Ultraviolet finite resummation of perturbative quantum gravity

Applications of Field Theory to non-Hermitian and Hermitian systems

13/09/24

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TRM, Class.Quant.Grav. [2401.02546]

Deser 1957, DeWitt 1964, Isham et al 1972, Perez 2001, 't Hooft 2010, Dvali et al 2014

Quantum gravity is not perturbatively renormalizable ...

$$S_{EH} = \int d^d x \, \mathcal{L}_{EH} \, , \qquad \mathcal{L}_{EH} = -2\sqrt{g}R/\kappa^2$$

$$\kappa = 2/M_{\text{Planck}} \, , \quad \kappa^2 = 32\pi G$$

$$g_{\mu\nu} = \delta_{\mu\nu} + \kappa H_{\mu\nu}$$

$$\mathcal{L}_{EH} = \partial H \partial H + \sum_{n=1}^{\infty} \kappa^n H^n \partial H \partial H$$

... requiring an infinite number of new couplings in loop corrections

But it also has another problem ...

$$S_{EH} = \int d^d x \, \mathcal{L}_{EH} \,, \qquad \mathcal{L}_{EH} = -2\sqrt{g} R/\kappa^2$$

$$\mathcal{Z} = \int \mathcal{D} g_{\mu\nu} \; \mathrm{e}^{-S_{EH}}$$ does not converge

Gibbons, Hawking, Perry '78

... suitably reinterpreted, it might solve the first problem

$$g_{\mu\nu} = \delta_{\mu\nu} + \kappa H_{\mu\nu}$$

$$h_{\mu\nu} + \tfrac{2}{d}\varphi\,\delta_{\mu\nu}$$

traceless

$$g_{\mu\nu} = \mathrm{e}^{2\kappa\varphi/d}\,\hat{g}_{\mu\nu}(h)$$

$$S_{EH} = -2\int d^dx\,\sqrt{\hat{g}}\,\mathrm{e}^{\kappa(d-2)\varphi/d}\left\{\frac{1}{\kappa^2}\hat{R} + \frac{(d-1)(d-2)}{d^2}\hat{g}^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi\right\}$$

We just need two key properties …

$$g_{\mu\nu} = \delta_{\mu\nu} + \kappa H_{\mu\nu}$$

$$h_{\mu\nu} + \tfrac{2}{d}\varphi\,\delta_{\mu\nu}$$

traceless

- Interactions terms are +ve exponentials of φ
- φ propagates with the wrong sign

$$g_{\mu\nu} = \mathrm{e}^{2\kappa\varphi/d}\hat{g}_{\mu\nu}(h)$$

$$S_{EH} = -2\int d^dx\,\sqrt{\hat{g}}\,\mathrm{e}^{\kappa(d-2)\varphi/d}\left\{\frac{1}{\kappa^2}\hat{R} + \frac{(d-1)(d-2)}{d^2}\hat{g}^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi\right\}$$

$$\int d^dx\,\left\{\frac{\alpha}{2}\delta^{\mu\nu}F_\mu F_\nu - \frac{1}{\kappa}\bar{c}^\mu Q F_\mu\right\}$$

De Donder:

$$F_\mu = \frac{1}{\kappa}\delta^{\alpha\beta}\left(\partial_\alpha g_{\beta\mu} - \frac{1}{2}\partial_\mu g_{\alpha\beta}\right) = \delta^{\alpha\beta}\partial_\alpha h_{\beta\mu} - \left(\frac{2}{d}-1\right)\partial_\mu\varphi + O(\kappa)$$

$$= \delta^{\alpha\beta}\mathrm{e}^{2\kappa\varphi/d}\left(\frac{1}{\kappa}\partial_\alpha\hat{g}_{\beta\mu} - \frac{1}{2\kappa}\partial_\mu\hat{g}_{\alpha\beta} + \frac{2}{d}\partial_\alpha\varphi\,\hat{g}_{\beta\mu} - \frac{1}{d}\partial_\mu\varphi\,\hat{g}_{\alpha\beta}\right)$$

$$Qg_{\mu\nu} = 2\kappa\partial_{(\mu}c^\alpha g_{\nu)\alpha} + \kappa c^\alpha\partial_\alpha g_{\mu\nu} = \kappa\,\mathrm{e}^{2\kappa\varphi/d}\left(2\partial_{(\mu}c^\alpha\hat{g}_{\nu)\alpha} + 2\frac{\kappa}{d}c^\alpha\partial_\alpha\varphi\,\hat{g}_{\mu\nu} + c^\alpha\partial_\alpha\hat{g}_{\mu\nu}\right)$$

Now, in momentum space the propagators for $h_{\mu\nu}$ and φ take the form (see *e.g.* [17]):

$$\langle \varphi(p)\, \varphi(-p) \rangle = \left(\frac{1}{\alpha} - \frac{d-1}{d-2} \right) \frac{1}{p^2}\,, \tag{2.10}$$

$$\langle h_{\mu\nu}(p)\, \varphi(-p) \rangle = \langle \varphi(p)\, h_{\mu\nu}(-p) \rangle = \left(1 - \frac{2}{\alpha} \right) \left(\frac{\delta_{\mu\nu}}{d} - \frac{p_\mu p_\nu}{p^2} \right) \frac{1}{p^2}\,, \tag{2.11}$$

$$\langle h_{\mu\nu}(p)\, h_{\alpha\beta}(-p) \rangle = \frac{\delta_{\mu(\alpha}\delta_{\beta)\nu}}{p^2} + \left(\frac{4}{\alpha} - 2 \right) \frac{p_{(\mu}\delta_{\nu)(\alpha}p_{\beta)}}{p^4} + \frac{1}{d^2} \left(\frac{4}{\alpha} - d - 2 \right) \frac{\delta_{\mu\nu}\delta_{\alpha\beta}}{p^2}$$
$$+ \frac{2}{d} \left(1 - \frac{2}{\alpha} \right) \frac{\delta_{\alpha\beta} p_\mu p_\nu + p_\alpha p_\beta \delta_{\mu\nu}}{p^4}\,. \tag{2.12}$$

Thus to ensure we have the key property that φ has a wrong-sign propagator, we just need to choose α outside the range $0 < \alpha < (d-2)/(d-1)$. Note that for all dimensions $d > 3$ this includes the popular Feynman – DeDonder gauge, $\alpha = 2$, where $h_{\mu\nu}$ and φ decouple

Preserve the exponentials ...

$$S = \frac{1}{2}\phi^A \Delta^{-1}_{AB}\phi^B + \epsilon S_I[\phi]$$

(Really, $\epsilon = 1$)

$$S_I \ni -2\sqrt{\hat{g}}\, \mathrm{e}^{\kappa(d-2)\varphi/d} \left\{ \frac{1}{\kappa^2}\hat{R} + \frac{(d-1)(d-2)}{d^2}\hat{g}^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi \right\}$$

$$-\frac{1}{2}(\partial_\mu h_{\alpha\beta})^2 + (\partial^\mu h_{\mu\nu})^2 + \frac{2}{d}(d-2)\varphi\partial^2_{\mu\nu}h^{\mu\nu} + 2\frac{(d-1)(d-2)}{d^2}(\partial_\mu\varphi)^2$$

$$\Gamma[\Phi] = \frac{1}{2}\Phi^A \Delta^{-1}_{AB}\Phi^B + \Gamma_I[\Phi]$$

$$\Gamma_I[\Phi] = \sum_{n=1}^{\infty} \epsilon^n \Gamma_n$$

$$\mathcal{P}_{ij} = \Delta^{AB} \frac{\partial}{\partial \Phi_i^B} \frac{\partial}{\partial \Phi_j^A}$$

$$\Gamma_1 = \mathrm{e}^{\mathcal{P}_{11}/2} S_I[\Phi]$$

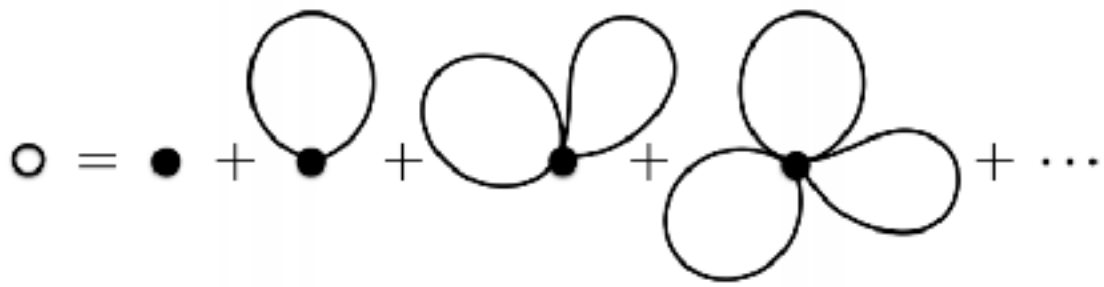


Figure 3.1: The classical interaction vertex is represented by the black dot and has any number of external legs (which are not drawn). Added to this is all its 1PI quantum corrections i.e. the sum over all tadpole corrections.

$$\Gamma_2[\Phi] = -\frac{1}{2}\left(\mathrm{e}^{\mathcal{P}_{12}} - 1 - \mathcal{P}_{12}\right)\Gamma_1^2$$

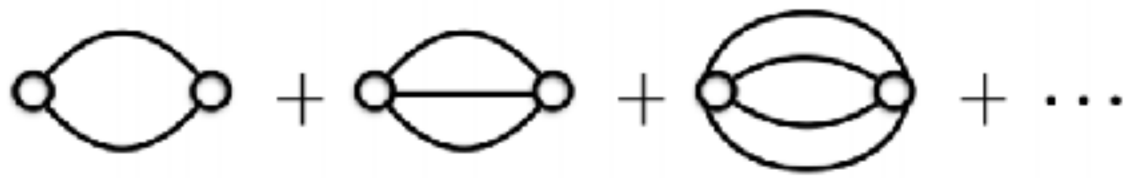
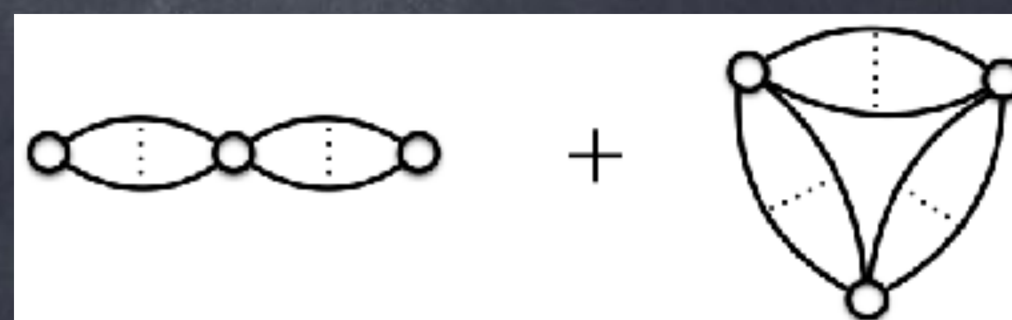
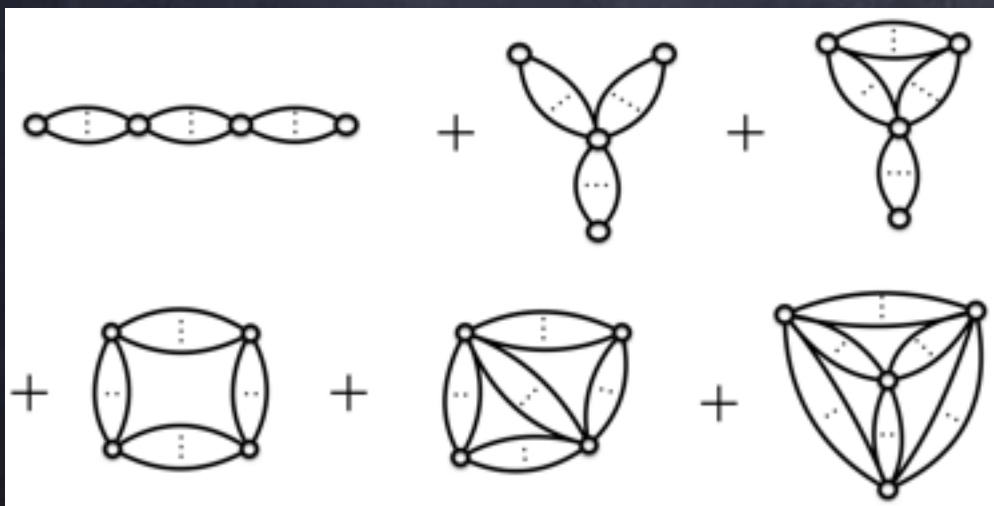


Figure 3.2: The second-order part, Γ_2, of the Legendre effective action, is given by an expansion over melonic Feynman diagrams, where the open circles are copies of Γ_1 as given in fig. 3.1. Again, these copies have any number of external legs which are not drawn.

$$\Gamma_3 = \frac{1}{2!}\left(\mathrm{e}^{\mathcal{P}_{12}} - 1 - \mathcal{P}_{12}\right)\left(\mathrm{e}^{\mathcal{P}_{23}} - 1 - \mathcal{P}_{23}\right)\Gamma_1^3 + \frac{1}{3!}\left(\mathrm{e}^{\mathcal{P}_{12}} - 1\right)\left(\mathrm{e}^{\mathcal{P}_{23}} - 1\right)\left(\mathrm{e}^{\mathcal{P}_{31}} - 1\right)\Gamma_1^3$$





Sum over φ corrections ...

$$\mathcal{P}_{ij} = \Delta^{AB}\frac{\partial}{\partial\Phi_i^B}\frac{\partial}{\partial\Phi_j^A}$$

$$\Gamma_1 = \mathrm{e}^{\mathcal{P}_{11}/2} S_I[\Phi]$$

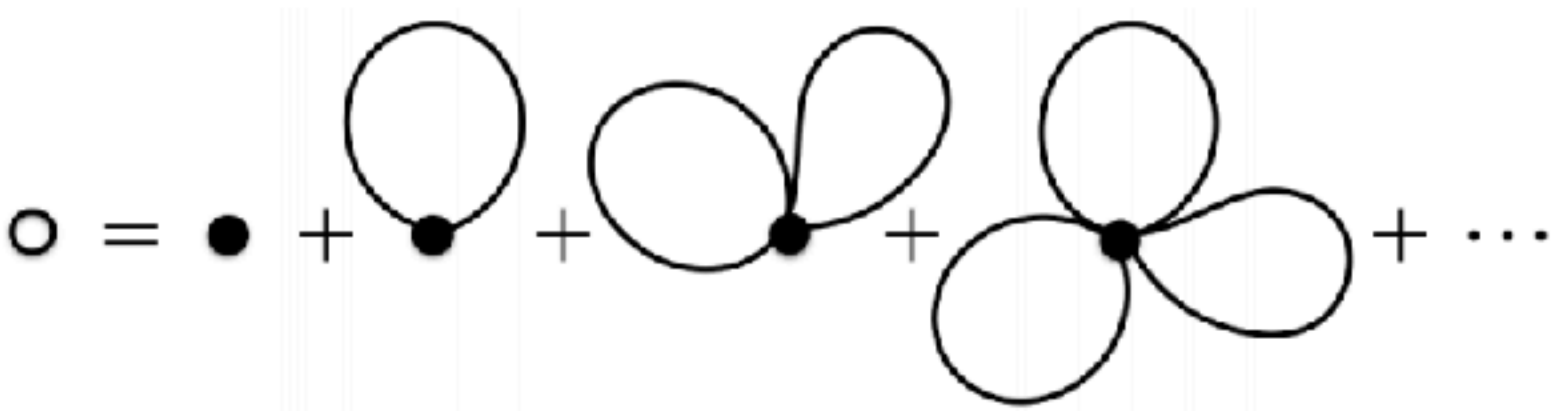


Figure 3.1: The classical interaction vertex is represented by the black dot and has any number of external legs (which are not drawn). Added to this is all its 1PI quantum corrections *i.e.* the sum over all tadpole corrections.

$$\Gamma_1 = S_I[\Phi]$$

... all massless tadpoles vanish in gauge inv regulator dim reg.

Sum over φ corrections ...

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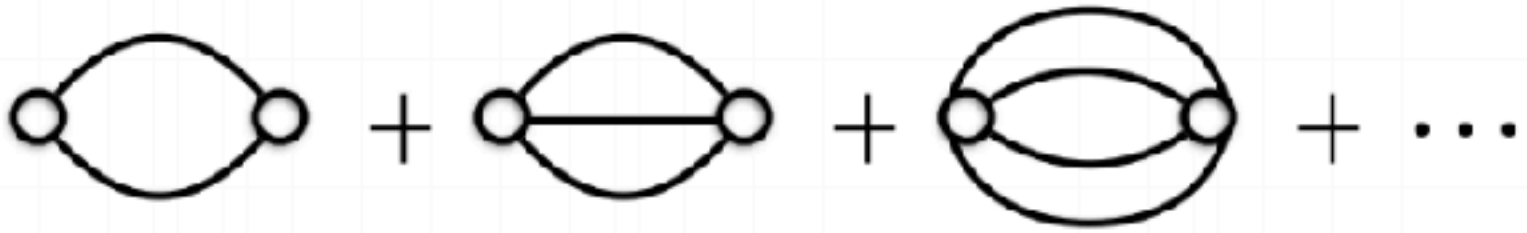

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$$\Gamma_1 = \sum_\alpha \int d^d x \, \mathrm{e}^{\kappa \beta_\alpha \varphi} \mathcal{L}_\alpha(\varphi, \partial\varphi) \;\; - \;\; \text{bilinear}$$

$$\beta_\alpha > 0$$

Sum over φ corrections ...

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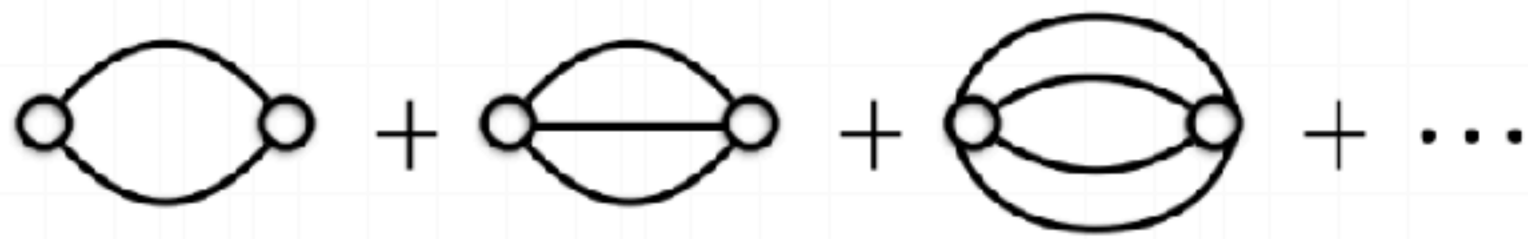

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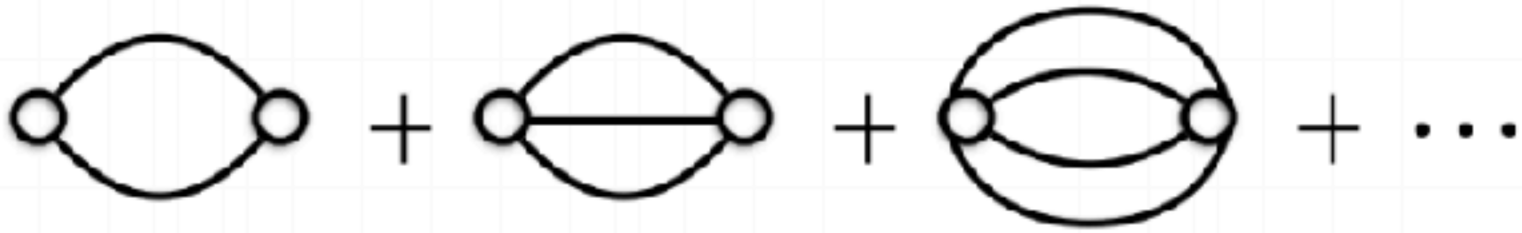

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$$\Gamma_1 = \sum_\alpha \int d^d x \, e^{\kappa \beta_\alpha \varphi} \mathcal{L}_\alpha(\varphi, \partial\varphi) \;\; - \;\; \text{bilinear}$$

$$\langle \varphi(0)\, \varphi(x) \rangle = -\xi^2 \Delta(x)$$

$$\Delta(x) = \int \frac{d^d p}{(2\pi)^d} \frac{e^{ip\cdot x}}{p^2} = \frac{\Gamma(d/2)}{2\pi^{d/2}(d-2)} \frac{1}{x^{d-2}}$$

Sum over φ corrections ...

$$\mathcal{P}_{ij} = \Delta^{AB}\frac{\partial}{\partial \Phi_i^B}\frac{\partial}{\partial \Phi_j^A}$$

$$\Gamma_2[\Phi] = -\frac{1}{2}\left(\mathrm{e}^{\mathcal{P}_{12}} - 1 - \mathcal{P}_{12}\right)\Gamma_1^2$$

$$\Gamma_1 = \sum_\alpha \int d^d x\, \mathcal{L}_\alpha\left(\frac{\partial}{\partial J_x}, \partial\frac{\partial}{\partial J_x}\right) \mathrm{e}^{(\kappa\beta_\alpha + J_x)\varphi(x)}\Big|_{J_x=0}$$

$$\Gamma_2[\Phi] = -\frac{1}{2}\,\mathrm{e}^{\hat{\mathcal{P}}_{12}} \sum_{\alpha_1,\alpha_2}\int d^d x_1 d^d x_2\, \mathcal{L}_{\alpha_1}\left(\frac{\partial}{\partial J_{x_1}}, \partial\frac{\partial}{\partial J_{x_1}}\right)\mathcal{L}_{\alpha_2}\left(\frac{\partial}{\partial J_{x_2}}, \partial\frac{\partial}{\partial J_{x_2}}\right)$$

$$\exp\left\{[\kappa\beta_{\alpha_1} + J_{x_1}]\varphi(x_1) + [\kappa\beta_{\alpha_2} + J_{x_2}]\varphi(x_2) - \xi^2[\kappa\beta_{\alpha_1} + J_{x_1}]\Delta(x_1 - x_2)[\kappa\beta_{\alpha_2} + J_{x_2}]\right\}\Big|_{J=0}$$

$$\Delta(x) = \int \frac{d^d p}{(2\pi)^d}\frac{\mathrm{e}^{ip\cdot x}}{p^2} = \frac{\Gamma(d/2)}{2\pi^{d/2}(d-2)}\frac{1}{x^{d-2}}$$

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$$\Gamma_2[\Phi] = -\frac{1}{2} e^{\hat{\mathcal{P}}_{12}} \sum_{\alpha_1, \alpha_2} \int d^d x_1 d^d x_2 \, \mathcal{L}_{\alpha_1} \left(\frac{\partial}{\partial J_{x_1}}, \partial \frac{\partial}{\partial J_{x_1}} \right) \mathcal{L}_{\alpha_2} \left(\frac{\partial}{\partial J_{x_2}}, \partial \frac{\partial}{\partial J_{x_2}} \right)$$

$$\exp \left\{ [\kappa \beta_{\alpha_1} + J_{x_1}] \varphi(x_1) + [\kappa \beta_{\alpha_2} + J_{x_2}] \varphi(x_2) - \xi^2 [\kappa \beta_{\alpha_1} + J_{x_1}] \Delta(x_1 - x_2) [\kappa \beta_{\alpha_2} + J_{x_2}] \right\} \Big|_{J=0}$$

Overall factor: $\exp \left\{ -\xi^2 \kappa^2 \beta_{\alpha_1} \beta_{\alpha_2} \Delta(x_1 - x_2) \right\}$

... which regularises any power of Δ

$$\Delta(x) = \int \frac{d^d p}{(2\pi)^d} \frac{e^{ip \cdot x}}{p^2} = \frac{\Gamma(d/2)}{2\pi^{d/2}(d-2)} \frac{1}{x^{d-2}}$$

As a concrete example we take the Einstein-Hilbert vertex (3.1) and compute the remaining J differentials in (3.11) to explicitly the complete resummation over purely φ quantum corrections:

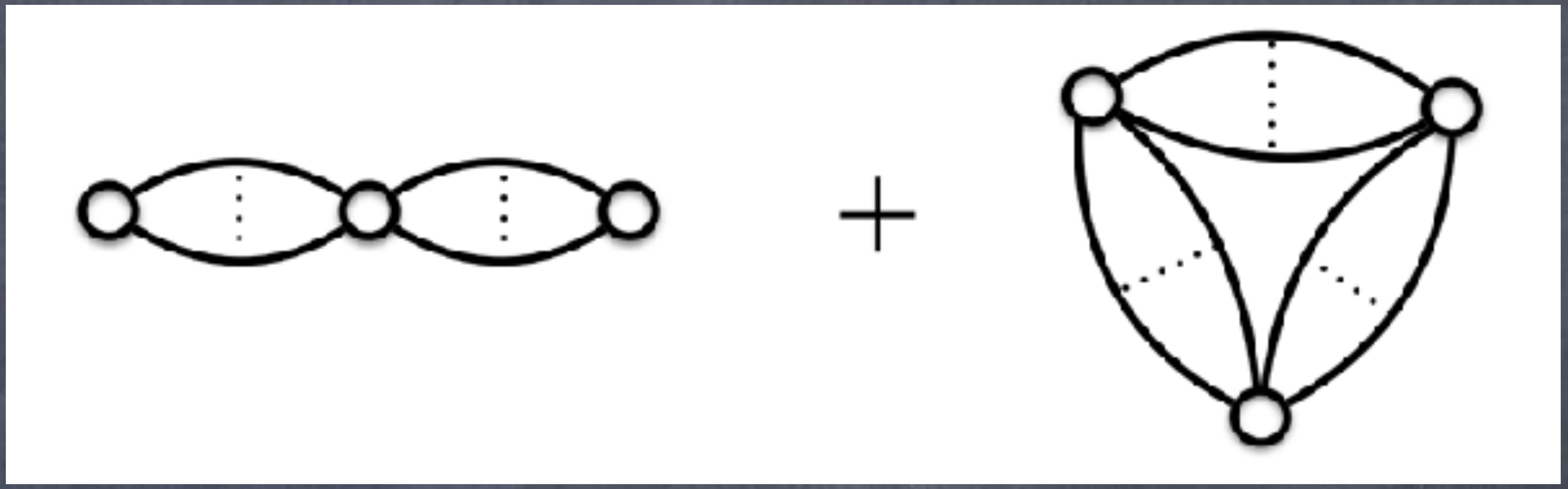
$$\begin{aligned}\Gamma_2 \ni\ & -2\,\mathrm{e}^{\hat{\mathcal{P}}_{xy}} \int d^dx d^dy\, \sqrt{\hat{g}(x)}\sqrt{\hat{g}(y)}\, \mathrm{e}^{\kappa\beta[\varphi(x)+\varphi(y)]}\, \mathrm{e}^{-\xi^2\beta^2\kappa^2\Delta} \Big\{ \\ & \frac{1}{\kappa^4}\hat{R}(x)\hat{R}(y) + 2\frac{\sigma}{\kappa^2}\hat{R}(x)\hat{g}^{\alpha\beta}(y)\partial^y_\alpha\left[\varphi(y)-\xi^2\beta\kappa\Delta\right]\partial^y_\beta\left[\varphi(y)-\xi^2\beta\kappa\Delta\right] \\ & +\sigma^2\hat{g}^{\mu\nu}(x)\,\hat{g}^{\alpha\beta}(y)\,\partial^x_\mu\left[\varphi(x)-\xi^2\beta\kappa\Delta\right]\partial^x_\nu\left[\varphi(x)-\xi^2\beta\kappa\Delta\right]\partial^y_\alpha\left[\varphi(y)-\xi^2\beta\kappa\Delta\right]\partial^y_\beta\left[\varphi(y)-\xi^2\beta\kappa\Delta\right] \\ & +4\xi^2\sigma^2\hat{g}^{\mu\nu}(x)\,\hat{g}^{\alpha\beta}(y)\,\partial^x_\mu\partial^x_\alpha\Delta\,\partial^x_\mu\left[\varphi(x)-\xi^2\beta\kappa\Delta\right]\partial^y_\beta\left[\varphi(y)-\xi^2\beta\kappa\Delta\right] \\ & +2\xi^4\sigma^2\hat{g}^{\mu\nu}(x)\,\hat{g}^{\alpha\beta}(y)\,\partial^x_\mu\partial^x_\alpha\Delta\;\partial^x_\nu\partial^x_\beta\Delta\Big\}\,, \end{aligned} \tag{3.13}$$

where $\Delta = \Delta(x-y)$, $\beta = (d-2)/d$, and for brevity we have also introduced $\sigma = (d-1)(d-2)/d^2$. It is tedious but straightforward to confirm that the same answer is arrived at by computing explicitly the Feynman diagrams in fig. 3.2 and then summing over them. We see again that, despite the products of (differentials of) propagators, as appear inside the braces above, all contributions are UV finite, thanks to the presence of $\mathrm{e}^{-\xi^2\beta^2\kappa^2\Delta}$.

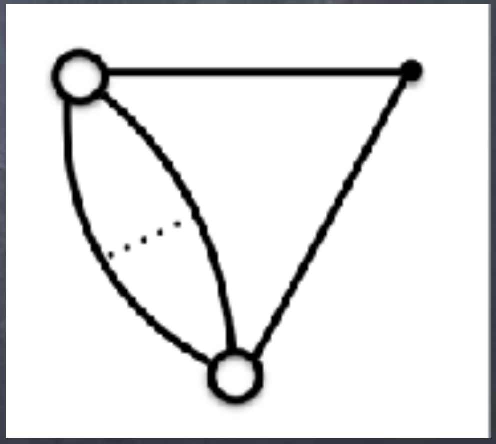
Sum over φ corrections ...

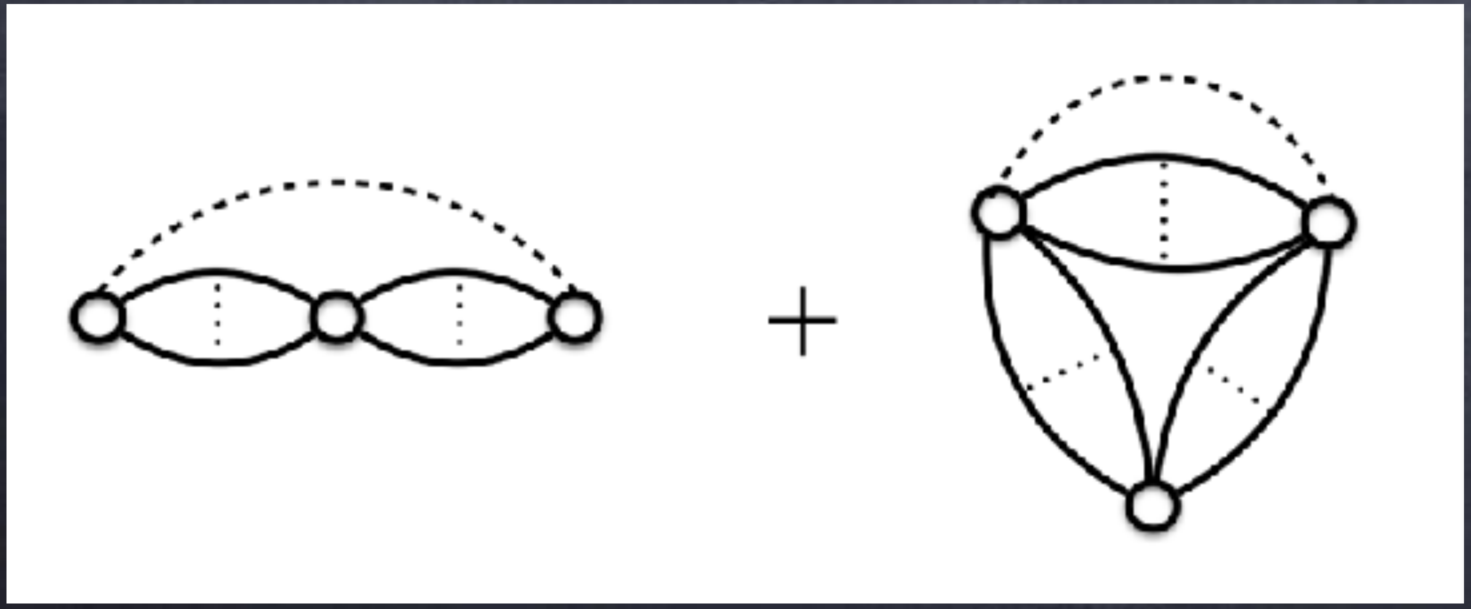
$$\mathcal{P}_{ij} = \Delta^{AB} \frac{\partial}{\partial \Phi_i^B} \frac{\partial}{\partial \Phi_j^A}$$

$$\Gamma_3 = \frac{1}{2!}\left(\mathrm{e}^{\mathcal{P}_{12}} - 1 - \mathcal{P}_{12}\right)\left(\mathrm{e}^{\mathcal{P}_{23}} - 1 - \mathcal{P}_{23}\right)\Gamma_1^3 + \frac{1}{3!}\left(\mathrm{e}^{\mathcal{P}_{12}} - 1\right)\left(\mathrm{e}^{\mathcal{P}_{23}} - 1\right)\left(\mathrm{e}^{\mathcal{P}_{31}} - 1\right)\Gamma_1^3$$



$$\exp\left\{-\xi^2 \beta_{\alpha_i}\beta_{\alpha_j}\kappa^2 \Delta(x_i - x_j)\right\}$$





Other interactions & fields ... $g_{\mu\nu} = \mathrm{e}^{2\kappa\varphi/d}\hat{g}_{\mu\nu}(h)$

Cosmological constant: $\sqrt{g} = \mathrm{e}^{\kappa\varphi}\sqrt{\hat{g}}$

Scalar:

$$\sqrt{g}g^{\mu\nu}\partial_\mu s\partial_\nu s + \sqrt{g}V(s) =$$
$$\sqrt{\hat{g}}\,\mathrm{e}^{\kappa(d-2)\varphi/d}\hat{g}^{\mu\nu}\partial_\mu s\partial_\nu s + \mathrm{e}^{\kappa\varphi}\sqrt{\hat{g}}V(s)$$

Fermion: $\mathrm{e}^{\kappa(d-1)\varphi/d}\sqrt{\hat{g}}\,\bar{\psi}i\hat{\gamma}^\mu\left(\hat{\nabla}_\mu + \frac{\kappa}{2d}(d-1)\partial_\mu\varphi\right)\psi + \mathrm{e}^{\kappa\varphi}\sqrt{\hat{g}}\,\bar{\psi}m\psi$

Vector: $\frac{1}{4}\sqrt{\hat{g}}\,\mathrm{e}^{\kappa(d-4)\varphi/d}\,\hat{g}^{\mu\sigma}\hat{g}^{\nu\lambda}F_{\mu\nu}F_{\sigma\lambda}$ ✗

But e.g. make A^ν the fundamental field:

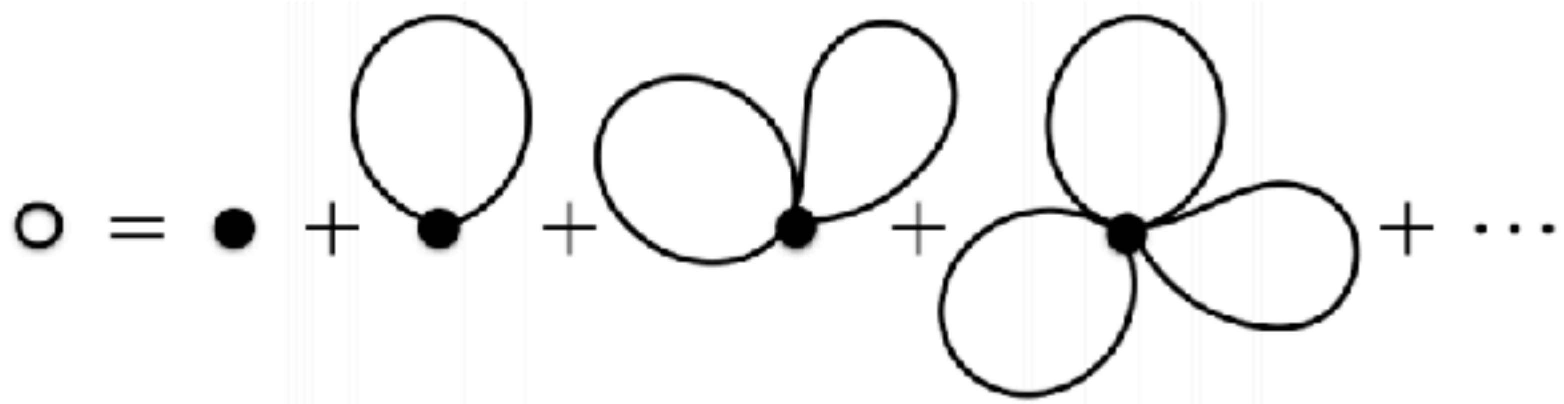
$$A_\mu = \mathrm{e}^{2\kappa\varphi/d}\hat{g}_{\mu\nu}A^\nu$$

Starobinsky: $\sqrt{g}R^2$ ✗ ... but use inflaton

Sum over φ corrections ...

$$\mathcal{P}_{ij} = \Delta^{AB} \frac{\partial}{\partial \Phi_i^B} \frac{\partial}{\partial \Phi_j^A}$$

$$\Gamma_1 = \mathrm{e}^{\mathcal{P}_{11}/2} S_I[\Phi]$$



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$$\Gamma_1 = S_I[\Phi]$$

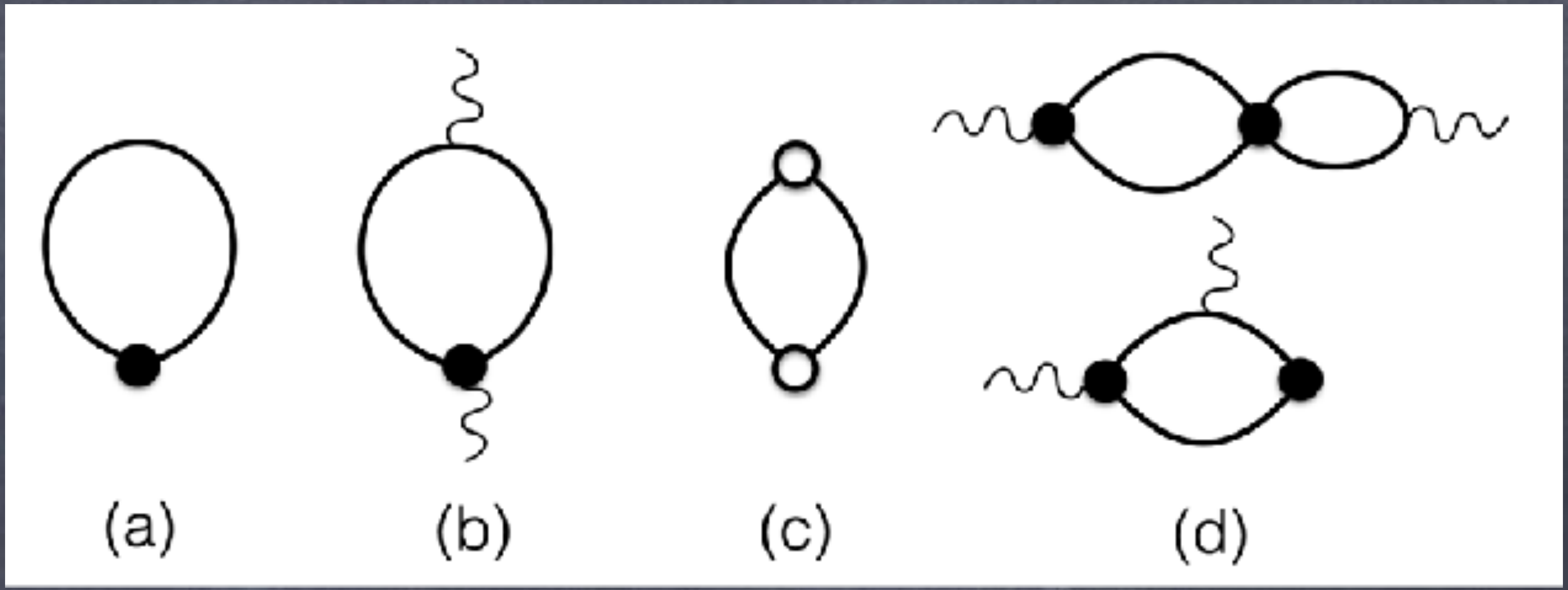
... but massive tadpoles don't vanish!

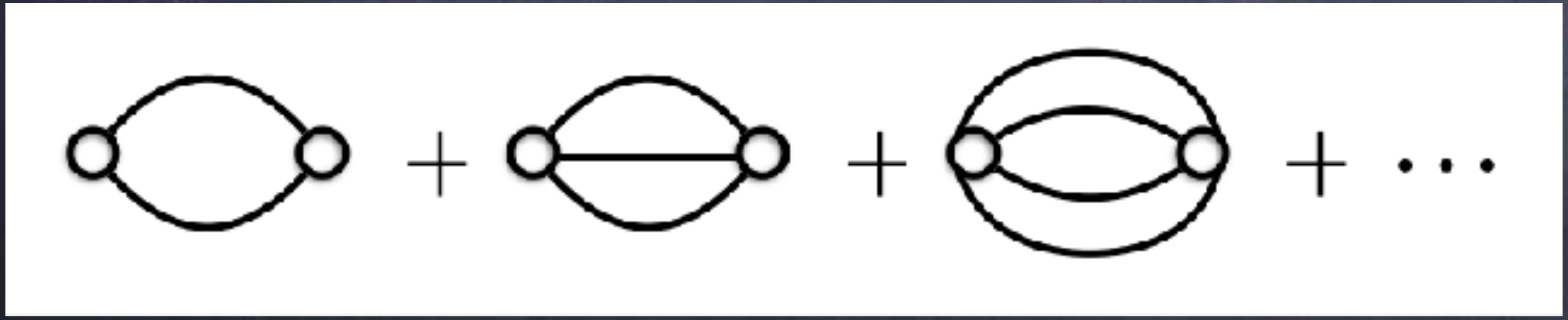
... discard e.g. by normal ordering.

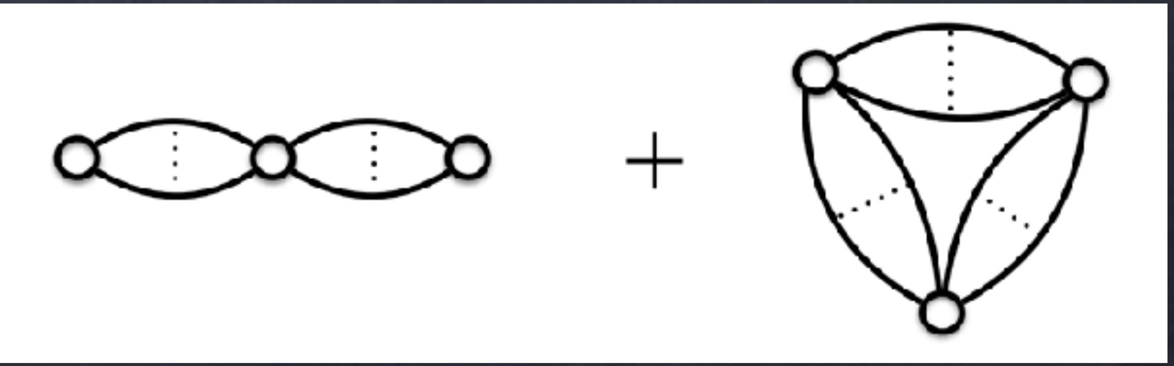
Diffeomorphism invariance ...

$$\mathcal{P}_{ij} = \Delta^{AB}\frac{\partial}{\partial\Phi_i^B}\frac{\partial}{\partial\Phi_j^A}$$

E.g. compare expanding around a background metric, using background covariant propagators.







Summary

- Can resum into a series of UV finite Γ_n
- Works also for matter fields with normal ordering
- But then no small parameter any more (Really, $\epsilon = 1$)
- Background diffeomorphism invariance: resum Γ_n?
- BRST invariance: resum Γ_n?

We can also take the limit in which ξ becomes arbitrary large, provided we also allow $\langle h_{\mu\nu}\varphi\rangle$ and $\langle c^\mu \bar{c}^\nu\rangle$ to become large, whilst nevertheless leaving the $\langle h_{\mu\nu}h_{\alpha\beta}\rangle$ propagator finite. Setting

$$\alpha = \sqrt{\frac{d-2}{2(d-1)}}\left(1+\frac{d^2\varepsilon^2}{2(d-1)(d-2)}+\frac{(d^2-\gamma)d^2\varepsilon^4}{8(d-1)^2(d-2)^2}\right),$$

$$\beta = -\sqrt{\frac{d-2}{2(d-1)}}\left(1+\frac{d\,\varepsilon^2}{2(d-1)(d-2)}+\frac{d(2d^2-[d+1]\gamma)\varepsilon^4}{16(d-1)^2(d-2)^2}\right), \tag{7.9}$$

where γ is a finite free gauge parameter, and $\varepsilon \ll 1$, yields $\xi^2 = 1/\varepsilon^2 + O(\varepsilon^2)$, together with:

$$\langle \varphi(p)\,\varphi(-p)\rangle = -\left(\frac{1}{\varepsilon^2}+O(\varepsilon^2)\right)\frac{1}{p^2} = -\frac{\xi^2}{p^2}, \tag{7.10}$$

$$\langle h_{\mu\nu}(p)\,\varphi(-p)\rangle = \langle \varphi(p)\,h_{\mu\nu}(-p)\rangle = \left(\frac{1}{\varepsilon^2}-\frac{2d^2+\gamma}{8(d-1)(d-2)}+O(\varepsilon^2)\right)\left(\frac{\delta_{\mu\nu}}{d}-\frac{p_\mu p_\nu}{p^2}\right)\frac{1}{p^2}, \tag{7.11}$$

$$\langle h_{\mu\nu}(p)\,h_{\alpha\beta}(-p)\rangle = \frac{\delta_{\mu(\alpha}\delta_{\beta)\nu}}{p^2}+\frac{2d}{d-2}\frac{p_{(\mu}\delta_{\nu)(\alpha}p_{\beta)}}{p^4}+\frac{\gamma-2d^2(d-2)}{2d^2(d-1)(d-2)}\frac{\delta_{\mu\nu}\delta_{\alpha\beta}}{p^2} \tag{7.12}$$

$$+\frac{2d(d-2)-\gamma}{2d(d-1)(d-2)}\frac{\delta_{\alpha\beta}p_\mu p_\nu+p_\alpha p_\beta\delta_{\mu\nu}}{p^4}+\frac{\gamma-2d(3d-4)}{2(d-1)(d-2)}\frac{p_\mu p_\nu p_\alpha p_\beta}{p^6}+O(\varepsilon^2),$$

$$\langle c^\mu(p)\,\bar{c}^\nu(-p)\rangle = \frac{\delta^{\mu\nu}}{p^2}+\left(\frac{2(d-2)}{d\varepsilon^2}-1+\frac{\gamma}{4d}+O(\varepsilon^2)\right)\frac{p^\mu p^\nu}{p^4}. \tag{7.13}$$