3D Hall-driven magnetic field evolution in neutron star crusts

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Lessons from 2D magnetic evolution

- Hall physics known to drive NS crustal evolution for magnetically-powered sources
  Pons & Geppert (2009), Vigano+ (2013), Gourgouliatos & Cumming (2014)
- Hall drift dominates magnetic reconnection
  - NS crust and magnetar magnetospheres
    Shepherd & Cassak (2010, 2012), Beloborodov (2009)
- NS crust: Myr decay // NS core: Gyr decay
  Goldreich & Reisenegger (1992), Rea+ (2012), Vigano+ (2013)
  Graber+ (2015), Elfritz+ (2016)

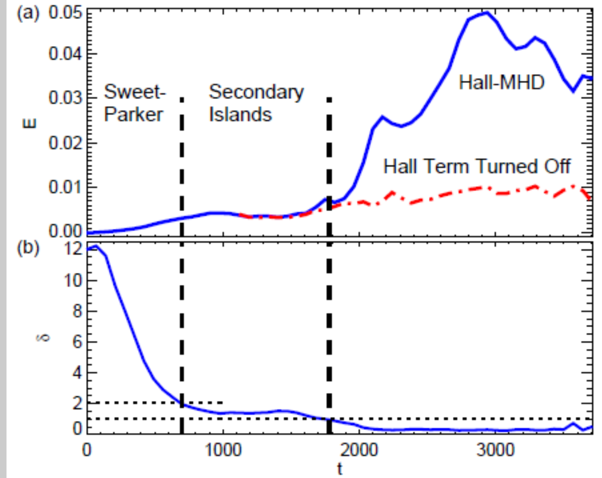


FIG. 1: (Color) (a) Reconnection rate E as a function of time t. The solid (blue) line is a Hall-MHD run. Dashed lines at $t \sim 700$ and 1780 indicate the onset of secondary islands and Hall reconnection, respectively. The dot-dashed (red) line shows E for a simulation restarted at $t = 1120$ with no Hall effect and $m_e = 0$. (b) Thickness δ of the dissipation region vs. t. Horizontal dotted lines mark predicted δ for the onset of secondary islands ($\delta \sim 2$) and Hall reconnection ($\delta \sim 1$).

Shepherd & Cassak (2010)

$$\tau_{\rm Ohm} = \frac{4\pi\sigma L^2}{c^2}$$

$$\tau_{\rm Hall} = \frac{4\pi e n_e L^2}{cB}$$

$$\tau_{{\rm Hall},l} = \frac{4\pi e n_e L^2}{l^2(l+1)^2 cB}$$

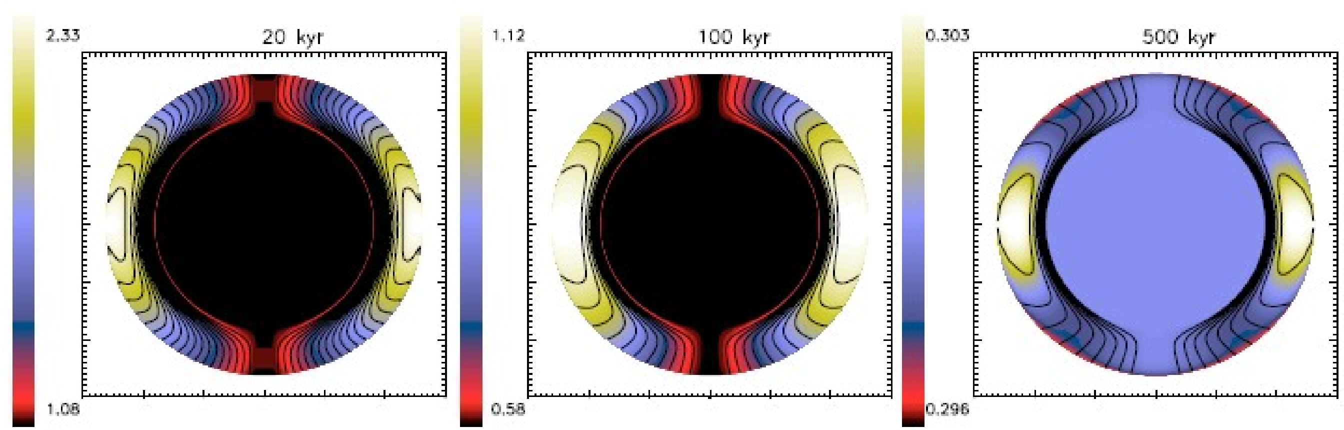
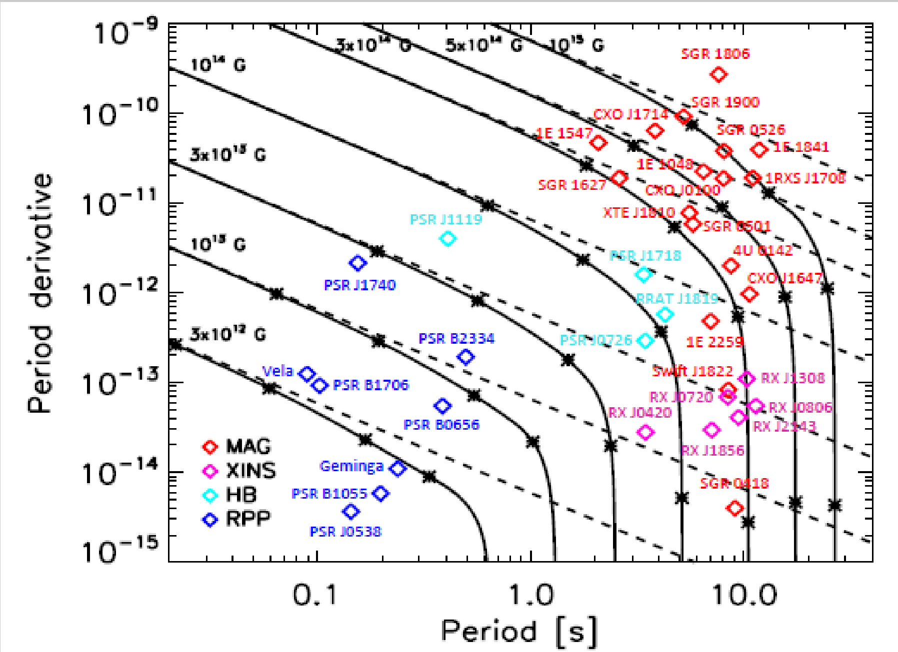


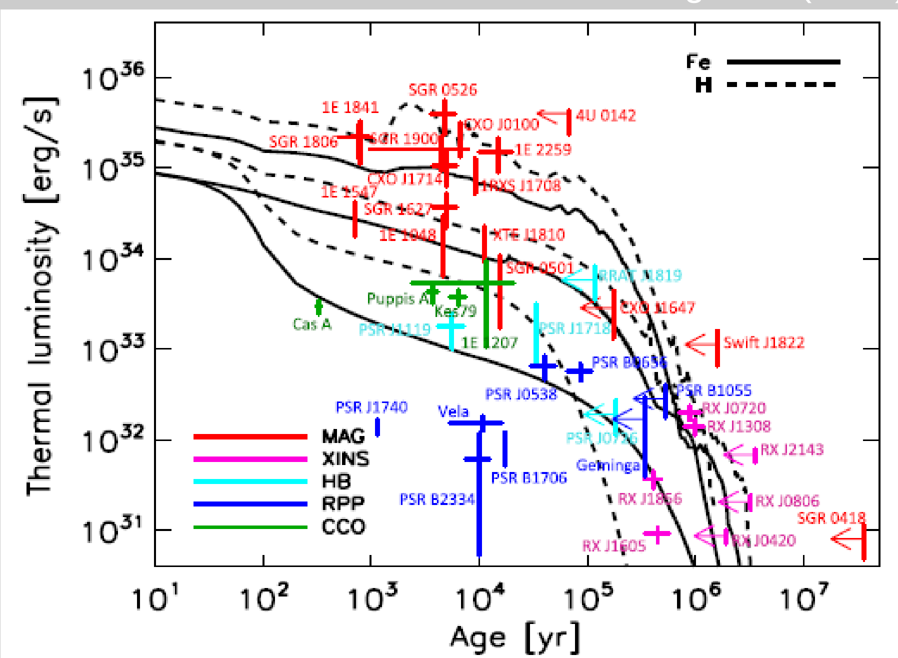
Fig. 4. Temperature distribution in the crust of a NS (i.e. *up to the bottom* of the envelope) at different ages. The initial MF is purely poloidal with (B_p = 10^{14} G). The field lines are also shown. The numbers on the color-scale on the left of each figure indicate the maximum and minimum values of the temperature (in units of 10^8 K) at each age. The crustal shell has been stretched a factor of 4 for clarity.

Pons & Geppert (2009)

2D and observations

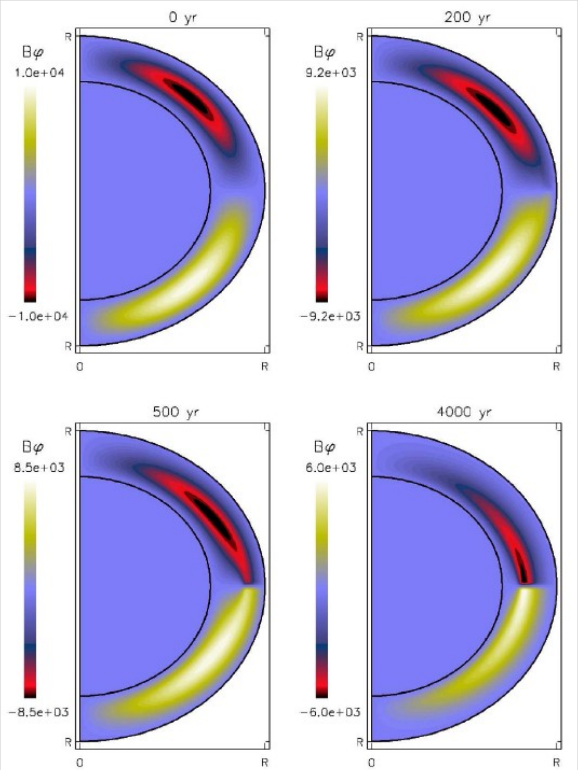


Viganò+ (2013)



- Fundamental observables determined by ICs, mass, composition
- Population synthesis distinguishes between birth of radio pulsars and magnetars

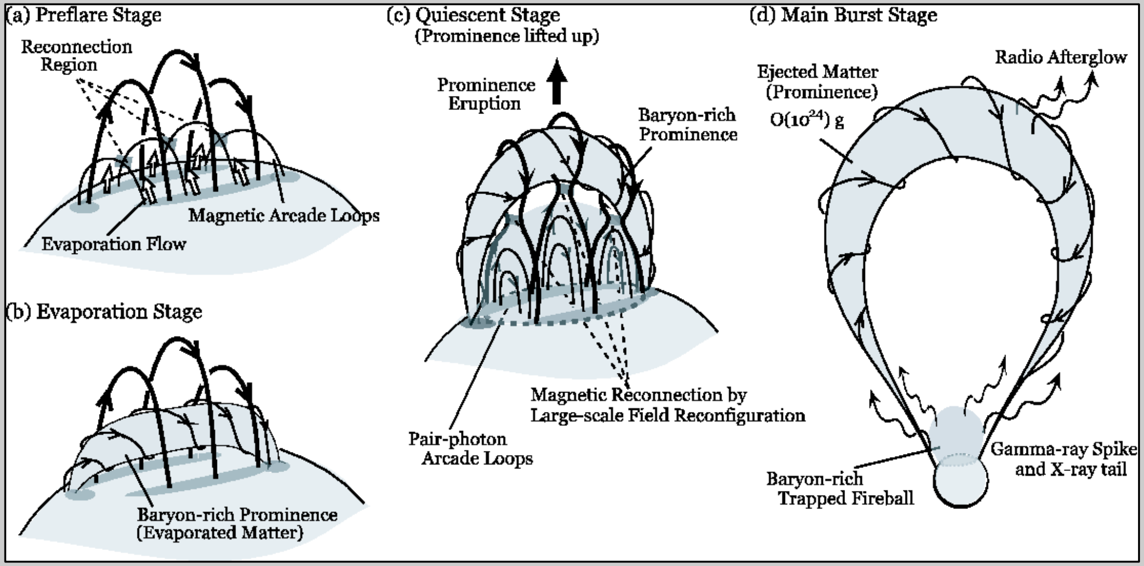
Gullón+ (2014, 2015), Rea+ (2015)



Viganò+ (2012)

Transitioning to 3D

- NS magneto-thermal-rotational-magnetospheric evolution is fundamentally a 3D problem
  - Creation and dynamics of hot spots, magnetic bundles



Masada+ (2010)

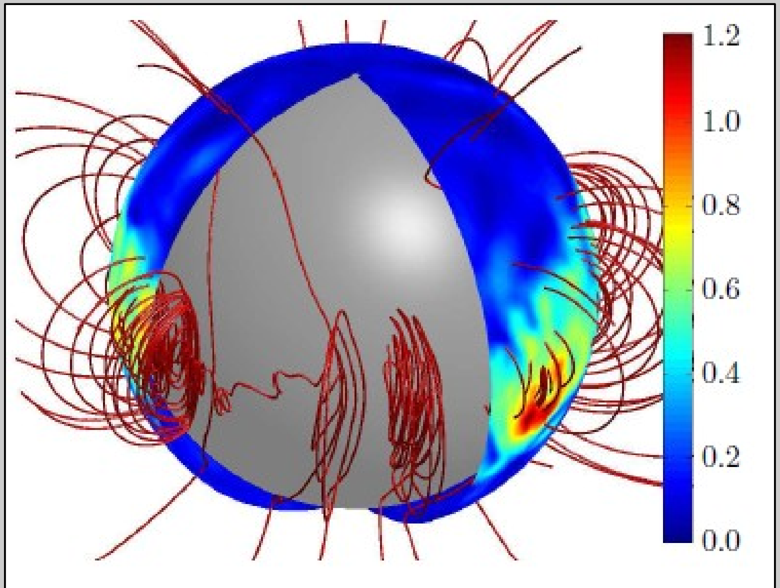


FIG. 5. Lines of the magnetic field generated by currents in the crust. The coloring indicates the ratio of magnetic pressure to breaking stress at the surface of the crust. A portion of the surface has been cut away to show fieldlines inside the crust, whose lower boundary is the gray sphere.

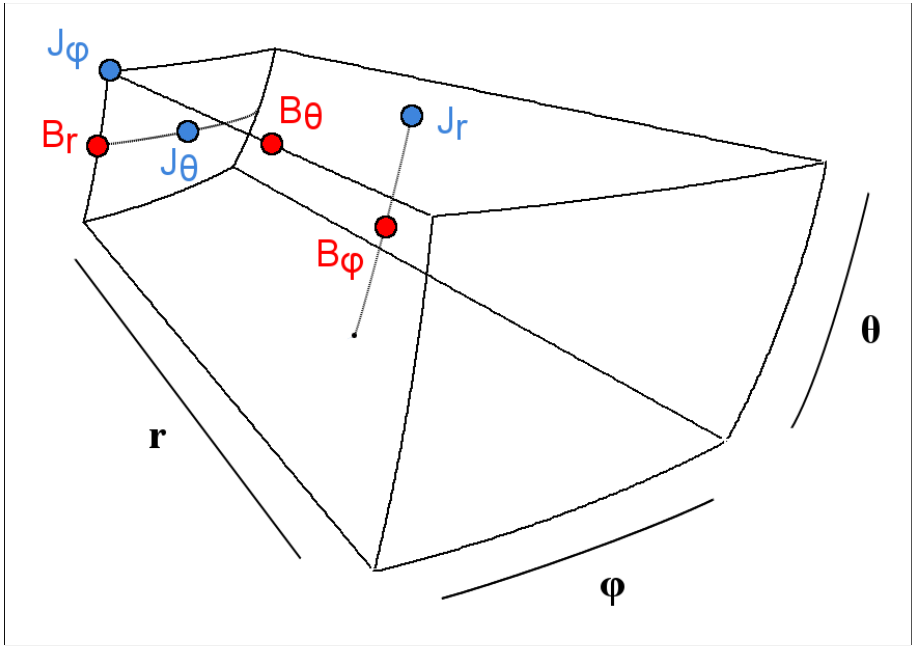
Wood & Hollerbach (2015)

- Hall effect known to exhibit fundamentally 3D dynamics in solar atmospheres, magnetospheres

3D mathematical formalism

- Obtain solutions to 3D magnetic induction equation
- Spatially staggered grid variables
- Exact solutions for Ohmic evolution
- Upwind scheme for Hall term
  - J, v_e determine wind direction

$$\partial_t \vec{B} = -c\vec{\nabla} \times \left(e^{\nu} \vec{E} \right)$$



$$\vec{E} = \frac{1}{\sigma}\vec{J} + \frac{1}{cen_e}\vec{J} \times \vec{B}$$

$$J_{\alpha}^{ijk} = \frac{c}{4\pi} \frac{e^{-\nu_j}}{S_{\alpha}^{ij}} \oint_{\partial S_{\alpha}^{ij}} e^{\nu_j} \vec{B} \cdot \mathrm{d}\vec{l}$$

$$B_{\alpha,\mathrm{n}}^{ijk} = B_{\alpha,\mathrm{n}-1}^{ijk} - \frac{c\Delta t}{S_{\alpha}^{ij}} \oint_{\partial S_{\alpha}^{ij}} e^{\nu_j} \vec{E} \cdot \mathrm{d}\vec{l}$$

3D mathematical formalism

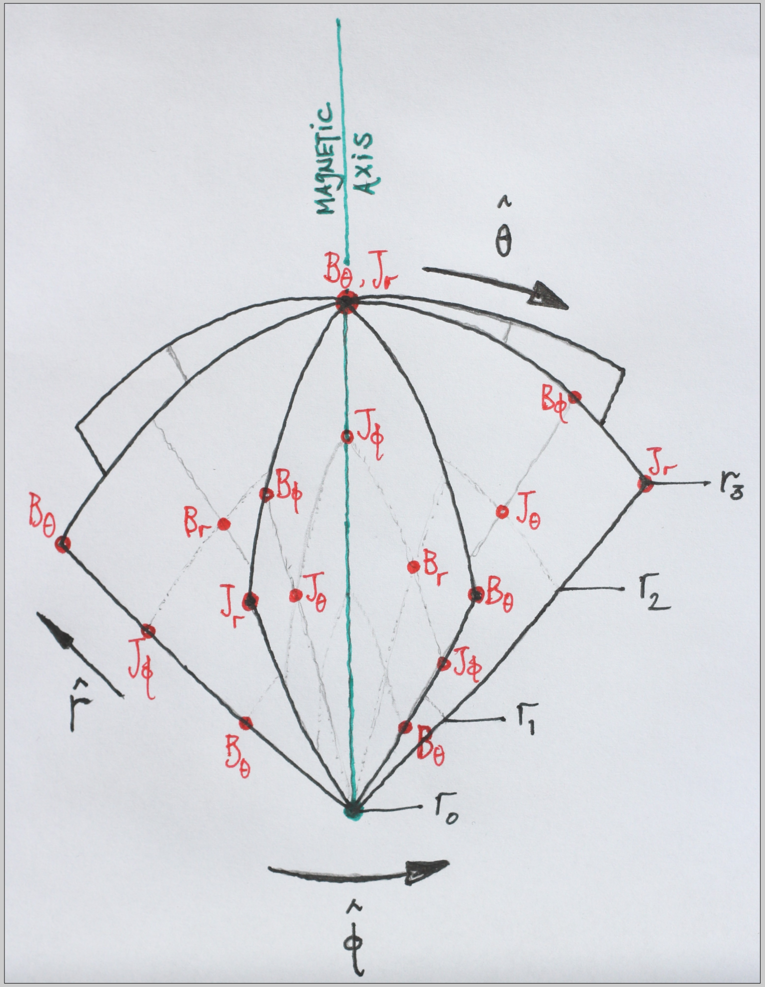
- Two-step time advance, validated in various 2D and 3D geometries
    - Toth+ (2008), Viganò+ (2012)

$$\vec{B} \rightarrow \vec{J} \rightarrow \vec{E}_{r,\theta} \rightarrow \vec{B}_{\phi} \rightarrow \vec{J}^{*}_{r,\theta} \rightarrow \vec{E}_{\phi} \rightarrow \vec{B}_{r,\theta}$$

- Non-trivial treatment of Burgers-like behavior and shock formation

- Adaptive timestep

- Special treatment of magnetic pole



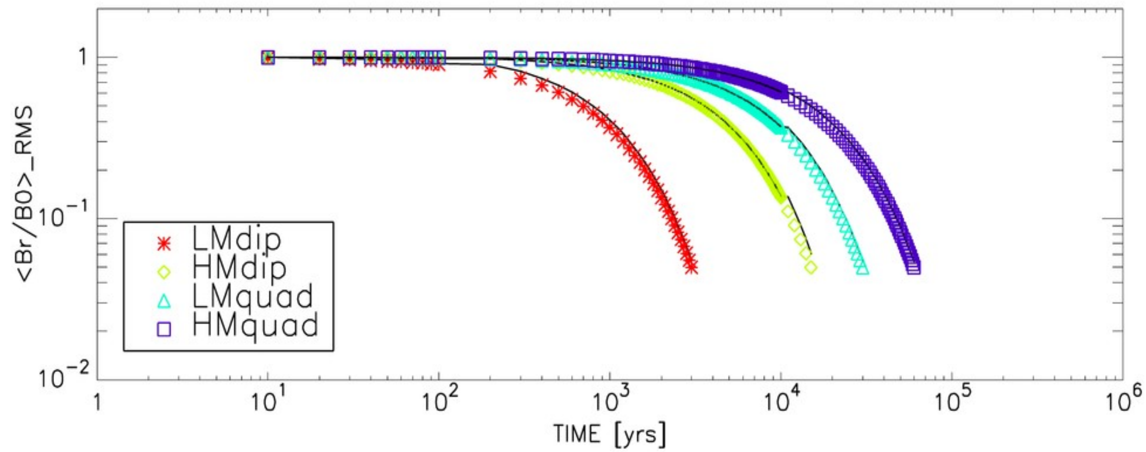
Boundary & initial conditions, EOS

- Inner boundary: superconducting core // analytic diffusion
- Outer boundary: vacuum decomposition // analytic diffusion
- Generalized multipole prescription in NS crust:
  - Aguilera+ (2008); Igoshev, Elfritz & Popov (in prep)
  - Does not imply Hall-neutral
  - Legendre polynomials and spherical Bessel functions

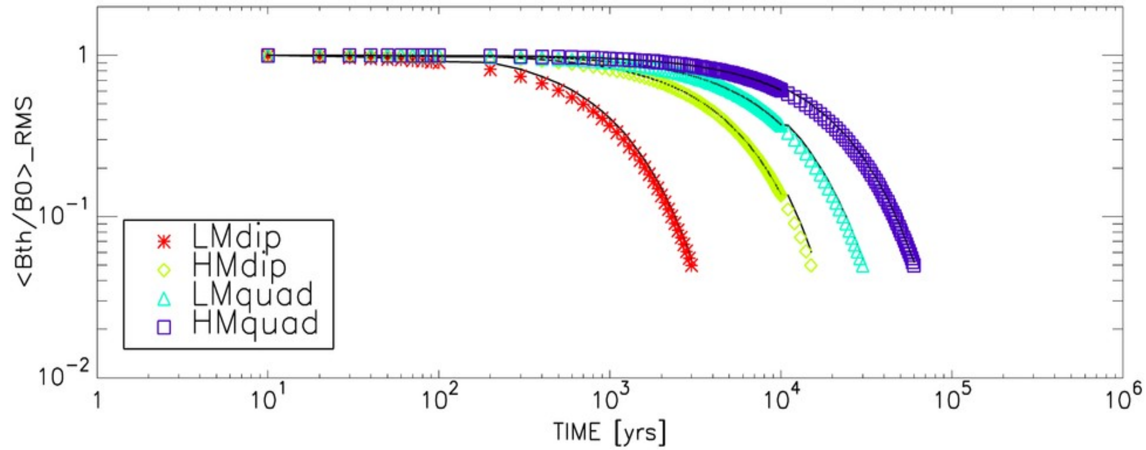
$$\boxed{\vec{\nabla} \times \vec{B} = \mu_l \vec{B}} \qquad \begin{aligned} B_r &= -\frac{\mu_l^2}{x^2} l(l+1) C_l \Gamma_l(x) P_l(\cos\theta) \\ B_\theta &= -\frac{\mu_l^2}{x} C_l \Gamma_l'(x) P_l'(\cos\theta) \\ A_\phi &= +\frac{\mu_l}{x} C_l \Gamma_l(x) P_l'(\cos\theta) \end{aligned}$$

  - General potential formulation : $\vec{B} = \vec{\nabla} \times \left(\vec{\nabla} \times \Phi \hat{r}\right) + \vec{\nabla} \times \Psi \hat{r}$
- Only simplified EOS parameters in this study
  - Constant conductivity, constant Hall pre-factor

Ohmic diffusion: basic analytic decay



$$< B_r/B_0 >_{RMS}^{t=t_n} = \sqrt{\frac{1}{N_{cells}} \sum_{r_j,\theta_i,\phi_k} \left(\frac{B_r^{ijk,n}}{B_{r,0}^{ijk}} \right)^2} \qquad < B_\theta/B_0 >_{RMS}^{t=t_n} = \sqrt{\frac{1}{N_{cells}} \sum_{r_j,\theta_i,\phi_k} \left(\frac{B_\theta^{ijk,n}}{B_{\theta,0}^{ijk}} \right)^2}$$



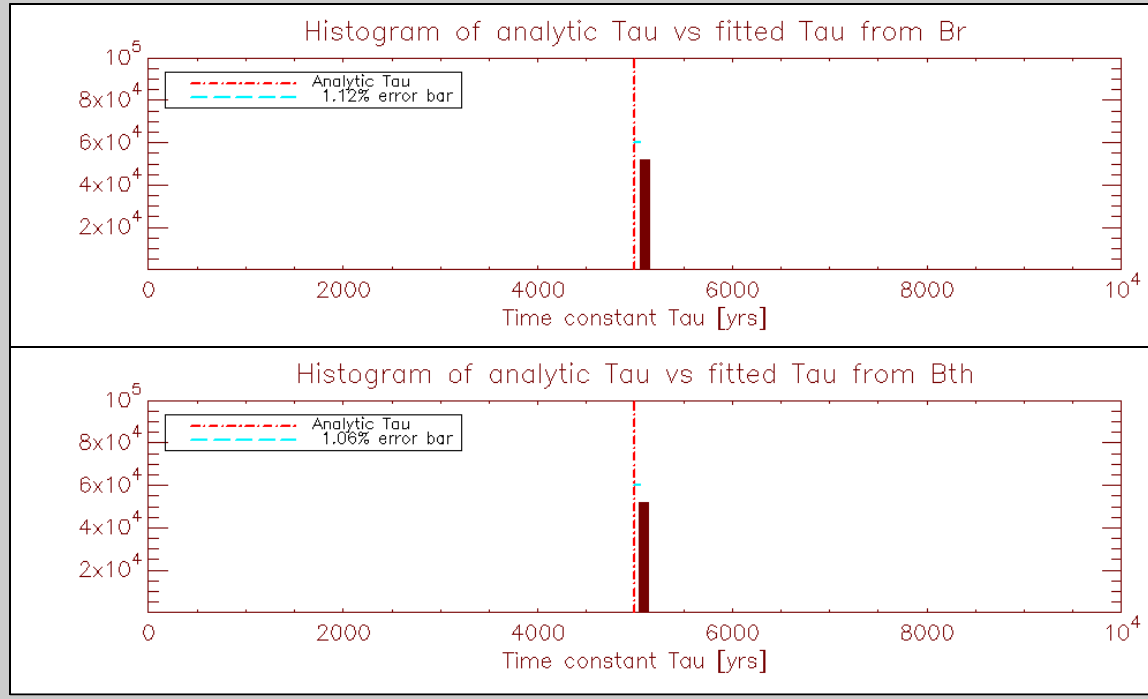
Model	R_{core} [km]	R_{NS} [km]	μ [km^{-1}]	η [km^2/Myr]	τ_{diff} [kyrs]	B_0^{pole} at t=0	B_ϕ at t=0
LM Dipole (1.40 $M_\odot$)	10.80	11.50	2.290	190.76	1.00	10^{14} G	0
HM Dipole (1.76 $M_\odot$)	10.77	11.18	3.879	13.293	5.00	10^{14} G	0
LM Quadrupole (1.40 $M_\odot$)	10.80	11.50	0.6280	253.53	10.0	10^{14} G	0
HM Quadrupole (1.76 $M_\odot$)	10.77	11.18	0.8230	73.815	20.0	10^{14} G	0

Table 1: Table illustrating model parameters for Ohmic diffusion tests. $\tau_{diff} = 1/\eta\mu^2$ is the analytic diffusion timescale. $N_{cells} \approx 10^5$.

Ohmic diffusion: basic analytic decay

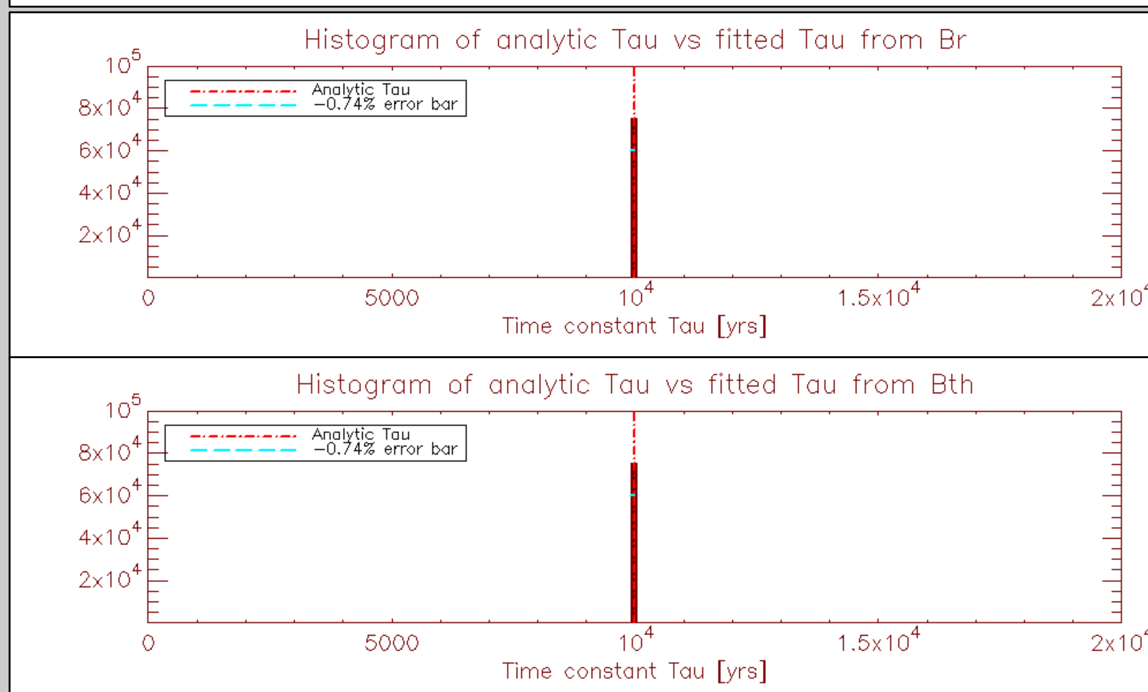
1.76M pure dipole

5kyr analytic decay timescale



1.40M pure quadrupole

10kyr analytic decay timescale



Ohmic diffusion: compressible toy model

- Force-free background, with toy azimuthal variation

$$B_{r,\theta} = B_{r,\theta}^{\mathrm{FF}}(1 + a \cdot \sin\theta \sin(m\phi))$$

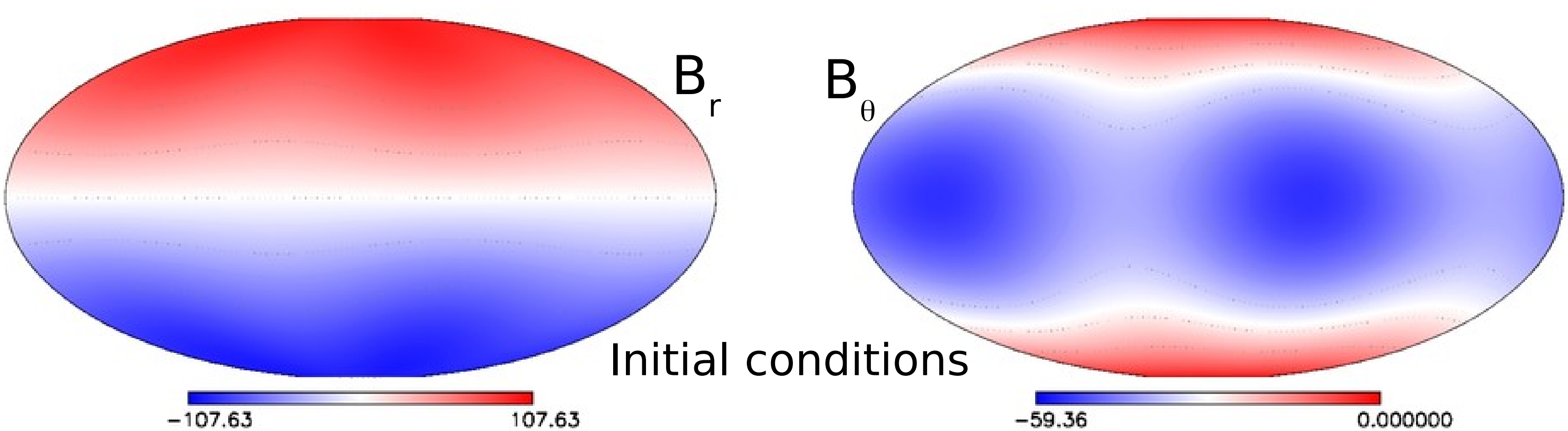
- $\vec{\nabla} \cdot \vec{B} \neq 0$ modifies the induction equation:

$$\partial_t \vec{B} = -\eta \vec{\nabla}\left(\vec{\nabla} \cdot \vec{B}\right) + \eta \nabla^2 \vec{B}$$

Azimuthal drift wave

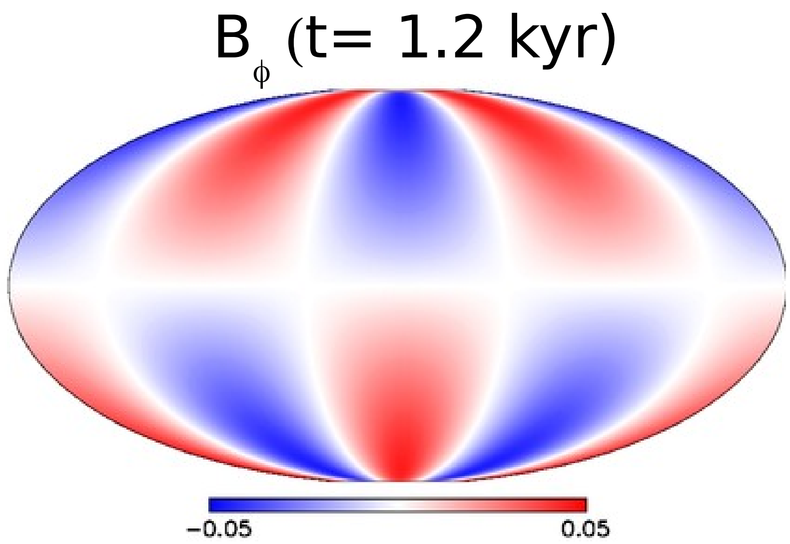
Normal Ohmic diffusion

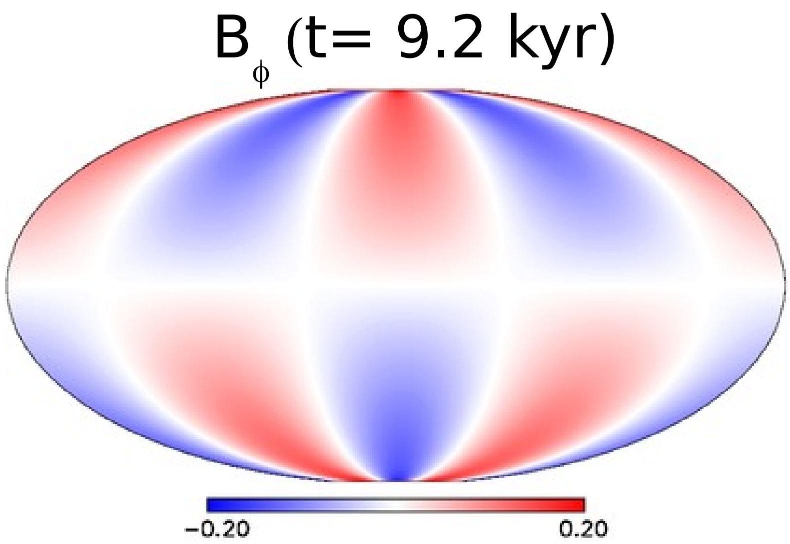
- Instability growth rate comparable to diffusion timescale



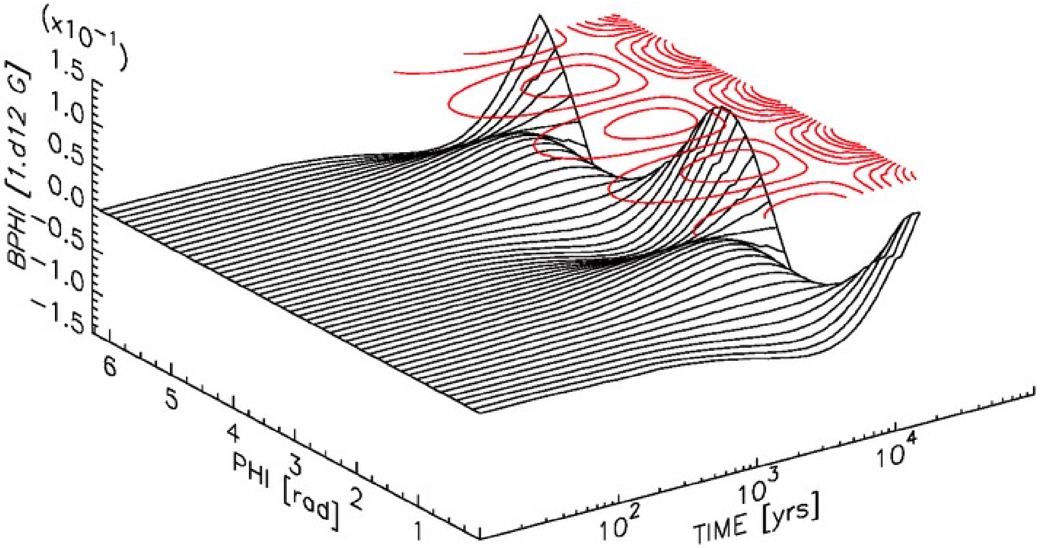
Initial conditions

Ohmic diffusion: compressible toy model

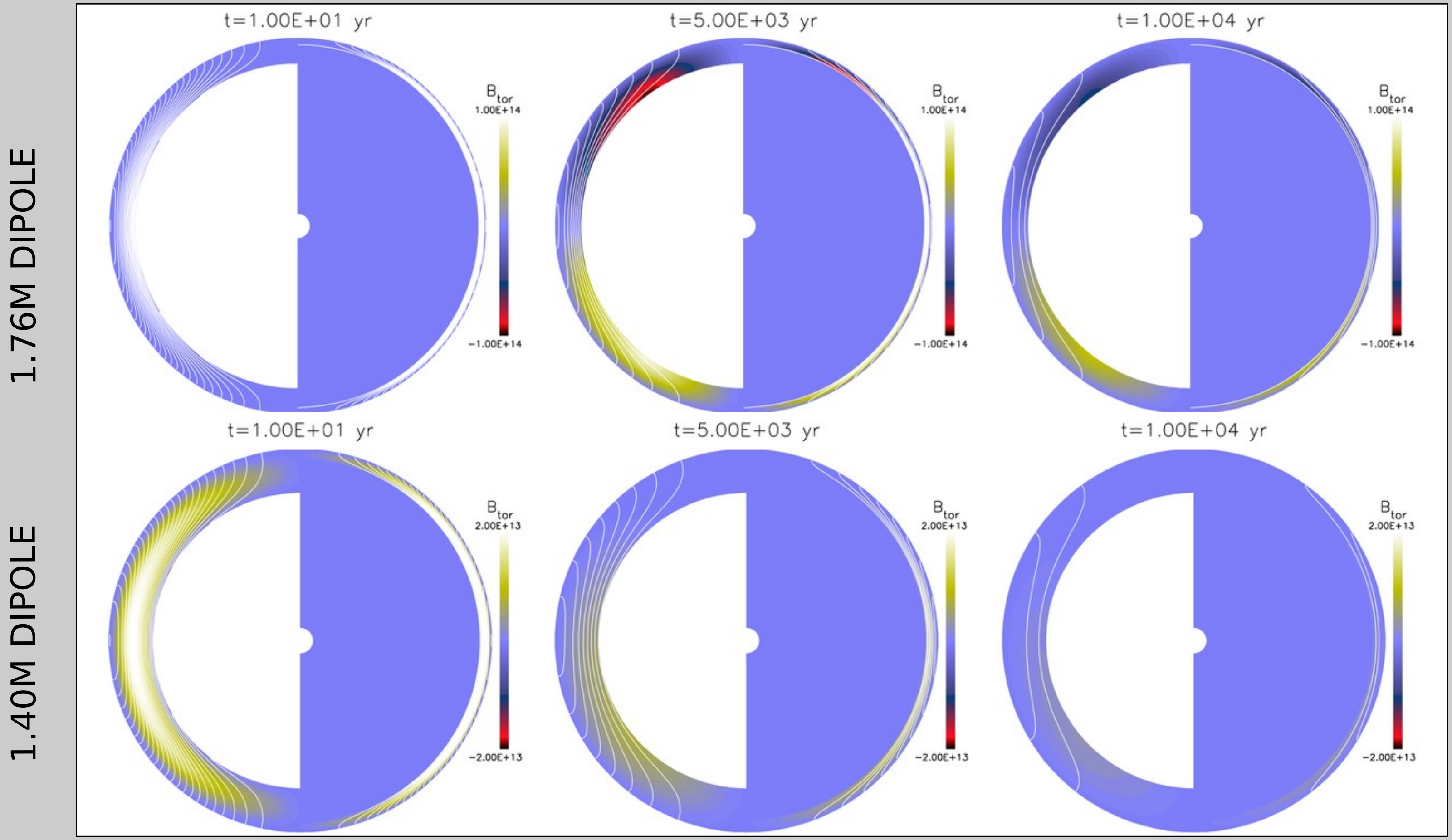




- Normal linear instability in azimuth
- Unstable drift of magnetic energy
- Fundamentally unstable!
  - Hall drift is stabilizing



Ideal Hall tests

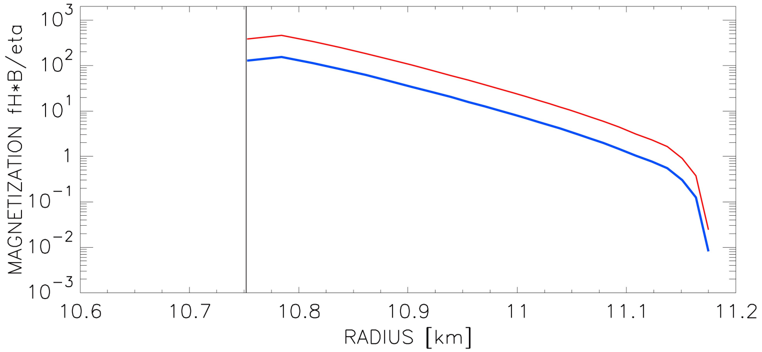


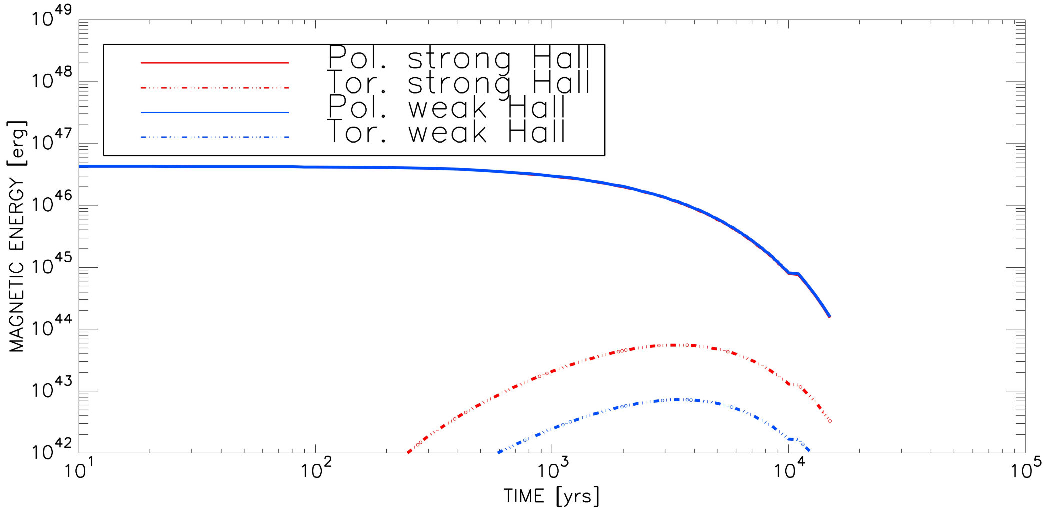
- Usual 2D evolution, no azimuthal variation induced

- 3 evolutionary phases:
  - initial cascade, toroidal saturation, late decay

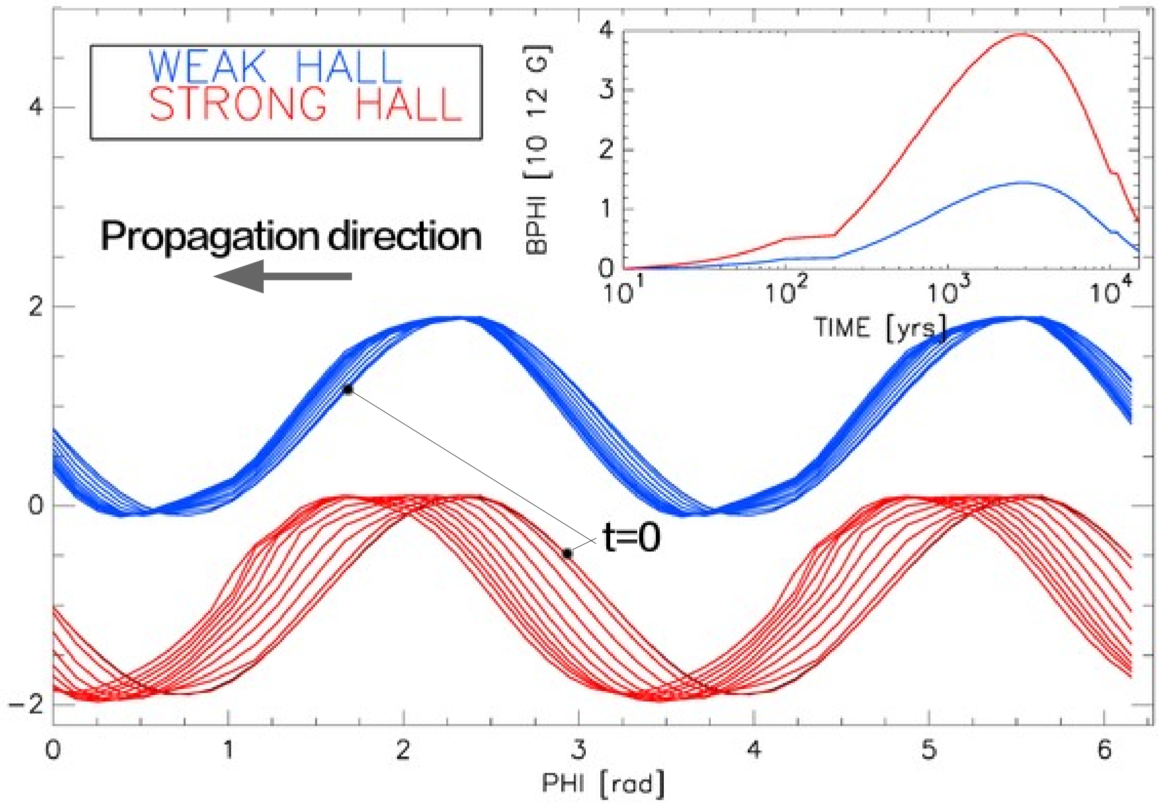
- Inclusion of toroidal field introduces no new effects

Toy Hall model with azimuthal variation

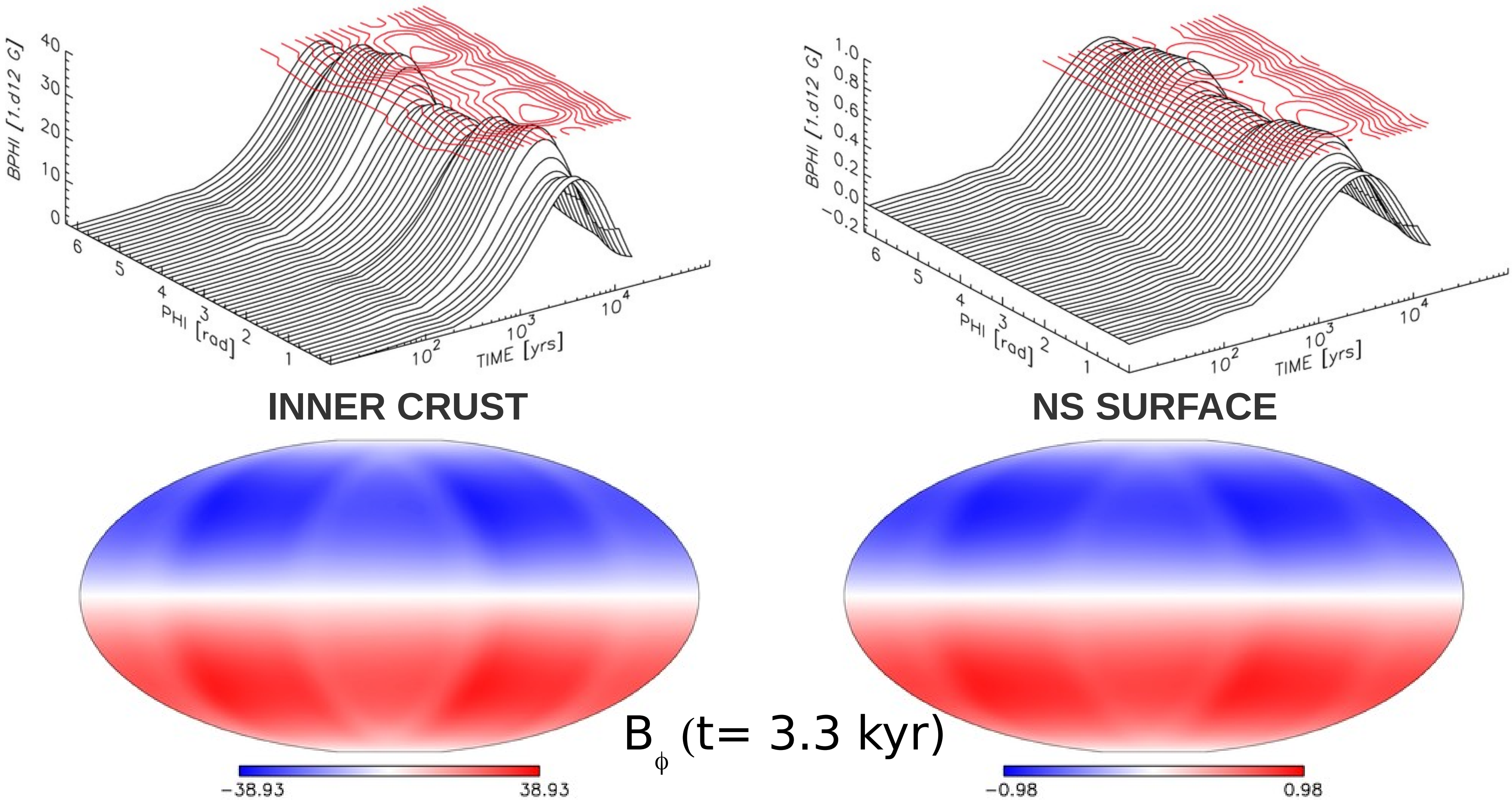




- Weak and strong: factor of 3 in f_H
- $f_H B/\eta >> 1$: Hall-dominated
- Onset of strong B_ϕ admits propagation of whistler mode
- Whistler phase speed $\sim f_H B_\phi$
- Hall cascade stabilizes unstable drift mode

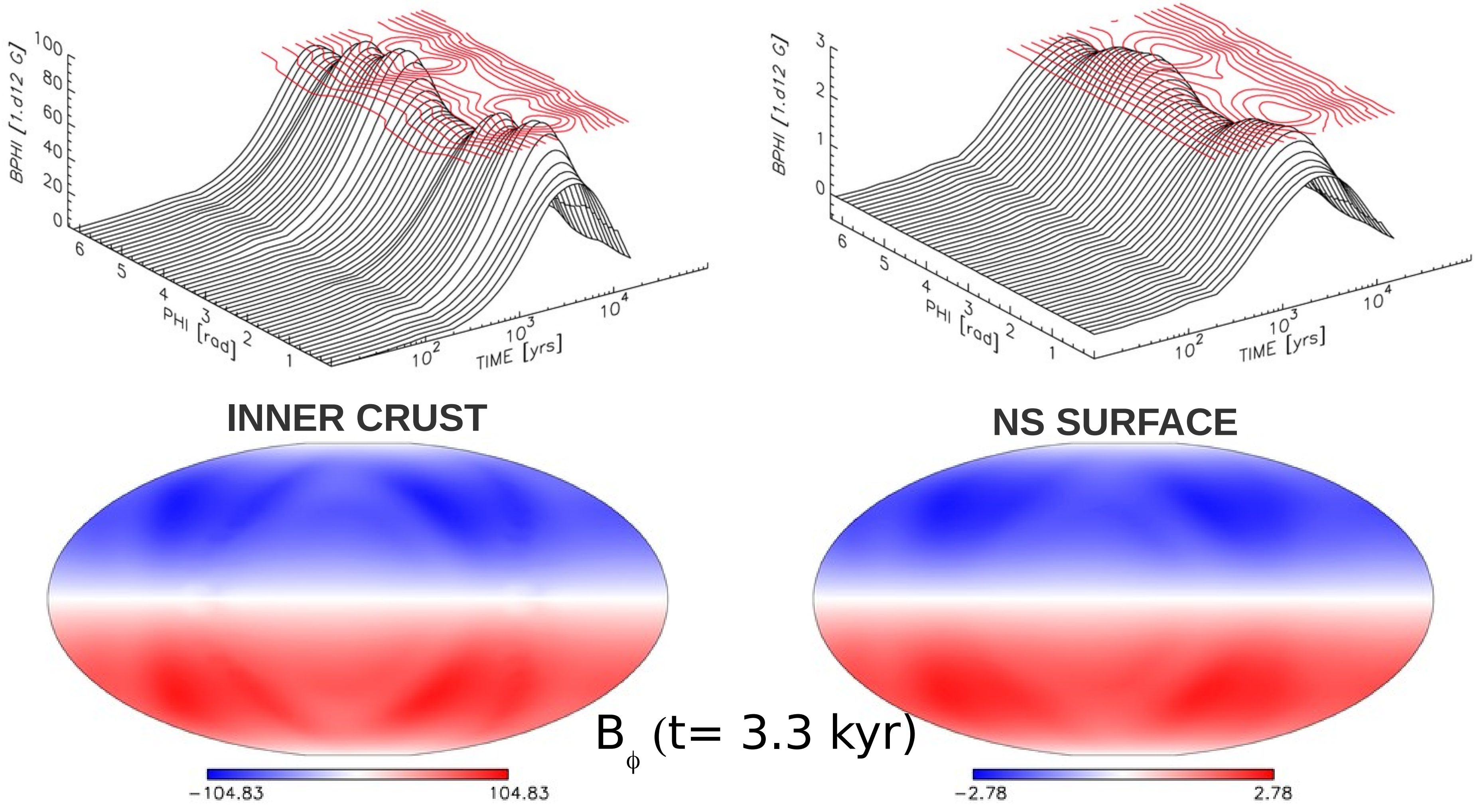


Weak Hall model with azimuthal variation



- Normal linear phase growth rate, depends on local field intensity
- Azimuthal field perturbation is function of radial location

Stronger Hall with azimuthal variation



- Generation of two-peak feature in innermost crust
- Whistler + drift instability co-propagating? Or simply whistler dispersion?

Conclusions / Moving forward

- First 3D extensions of Pons, Viganò+ magneto-thermal model
  - Also see Wood & Hollerbach (2015), Gourgouliatos+ (2016)
- Validation of fundamental physical evolution
  - Thus far, benchmarking against known 2D evolution when possible
- Inclusion of self-consistent thermal evolution
  - Analytic studies first!
  - Electrical conductivity has strong temperature dependence
- Nonlinear whistler propagation in 3D
- 'Smearing' the physics with drift instabilites
  - $\nabla\eta$ and ∇f_H can drive continuous bulk energy transfer
- 3D Hall cascade vs instability
  - Rheinhardt & Geppert (2002), Wareing & Hollerbach (2009)
- Coupled NS core field evolution
- Improvements to B-field advance algorithm