A MODEL OF DARK MATTER AND ENERGY

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Introduction

A well-known question in theoretical cosmology concerns the cosmological constant: why does it have the value observed?

This constant, Λ, has the dimensions of a density and observations suggest that its value is of order

$$\Lambda = +\, O\left((meV)^4\right). \qquad (1)$$

It is sometimes stated that this value is surprisingly small, and ingenious explanations have been offered for its alleged smallness.

Conventional wisdom is that the energy make-up of the universe is approximately 5% normal matter, 25% dark matter and 70% dark energy where dark energy is responsible for the observed accelerated expansion.

In the present talk, we discuss a model in which both of these issues are addressed in a novel way. The cosmological constant naturally appears with a value consistent with Eq.(1) without fine-tuning. Our calculations will all be carried out with the Planck constant $\hbar$. and hence the Planck mass $M_{Planck} \equiv \sqrt{\hbar G/c}$, set equal to zero. In other words, Eq.(1) has no connection at all with quantum mechanics.

How can it be, that Eq.(1) has no connection to the Planck mass? That sounds crazy! We cannot provide a quick and sharp answer to this question. Instead, we shall begin by providing a broad-brush discussion of our model which may hint at the answer, and also prepare the reader for some of our later results which may seem, at first sight, to be counterintuitive.

In the model there are four different ranges of length scale for each of which the ruling forces are different. At the largest scale (the universe) it is electromagnetic; at the second scale (clusters, galaxies, planetary systems) it is gravitational; at the third scale (molecules, atoms) it is electromagnetic again; only at the fourth and smallest scale (nucleons, quarks and leptons) is electromagnetism joined by strong and weak interactions. Note that the first and third scales, dominated by electromagnetism "sandwich" the second scale, dominated by gravitation. It seems conceivable then that, within this cosmology, careful study of the EM$\rightarrow$ Gravity $\leftarrow$EM matching across the 1st$\rightarrow$2nd$\leftarrow$ 3rd ranges of mass scales could shed light on how to make a theory of gravity beyond Einstein.

As for the energy make up, in our model it is 5% normal matter and 95% dark matter, with 0% dark energy. Dark energy is replaced by a 70% part in the 95% dark matter, a part which is composed of electrically-charged, extremely-massive primordial black holes.

The visible universe

In this section, we establish the mean mass density of our model universe.

Its present co-moving radius is 14 Gpc which translates into a volume

$$V_U \sim 4 \times 10^{89} cm^3 \sim \left(5 \times 10^{91} (meV)^3\right)^{-1} \tag{2}$$

We shall adopt as the mass M_U of the model universe

$$M_U \equiv 10^{23} M_\odot \sim 2 \times 10^{92} meV \tag{3}$$

where we have used the approximation $M_\odot = 2 \times 10^{30}$ kg.

The density ρ_U in the model is therefore

$$\rho_U = \left(\frac{M_U}{V_U}\right) \sim (\sqrt{2}meV)^4 \tag{4}$$

which, by comparison with Eq.(1), shows that

$$\rho_U = O(\Lambda) \tag{5}$$

meaning that the mean mass density of our model universe is within one order of magnitude of the cosmological constant observed in the real universe.

We note that the above values of M_U and ρ_U are for inclusion only of Old Dark Matter. Both will be exactly doubled when we add New Dark Matter, but the order of magnitude statement in Eq.(5), the only thing actually used in all of our ensuing discussion, will remain valid.

Old dark matter

For the dark matter in galaxies and clusters, we assume the dark matter constituents are PIMBHs (IM=Intermediate Mass) with masses in the range

$$100M_\odot < M_{PIMBH} < 10^5 M_\odot \quad (6)$$

This assumption could be tested using microlensing of the stars in the Magellanic Clouds, for which a precursor is the MACHO Collaboration. They discovered light curves corresponding to PIMBH masses only up to $\sim 10M_\odot$. Checking the mass range in Eq.(6) cannot be done quickly as the light curve duration is ~ 2 years for $M_{PIMBH} = 100M_\odot$ and increases to ~ 60 years for $M_{PIMBH} = 10^5 M_\odot$.

In our specific model of Old Dark Matter, we take 10^{21} PIMBHs, each with mass $100 M_\odot$.

We include as Old Dark Matter also the supermassive black holes observed in galactic centres. For these we assume a mass range

$$10^6 M_\odot < M_{PSMBH} < 10^{11} M_\odot \quad (7)$$

and for these, we assume they all are primordial on the basis that there seems to be insufficient time for stellar-collapse black holes to reach the mass range in Eq.(7) by accretion and merging.

In the model we take, for simplicity, PSMBHs all with mass $10^7 M_\odot$, and one in each of the 10^{11} galaxies. In other words, the total number n_{PSMBH} of PSMBHs in the visible universe is $n_{PSMBH} = 10^{11}$.

New dark matter ≡ Dark energy

We follow the suggestion made in 2022 that there exist a number of PEMBHs (EM=Extremely Massive) where

$$10^{12}M_{\odot} < M_{PEMBH} < 10^{22}M_{\odot} \qquad (8)$$

We shall take for definiteness $M_{PEMBH} = 10^{12}M_{\odot}$ and thus, according to the semi-empirical rule enunciated in 2022, they each carry negative charge $Q_{PEMBH} \simeq -4 \times 10^{32}$ Coulombs. We shall take a number of PEMBHs

$$n_{PEMBH} = 10^{11}. \qquad (9)$$

Their total mass is therefore $M_{total}(PSMBH) = 10^{23}M_{\odot}$.

We are neglecting spin, so each charged black hole is described by a Reissner-Nördstrom (RN) metric

$$ds^2 = f(r)dt^2 - f(r)^{-1}dr^2 - r^2 d\theta^2 - r^2 \sin^2\theta d\phi^2 \tag{10}$$

where

$$f(r) \equiv \left(1 - \frac{r_S}{r} + \frac{r_Q}{r^2}\right). \tag{11}$$

with

$$r_S = 2GM \qquad r_Q = Q^2 G \tag{12}$$

The horizon(s) of the RN metric are where

$$f(r) = 0 \tag{13}$$

which gives

$$r_\pm = \frac{1}{2}\left(r_S \pm \sqrt{r_S^2 - 4r_Q^2}\right) \tag{14}$$

For $2r_Q < r_S$, $Q < M$, there are two horizons. When $2r_Q = r_S$, $Q = M$ the RN black hole is extremal and there is only one horizon.

If $2r_Q > r_S$, $Q > M$, the RN black hole is super-extremal, there is no horizon at all and the $r = 0$ singularity is observable to a distant observer. This is known as a naked singularity.

All of the PEMBHs are super-extremal RN black holes so our model universe contains precisely a hundred billion naked singularities. We are aware of the cosmic censorship hypothesis which would allow only zero naked singularities but that hypothesis is, to our knowledge, unproven so we believe this is not a fatal flaw.

The suggestion in 2022, which the present talk will support, is that the Coulomb repulsion between PEMBHs could explain the observed accelerating expansion of the universe. If so, it must lead to a negative pressure as one requirement and we shall show that this actually occurs.

More importantly, we shall show that the magnitude of this pressure is consistent with the observed equation of state associated with the cosmological constant.

Given the size of the visible universe discussed in the introduction, it is straightforward to estimate that the mean separation $\bar{L}$ of the PEMBHs is a few parsecs while their Schwarzschild radius is $r_S \sim 0.1$ pc, The wide separation, with $r_S/\bar{L} \sim 10^{-7}$, implies that an expansion in $1/r$ is rapidly convergent and this fact will simplify our derivation of the pressure.

To derive the pressure we shall need to evaluate the gravitational stress-energy pseudotensor

$$T_{\mu\nu}^{(GRAV)} = -\frac{1}{8\pi G}\left(G_{\mu\nu} + \Lambda g_{\mu\nu}\right) + \frac{1}{16\pi G(-g)}\left((-g)(g_{\mu\nu}g_{\alpha\beta} - g_{\mu\alpha}g_{\nu\beta})\right)_{,\alpha\beta}$$

where the final subscripts represent simple partial derivatives. We shall need also to evaluate the electromagnetic counterpart

$$T_{\mu\nu}^{(EM)} = F_{\mu\alpha}g^{\alpha\beta}F_{\mu\beta} - \frac{1}{4}g_{\mu\nu}F^{\alpha\beta}F_{\alpha\beta}. \quad (15)$$

Let us begin with $T_{\mu\nu}^{(GRAV)}$. This calculation involves taking up to the second derivatives of the metric.

For the ubiquitous function $f(r)$:

$$\frac{\partial}{\partial r} f(r) \sim O(1/r^2) \qquad \frac{\partial^2}{\partial^2 r} f(r) \sim O(1/r^4) \tag{16}$$

$$\frac{\partial}{\partial r} (f(r))^2 \sim O(1/r^2) \quad \frac{\partial^2}{\partial^2 r} (f(r))^2 \sim O(1/r^4) \tag{17}$$

$$\frac{\partial}{\partial r} (f(r))^{-1} \sim O(1/r^2) \quad \frac{\partial^2}{\partial^2 r} (f(r))^{-1} \sim O(1/r^4) \tag{18}$$

$$\frac{\partial}{\partial r} (f(r))^{-2} \sim O(1/r^2) \quad \frac{\partial^2}{\partial^2 r} (f(r))^{-2} \sim O(1/r^4) \tag{19}$$

It is not difficult to check, using these derivatives, that the 1st, 3rd and 4th terms in Eq.(15) all fall off as $O(1/r^2)$ and that only the 2nd term does not.

Turning to $T_{\mu\nu}^{(EM)}$, and using the fact that the gauge potential for an RN black hole is

$$A_\mu = \left(\frac{Q}{r}, 0, 0, 0\right) \qquad (20)$$

we can straightforwardly see that both terms in Eq.(15) fall off as $O(1/r^2)$.

Collecting results for the two pieces, gravitational and electromagnetic, of the stress-energy tensor we deduce that

$$T_{\mu\nu}^{(GRAV)} + T_{\mu\nu}^{(EM)} = -\left(\frac{\Lambda}{8\pi G}\right) g_{\mu\nu} + O(1/r^2). \qquad (21)$$

The cosmological constant is predicted to be

$$\Lambda \sim + O\left((meV)^4\right). \qquad (22)$$

which is consistent with its observed value.

Since the stress-energy tensor is proportional to the metric tensor, and bearing in mind the rapid convergence of the $(1/r)$ expansion, the equation of state is predicted to be

$$\omega = P/\rho = -1 \pm O(10^{-14}) \tag{23}$$

extremely close to the value when the cosmological term in the generalised Einstein tensor is proportional to the metric.

Assuming an FLRW metric for the visible universe, and zero curvature, the Friedmann equation is, ignoring radiation,

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{\Lambda}{3} + \frac{8\pi}{3}\rho_{matter} \tag{24}$$

so that the PEMBHs of the New Dark Matter provide a cosmological component indistinguishable, as far as the expansion properties of the visible universe are concerned, from what was called dark energy. Hence: New Dark Matter $\equiv$ Dark Energy.

Entropy

Of the known constituents in the universe the entropy is dominated by the supermassive black holes, PSMBHs at the galactic centres. In our model there are 10^{11} galaxies each with a $10^7 M_\odot$ supermassive black hole at its centre. Using the well-known PBH entropy formula for black holes

$$\left(\frac{S}{k}\right)_{BH} (\eta M_\odot) \simeq 10^{78}\eta^2 \qquad (25)$$

the PSMBHs are seen to contribute

$$S/k(PSMBHs) \sim 10^{103}. \qquad (26)$$

to the entropy of the universe.

The holographic maximum entropy of the universe is its surface area in units of $L_{Planck}^2 \sim 10^{-66} cm^2$, namely

$$S/k(holog) \sim \frac{4\pi(14Gpc)^2}{L_{Planck}^2} \sim 10^{123} \quad (27)$$

so that the PSMBH entropy in Eq.(26) falls far short of this, by some twenty orders of magnitude.

For the dark matter in galaxies necessary to explain observed rotation curves, we include PIMBHs. We take 10^{11} of them each with mass $M_{PIMBH} = 100 M_{\odot}$ which leads to

$$S/k(PIMBHs) \sim 10^{103} \quad (28)$$

which, by coincidence, approximately equals the entropy in Eq.(26) from PSMBHs.

A significant increase towards the holographic bound arises when we add the new dark matter, the PEMBHs. If we take 10^{11} PEMBHs each with mass $10^{12}M_{\odot}$ the entropy is increased by ten orders of magnitude to

$$S/k(PEMBHs) \sim 10^{113}. \qquad (29)$$

which, if the aim is to saturate the holographic limit, is a considerable improvement.

We are unaware of any compelling argument why the holographic entropy bound for the universe should be saturated by its contents, other than that it makes the mathematics, and hence the physical universe, appear to be more beautiful.

We can achieve saturation by increasing the PEMBH mass to $10^{12+p} M_{\odot}$ and reducing correspondingly the number to $n_{PEMBH} = 10^{11-p}$ in order to arrive at the revised estimate

$$S/k(PEMBHs) \sim 10^{113+p} \qquad (30)$$

so that $p = 10$ gives the required additional ten orders of magnitude to achieve saturation. At first sight, a black hole with mass $10^{22} M_{\odot}$ sounds ridiculous, but dark matter is so mysterious that we should not leave any stone unturned.

Time Dependence

Our discussion has focused on the present cosmological time $t = t_0 \simeq 13.8$ Gy and already provided some counterintuitive ideas such as that at the largest cosmological distances, *e.g.* greater than 1 Gpc, the dominant force is electromagnetism rather than gravitation. The production mechanism for PBHs in general is not well understood, and for the cPEMBHs we shall make the simplifying assumption that they are first formed when the accelerated expansion begins at $t = t_{DE} \sim 9.8$ Gy. For the expansion before t_{DE} we shall assume that the ΛCDM model is approximately accurate.

The subsequent expansion in the charged dark matter cPEMBH model will in the future depart markedly from the ΛCDM case. We can regard this as advantageous because the future fate of the universe in the conventional picture does have certain distasteful features in terms of the extroverse, as we briefly review.

In the ΛCDM model the introverse, or what is also called the visible universe, coincides with the extroverse at $t = t_{DE} \sim 9.8$ Gy with the common radius

$$R_{EV}(t_{DE}) = R_{IV}(t_{DE}) = 39Gly \qquad (31)$$

The introverse expansion is limited by the speed of light and its radius increases from Eq. (31) to 44 Gly at the present time $t = t_0$ and asymptotes to

$$R_{IV}(t \to \infty) = 58Gly \qquad (32)$$

The extroverse expansion is exponential and superluminal. Its radius increases from its value 39 Gly in Eq. (31) to 52 Gly at the present time $t = t_0$ and grows without limit so that after a trillion years it attains the *extremely* large value

$$R_{EV}(t = 1Ty) = 9.7 \times 10^{32} Gly. \qquad (33)$$

This future for the ΛCDM scenario seems distasteful because the introverse becomes of ever decreasing, and eventually vanishing, significance, relative to the extroverse. At $t = 1$Ty, all other galaxies would have exited from the visible universe so that cosmology would no longer be possible.

Electrically neutral PEMBHS were first considered, with a different acronym SLABs, by Carr *et al* in 2021.

We shall for simplicity assume that the cPEMBHs are all formed between $t = t_{DE} \sim 9.8$ Gy and $t_0 \sim 13.8$Gy. The Friedmann equation ignoring radiation, during this time window, is

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{\Lambda}{3} + \frac{8\pi G}{3}\rho_{matter} \qquad (34)$$

where Λ is the cosmological constant generated by the Coulomb repulsion between the cPEMBHs. From Eq.(34), with $a(t_0) = 1$ and constant Λ, we would predict that

$$a(t \to \infty) \sim exp\left(\sqrt{\frac{\Lambda}{3}}(t - t_0)\right) \qquad (35)$$

However, in the case of charged dark matter, with no dark energy, we must re-write Eq, (34) as

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho_{cPEMBHs} + \frac{8\pi G}{3}\rho_{matter} \tag{36}$$

in which

$$\rho_{matter}(t) = \frac{\rho_{matter}(t_0)}{a(t)^3} \tag{37}$$

where matter includes both normal matter and the uncharged dark matter.

Of special interest in the present discussion is the expected future behaviour of the charged dark matter

$$\rho_{cPEMBHs}(t) = \frac{\rho_{cPEMBHs}(t_0)}{a(t)^3} \tag{38}$$

so that comparison of Eq.(34) and Eq.(36) suggests that the cosmological constant is predicted to decrease from its present value.

More specifically, we find that asymptotically the scale factor will behave as if matter-dominated and the cosmological constant will decrease at large future times as a power

$$a(t \to \infty) \sim t^{\frac{2}{3}} \qquad \Lambda(t \to \infty) \sim t^{-2}. \quad (39)$$

so that a trillion years in the future $\Lambda(t)$ will have decreased by some four orders of magnitude relative to $\Lambda(t_0)$.

According to the ΛCDM model, we live at a special time in cosmic history because of the density coincidence between dark matter and dark energy. In the case of charged dark matter replacing dark energy, the present era is even more special because the striking accelerated expansion, discovered in 1998, is a temporary phenomenon at around the present time. Acceleration began about 4 Gy ago at $t_{DE} = 9.8Gy = t_0 - 4Gy$.

Let us discuss the future time evolution of the introverse and extroverse in the case of charged dark matter. For the introverse, nothing changes from the ΛCDM case. After a trillion years, the introverse radius will be at its asymptotic value $R_{IV} = 58 Gly$, as stated in Eq.(32). By contrast, the future for the extroverse is very different for charged dark matter. WIth the growth $a(t) \propto t^{\frac{2}{3}}$ we find that the radius of the extroverse at $t = 1$ Ty is

$$R_{EV}(t = 1 Ty) \sim 900 Gly \tag{40}$$

to be compared with the corresponding huge value 9.7×10^{32} Gly predicted by the ΛCDM model. This means that if there still exist humans in the Solar System, or at least in the Milky Way, their view of the distant universe will include many billions of galaxies.

In the ΛCDM case, a hypothetical observational cosmologist, one trillion years in the future, could observe only the Milky Way and objects such as the Magellanic Clouds which are gravitationally bound to it, so that cosmology could become an extinct science. In the case of charged dark matter, for comparison, the time dependence will allow about 180 billion out of a present trillion galaxies to remain observable at $t = 1Ty$ so that the view of the universe at that distant future time will look quite similar to the view at the present and will provide a conducive environment for cosmology, a trillion years in the future.

Discussion

Admittedly we have studied only a simplified model of the visible universe but it is sufficiently realistic to take away two lessons: (1) Dark energy does not exist as a separate entity. It was a misidentification of one part of dark matter which is composed of electrically-charged extremely-massive primordial black holes. (2) The observed magnitude of the cosmological constant $|\Lambda| \sim (meV)^4$ is not surprisingly small but is closely equal to the mean mass density of the universe. Distinguished authors have provided clever explanations for an alleged smallness. They assume, however, that the theory of gravity beyond Einstein is based on quantum theories of fields or strings, although we do not yet know this as an established fact.

Our first argument for the existence of PEMBHs arose from saturating the holographic maximum entropy bound. This would be more compelling if the entropy of the universe were shown to be, as we suspect, a truly meaningful and useful concept. Regrettably this has not really happened, because it presently lacks a sufficiently rigorous underlying mathematical basis.

Our second argument favouring PEMBHs is that they can replace the notion of dark energy which was a term inserted into the Friedmann equation to parametrise the observed accelerating expansion. Dark energy required repulsive gravity which contradicted the fact that gravity is always attractive.

It seems more natural to ascribe the accelerated expansion to the other long-range force, electromagnetism. The same-sign charges of our PEMBHs can provide sufficient Coulomb electromagnetic repulsion. As we have shown, it can generate a cosmological constant of the correct magnitude and the observed equation of state with high accuracy.

At first sight, our model appeared counter-intuitive because at the largest length scales it seemed obvious to assume that gravitation dominates. But this assumption need not be true in Nature. In our model we actually have different regimes of length scales, and only in the one characterising the Solar System, galaxies and clusters, does gravity dominate while, at larger, truly-cosmological scales, electromagnetism takes over from gravity again.

Based on our model and the precision of the wide-separation expansion in $1/r$ we have predicted that the equation of state associated with the cosmological constant to be $\omega = -1$ to $O(10^{-14})$ accuracy and therefore, since CMB perturbations are much larger, this deviation from $\omega = -1$ is likely long to remain unobservable experimentally. If and when there emerges a convincing (quantum?)gravity theory beyond Einstein, it could possibly give further corrections to the equation of state. We suspect, however, that such corrections, if they exist, would be even smaller than the error arising from the wide-separation approximation.

Thank you for your attention