# Exploring the landscape of fermionic theories at large *N*

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*based on work with Daniel F. Litim*
*2207.10115, 2212.06815, 2311.16246, 2406.00100, ...*

**Applications of field theory to Hermitian and non-Hermitian systems @ KCL, September 2024**

UNIVERSITY OF SUSSEX

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**strong interactions**

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interacting 3d CFTs

$$\bar{\psi}_a \partial\!\!\!/ \psi_a + \frac{1}{2} G (\bar{\psi}_a \psi_a)^2$$

Wilson ’73; Gross, Neveu ’74; Parisi ’75; Gawędzki, Kupiainen ’85; Rosenstein, Warr, Park ’88;
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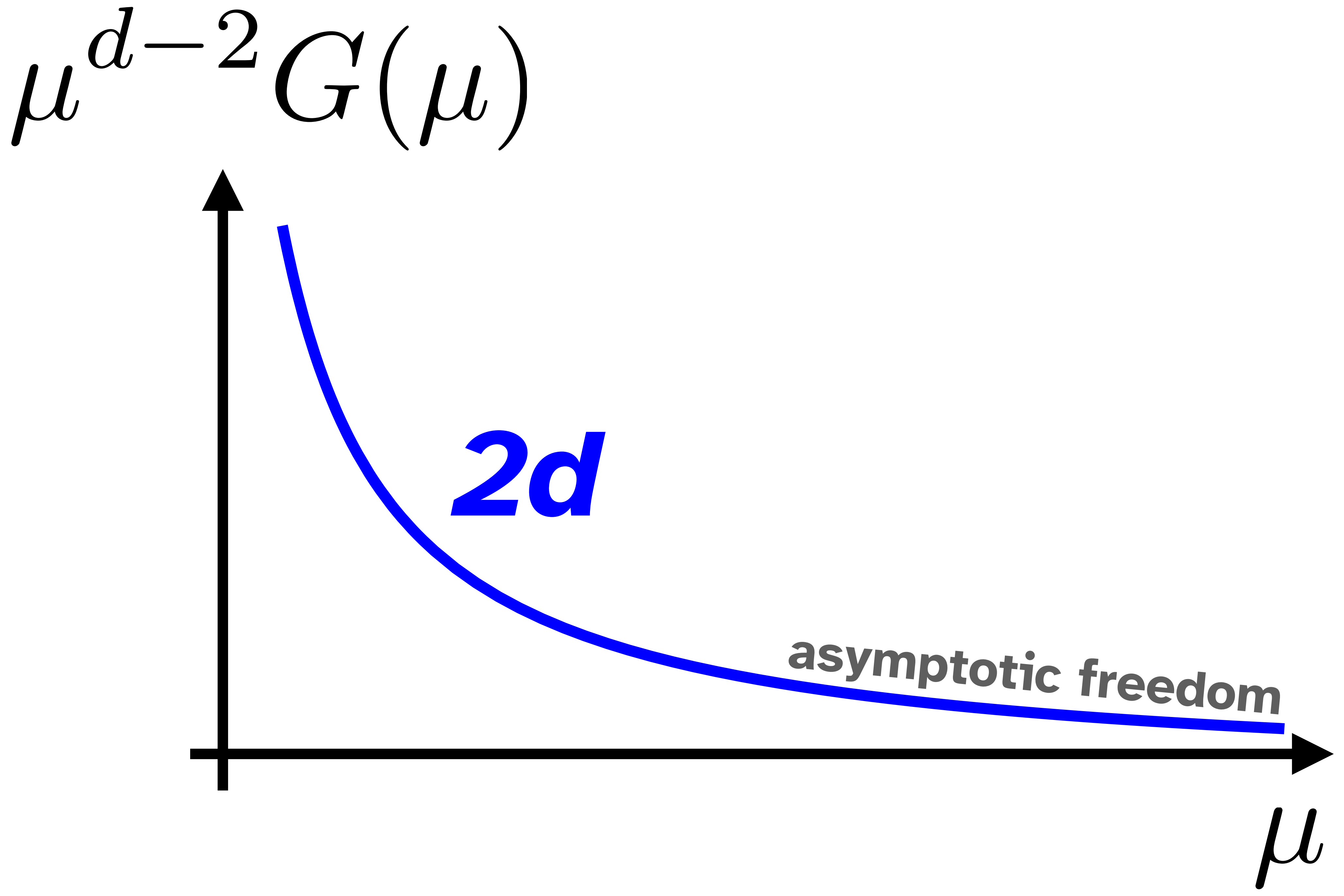


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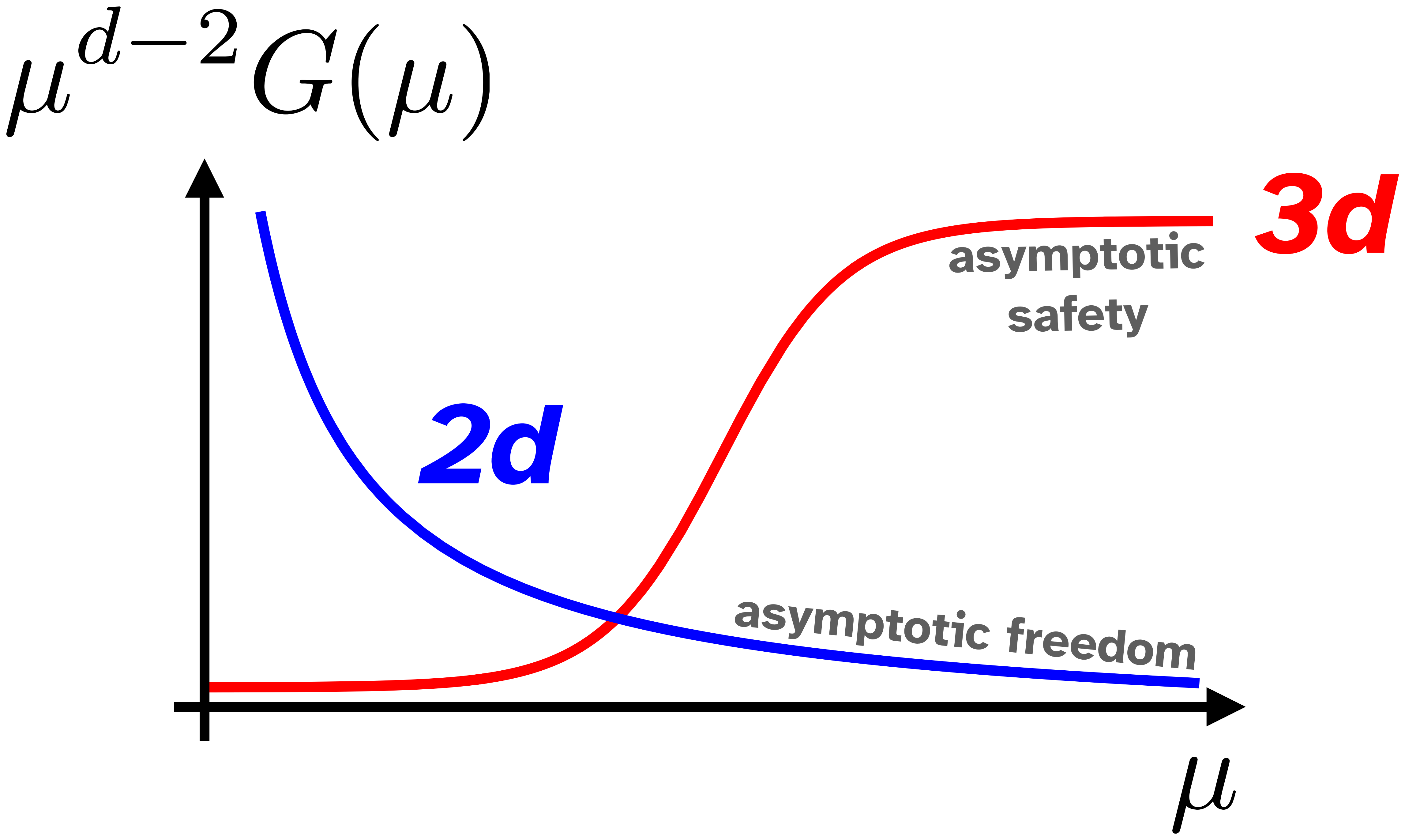


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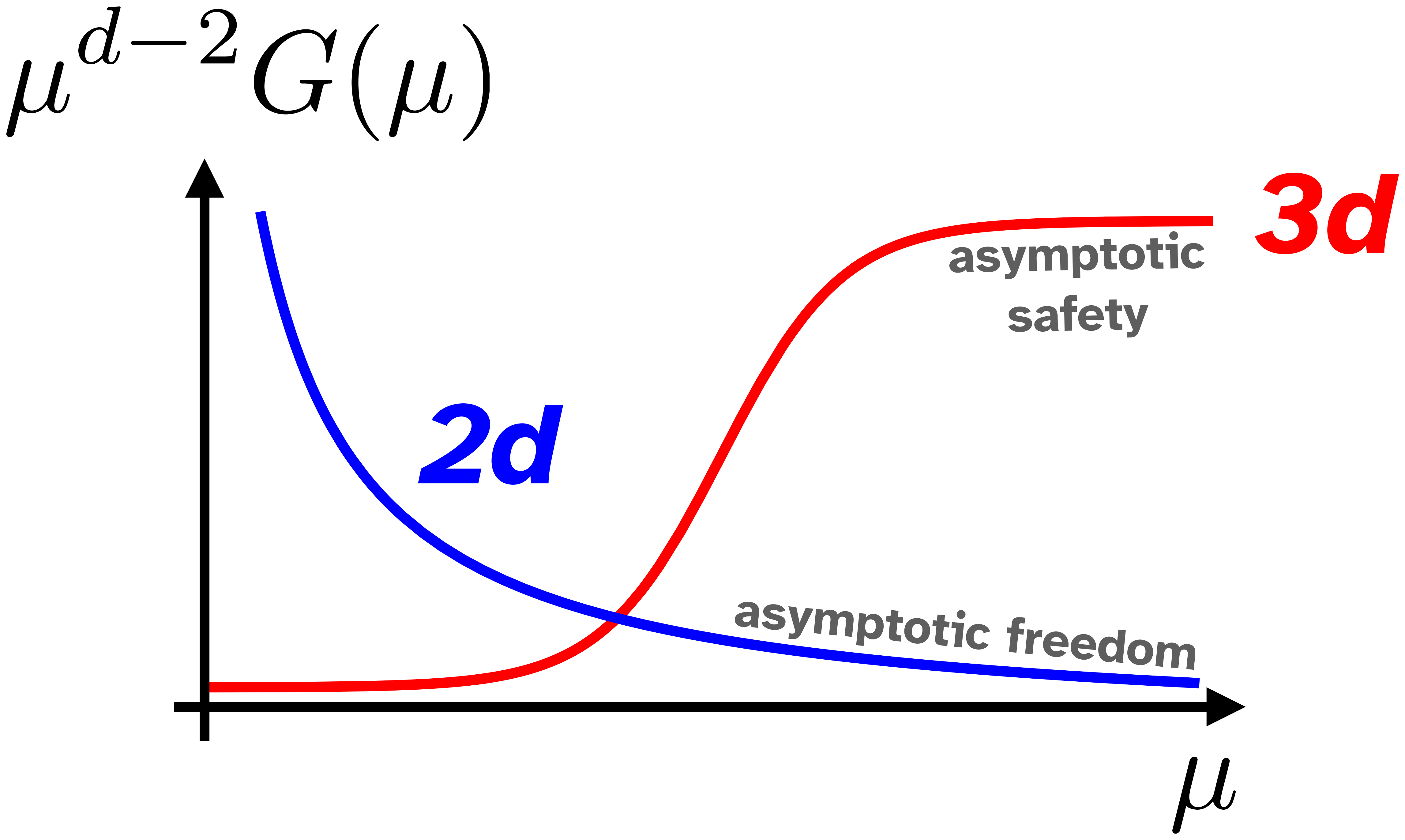


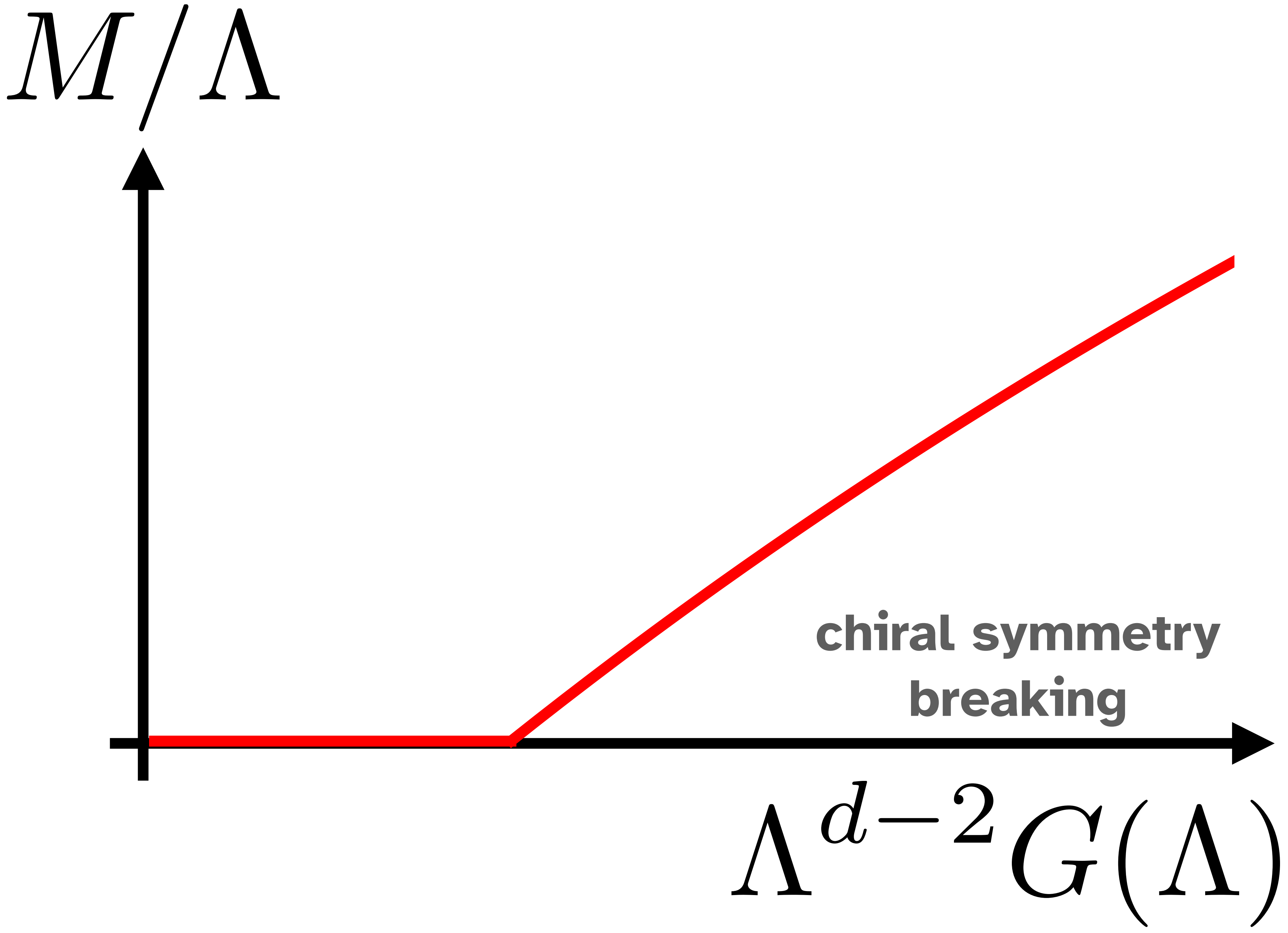


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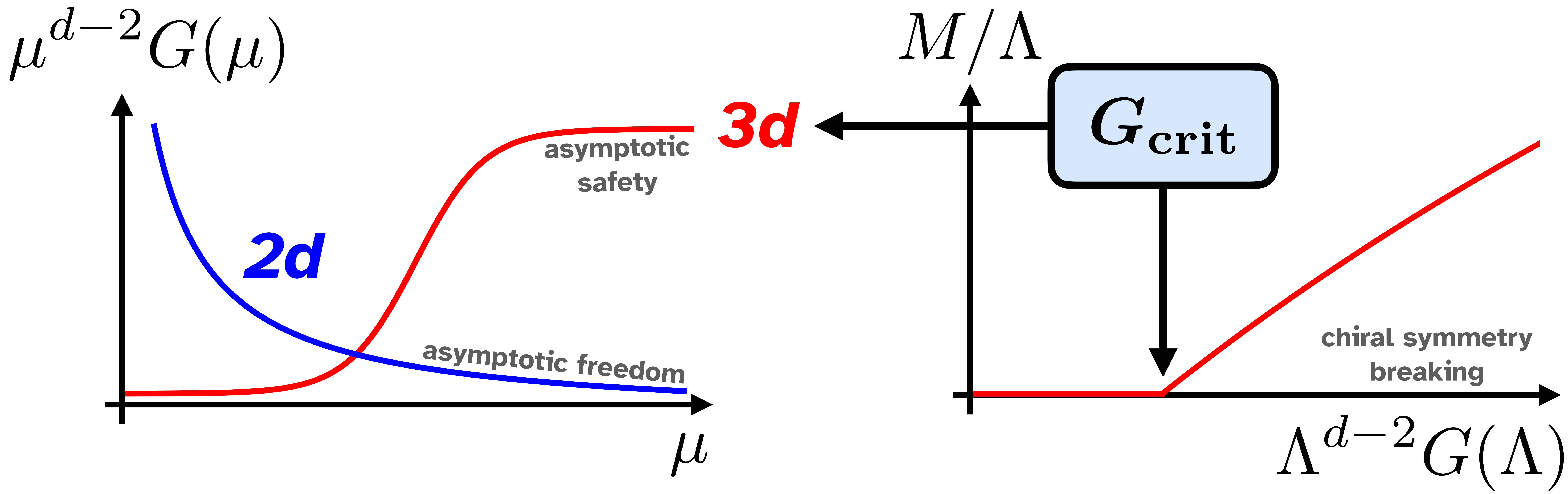


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# Renormalisation group

$$S[\psi, \bar{\psi}] \to S[\psi, \bar{\psi}] + \int_q \bar{\psi}_a(q)\, R_k(q)\, \psi_a(q)$$

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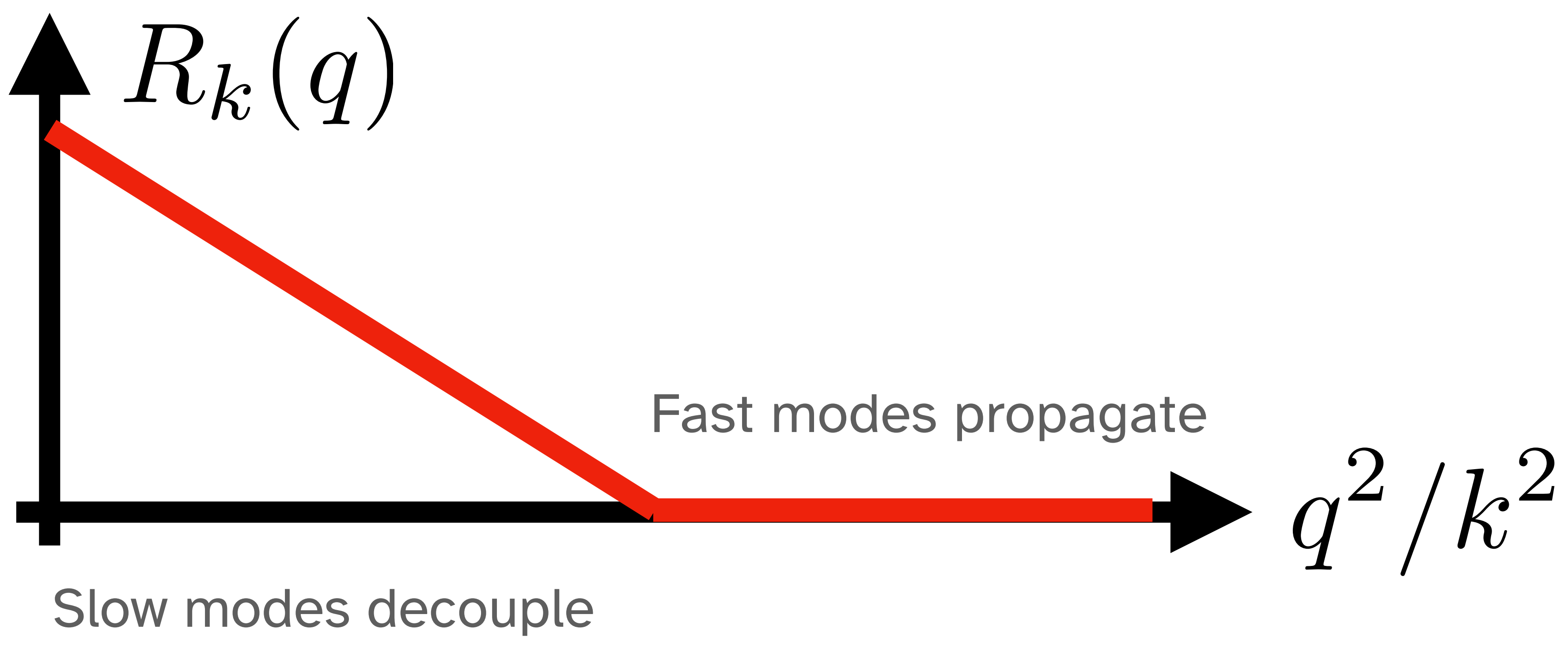

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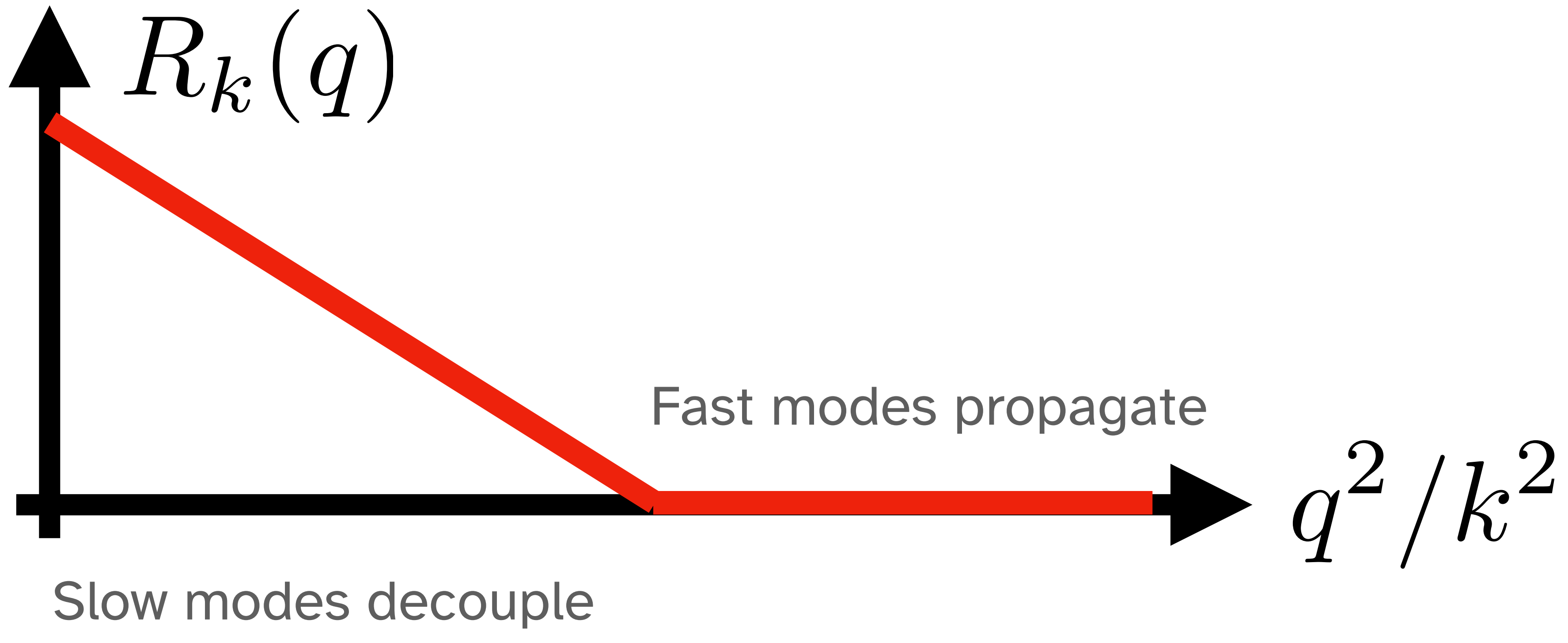


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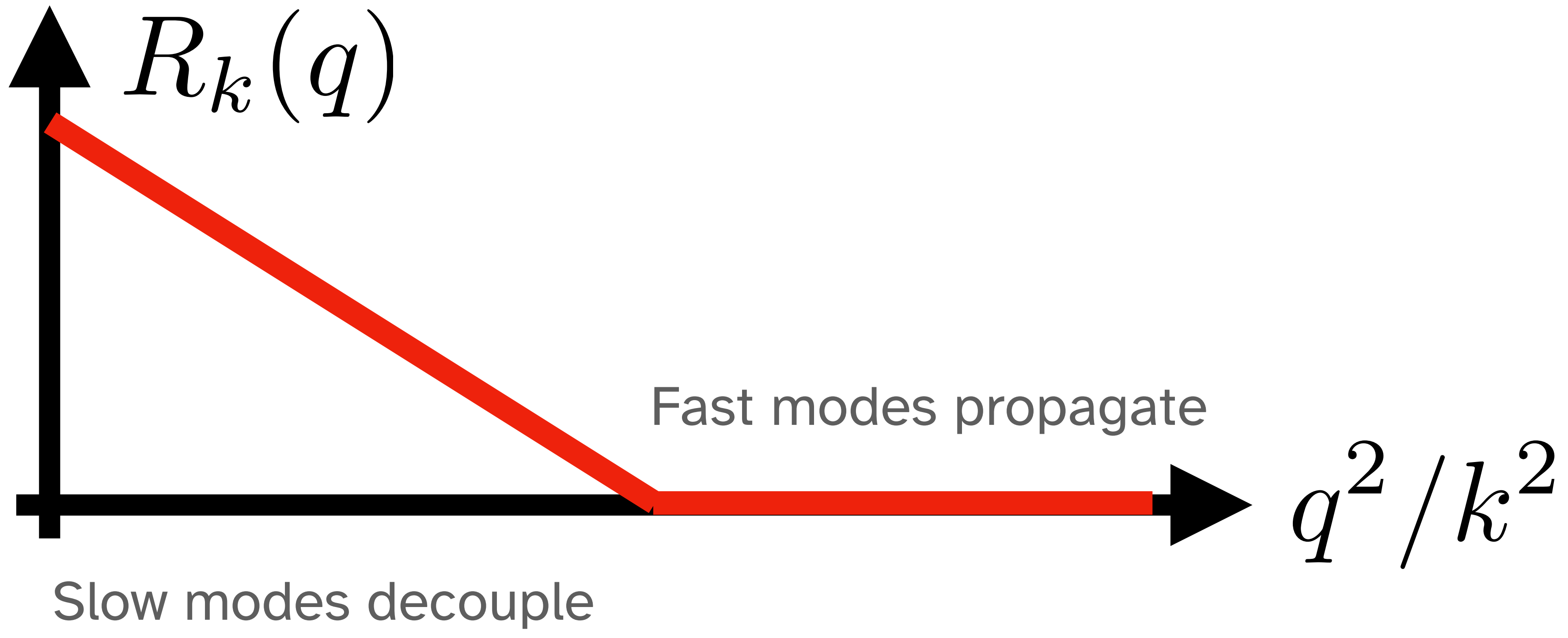


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**UV** → **IR**

$$k = \Lambda : S_\Lambda$$

$$k = 0 : \Gamma_{\mathrm{1PI}}$$

# Renormalisation group

$$S[\psi,\bar{\psi}] \to S[\psi,\bar{\psi}] + \int_q \bar{\psi}_a(q)\, R_k(q)\, \psi_a(q)$$

$$\partial_t \Gamma_k = \frac{1}{2}\mathrm{STr}\left\{[\Gamma_k^{(2)} + R_k]^{-1} \cdot \partial_t R_k\right\}$$

Wetterich ’93; Morris ’94; Ellwanger ‘94

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↑ span Clifford algebra

$$\mathbb{1}\,,\ \gamma^\mu\,,\ \gamma^5\,,\ \ldots$$

# Large *N*

Exact solutions of form:

$$\Gamma_k[\psi, \bar{\psi}] = \int_x \bar{\psi}_a \partial\!\!\!/ \psi_a + F_k[J]$$

D'Attanasio, Morris '97; CCH, Litim, *in prep.*

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| | |
|---|---|
| $f(J)\, J \partial^n J$ | ✓ |
| $f(J)\, (\bar{\psi} \not{\partial} \psi)^n$ | ✗ |

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All contributions to pointlike terms sourced by:

$$\Gamma_k[\psi,\bar{\psi}] = \int_x \left\{\bar{\psi}_a \partial\!\!\!/ \psi_a + V_k(J)\right\}$$

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**fermionic local potential approximation**

Jakovác, Patkós, '13; Jakovác, Patkós, Pósfay '14
Aoki, Kumamoto, Sato '14

# Pointlike interactions

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RG integration measure:

$$\mathrm{d}K(q) = \frac{\partial_t r(q^2/k^2)}{1 + r(q^2/k^2)} \frac{\mathrm{d}^d q}{(2\pi)^d}$$

$$R_k(q) = i q_\mu \gamma^\mu \, r\big(\tfrac{q^2}{k^2}\big)$$

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Mass terms are 1/N protected:

$$\partial_t m_\psi = (\dots) m_\psi + \mathcal{O}(1/N)$$

cf. technical naturalness *('t Hooft '80)*
but also applies if no symmetry protecting *(CCH, Litim, 2406.00100)*

# Pointlike interactions

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**S²:** $(\bar{\psi}_a \psi_a)^2$

**V²:** $(\bar{\psi}_a \gamma^\mu \psi_a)(\bar{\psi}_b \gamma_\mu \psi_b)$

**P²:** $(\bar{\psi}_a \gamma^5 \psi_a)^2$

**A²:** $(\bar{\psi}_a \gamma^\mu \gamma^5 \psi_a)(\bar{\psi}_b \gamma_\mu \gamma^5 \psi_b)$

# Pointlike interactions

**S²:** $\partial_t \lambda = (d-2)\,\lambda + 2d_\gamma N\, C_d[r]\,\lambda^2$

**P²:** $\partial_t \lambda_5 = (d-2)\,\lambda_5 - 2d_\gamma N\, C_d[r]\,\lambda_5^2$

**V²:** $\partial_t g \;= (d-2)\,g \;+ 2d_\gamma N\left(\frac{2}{d}-1\right) C_d[r]\,g^2$

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$$C_d[r] = -\Omega_d \int_0^\infty \mathrm{d}y\, \frac{y^{d/2-1}\, r'(y)}{(1+r(y))^3}$$

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Asymptotic freedom in 2d
cf. *Gross, Neveu '74*

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Conformal manifold in 2d
cf. *Dashen, Frishman '75*

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UV fixed points in 3d:

$$\Delta_{4\mathrm{F}}\big|_{\mathrm{UV}} = 2$$

$$\Delta_{4\mathrm{F}}\big|_{\mathrm{IR}} = 4$$

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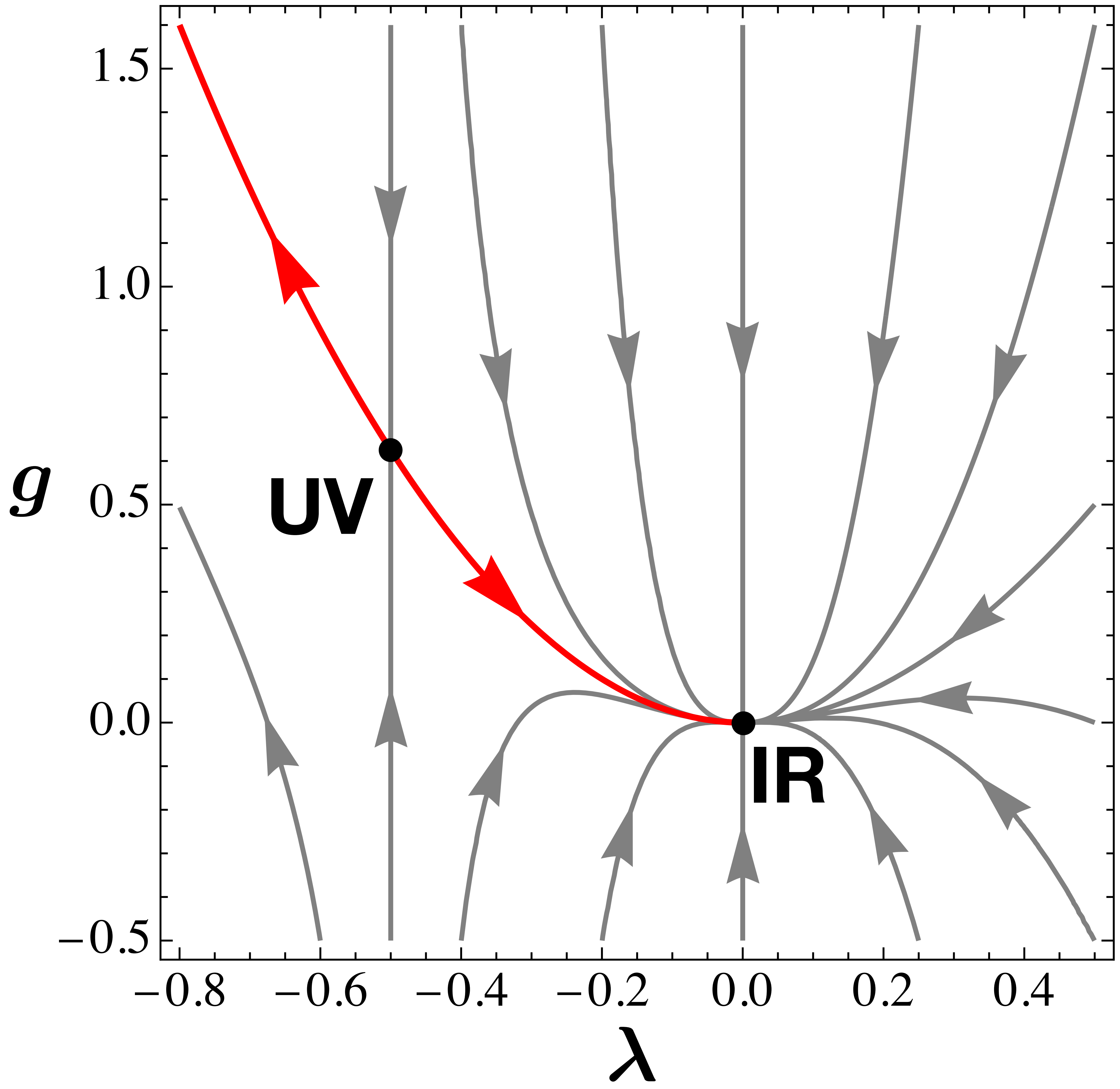
# Derivative interactions

$\boldsymbol{\lambda} : (\bar{\psi}_a \psi_a)^2$

$\boldsymbol{g} : \partial_\mu(\bar{\psi}_a \psi_a)\partial^\mu(\bar{\psi}_b \psi_b)$



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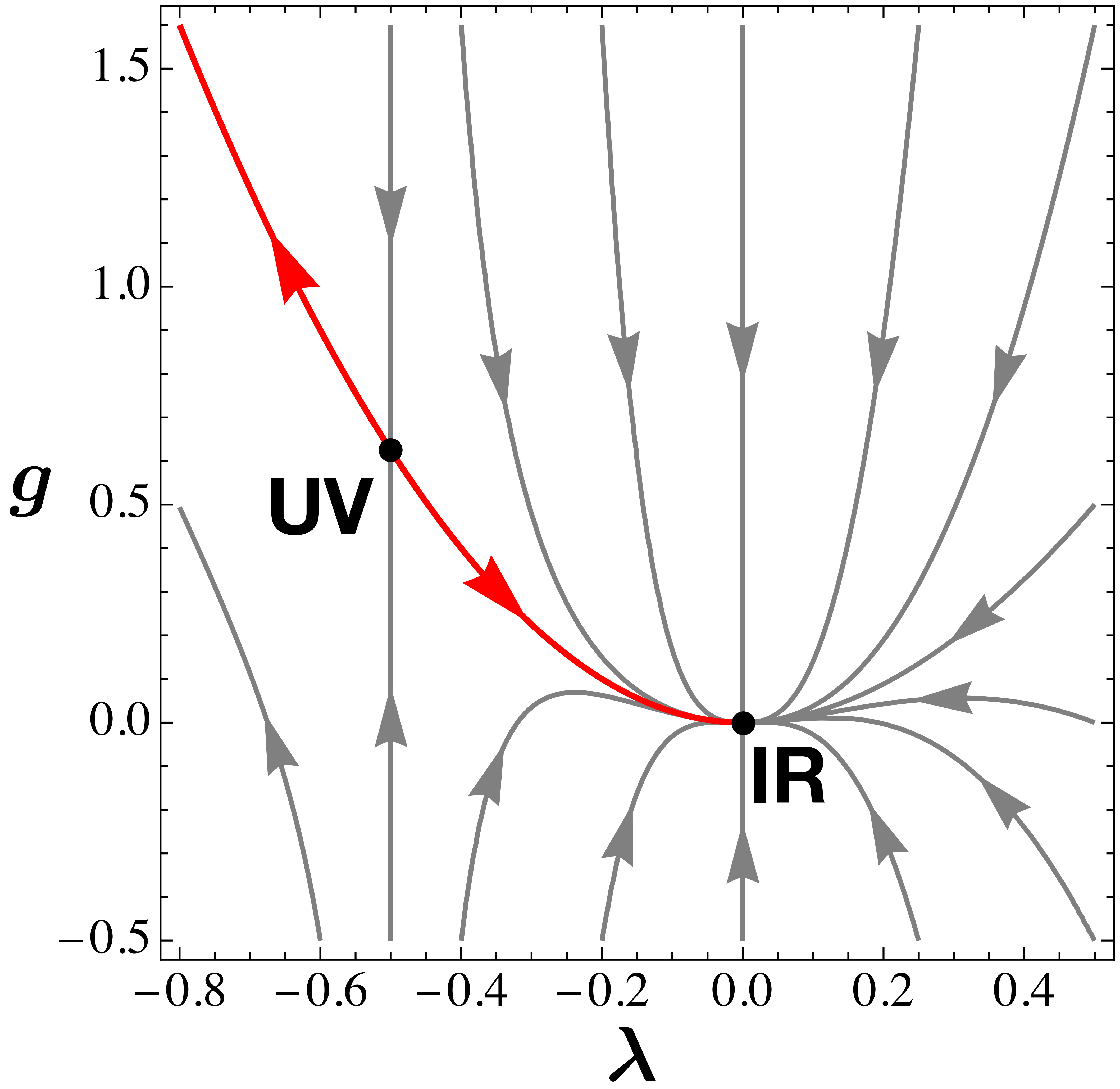

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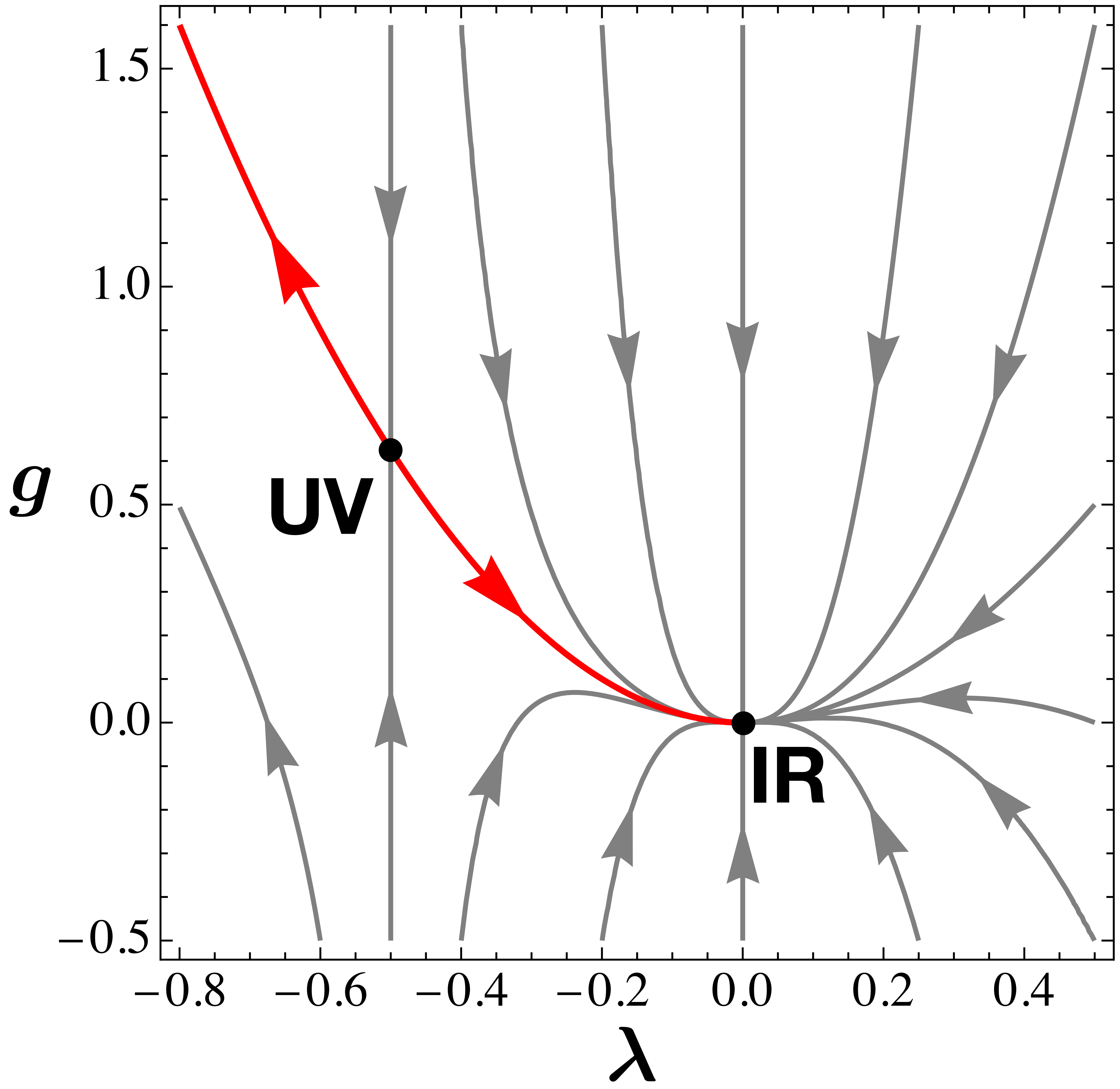
$$\lambda^* = \frac{2-d}{2}$$

$$g^* = \frac{(d-2)(3d-4)}{8(4-d)}$$



# Gross-Neveu theories

$$V_k \;\to\; V_k(\bar{\psi}_a \psi_a)$$

$$v(z,t) = k^{-d} V_k(k^{d-1} z)$$

$$\partial_t v = -3v + 2z\partial_z v - \frac{1}{1+(\partial_z v)^2}$$

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Fixed points:

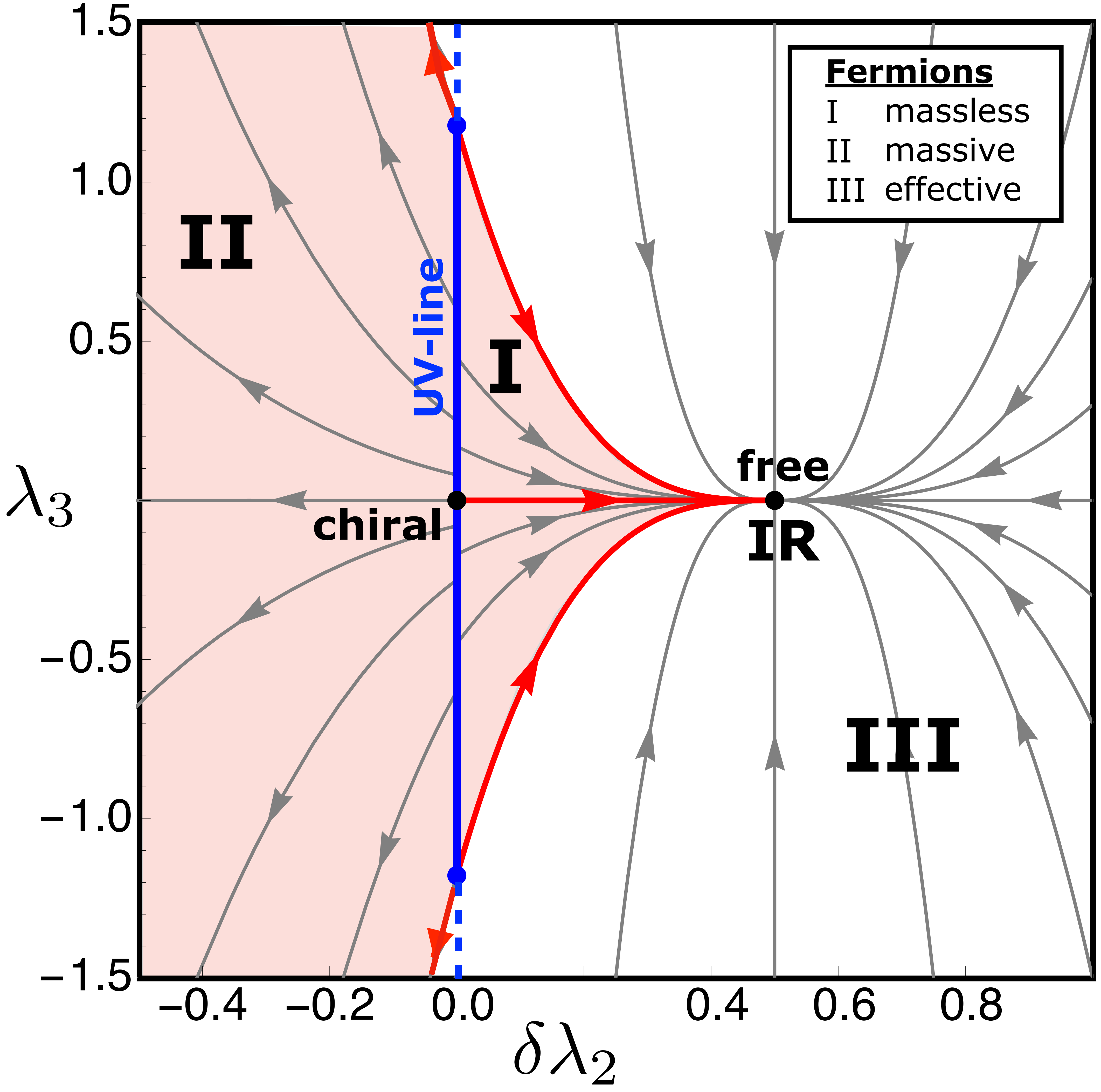
$$z = -2v'_* + 4(v'_*)^2 \left[\lambda_3^* - \frac{\frac{1}{4}v'_*}{1+(v'_*)^2} - \frac{3}{4}\arctan(v'_*)\right]$$

# Six-fermion interactions

$$\partial_t \lambda_2 = \lambda_2 \left(1 + 2\lambda_2\right)$$
$$\partial_t \lambda_3 = 3\lambda_3 \left(1 + 2\lambda_2\right)$$

$$\lambda_2^* = -\tfrac{1}{2}$$
$$\lambda_3^* = \text{free parameter}$$

CCH, Litim, 2207.10115



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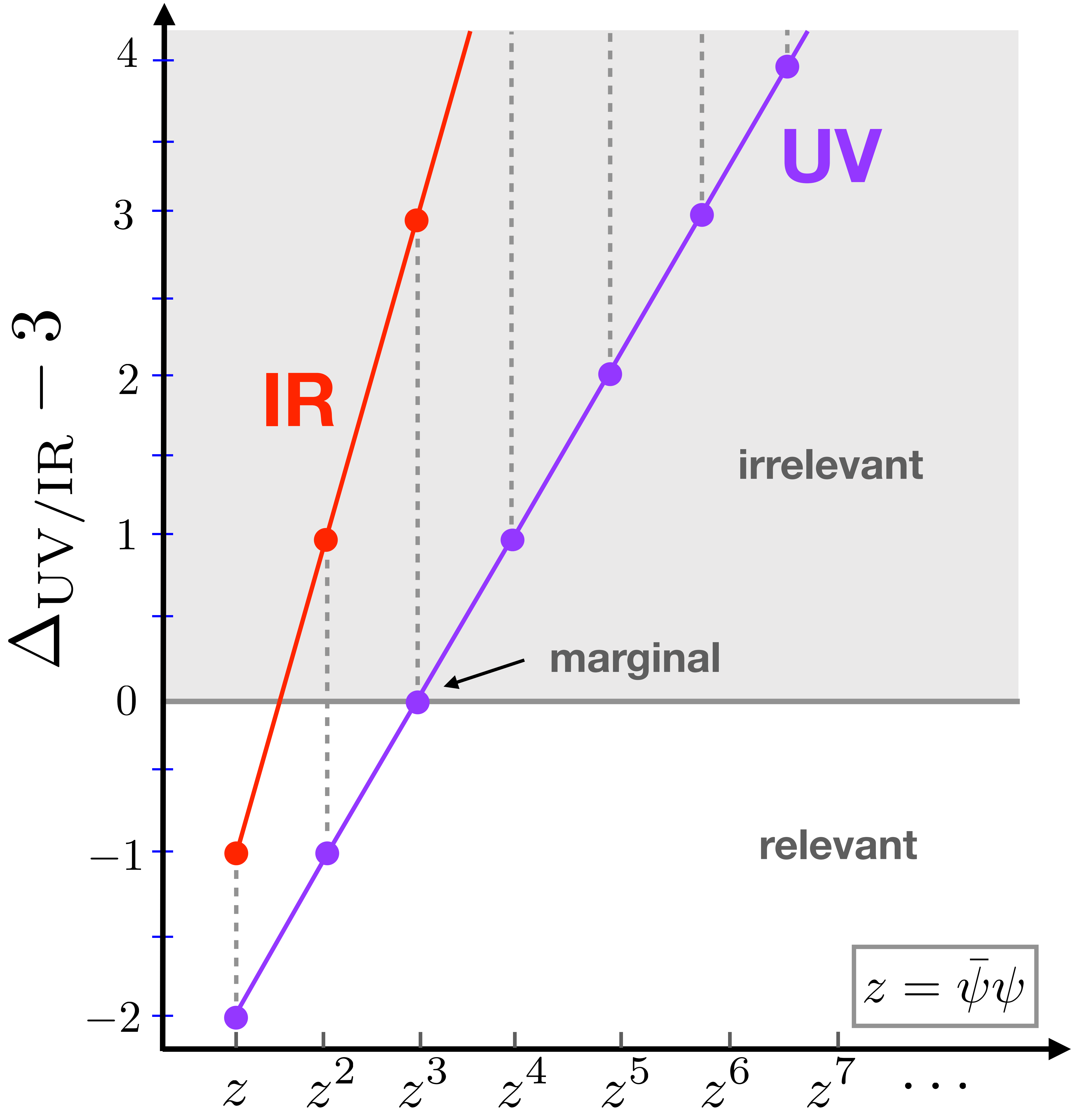


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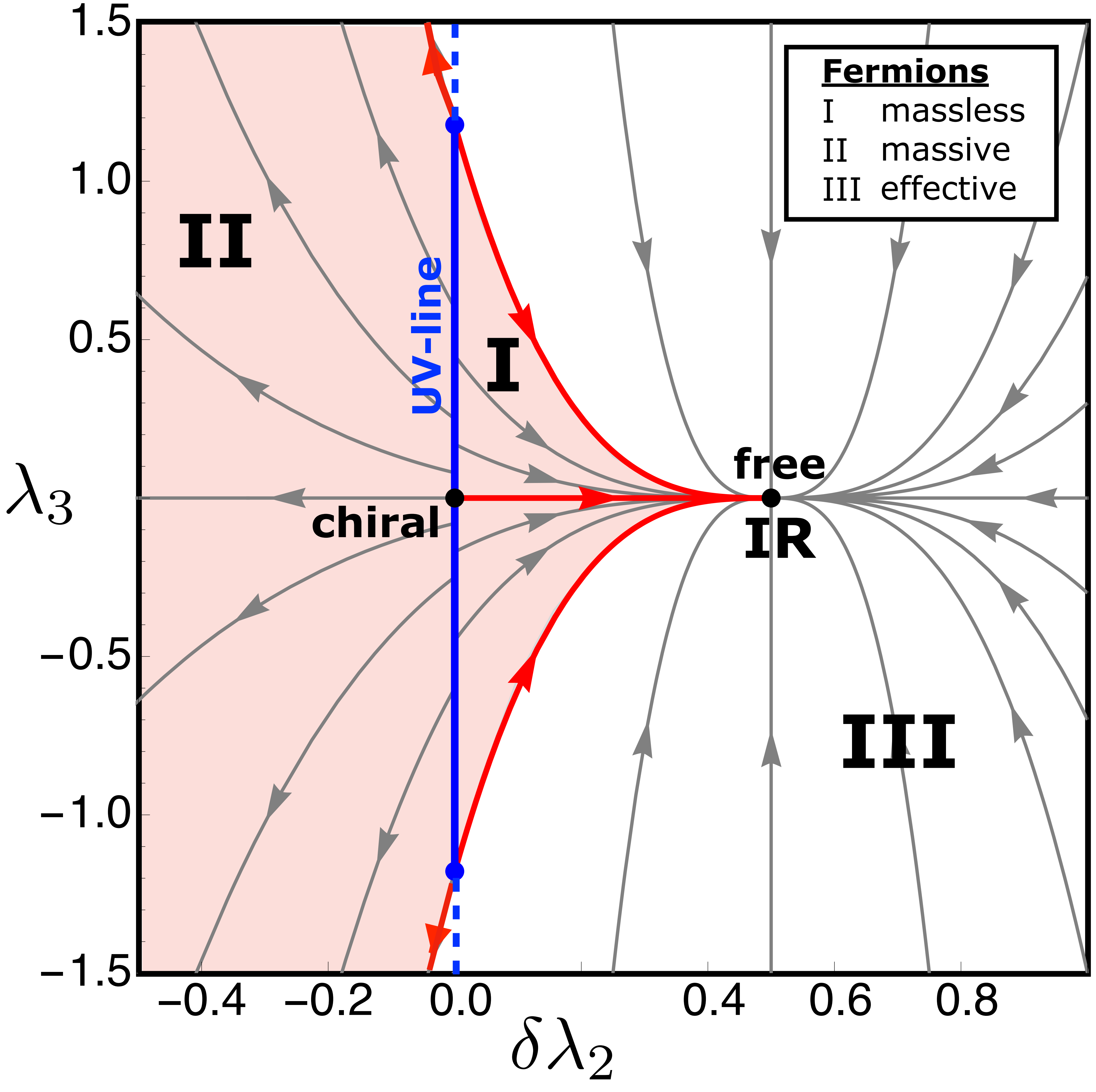

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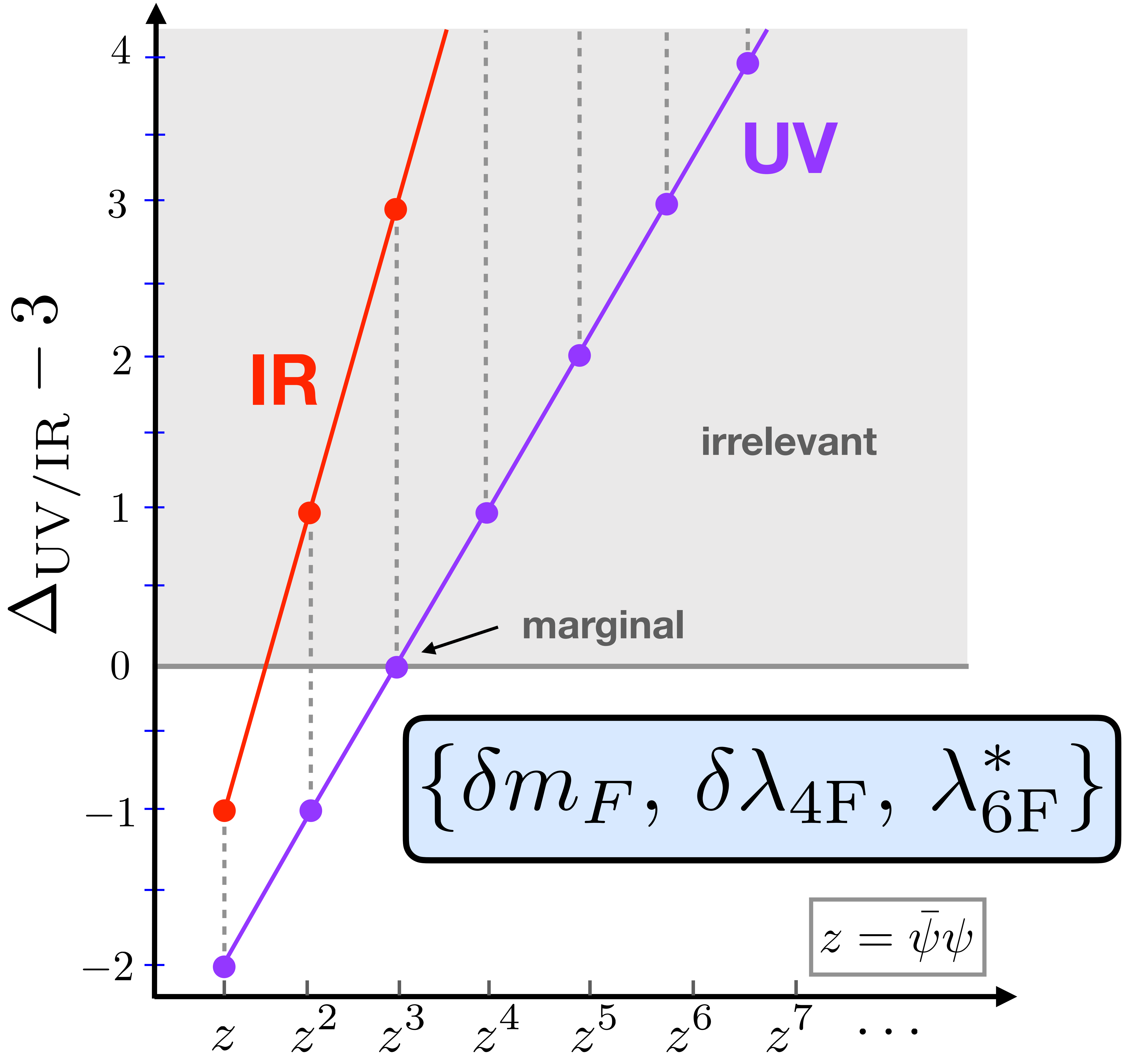


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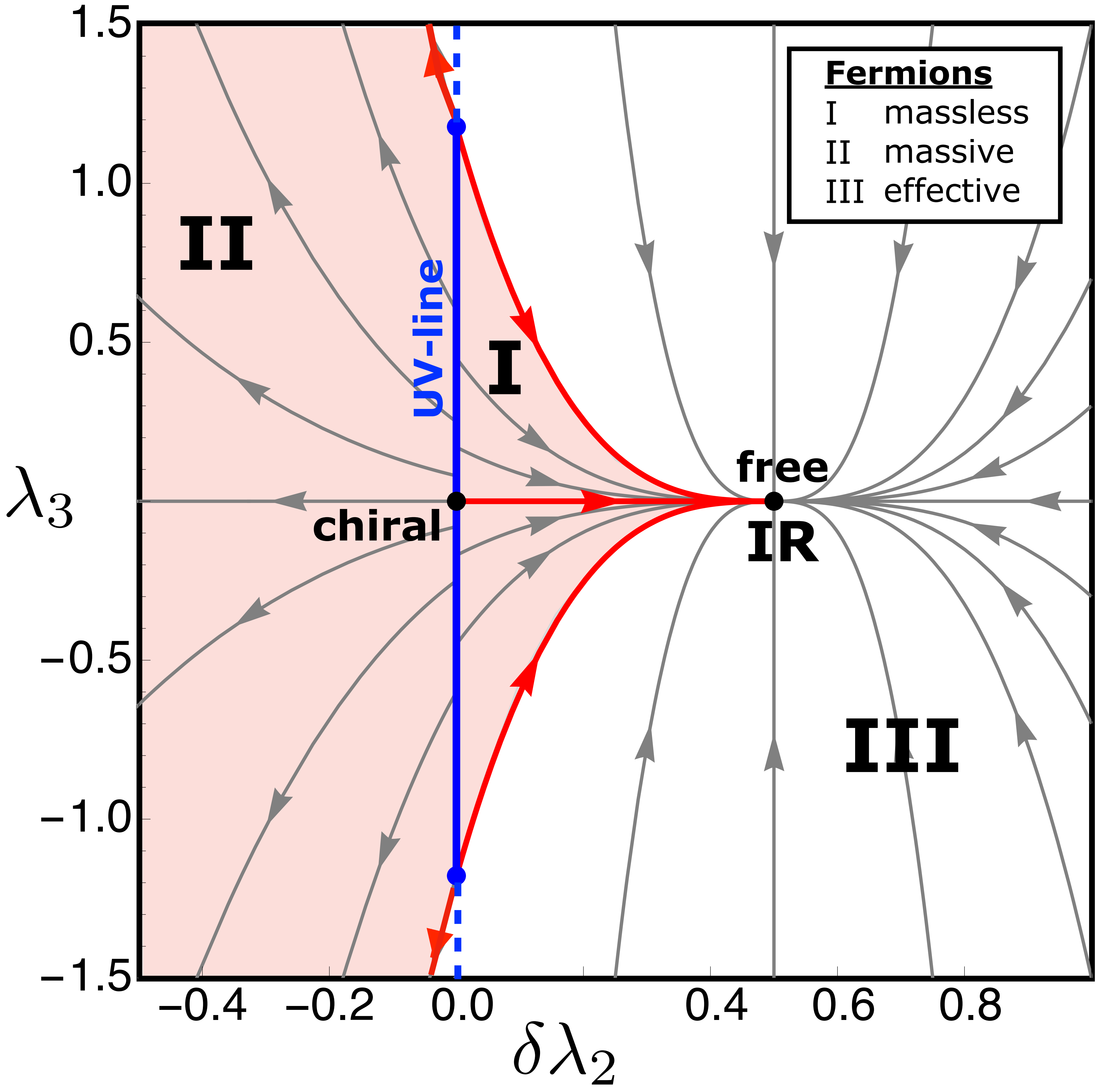

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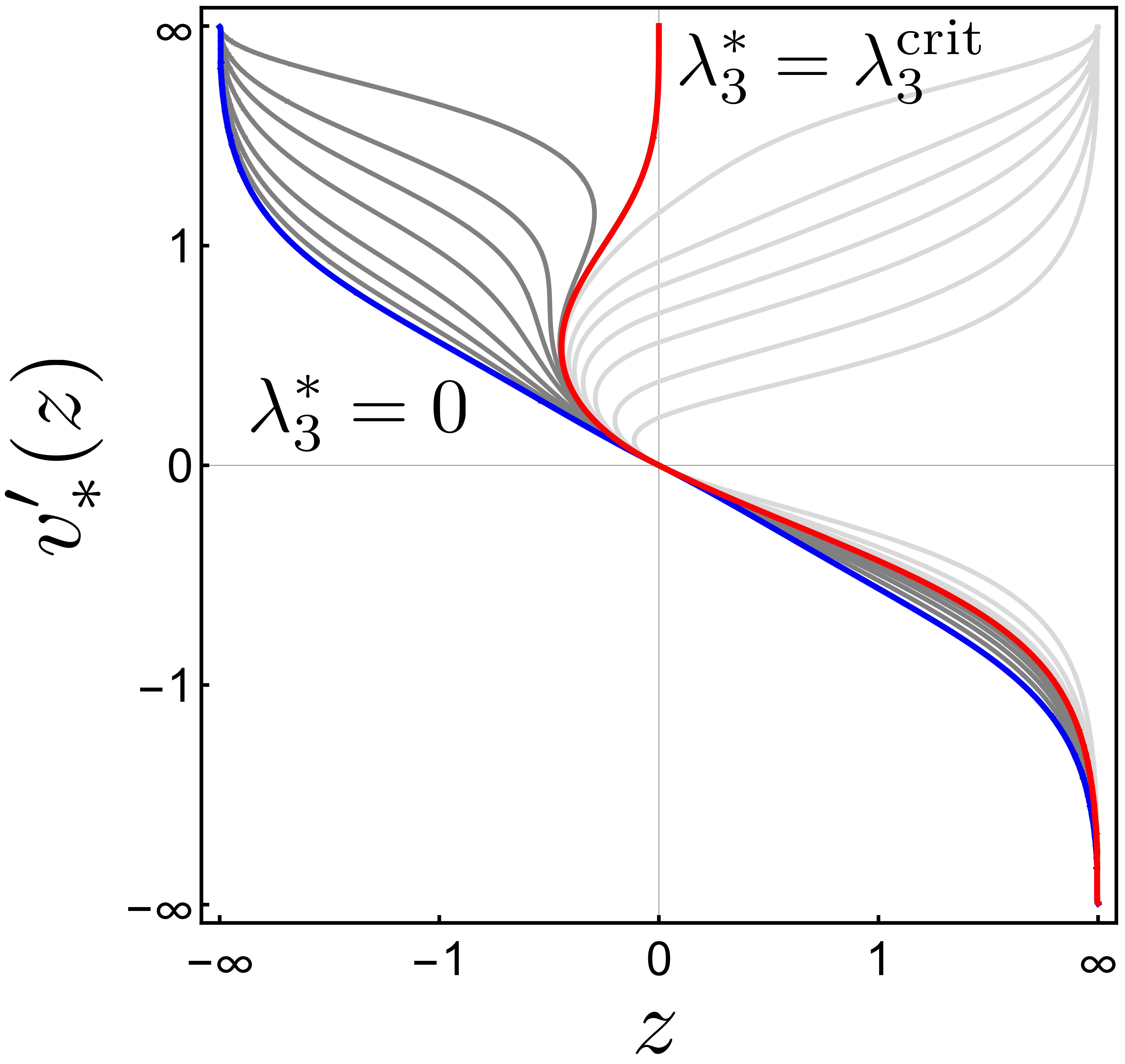


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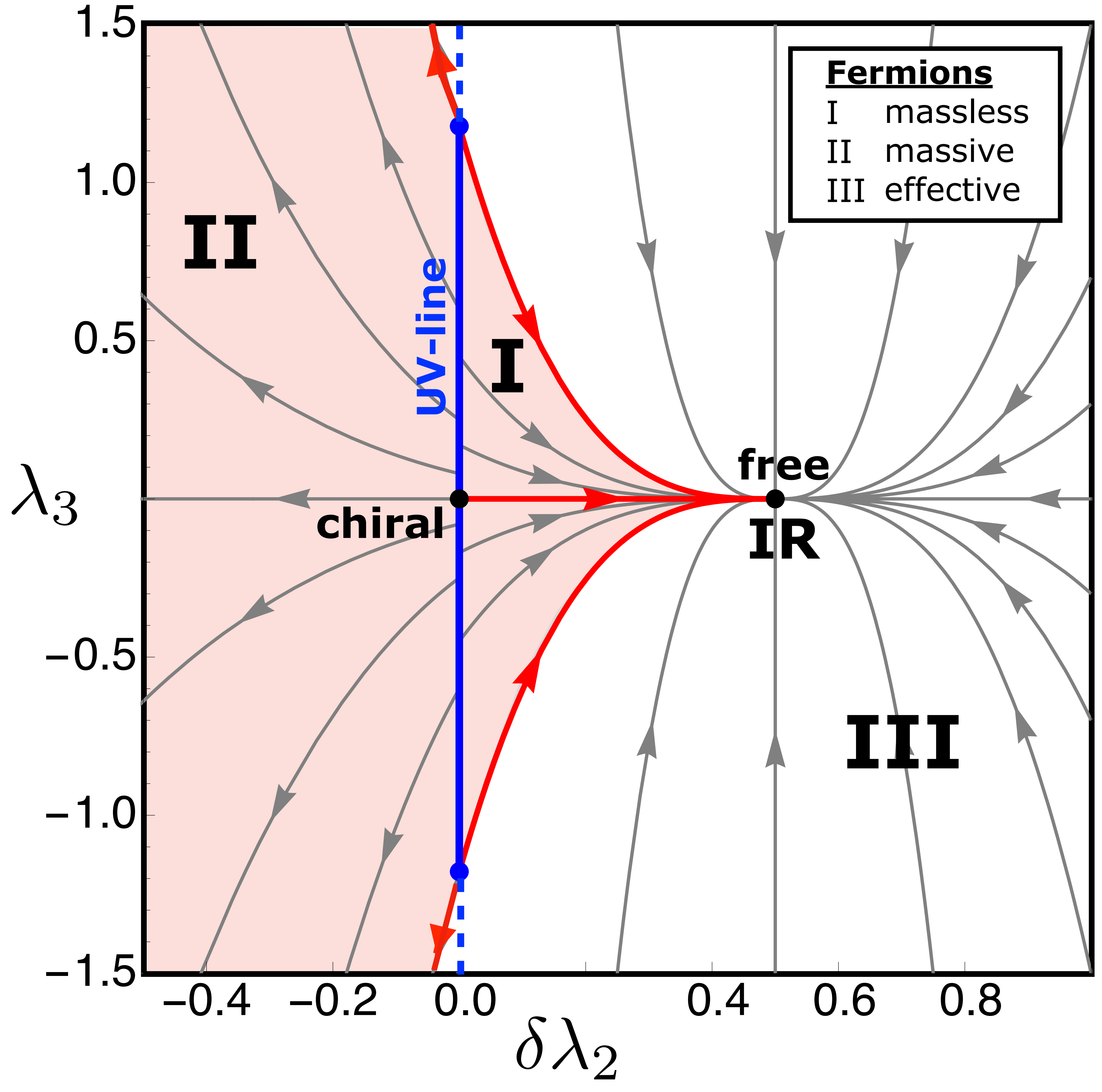

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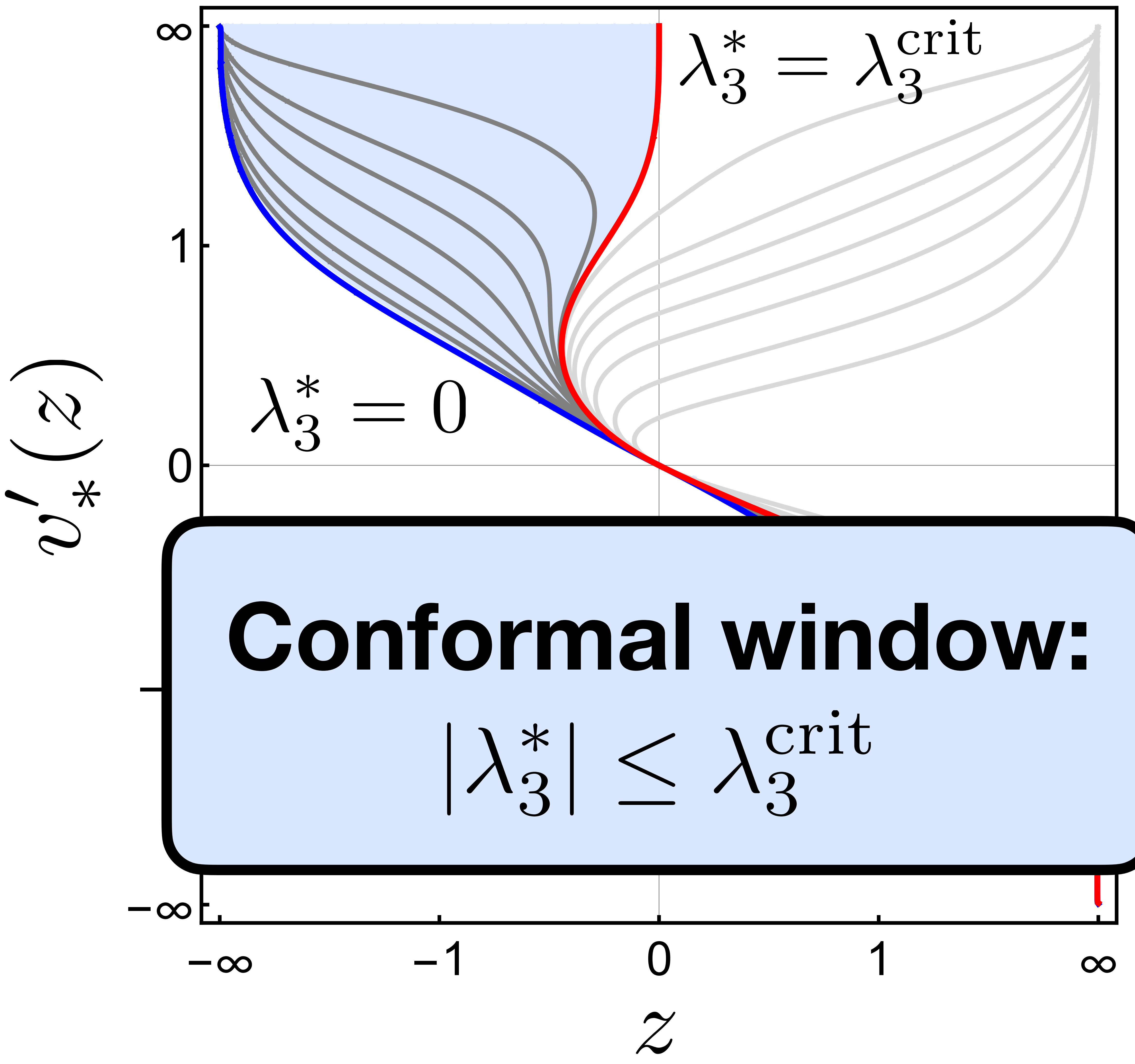
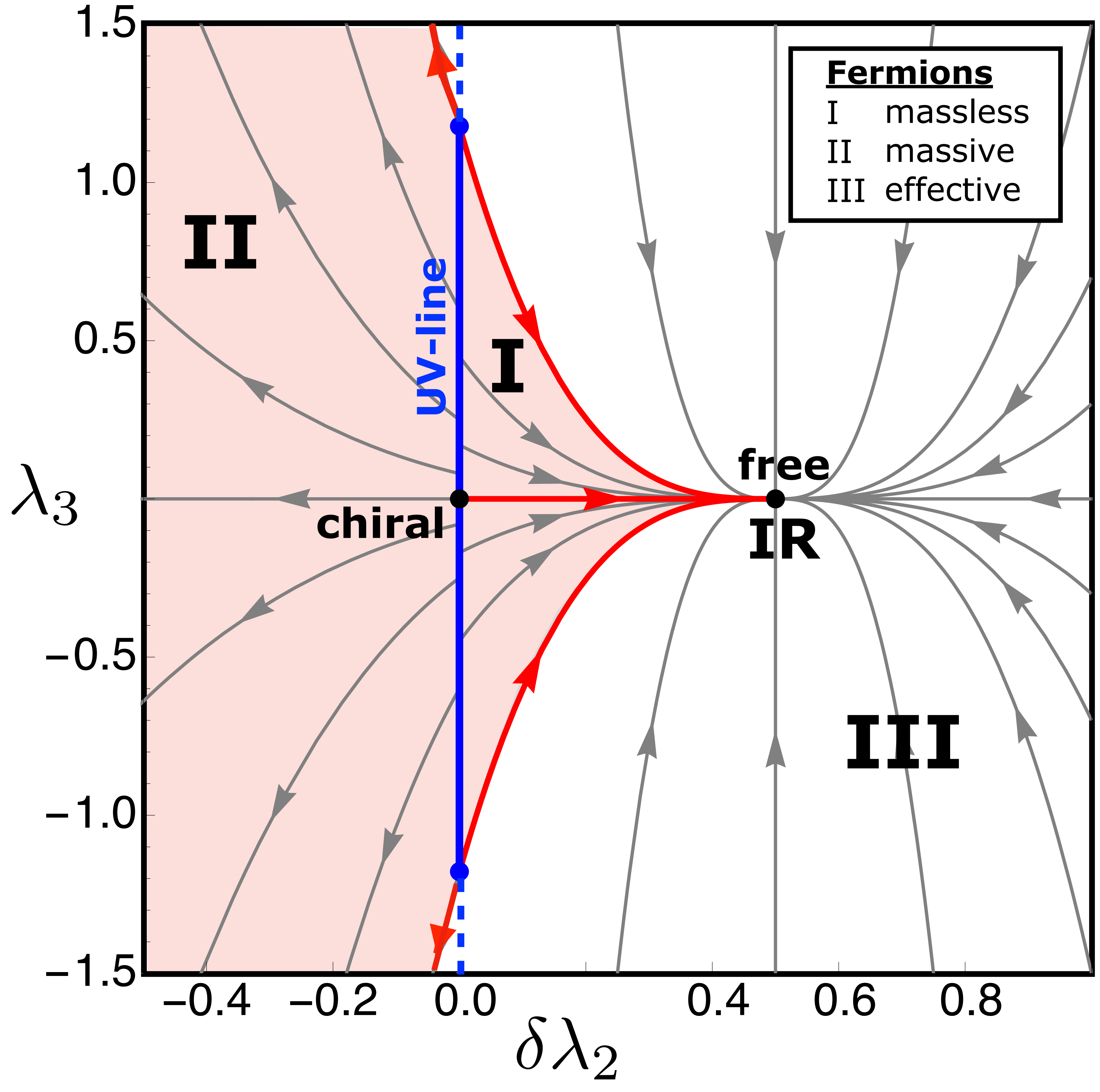


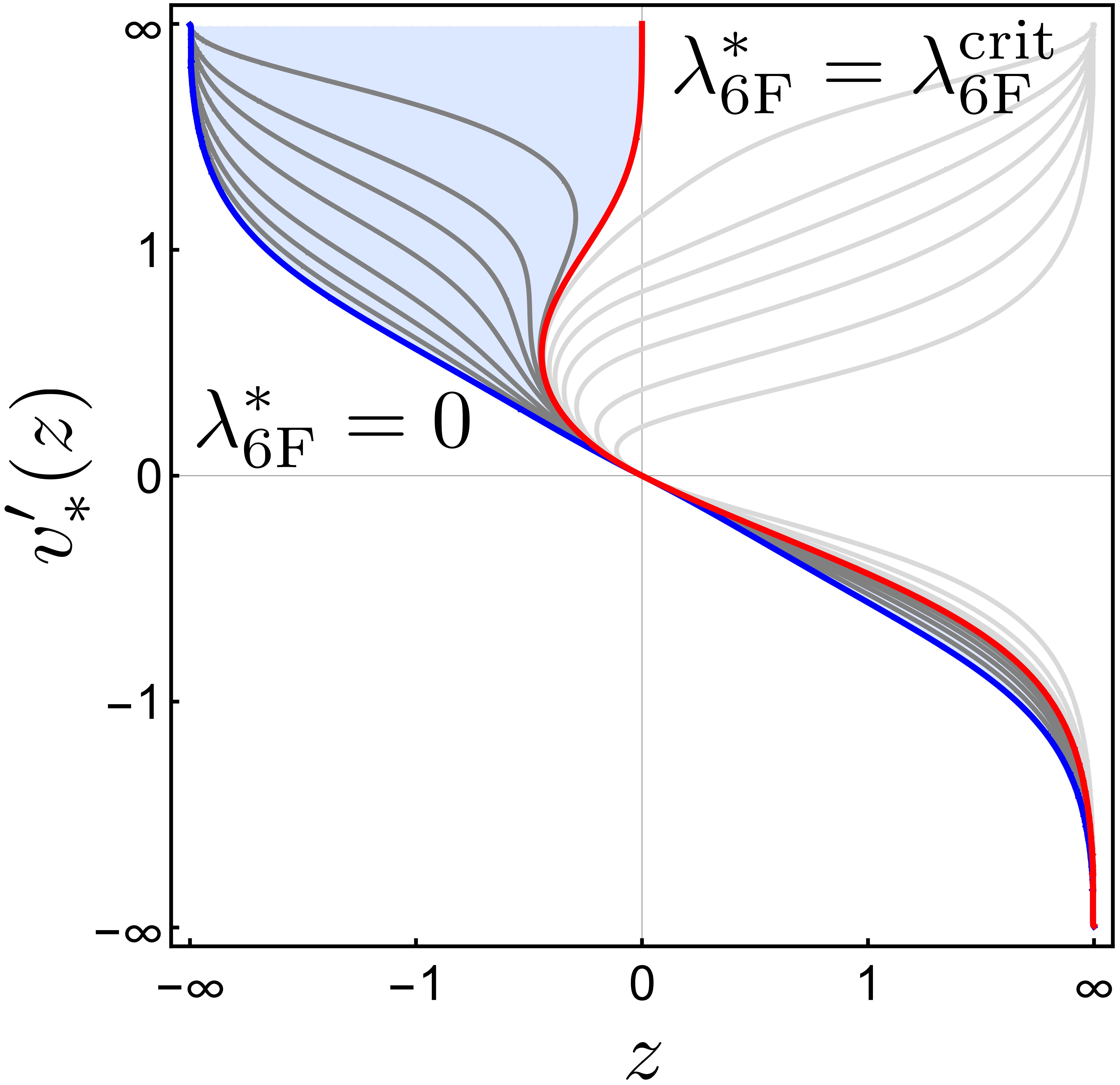

**Conformal window:**

$|\lambda_3^*| \leq \lambda_3^{\rm crit}$



Physical fermion mass

$$M_F = \lim_{k\to 0} k v'_*(0)$$



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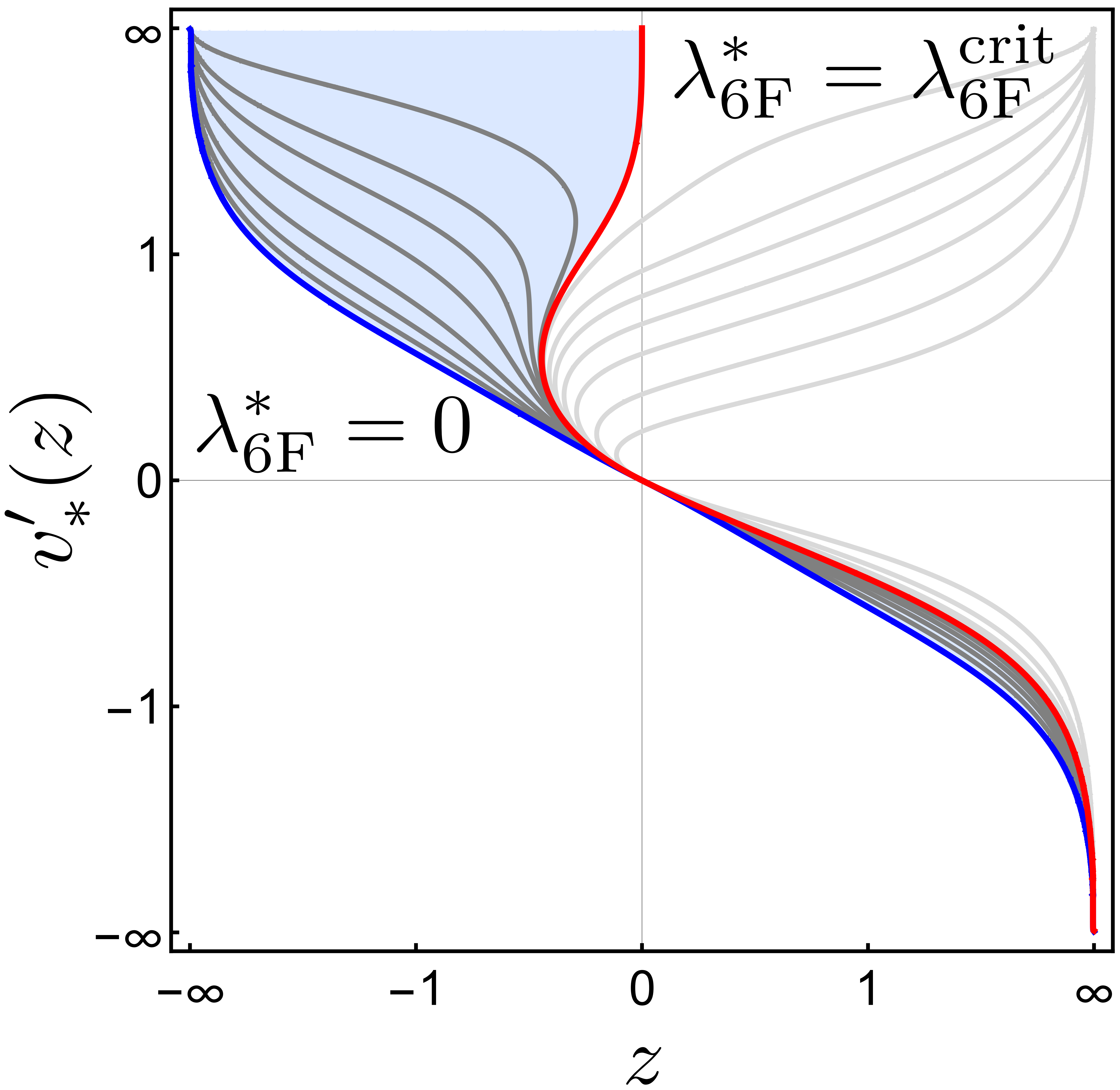
CCH, Litim, 2212.06815



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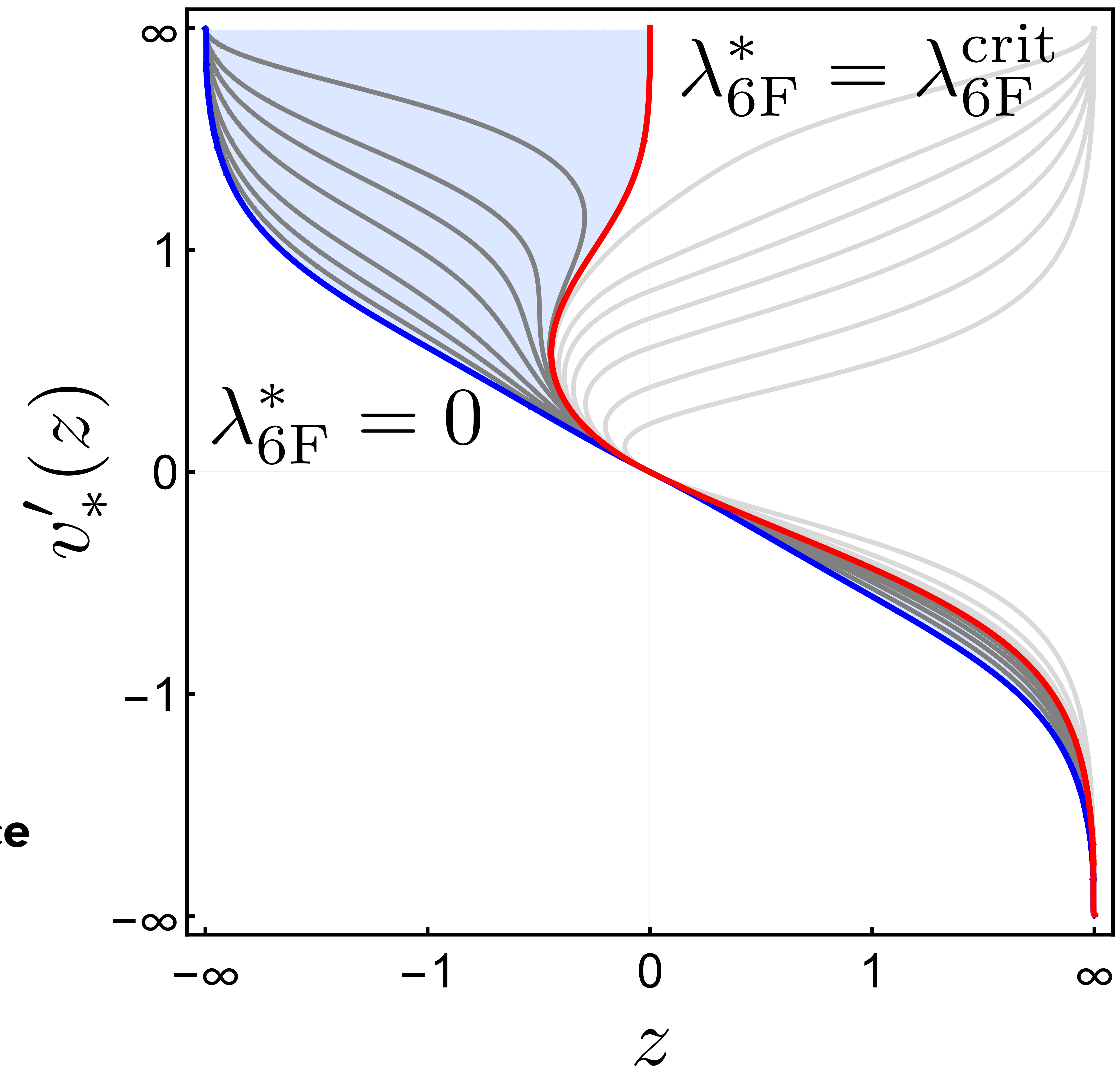
CCH, Litim, 2212.06815

**Spontaneous breaking of scale invariance**

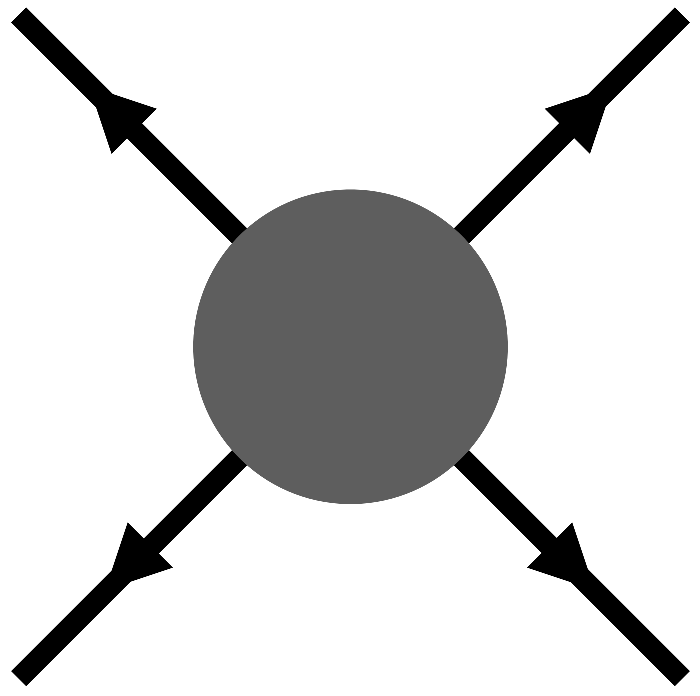
***cf. O(N) model:***
Bander, Bardeen, Moshe, PRL ’83
David, Kessler, Neuberger, PRL ’85
Litim, Marchais, Mati, 1702.05749



# Dilaton physics

Work in prep. with D. Litim & R. Zwicky

Fermions massive but correlators show massless pole:



$$\sim \frac{1}{p^2}$$

Match to dilaton EFT: extract decay constant, induced mass, ...

**In strong interactions & QCD:**
Ellis ’70, ’71; Crewther ’70, ’71
Crewther, Tunstall 1203.1321, 1510.01322
Zwicky 2306.06752, 2312.13761

**In strongly coupled gauge theories:**
LSD collaboration 2306.06095
Hasenfratz, Peterson 2402.18038
LatKMI collaboration 1610.07011

**In Higgs physics:**
Goldberger, Grinstein, Skiba 0708.1463
Bellazzini, Csaki, Serra, Terning 1209.3299
Csaki, Grojean, Terning 1512.00468

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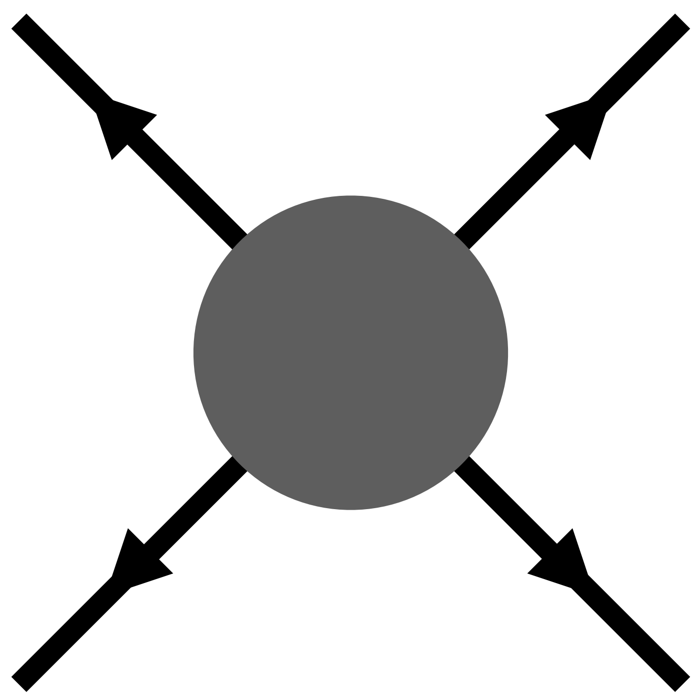

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**Possible extensions to finite N:** Semenoff, Stewart, 2402.09646, 2408.04855

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Zwicky 2306.06752, 2312.13761

**In strongly coupled gauge theories:**
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- FRG natural tool to explore space of fermionic QFTs
- No need for “bosonisation”
- Six-fermion interactions can be marginal in 3d
- 6F theories offer testing ground for dilaton physics
- Implications for finite N: condensed matter systems?