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Neutron stars in alternative theories of gravity – models, astrophysical implications and universal relations

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Plan of the talk:

- Alternative theories of gravity
- Equilibrium neutron-star solutions: static and rapidly rotating
- Astrophysical implications

Alternative Theories of Gravity

Motivation and Overview

Alternative theories of gravity: Motivation

Motivation for modifying General Relativity

Theory

Theories trying to unify all the interactions: Kaluza-Klein theories, higher dimensional gravity, etc.

Quantum corrections in the strong field regime

Studying alternative theories of gravity can give us a deeper insight in GR itself

Observations

Dark energy and dark matter does not fit well in the standard GR framework

The strong field regime of gravity is essentially unconstrained

Alternative theories of gravity: Motivation

- There is a very wide range of alternative theories of gravity constructed from different generalizations/modifications of Einstein's theory.

- We will concentrate on the most natural and widely used generalizations:

    - Scalar-tensor theories of gravity
    - $f(R)$ theories of gravity

- They are in agreement with all the observations and do not posses any intrinsic problems.

- Widely used as an alternative explanation of the dark energy phenomena.

- Scalar-tensor theories can be consider as an Einstein theory of gravity but with variable gravitational constant.

Scalar-tensor theories

- **Essence:** one or several scalar field that can be viewed as mediators of the gravitational interaction in addition to the spacetime metric
- **Action:**

Jordan frame
Physical one

$$S = \frac{1}{16\pi G_*}\int d^4x\sqrt{-\tilde{g}}\left[F(\Phi)\tilde{R} - Z(\Phi)\tilde{g}^{\mu\nu}\partial_\mu\Phi\partial_\nu\Phi - 2U(\Phi)\right] + S_m\left[\Psi_m;\tilde{g}_{\mu\nu}\right]$$

✓ Conformal transformation of the metric
✓ Redefinition of the scalar field

Einstein frame
Much simpler!

$$S = \frac{1}{16\pi G_*}\int d^4x\sqrt{-g}\left(R - 2g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi - 4V(\varphi)\right) + S_m[\Psi_m;\boxed{A^2(\varphi)g_{\mu\nu}}]$$

The price we pay for simplicity: Explicit coupling between the matter and the scalar field

$f(R)$ theories

- **Motivation:** widely used as un alternative explanation of the accelerated expansion of the universe

- **Studied mainly at cosmological scales**, but every theory of gravity should pass via the observations at astrophysical scale too

- **Action:** $$\boxed{S = \frac{1}{16\pi G}\int d^4x\sqrt{-g}\,f(R) + S_{\rm matter}(g_{\mu\nu},\chi),}$$

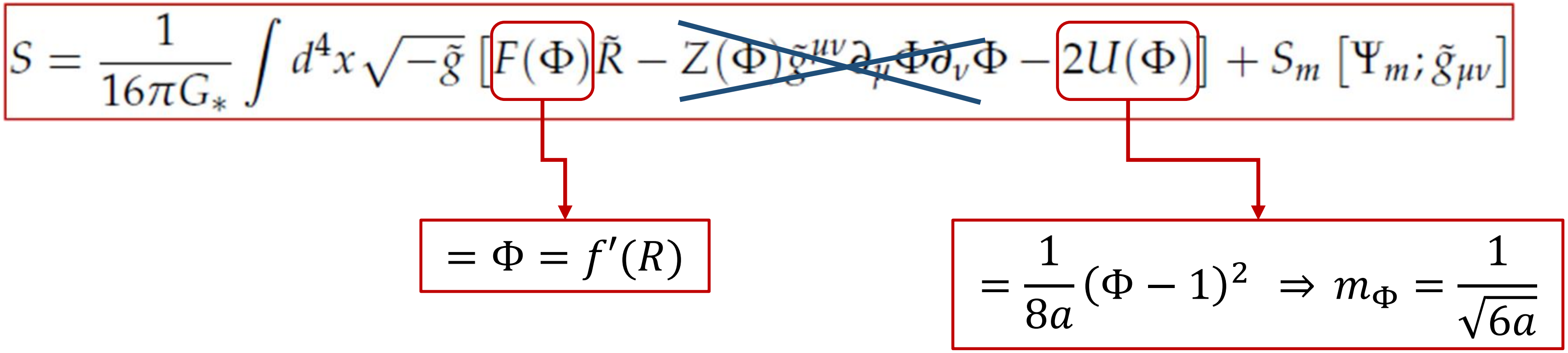
- Free of tachyonic instabilities and the appearance of ghosts when:

$$\boxed{\frac{d^2 f}{dR^2} \geq 0, \qquad \frac{df}{dR} > 0}$$

- **Mathematical treatment** of the problem: $f(R)$ theories are mathematically equivalent to a particular class of massive scalar-tensor theories.

$f(R)$ theories

- **Example:** R^2 gravity ($f(R) = R + aR^2$)



$$S = \frac{1}{16\pi G_*}\int d^4x\sqrt{-\tilde{g}}\left[\boxed{F(\Phi)}\tilde{R} - \xcancel{Z(\Phi)\tilde{g}^{\mu\nu}\partial_\mu\Phi\partial_\nu\Phi} - \boxed{2U(\Phi)}\right] + S_m\left[\Psi_m;\tilde{g}_{\mu\nu}\right]$$

$$= \Phi = f'(R)$$

$$= \frac{1}{8a}(\Phi - 1)^2 \;\Rightarrow\; m_\Phi = \frac{1}{\sqrt{6a}}$$

Field equations in STT (Einstein frame)

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi G_* T_{\mu\nu} + 2\partial_\mu \varphi \partial_\nu \varphi - g_{\mu\nu} g^{\alpha\beta} \partial_\alpha \varphi \partial_\beta \varphi - 2V(\varphi) g_{\mu\nu}$$

$$\nabla^\mu \nabla_\mu \varphi = -4\pi G_* k(\varphi) T + \frac{dV(\varphi)}{d\varphi}.$$

These equations have to be supplemented with:

- Equation for hydrostatic equilibrium
- Equation of state of the nuclear matter

Equilibrium neutron star solutions

Static and rapidly rotating case

Scalar-tensor theories with massless scalar field

$$S = \frac{1}{16\pi G_*}\int d^4x\sqrt{-g}\left(R - 2g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi - \cancel{4V(\varphi)}\right) + S_m\left[\Psi_m; \boxed{A^2(\varphi)}g_{\mu\nu}\right]$$

Coupling function $\alpha(\varphi) = \frac{d\, lnA(\varphi)}{d\ \varphi}$

- The coupling function can be expanded as $\boldsymbol{\alpha(\varphi) = \alpha_0 + \beta\varphi +}$ **higher order terms**

  1. $\boldsymbol{\alpha(\varphi) = \alpha_0}$
     - Equivalent to the Brans-Dicke theory.
     - Differs from GR in the weak field regime.
     - Neutron stars have nontrivial scalar field for every $\alpha_0 \neq 0$

  2. $\boldsymbol{\alpha(\varphi) = \beta\varphi}$
     - Equivalent to GR in the weak field regime.
     - Can differ significantly when strong fields are considered.
     - Nonuniqueness of the neutron star solutions can exist – one solution with trivial scalar field and one or several others with nontrivial scalar field.

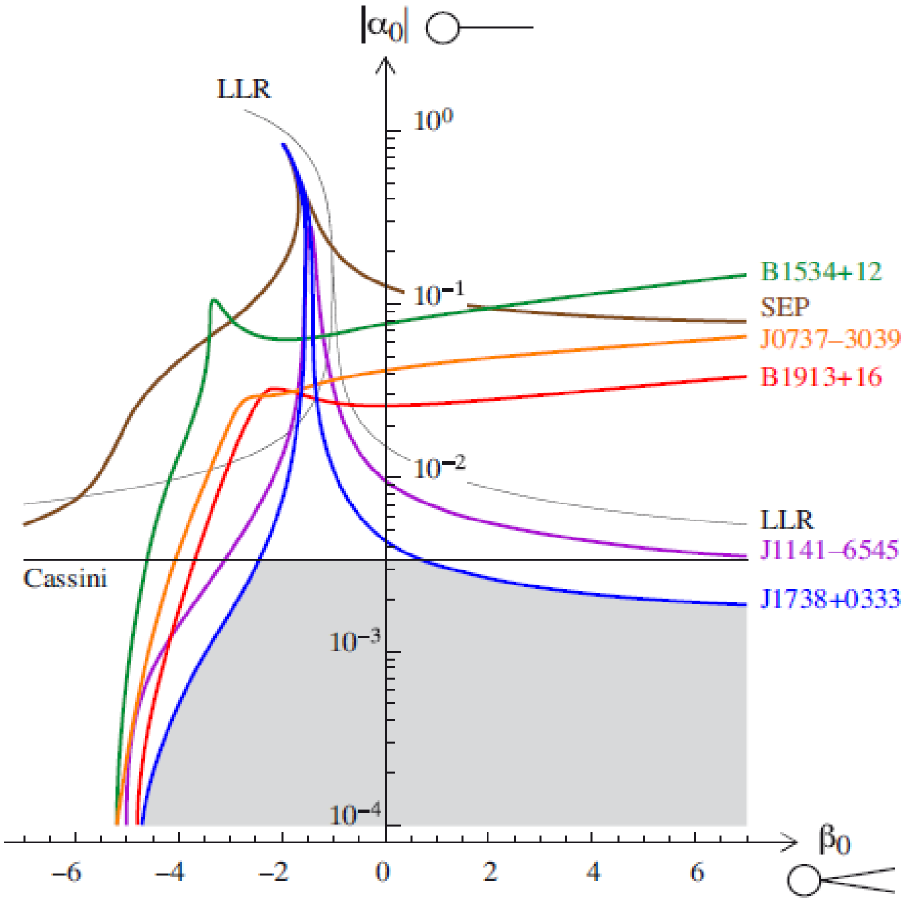
- **Higher order terms** in $\alpha(\varphi)$ lead to qualitatively similar results

Equilibrium neutron star solutions: STT

Observational constraints

$\alpha_0 < 0.004$ and $\beta > -4.5$

(Damour & Esposito-Farese (1996,1998), Will (2006), Freire et al (2012), Antoniadis et al (2013))



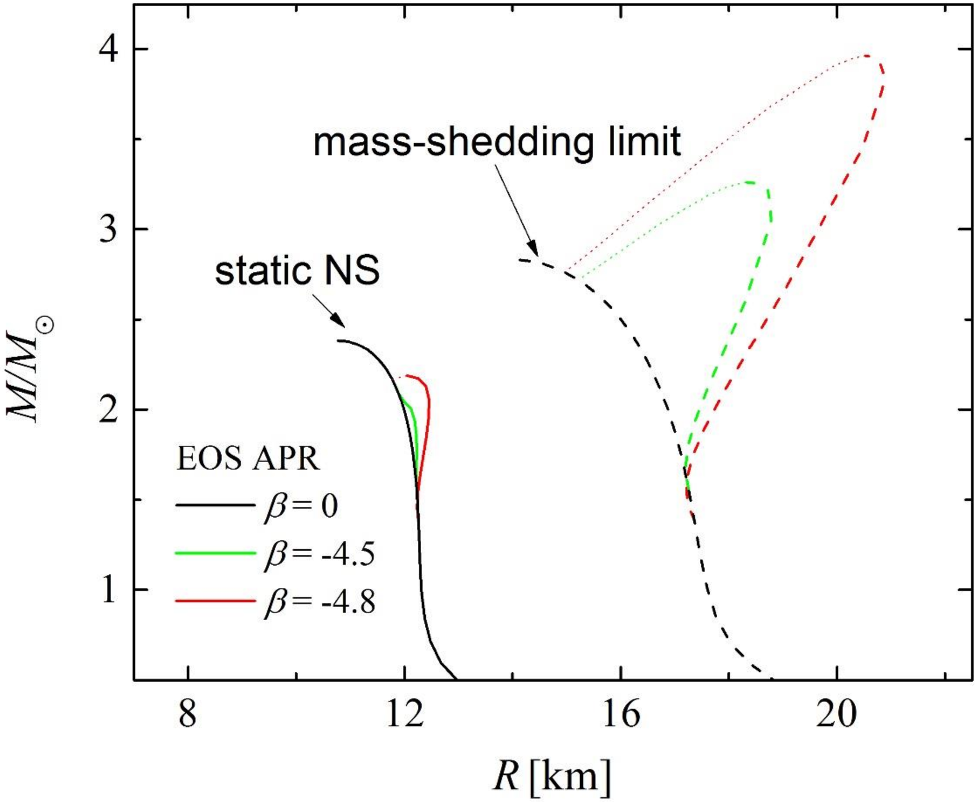
Freire et al (2012)

- Scalarized solutions exist only for $\beta < -4.35$ in the static case and $\beta < -3.9$ in the rapidly rotating case.

Equilibrium neutron star solutions: STT

- **Scalarization of neutron stars** in the second class of scalar-tensor theory was considered for the first time by Damour&Esposito-Farese (1993)
- **Slow rotation approximation** was also considered (Damour&Esposito-Farese (1996), Sotani (2012), Pani&Berti(2014)).
- **Rapid rotation** – changes the picture significantly (Doneva et al (2014))

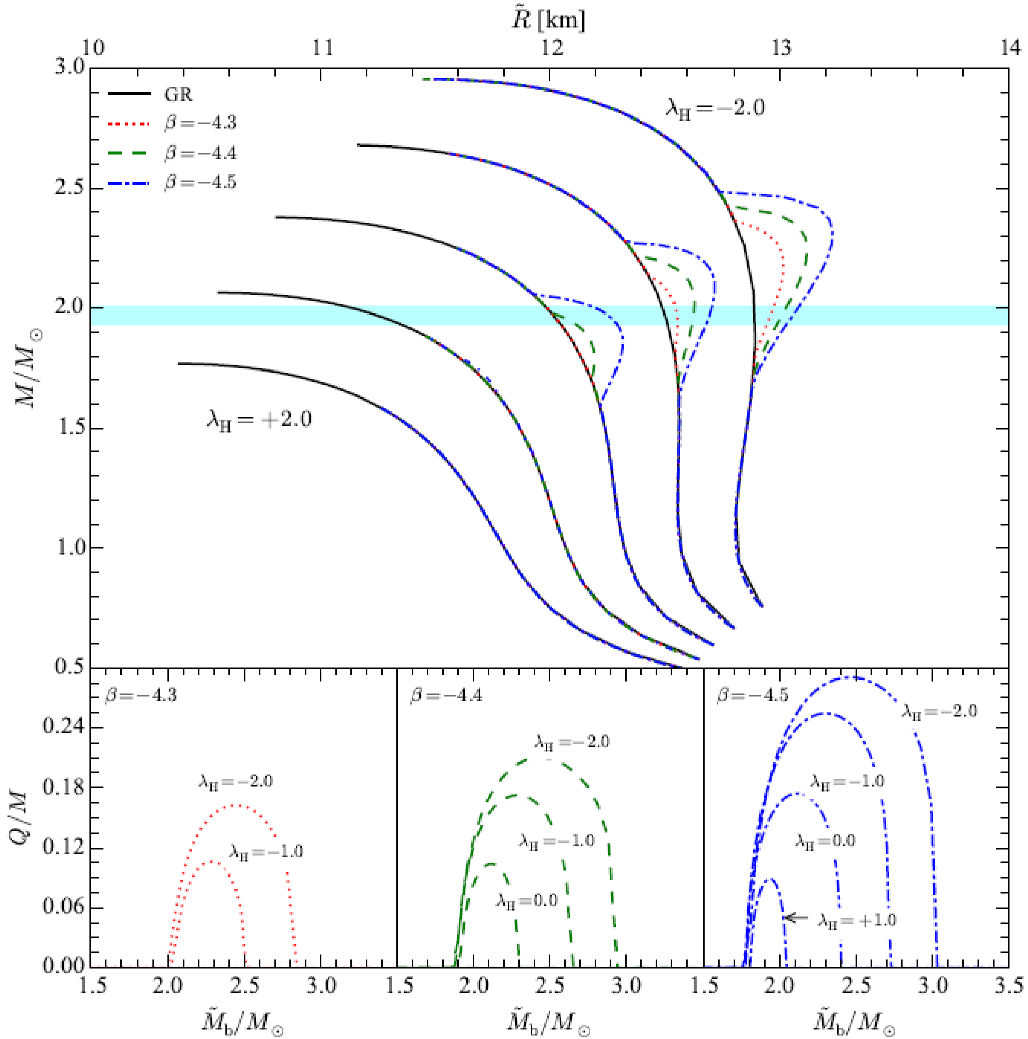
Coupling function $\alpha(\varphi) = \beta\varphi$



- Scalarization possible also for **positive β** and negative trace of the energy momentum tensor. Possible for stiff EOS and very massive stars, not fully studied yet (Mendes (2015), Mendes&Ortiz(2016), Palenzuela&Liebling(2015))
- **Tensor-multi-scalar theories** (Horbatsch et al (2015)) – new interesting phenomena, still in development.

Equilibrium neutron star solutions: STT

- Anisotropic scalar-tensor neutron stars (Silva et al (2015)) – the deviations from GR are magnified significantly for strong degree of anisotropy



Scalar-tensor theories with MASSIVE scalar field

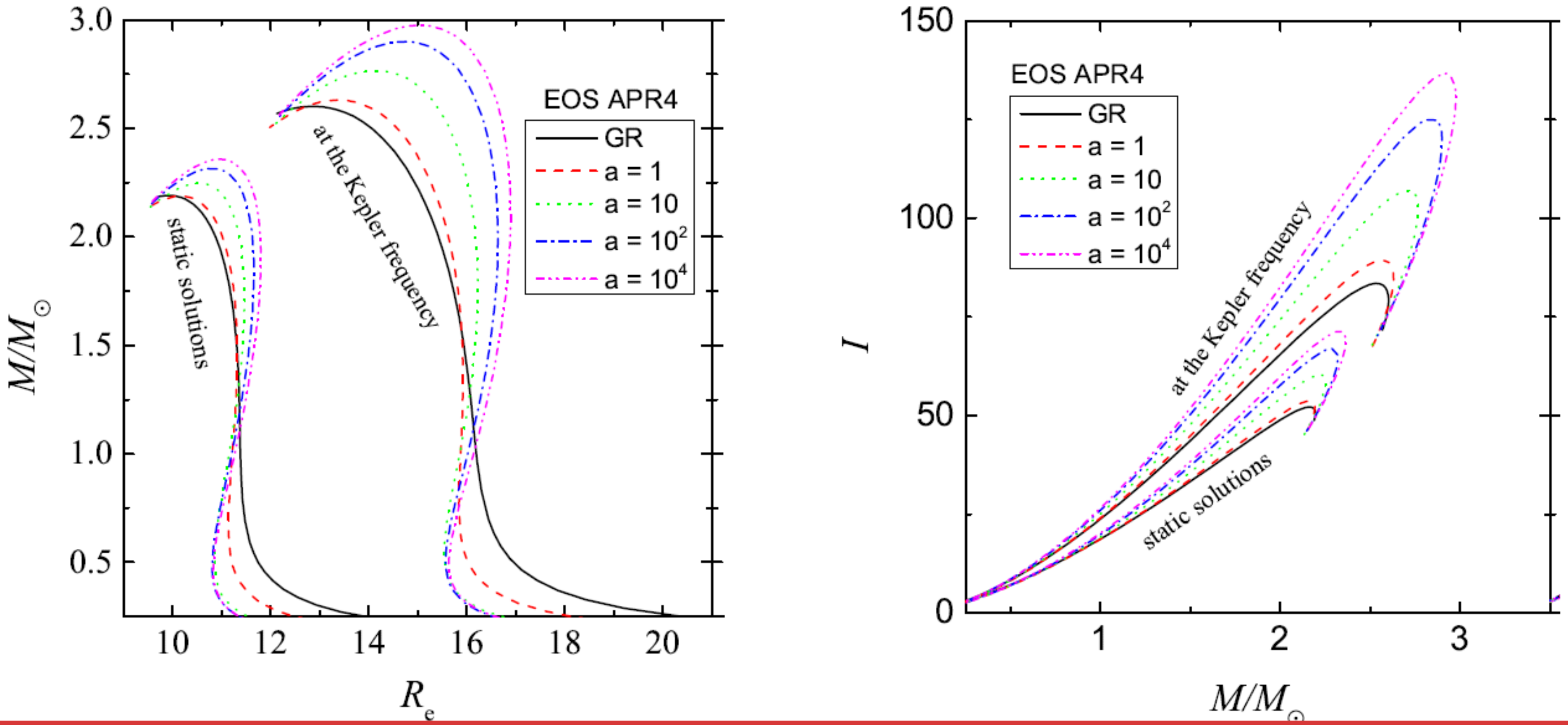
FOLLOWS NEXT!

$\boldsymbol{f(R)}$ theories of gravity

- We will concentrate on the $\boldsymbol{R^2}$ **gravity** ($f(R) = R + a\,R^2$) case, that is expected to give the dominant contribution at astrophysical scales.
- **Pertubative approach**, assuming that a is a small number, (Cooney, DeDeo, Psaltis (2010)) widely used in the past, but recently it was shown to be **misleading** (Yazadjiev, Doneva, Kokkotas, Staykov (2014))
- **Observational constraints** – the most severe coming from the Gravity Probe B experiments $a < 2.5 \times 10^5$ (or $a < 5 \times 10^{11} m^2$ in physical units).
- The **scalar-tensor representation** of $f(R)$ theories is commonly employed.
- The field equation for the Ricci scalar curvature (or equivalently the scalar field) is stiff which poses a computational difficulty.

Equilibrium neutron star solutions

- **Non-perturbative approach**: reported in Babichev&Langlois(2010), Jaime et al (2011), and the first detailed study of realistic NS models was done in Yazadjiev, Doneva, Kokkotas, Staykov (2014)
- **Rotating models** are also studied (Staykov et al (2014), Yazadjiev et al (2015))
- **Non-negligible deviation** for the allowed values of a. The **moment of inertia** is very sensitive and can be used to set constraints on the parameters.



Astrophysical implications

Astrophysical implications

- **Final goal** – test the strong field regime of gravity via neutron star observations and impose constraints on the alternative theories

- **Obstacles**:
  - Accuracy of observations
  - Accurate models of the observed phenomena
  - EOS uncertainty

- Ways out:
  - Deviation from GR stronger than the EOS uncertainty for the allowed range of parameters
  - EOS independent relations

Astrophysical implications

Possible approaches for testing alternative theories of gravity

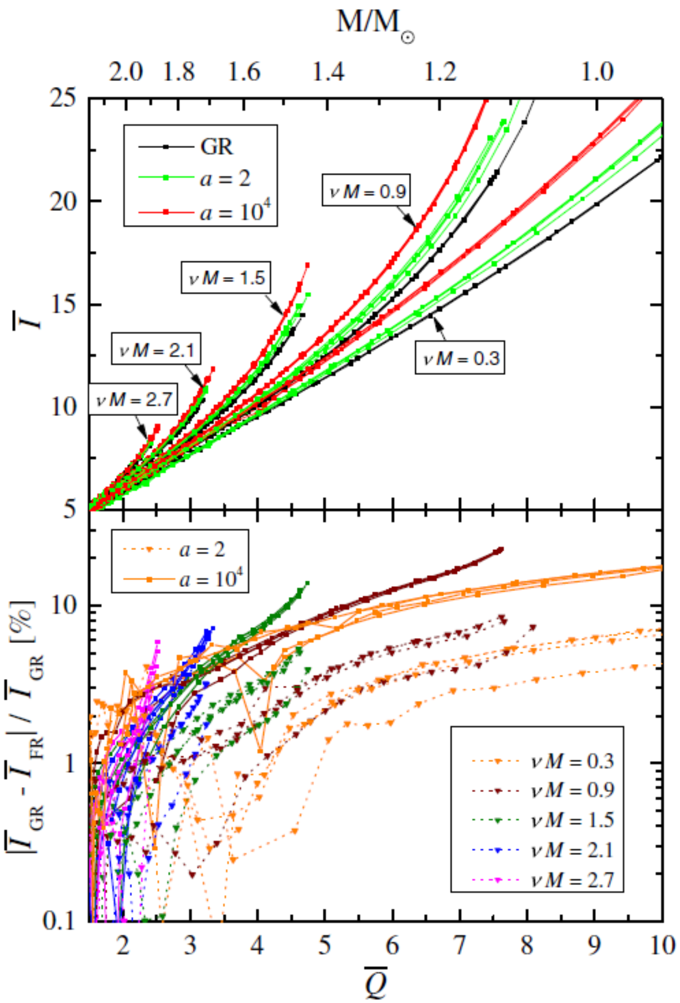
- Direct observation of the mass and radius.
- Observations of the moment of inertia: applicable for example for $f(R)$ theories Staykov at al (2014) and Eddington inspired gravity Pani, Cardoso, Delsate (2011)
- Quasiperiodic oscillations DeDeo&Psaltis(2004), Doneva et al (2014), Staykov, Doneva, Yazadjiev (2015)
- The redshift of surface spectral lines in X-rays and γ-rays DeDeo&Psaltis(2003)
- Gravitational wave emission of oscillating neutron stars
- Neutron star mergers
- Universal relations

Universal relations

- **EOS independent relations** between the properties, including the oscillation spectrum, of neutron stars. Normally a proper normalization of the quantities is required.
- Very convenient way to **circumvent the EOS uncertainty**.
- Attracted particular attraction with the paper of Yagi&Yunes (2013)
- We will focus on the **I-Love-Q relations** as a particular example but many other universal relations exist (Lattimer&Schutz(2005),Yagi et al (2014), AlGendy&Morsink(2014), Breu&Rezzolla(2016))
- **General idea** for **testing the strong field** regime of gravity: if the two parameters that enter in a universal relation are measured independently, then a possible deviation from the GR EOS independent relations can be measured.

Astrophysical implications

Example R^2 theories:



- I-Love-Q relations: appreciable deviations from GR only for some alternative theories of gravity (**dynamical Chern-Simons gravity** Yagi&Yunes(2013), $\boldsymbol{f(R)}$ gravity Doneva, Yazadjiev, Kokkotas (2015), **massive STT** Yazadjie,Doneva in preparation)

- Most of the studied alternative theories of gravity give only marginal deviations from GR (eg. Sham, Lin, Leung(2014); Kleinhaus, Kunz, Mojica (2014), Pani, Berti (2014), Pappas, Sotiriou (2015)).

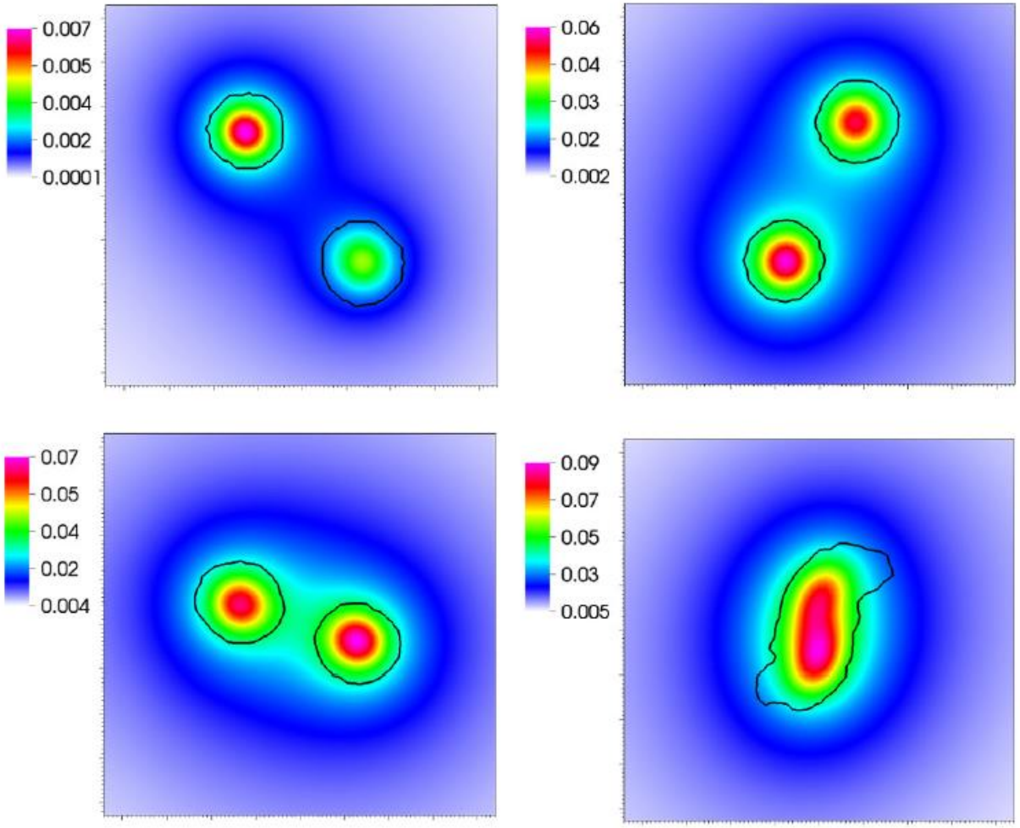
- **Unnormalized relations** STILL differ significantly from GR. Solution:
  - ➢ Different normalization
  - ➢ Different universal relation

- Strong point: there relations are also theory independent up to a good extend that might have different application.

Dynamical scalarization – NS mergers

- Even if the two NS are not scalarized when separated, in close binary system they **develop strong scalar field**.

Coupling function $\boldsymbol{\alpha(\varphi) = \beta\varphi}$



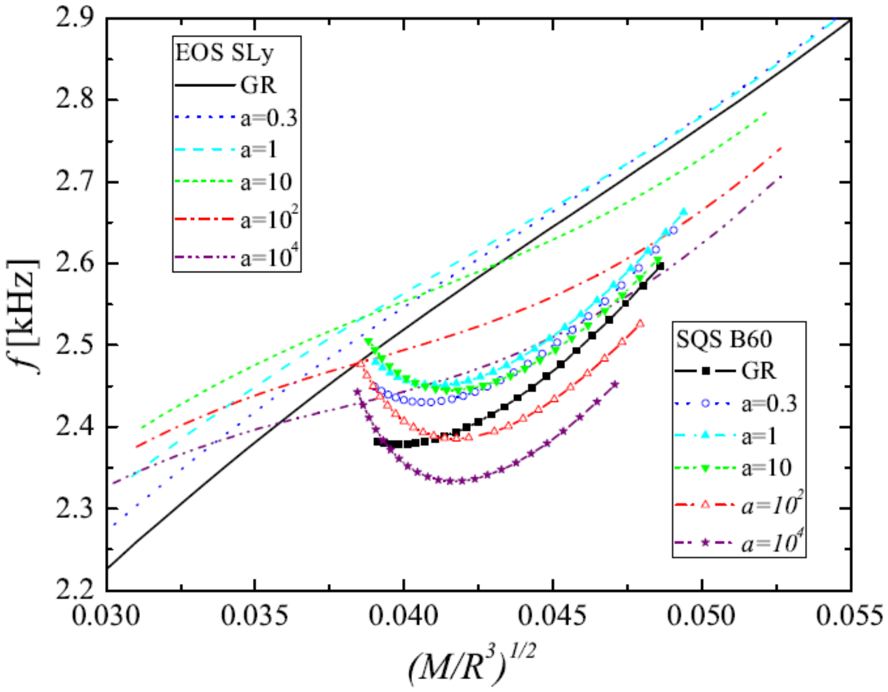
- The effects of **spontaneous** scalarization, **induced** scalarization and **dynamical scalarization** can be distinguished.
- The observational signature of the scalarized merging neutron stars has been studied in Barause et al (2013), Palenzuela et al (2014), Shibata et al (2014), Sampson (2014), Taniguchi et al (2015).

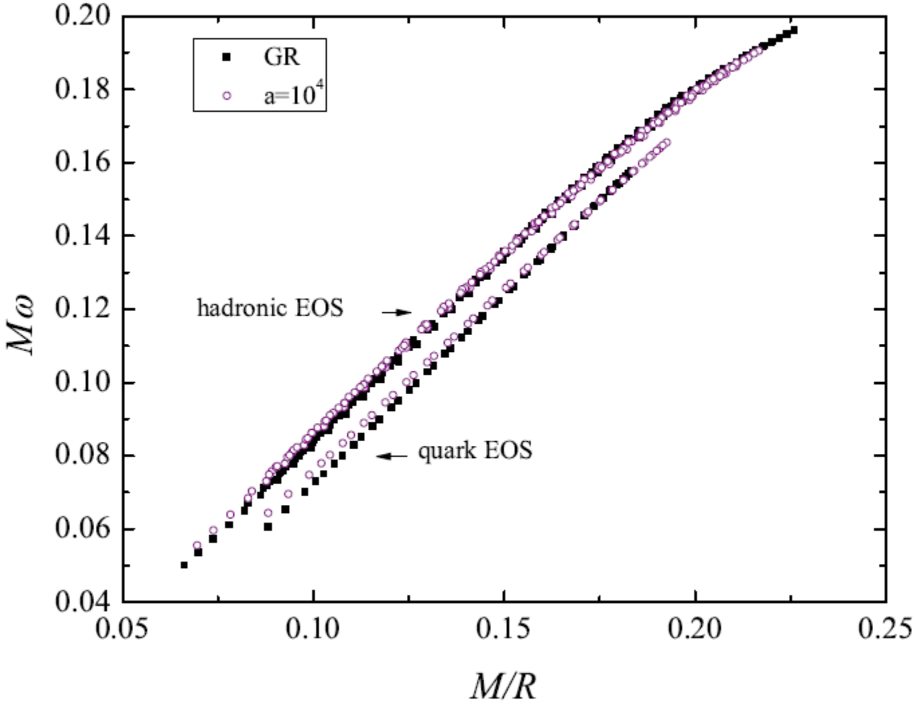
Neutron star oscillations

- The study was **initiated** with the work of Sotani&Kokkotas (2004) for f- and p-modes in STT.
- The main idea is to constrain the deviations from GR using the emitted gravitational wave signal or in some cases electromagnetic signal, related to neutron star oscillations
- Several alternative theories studied until now – STT Sotani&Kokkotas (2004), Silva et al (2014), TeVeS Sotani (2010, 2011, 2009), $f(R)$ Staykov et al (2015), Einstein-Gauss-Bonnet-dilaton gravity Blázquez-Salcedo et al (2016)
- Fundamental f-modes, torsional modes, w-modes and others are studied. In many cases the Cowling approximation is employed.

Asteroseismology relations in R^2 theories

- ***f*-mode** oscillation frequencies, nonrotating case
- Quite **EOS independent** with suitable choice of normalization





Staykov et al (2015)

Conclusions

- Neutron stars in alternative theories of gravity can have significantly different properties compared to their general relativistic counterparts.
- Rotation can magnify the deviations and lead to new observational consequences.
- A further study of the astrophysical implications is required in order to check what are the most promising astrophysical implications.
- Further info: Berti et al (2015)

Thank you!