

FRG Approach to Nuclear Matter in Extreme Conditions



- [arXiv:1604.01717](https://arxiv.org/abs/1604.01717) [hep-th]
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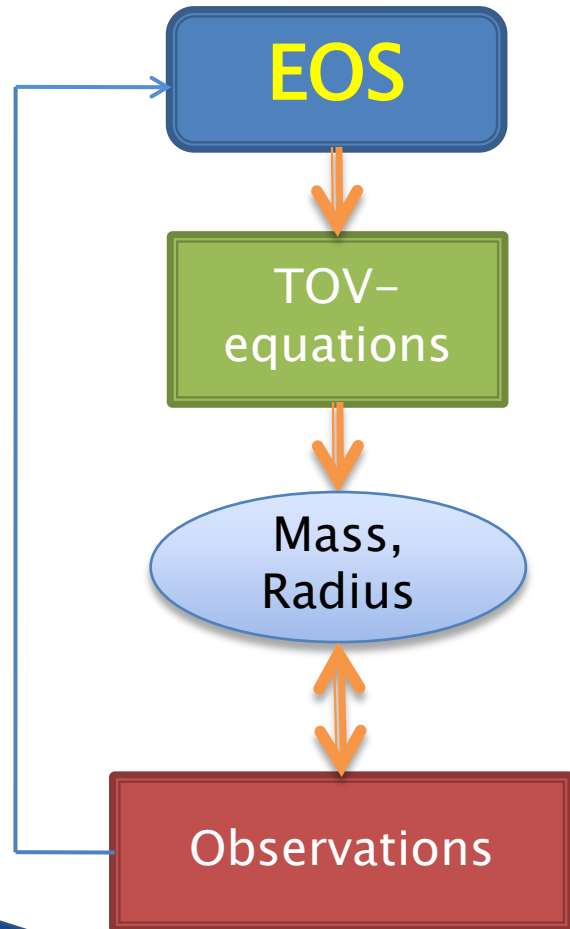
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Outline

1. Introduction to FRG
2. Fermi-gas model at finite temperature
3. Solving the Wetterich-equation at finite chemical potential
4. Comparison between FRG results and other methods

Motivation for Using FRG



What are the effects of **quantum fluctuations** on the Equation of State (EOS) ?

What is the difference between the same parameters in mean field and quantum fluctuations included ?

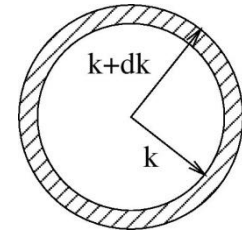
- **Compressibility** (important for neutron star mass!)
- Binding energy
- Surface tension of nuclear matter

FRG is a general method to take quantum fluctuations into account.

Introduction to FRG

- The generator functional is modified with a regulator term to enable gradual momentum integration

- The action becomes scale dependent
- The regulator acts as a mass term and suppresses fluctuations under scale k



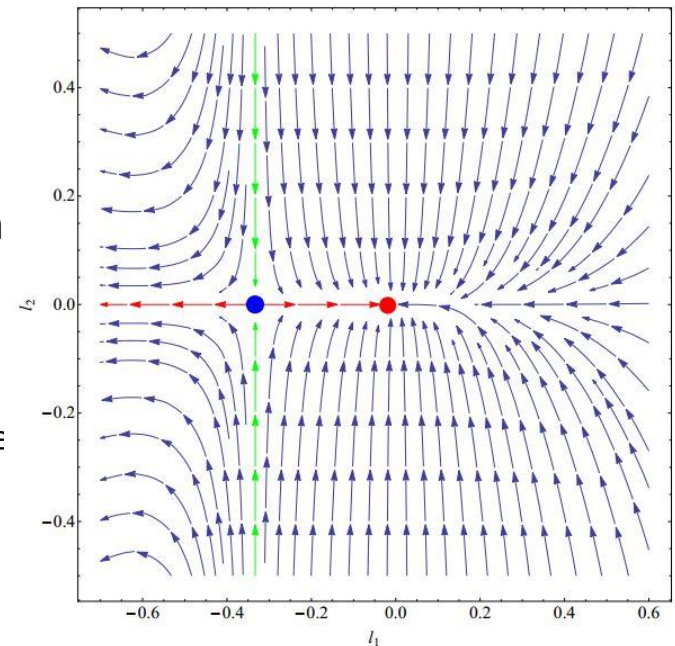
$$Z_k[J] = \int \left(\prod_a d\Psi_a \right) e^{-S[\Psi] - \frac{1}{2} R_{k,ab} \Psi_a \Psi_b + \Psi_a J_a}$$

- The **effective action** of the theory is defined as the Legendre-transformation of the Schwinger-functional.

$$\Gamma_k[\psi] = \sup_J (\psi_a J_a - W[J]) - \frac{1}{2} R_{k,ab} \psi_a \psi_b$$

- Scale dependence of the effective action is given by the
 - Solving the equation yields a trajectory in theory space
 - (one point in theory space is characterized by the coupling constants of the theory)
 - Integrating the equation from k_{max} to $k=0$ gives the infrared effective action containing all quantum fluctuations

$$\partial_k \Gamma_k = \frac{1}{2} Str \left[(\partial_k R_k) \left(\Gamma_k^{(1,1)} + R_k \right)^{-1} \right]$$



Functional Renormalization Group (FRG)

- ▶ General non-perturbative method to determine the effective action of a system.
 - Scale dependent effective action (k scale parameter)

$$\partial_k \Gamma_k = \frac{1}{2} \int dp^D \text{STr} \left[\frac{\partial_k R_k}{\Gamma_k^{(2)} + R_k} \right]$$

Wetterich equation

$k=\Lambda$
Classical
action



Integration

$k=0$
*Quantum
fluctuations
included*

Functional Renormalization Group (FRG)

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$$\partial_k \Gamma_k = \frac{1}{2} \int dp^D \text{STr} \left[\frac{\partial_k R_k}{\Gamma_k^{(2)} + R_k} \right]$$

Need an ansatz for the integration


**Not necessarily
perturbative ansatz!**

$$\Gamma_k = \sum_{l=1}^{l=N} \frac{g_l(k)}{l!} \hat{O}_l$$

**Scale
dependent
coupling**

Functional Renormalization Group (FRG)

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$$\partial_k \Gamma_k = \frac{1}{2} \int dp^D \text{STr} \left[\frac{\partial_k R_k}{\Gamma_k^{(2)} + R_k} \right]$$


Regulator:

- determines the modes present on scale k
- physics is regulator independent

Interacting Fermi-gas model

Ansatz for the effective action:

$$\Gamma_k[\varphi, \psi] = \int d^4x \left[\bar{\psi} (i\partial - g\varphi) \psi + \frac{1}{2} (\partial_\mu \varphi)^2 - U_k(\varphi) \right]$$

Fermions : $m=0$, **Yukawa-coupling** generates mass

Bosons: the **potential** contains self interaction terms

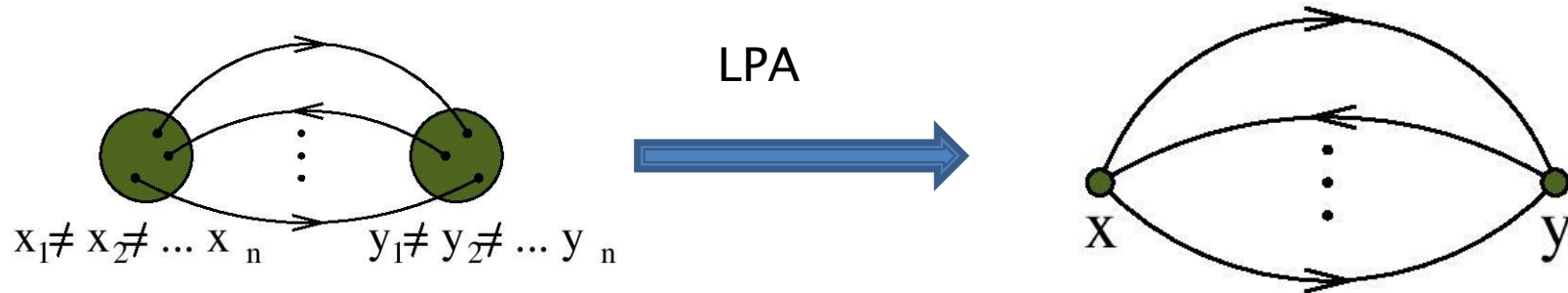
We study the scale dependence of the potential only!!

Local Potential Approximation (LPA)

What does the ansatz exactly mean ?

LPA is based on the assumption that the contribution of these two diagrams are close.

(momentum dependence of the vertices is suppressed)



This implies the following ansatz for the effective action:

$$\Gamma_k [\psi] = \int d^4x \left[\frac{1}{2} \psi_i K_{k,ij} \psi_j + U_k (\psi) \right]$$

Interacting Fermi-gas at finite temperature

Ansatz for the effective action:

$$\Gamma_k[\varphi, \psi] = \int d^4x \left[\bar{\psi} (i\partial - g\varphi) \psi + \frac{1}{2} (\partial_\mu \varphi)^2 - U_k(\varphi) \right]$$



Wetterich -equation

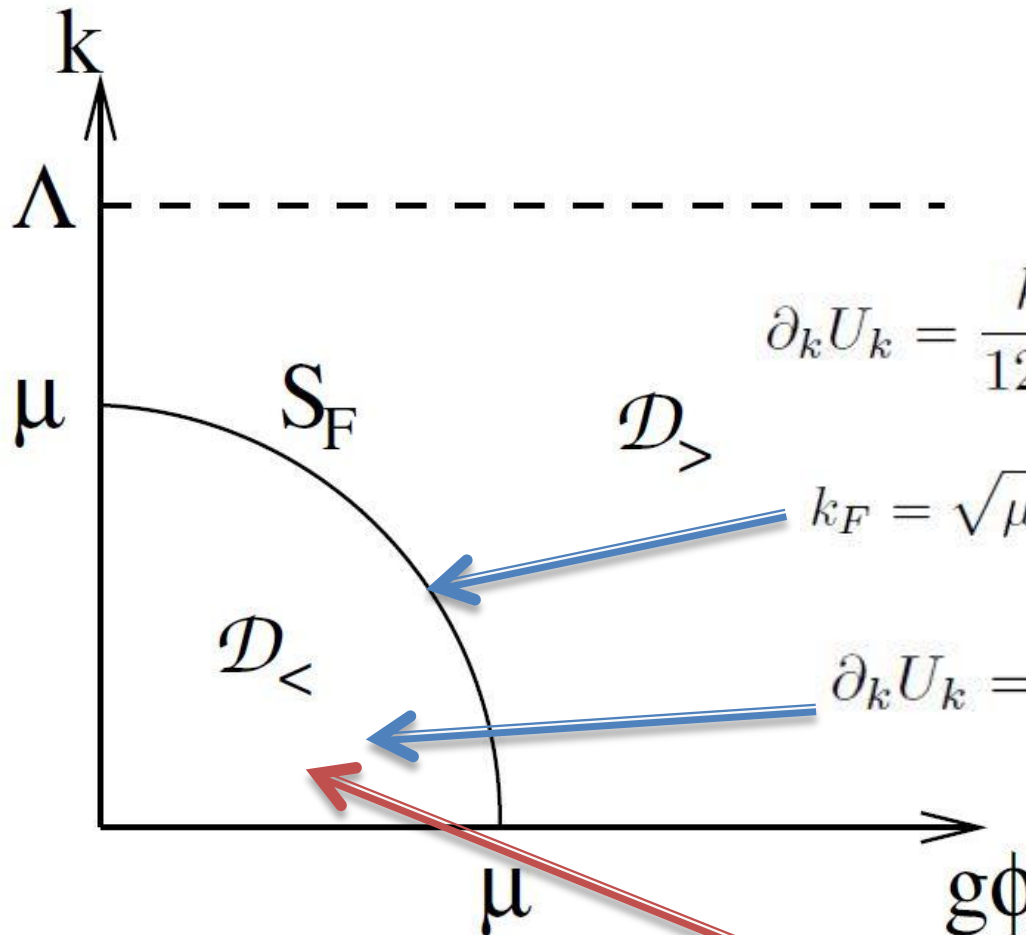
$$\partial_k U_k = \frac{k^4}{12\pi^2} \left[\underbrace{\frac{1 + 2n_B(\omega_B)}{\omega_B}}_{\text{Bosonic part}} + 4 \underbrace{\frac{-1 + n_F(\omega_F - \mu) + n_F(\omega_F + \mu)}{\omega_F}}_{\text{Fermionic part}} \right]$$

$$U_\Lambda(\varphi) = \frac{m_0^2}{2} \varphi^2 + \frac{\lambda_0}{24} \varphi^4 \quad \omega_F^2 = k^2 + g^2 \varphi^2 \quad \omega_B^2 = k^2 + \partial_\varphi^2 U \quad n_{B/F}(\omega) = \frac{1}{1 \mp e^{-\beta\omega}}$$

Interacting Fermi-gas at zero temperature

$$T=0, \mu \neq 0 \implies n_F(\omega) \rightarrow \Theta(-\omega)$$

We have two equations for the two values of the step function each valid on different domain



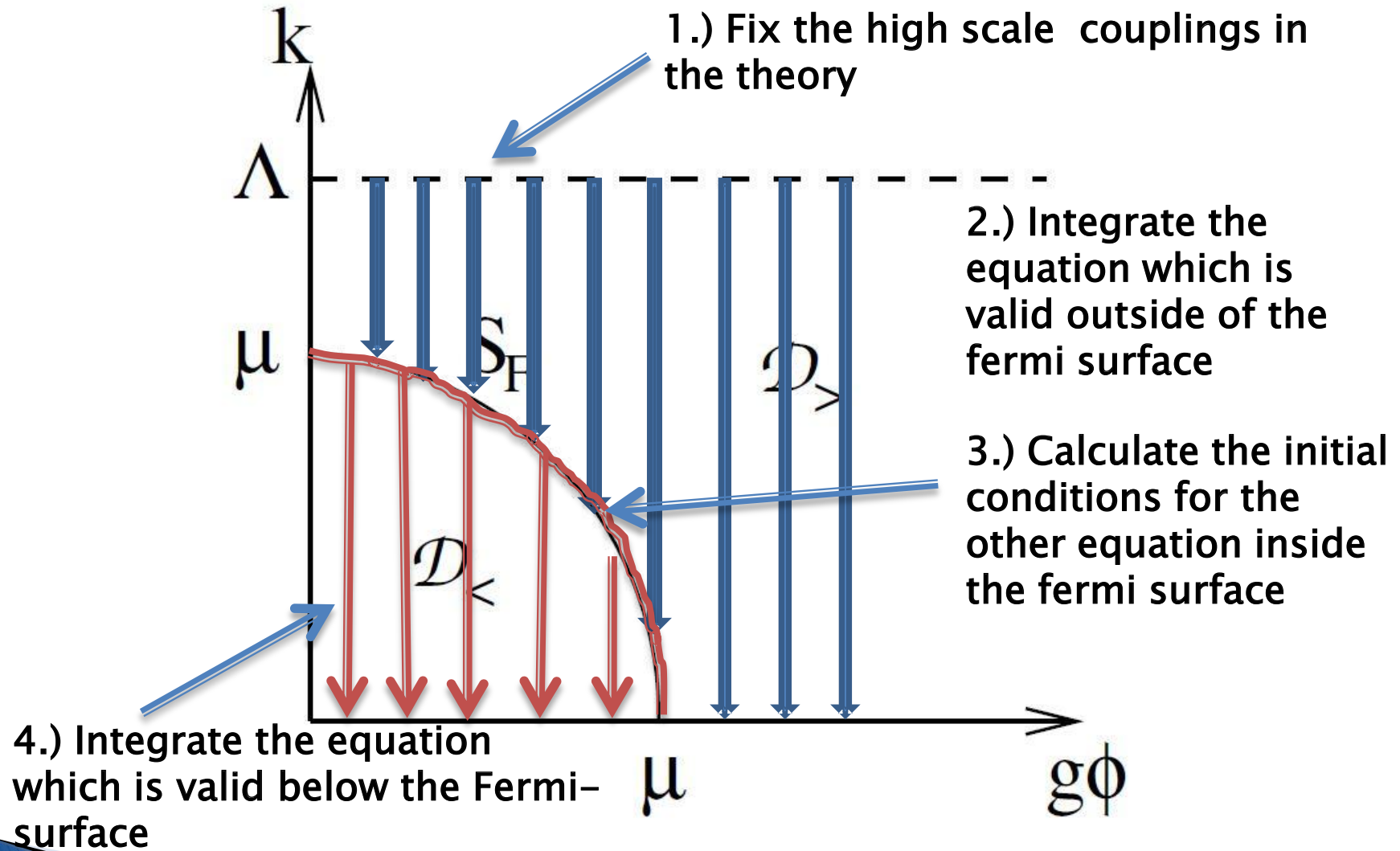
$$\partial_k U_k = \frac{k^4}{12\pi^2} \left[\frac{1}{\omega_B} - \frac{4}{\omega_F} \right]$$

$$k_F = \sqrt{\mu^2 - g^2 \varphi^2}, \text{ Fermi-surface}$$

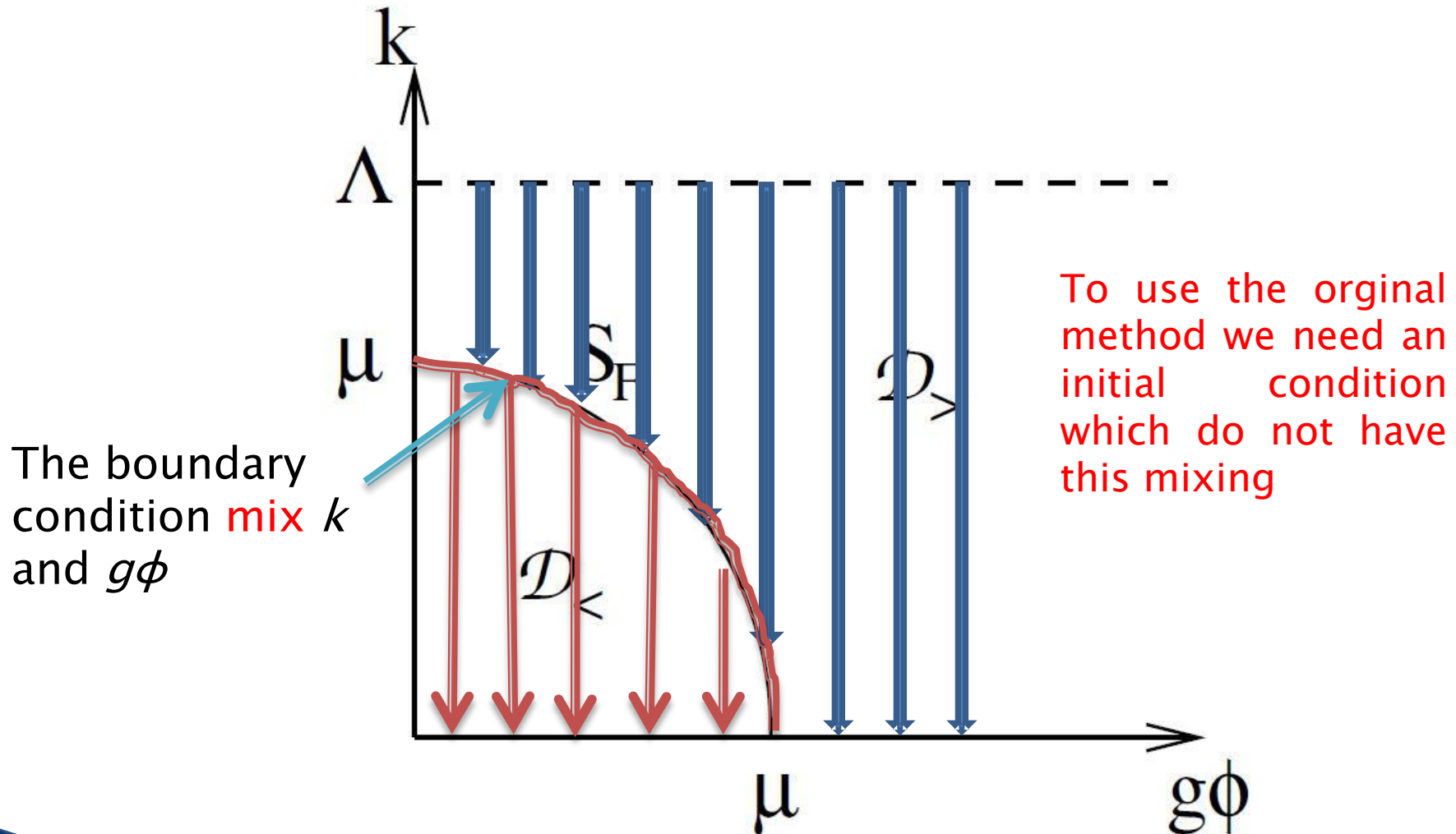
$$\partial_k U_k = \frac{k^4}{12\pi^2} \frac{1}{\omega_B}$$

Fermionic vacuum fluctuations and thermodynamic fluctuations cancel

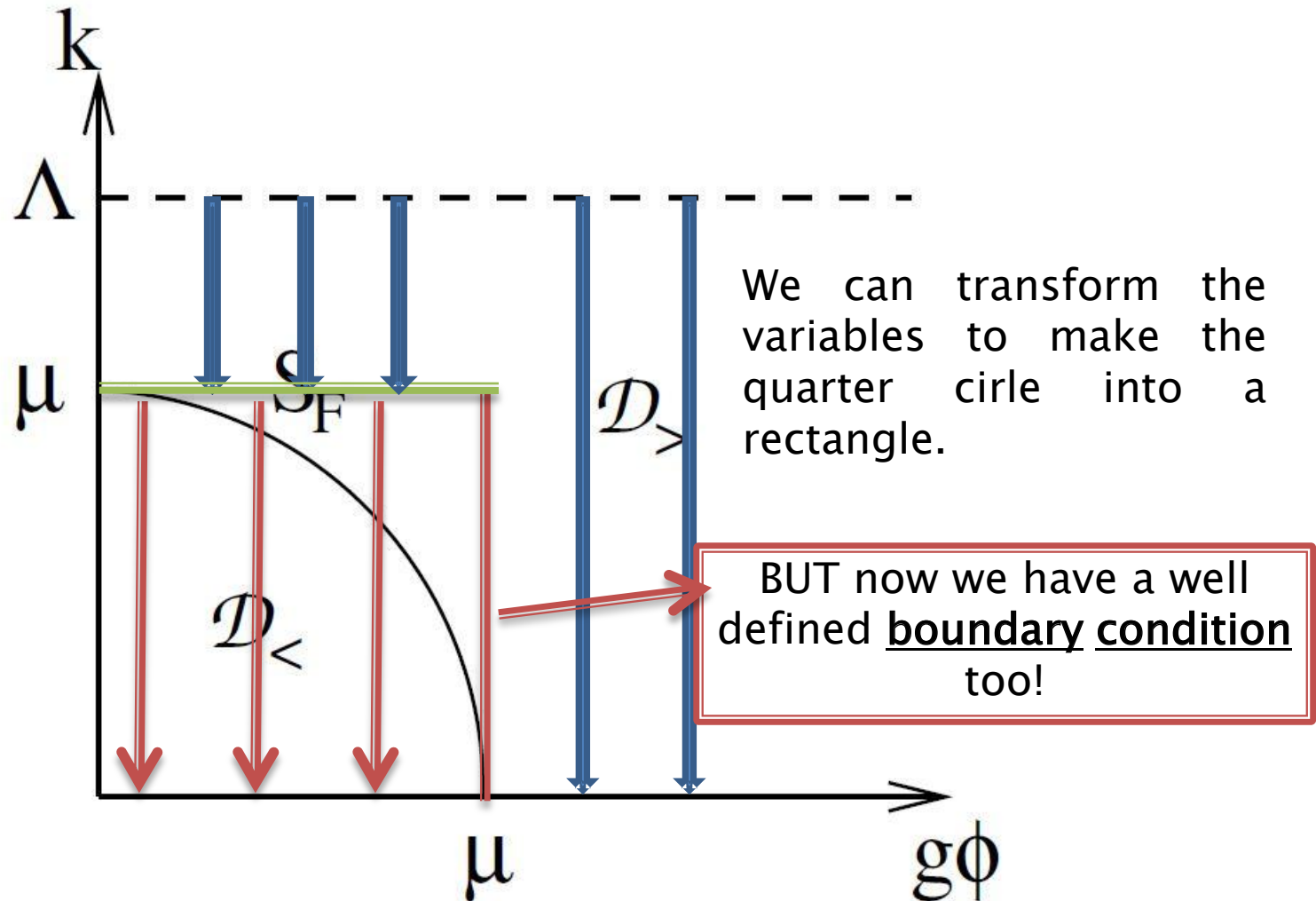
Integration of the Wetterich-equation



BUT...



Transform the variables



The transformed equation

- ▶ Circle–rectangle transformation: $(k, \varphi) \mapsto (x, y) \quad x = \varphi_F(k), \quad y = \frac{\varphi}{x}$
- ▶ Transformation of the potential: $\tilde{U}(x, y) = V_0(x) + \tilde{u}(x, y)$
- ▶ The transformed Wetterich–equation:



Boundary condition
at Fermi–surface

$$x\partial_x\tilde{u} = -xV_0' + y\partial_y\tilde{u} - \frac{g^2(kx)^3}{12\pi^2} \frac{1}{\sqrt{(kx)^2 + \partial_y^2\tilde{u}}}$$

- ▶ And the new boundary conditions:

$$\tilde{u}(x = 0, y) = \tilde{u}(x, y = \pm 1) = 0.$$

Solution by orthogonal system

- ▶ Solution is expanded in an **orthogonal basis** to accommodate the strict boundary condition in the transformed area

$$\tilde{u}(x, y) = \sum_{n=0}^{\infty} c_n(x) h_n(y) \quad h_n(1) = 0 \quad \int_0^1 dy h_n(y) h_m(y) = \delta_{nm}$$

- ▶ The **square root** in the Wetterich-equation is also expanded:

$$x c'_n(x) = \int_0^1 dy h_n(y) \left[-x V'_0 + y \partial_y \tilde{u} - \frac{g^2 (kx)^3}{12\pi^2} \underbrace{\sum_{p=0}^{\infty} \binom{-1/2}{p} \frac{(\partial_y^2 \tilde{u} - M^2)^p}{\omega^{2p+1}}}_{\text{Expanded square root}} \right]$$

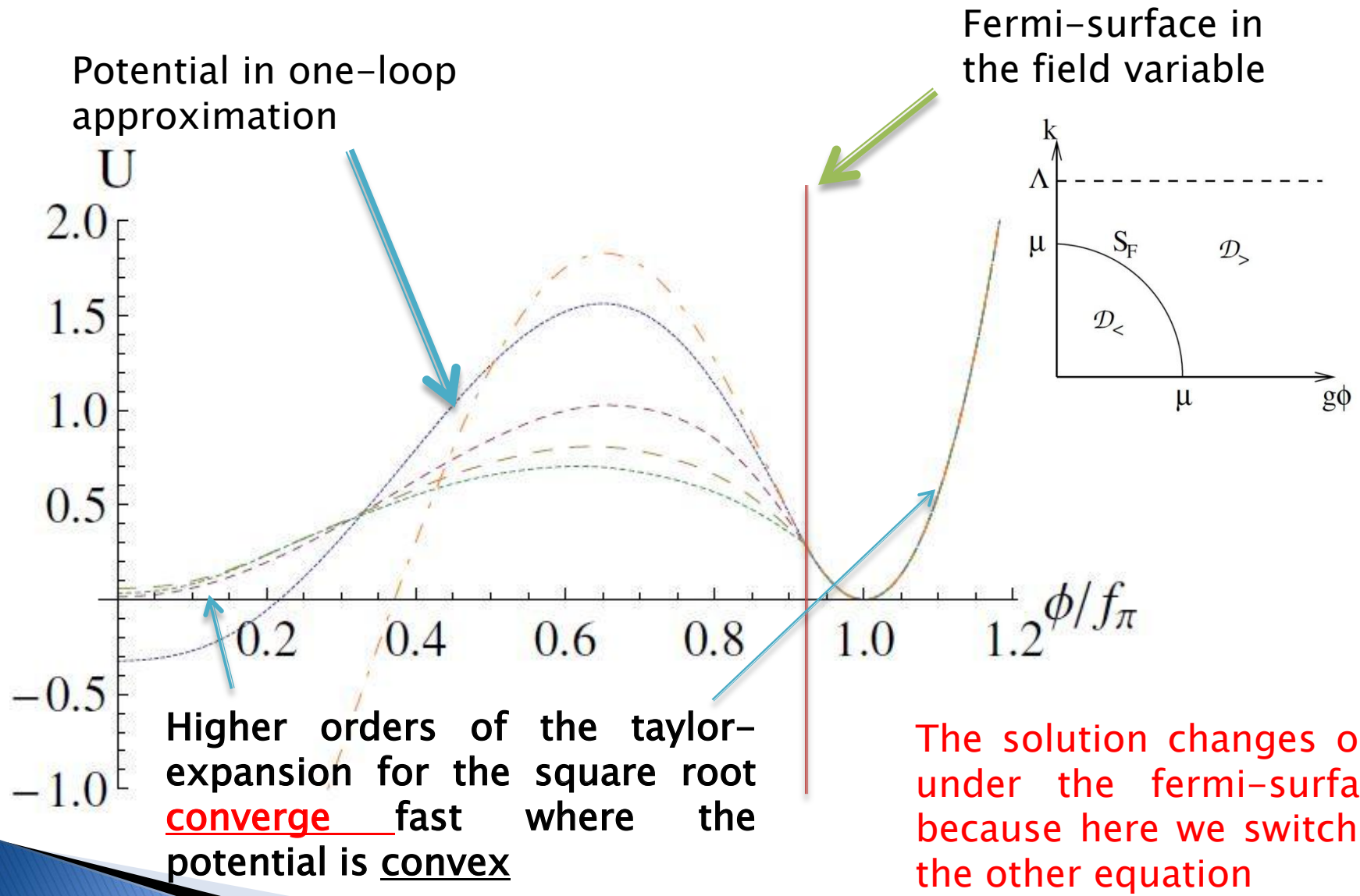
Where: $\omega^2 = (kx)^2 + M^2$

Expanded square root

We use harmonic base

$$h_n(y) = \sqrt{2} \cos q_n y, \quad q_n = (2n + 1) \frac{\pi}{2}$$

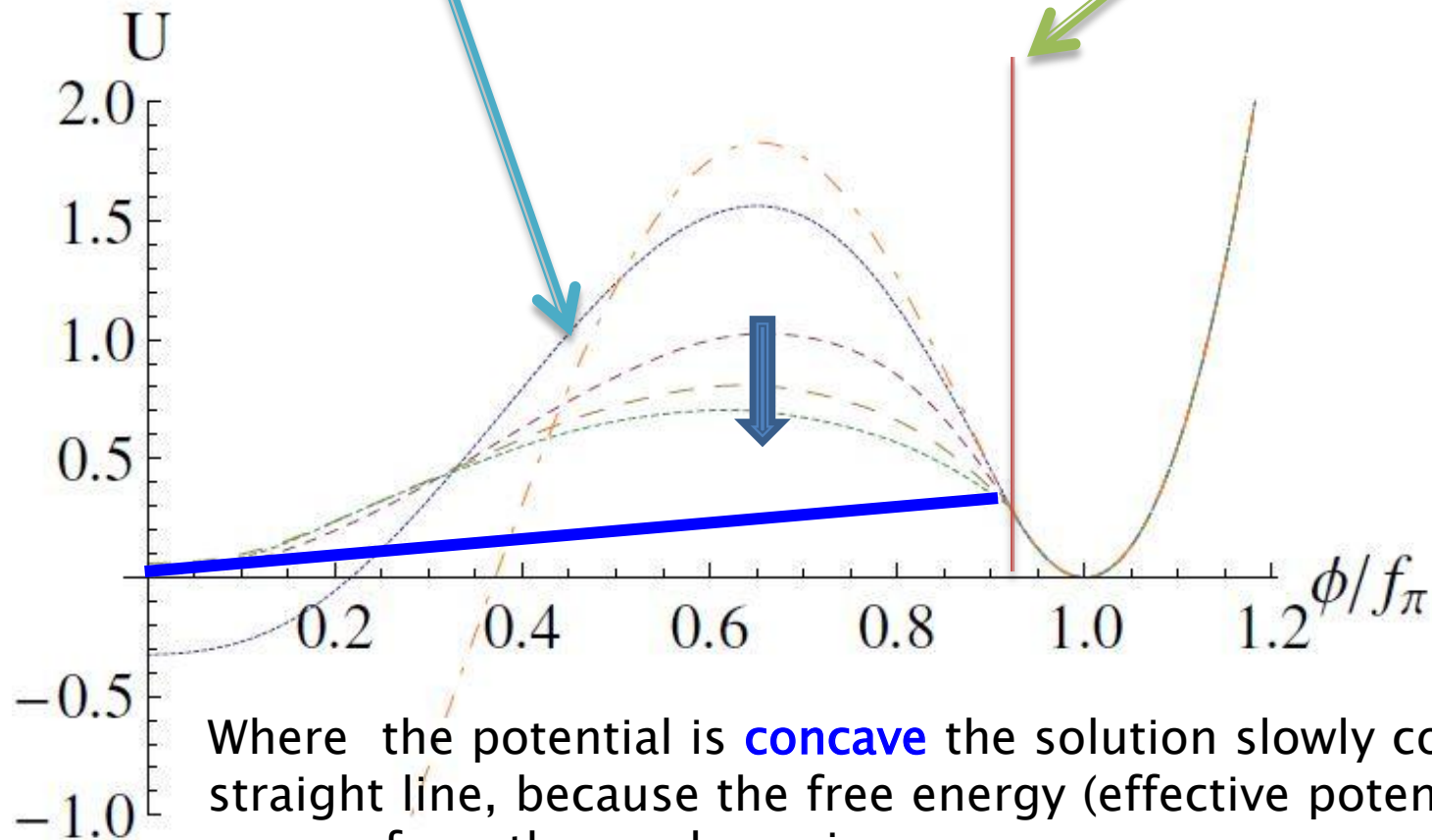
Results-I



Results-I

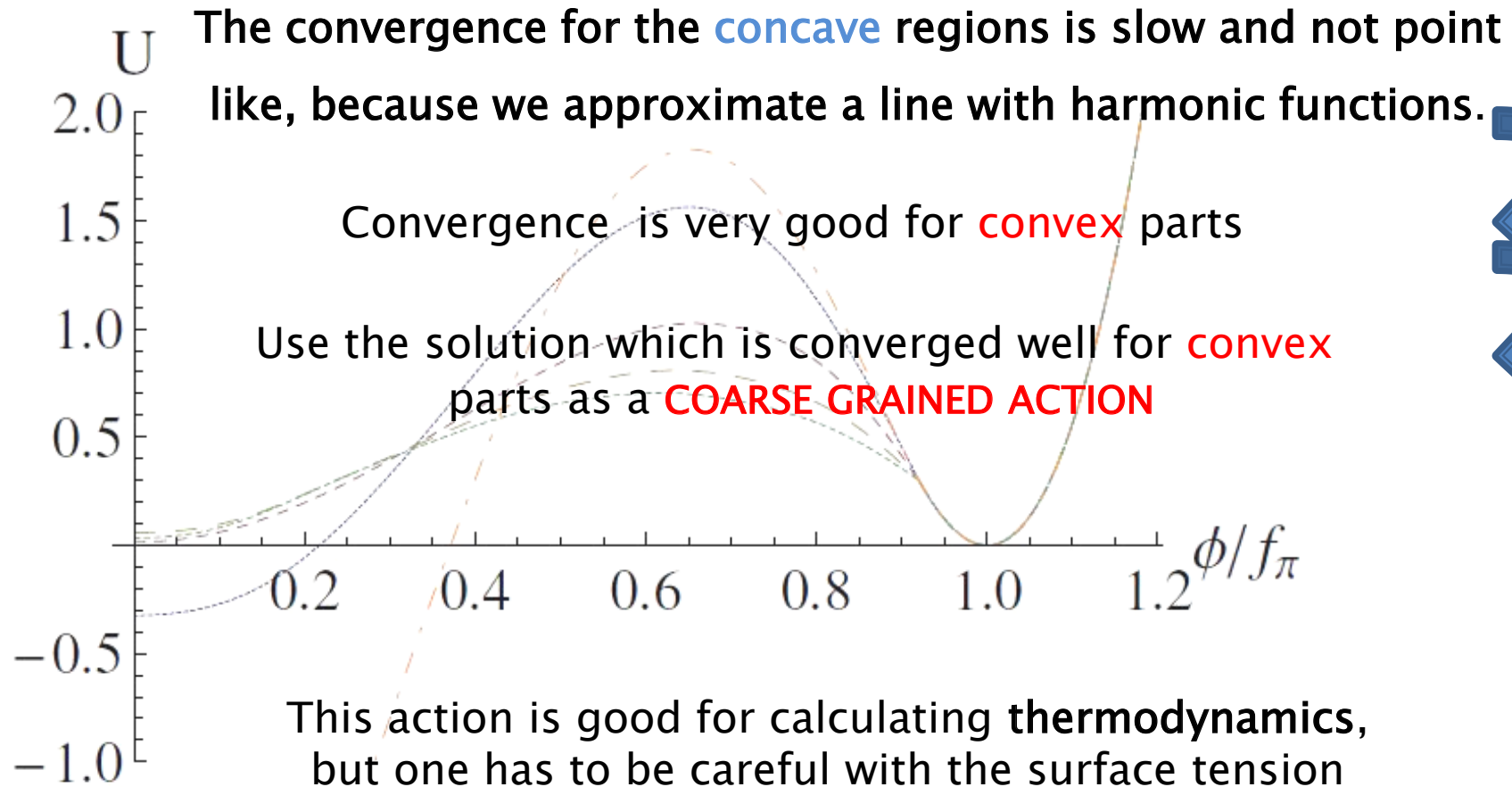
Potential in one-loop approximation

Fermi-surface in the field variable



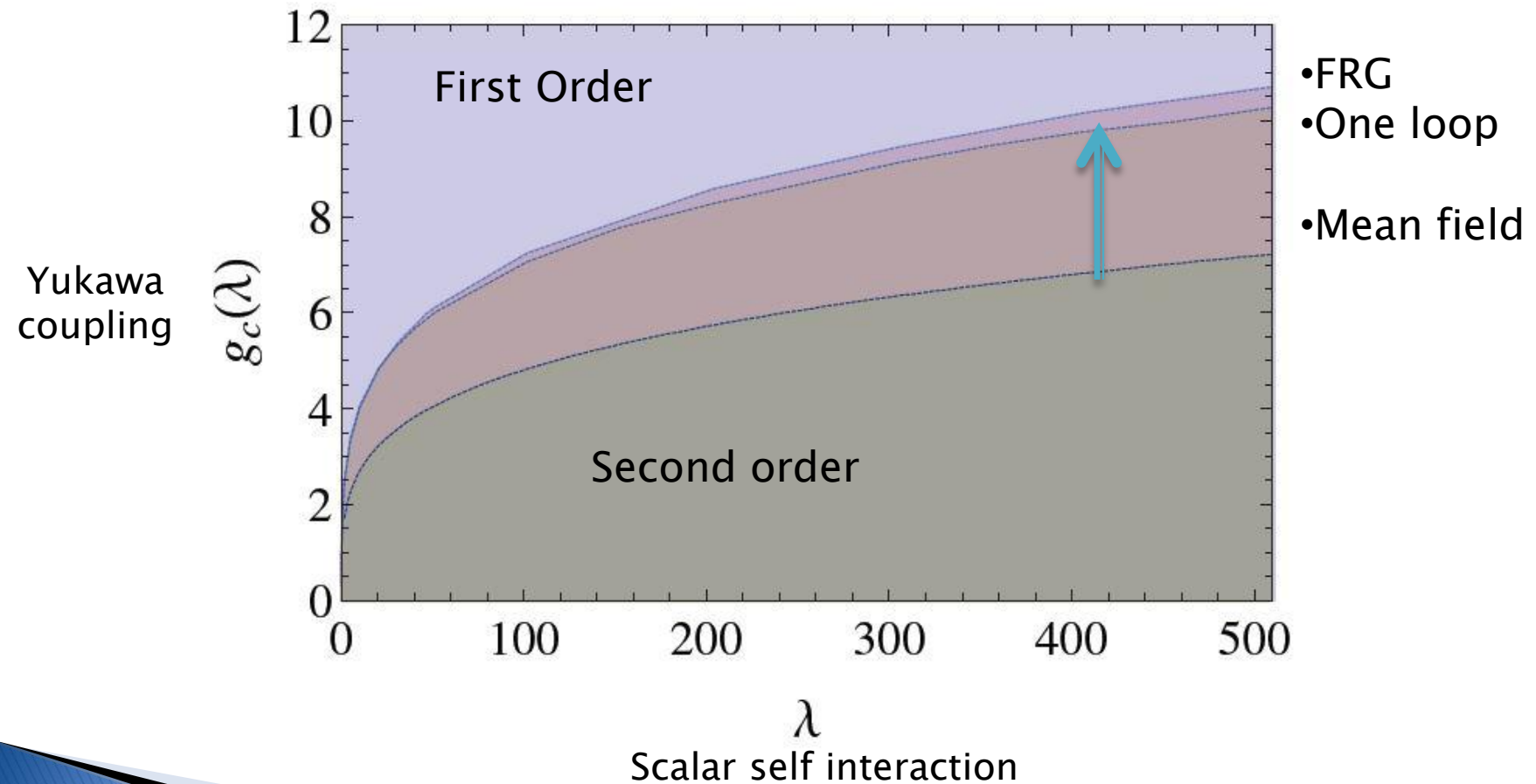
This is the **Maxwell construction**.

Results-I



Results-II

Phase structure of the interacting Fermi-gas model



Application

Effective model for the chiral phase transition

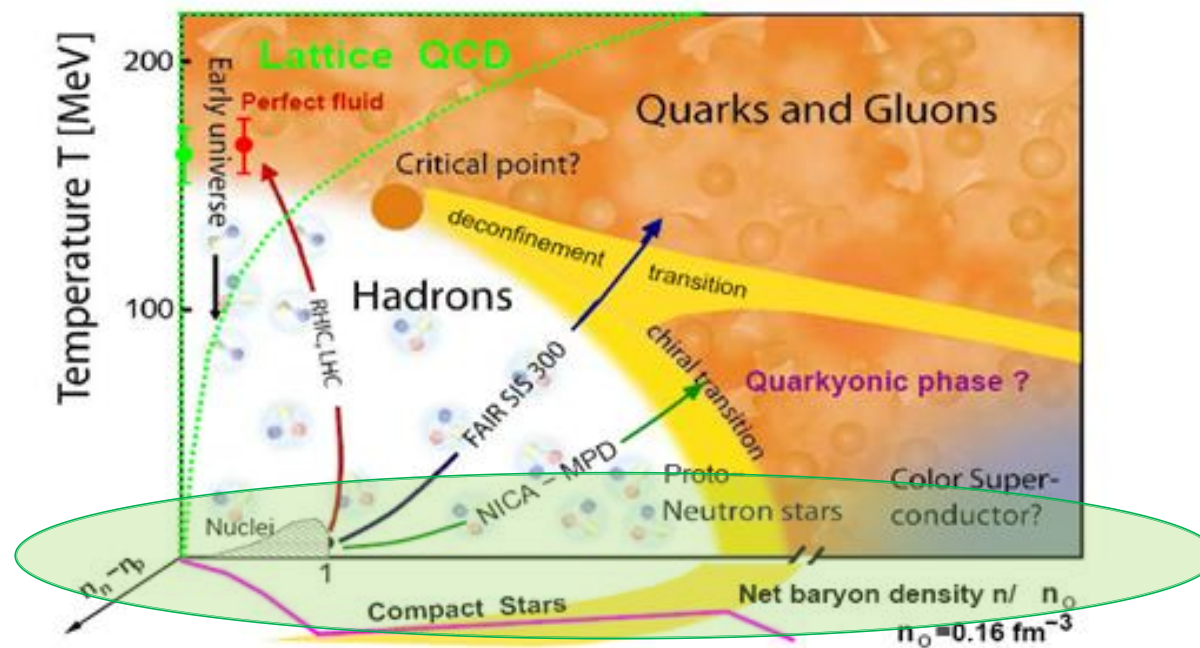
We can study the hardly accesible parts of the QCD phase diagram.

- ▶ High density
- ▶ High chemical potential
- ▶ Low temperature

Neutron star equation of state



Gravity -wave sources!



Thank you for the attention !

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