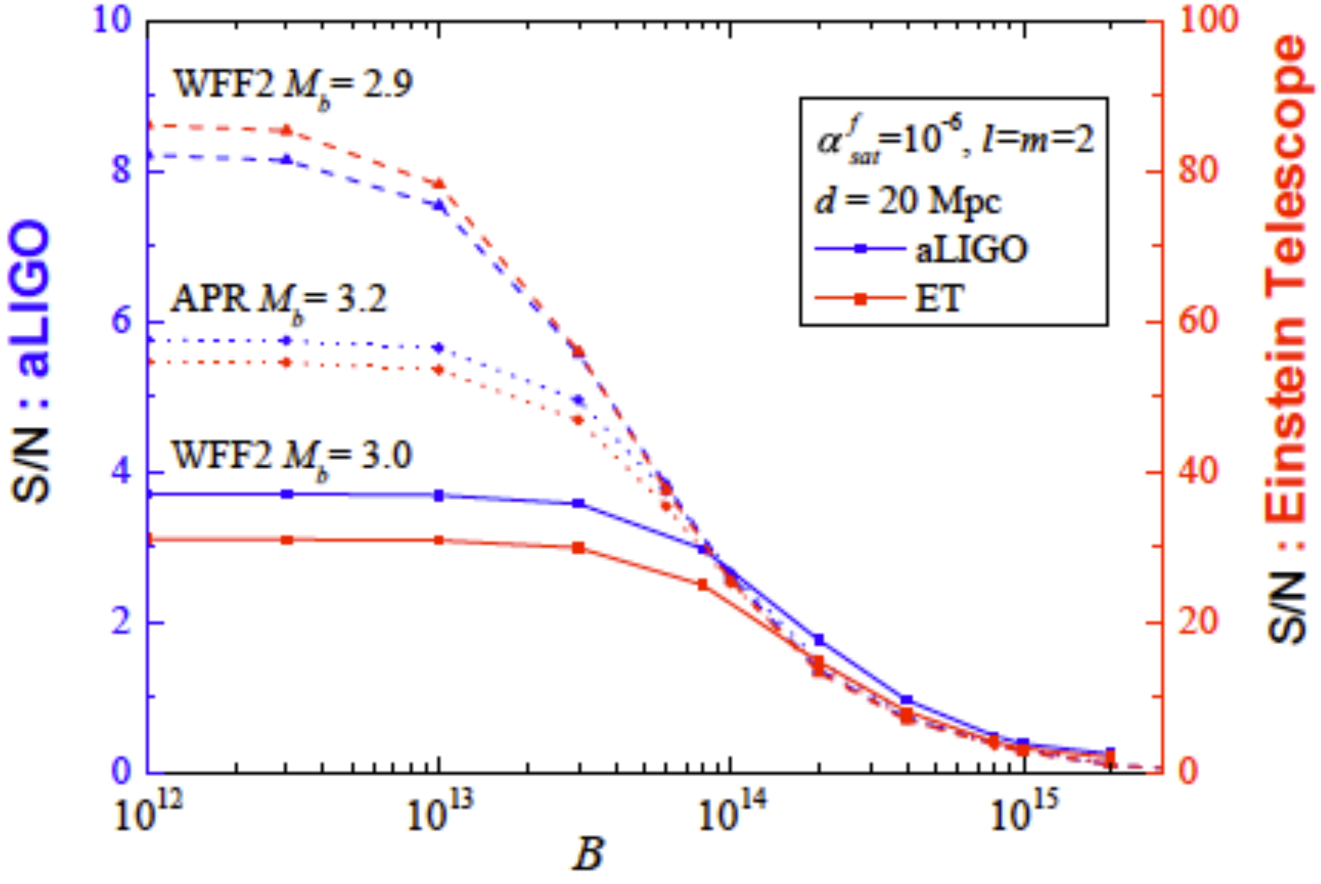


f-mode: detectabilty

- Recent work on the *f*-mode's *saturation energy* (due to non-linear mode couplings) suggests:

$$E_{\rm mode} \lesssim 10^{-6} M c^2$$

- Post-merger remnants *could be detectable* by ET (or by aLIGO, but this would require much shorter distances than those associated with observed sGRBs).

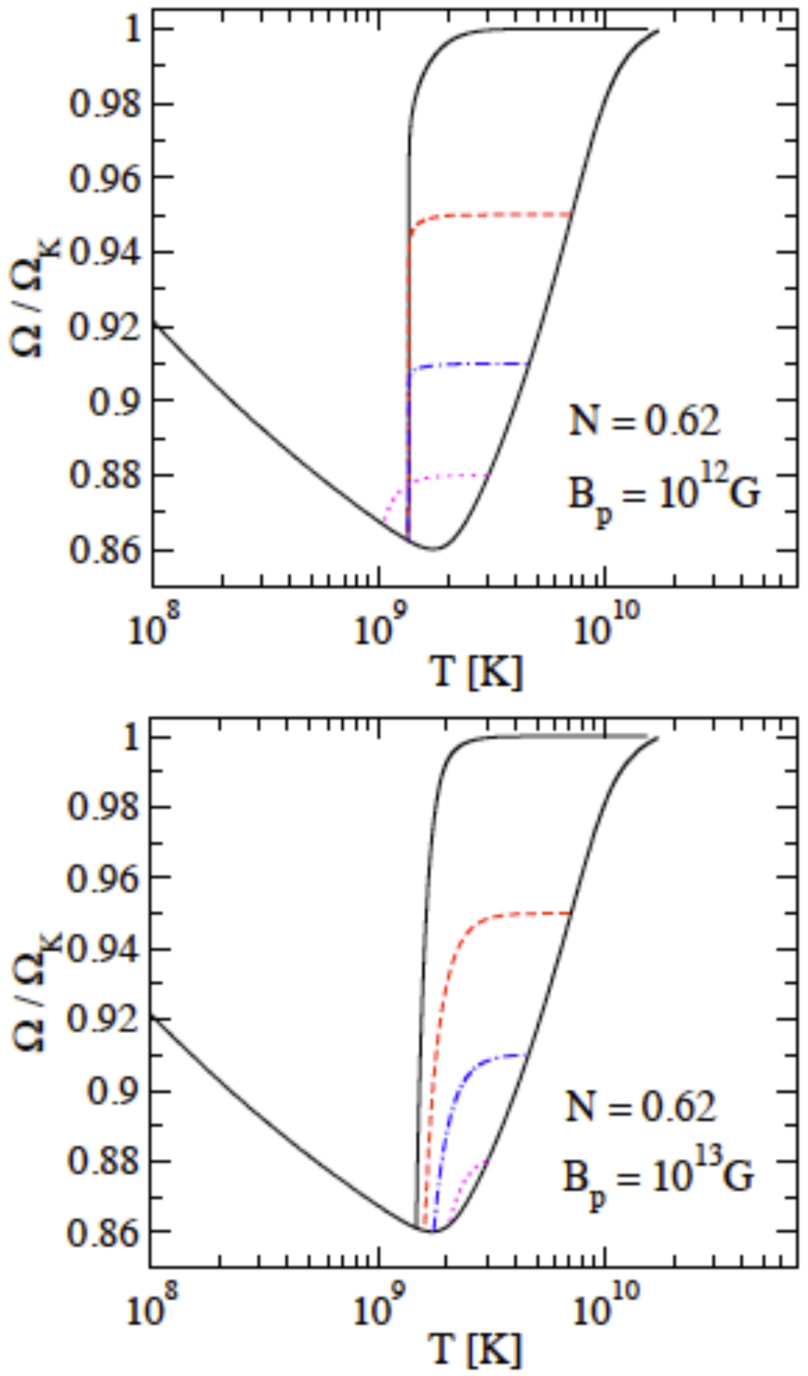


[Doneva et al. 2015]

- The unstable *f*-mode competes against the magnetic dipole spin down.

f-mode instability: evolutions

- Newborn neutron stars: coupled spin-temperature evolution, under the combined action of an unstable *f*-mode and magnetic dipole spindown.
- For realistic saturation amplitudes, the mode grows and decays before (say) the onset of superfluidity.
- Figure: $\ell = m = 4$ mode, $M = 1.98\,M_\odot$



[Passamonti et al. 2013]

(some) Theory assignments

- r-mode:
  - ✓ Detailed modelling of crust-core boundary (B-field, finite thickness, the effect of superfluidity & superconductivity).
  - ✓ Resonances between the r-mode and other modes.
- *f*-mode:
  - ✓ GR calculations without the Cowling approximation.
  - ✓ GR calculation of saturation energy.

Outlook

Prospects for detection of GWs from unstable modes are not too optimistic, but both mechanisms remain viable sources for aLIGO and ET-calibre GW observatories.