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Running Vacuum and the Cosmological Constant Problem + a challenging solution to the Cosmological Tensions

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Guidelines of the Talk

- Vacuum energy and the CC Problem
- Running Vacuum in QFT and beyond
- RVM and the ΛCDM tensions: H_0 and σ_8
- Composite Dark Energy and cosmic tensions
- ``Phantom matter'': a challenging solution
- Conclusions

$\Lambda\uparrow$

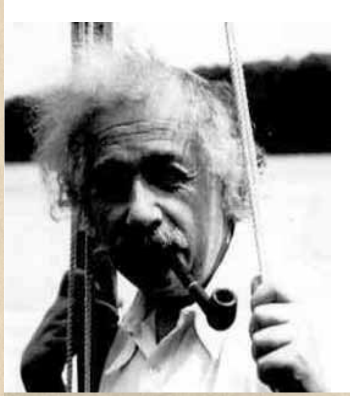
Kosmologische Betrachtungen zur allgemeinen Relativitätstheorie.

Von A. Einstein.

$\Lambda\downarrow$

Zum kosmologischen Problem der allgemeinen Relativitätstheorie.

Von A. Einstein.



14 yrs later !!

➢ The old CC problem as a fine tuning problem

The CC problem stems from realizing that the effective or physical vacuum energy is the sum of two terms:

$$\boxed{\rho_{\Lambda\text{phys}} = \rho_{\Lambda\text{vac}} + \rho_{\Lambda\text{ind}}}$$

$$S_{EH} = \frac{1}{16\pi G_N}\int d^4x\sqrt{|g|}\,(R - 2\Lambda_{\text{vac}}) = \int d^4x\sqrt{|g|}\left(\frac{1}{16\pi G_N}R - \rho_{\Lambda\text{vac}}\right)$$

$$\boxed{\rho_{\Lambda\text{vac}} = \frac{\Lambda}{8\pi G_N}}$$

Vacuum bare term in Einstein eqs.

$$R_{ab} - \frac{1}{2}g_{ab}R = -8\pi G_N\left(\langle\tilde{T}_{ab}^{\varphi}\rangle + T_{ab}\right) = -8\pi G_N\, g_{ab}\,(\rho_{\Lambda\text{vac}} + \rho_{\Lambda\text{ind}} + T_{ab})$$

Quantum effects $\Rightarrow$ $\rho_{\Lambda\text{ind}} = \langle V(\varphi)\rangle + \text{ZPE}$

...of quantum "bubles" !!

e^+ e^-

e^+ e^-

e^+ e^-

e^+ e^-

e^+ e^-

e^+ e^-

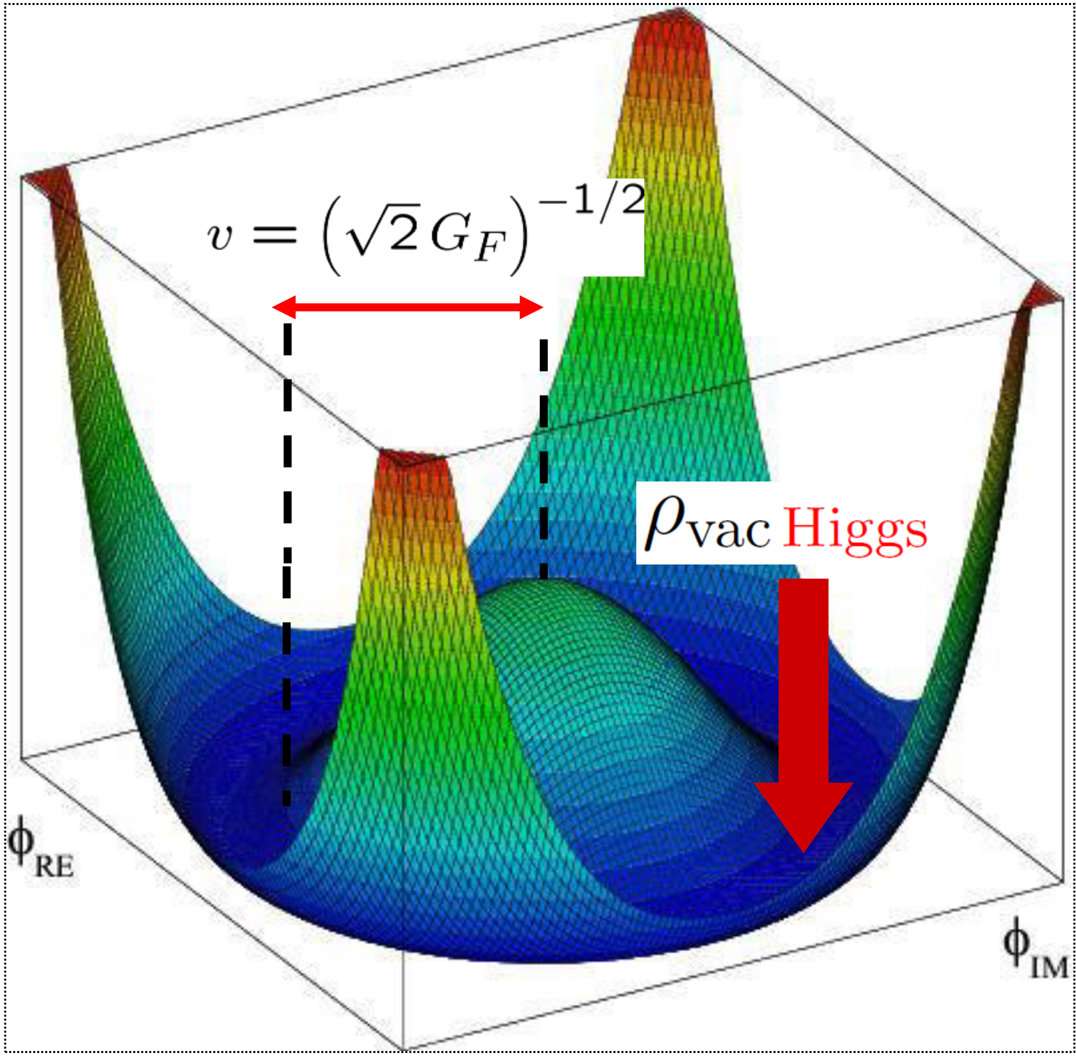
e^+ e^-

e^+ e^-

$$\sum_k \frac{1}{2} \hbar \omega_k$$

the quantum vacuum is full!

Higgs Potential → Vacuum Energy



$$G_F/\sqrt{2} = g^2/8M_W^2$$

$$V(\varphi) = \frac{1}{2}\, m^2\, \varphi^2 + \frac{1}{4!}\, \lambda\, \varphi^4$$

$$m^2 < 0 \quad \Rightarrow$$

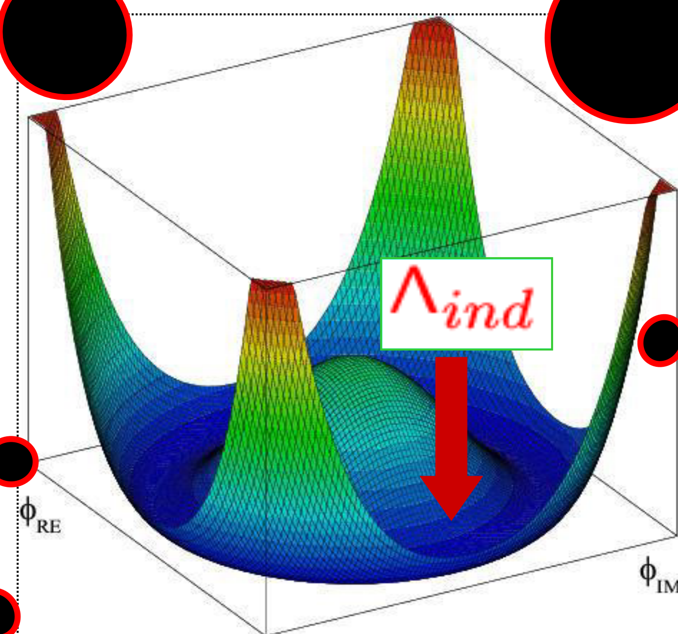
$$v \equiv \langle \varphi \rangle = \sqrt{\frac{-6\, m^2}{\lambda}}$$

$$\langle V(\varphi) \rangle = -\tfrac{1}{8}\, M_{\mathcal{H}}^2\, v^2$$
$$\sim -10^8 \; GeV^4$$

$$\boxed{\begin{aligned} M_W &= \tfrac{1}{2}\, g\, v \\ M_Z &= M_W / \cos\theta_w \\ m_f &= \lambda_f\, v \end{aligned}}$$

Vacuum energy = bubbles + SSB

$$\frac{1}{2}\hbar\omega_k$$



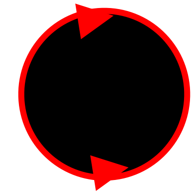
Out of desperation

Beyond Λ...

Quintessence and all that...

Zero-point energy in quantum field theory in flat spacetime

$$V_{\rm ZPE}(P) = \hbar\, V_P^{(1)} + \hbar^2\, V_P^{(2)} + \hbar^3\, V_P^{(3)} + \ldots.$$



Real scalar field, one loop:

$$V_P^{(1)} = (1/2)\sum_{\mathbf{k}} \hbar\omega_{\mathbf{k}} = (1/2)\sum_{\mathbf{k}} \hbar\sqrt{\mathbf{k}^2+m^2} \;\rightarrow$$

$$= \frac{1}{2}\int \frac{d^3k}{(2\pi)^3}\sqrt{\mathbf{k}^2+m^2} = \frac{1}{4\pi^2}\int_0^{\Lambda_{\rm UV}} dk\, k^2\sqrt{k^2+m^2}$$

$$= \frac{\Lambda_{\rm UV}^4}{16\pi^2}\left(1+\frac{m^2}{\Lambda_{\rm UV}^2}-\frac{1}{4}\frac{m^4}{\Lambda_{\rm UV}^4}\ln\frac{\Lambda_{\rm UV}^2}{m^2}+\cdots\right),$$

renormalization

$$\boxed{\rho_{\rm vac}^{(1)} = \rho_\Lambda(\mu) + V_{\rm ZPE}^{(1)}(\mu) = \rho_\Lambda(\mu) + \frac{m^4\,\hbar}{4\,(4\,\pi)^2}\left(\ln\frac{m^2}{\mu^2}-\frac{3}{2}\right)}$$

➢ Origin of the CCP in QFT terms

i)

$$\rho_{\Lambda \mathrm{ind}} = \langle V(\varphi)\rangle + \mathrm{ZPE}$$ is too large !!

and

ii)

$$\beta_{\rho_{\mathrm{vac}}} \propto m^4$$

Λ runs too fast !!

➢ The road to RVM: suppose, instead, that…

i) Vacuum can have effective EoS other than $w = -1$

ii) (Adiabatically) Renormalized VED at scale M adopts the form

$$\rho_{\rm vac}(M) = \rho_\Lambda(M) + f(m, M) + b(m, M)H^2 + \mathcal{O}(H^4)$$

and

iii) $$\beta_{\rho_{\rm vac}} = M\frac{\partial \rho_{\rm vac}(M)}{\partial M} \propto m^2 H^2$$

$$\beta_{\rho_{\rm vac}} \propto \cancel{m^4}$$

Λ no longer runs fast !!

➢ Beta Function of the VED in RVM

$$\beta_{\rho_{\rm vac}} = M\frac{\partial\rho_{\rm vac}(M)}{\partial M}$$

C. Moreno-Pulido, JSP (EPJC 2020,2022a,b)

(bosonic case)

$$\boxed{= \left(\xi - \frac{1}{6}\right)\frac{3H^2}{8\pi^2}\left(M^2 - m^2\right)}$$

$$+ \left(\xi - \frac{1}{6}\right)^2 \frac{9\left(\dot{H}^2 - 2H\ddot{H} - 6H^2\dot{H}\right)}{8\pi^2}$$

(Higher order, negligible for current universe)

$$\beta_{\rho_{\rm vac}} \propto \cancel{m^4}$$

and $\beta_{\rho_{\rm vac}} = 0$ in **Minkowski spacetime** !!

Cosmic scale setting in RVM

- In FLRW universe we set $M = H$ at each còsmic epoch what else more natural?...

(consistent with very recent studies of the RVM from lattice QG:
Dai, Freeman, Laiho,Schiffer and Unmuth-Yockey, arXiv:2408.08963 [hep-lat])

- Similar to ordinary gauge teories, such as QED, EW, QCD...
e.g. $\mu = M_Z$ at LEP.

- From the above considerations we expect that:

$$\boxed{\rho_{\rm vac}(H) - \rho_{\rm vac}(H_0) \propto m^2\,(H^2 - H_0^2) + \mathcal{O}(H^4)}$$

➢ VED evolution

Explicit calculation indeed yields C. Moreno-Pulido, JSP (EPJC 2020,2022a,b)

for the current universe

*

$$\boxed{\rho_{\rm vac}(H) = \rho^0_{\rm vac} + \frac{3\nu_{\rm eff}(H)}{8\pi}\, m^2_{\rm Pl}\,(H^2 - H_0^2) + \mathcal{O}(H^4)}$$

where $\rho^0_{\rm vac} \equiv \rho_{\rm vac}(H_0)$, and (for nonminimally coupled scalar fields),

$$\boxed{\nu_{\rm eff}(H) \equiv \frac{1}{2\pi}\left(\xi - \frac{1}{6}\right)\frac{m^2}{m^2_{\rm Pl}}\left(-1 + \ln\frac{m^2}{H^2} - \frac{H_0^2}{H^2 - H_0^2}\ln\frac{H^2}{H_0^2}\right)}$$

naturally small parameter

(β -function coefficient) **!**

RVM structure !!

(J. Solà, 2007,2011,2013,2014, 2016, 2021)

(Recent review: Phil.Trans.Roy.Soc.Lond.A 380 (2022) 20210182)

Effectively,

$$\boxed{\nu_{\rm eff} \simeq \epsilon \ln\frac{m^2}{H_0^2}} \qquad \boxed{\epsilon \equiv \frac{1}{2\pi}\left(\xi - \frac{1}{6}\right)\frac{m^2}{m^2_{\rm Pl}}}$$

* Formally the same $\rho_{\rm vac}$ structure as in lattice QG (Dai et al., arXiv:2408.08963 [hep-lat])

➢ Combined contribution Bosons+Fermions

$$S_\phi = -\int d^4x\sqrt{-g}\left(\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi + \frac{1}{2}\left(m_\phi^2 + \xi R\right)\phi^2\right)$$

$$S_\psi(x) = -\int d^4x\sqrt{-g}\left[\frac{1}{2}i\left(\bar{\psi}\underline{\gamma}^\mu\nabla_\mu\psi - \left(\nabla_\mu\bar{\psi}\right)\underline{\gamma}^\mu\psi\right) + m_\psi\bar{\psi}\psi\right]$$

C. Moreno-Pulido, JSP & S.Cheraghchi
arXiv:2301.05205[gr-qc] (EPJC)

$$\boxed{\rho_{\rm vac}(H) = \rho_{\rm vac}(H_0) + \frac{3\nu_{\rm eff}}{8\pi}m_{\rm Pl}^2(H^2 - H_0^2)}$$

$$\boxed{\nu_{\rm eff} = \frac{1}{2\pi}\left[\sum_{j=1}^{N_{\rm s}}\left(\xi_j - \frac{1}{6}\right)\frac{m_{\phi_j}^2}{m_{\rm Pl}^2}\ln\frac{m_{\phi_j}^2}{H_0^2} - \frac{1}{3}\sum_{\ell=1}^{N_{\rm f}}\frac{m_{\psi_\ell}^2}{m_{\rm Pl}^2}\ln\frac{m_{\psi_\ell}^2}{H_0^2}\right]}$$

SOME DETAILS OF THE QFT CALCULATIONS…

Comprehensive details given in the following 4 papers and review:

C. Moreno-Pulido, JSP, arXiv:2005.03164 [gr-qc] (EPJC 2020)

C. Moreno-Pulido, JSP, arXiv:2201.05827 [gr-qc] (EPJC 2022)

C. Moreno-Pulido, JSP, arXiv:2207.07111 [gr-qc] (EPJC 2022)

C. Moreno-Pulido, JSP, arXiv:2301.05205 [gr-qc] (EPJC 2023)

JSP, arXiv:2203.13757 [gr-qc] (Phil.Trans.Roy.Soc.Lond.A 2022)

Adiabatic renormalization of the **VED** in QFT in a FLRW background: absence of quartic mass terms

C. Moreno-Pulido and JSP arXiv:2005.03164 (EPJ-C)

- The gravitational field equations read

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G_N T_{\mu\nu}^{matter},$$

where Λ is the Cosmological constant, with energy density $\rho_\Lambda \equiv \Lambda/(8\pi G_N)$. (this is not yet the physical VED)

Consider a toy-model (but non-trivial) calculation of the VED.

⇓

- We will suppose that there is only one matter field contribution to the EMT in $T_{\mu\nu}^{matter}$ in the form of a real scalar field, ϕ.

(nonminimal coupling ξ)

$$S[\phi] = -\int d^4x\sqrt{-g}\left(\frac{1}{2}g_{\mu\nu}\partial_\nu\phi\partial_\mu\phi + \frac{1}{2}(m^2+\xi R)\phi^2\right)$$

(no SSB contribution!)

- The Energy-Momentum tensor (EMT) associated to the scalar field is

$$T_{\mu\nu}(\phi) = (1-2\xi)\,\partial_\mu\phi\partial_\nu\phi + \left(2\xi - \frac{1}{2}\right) g_{\mu\nu}\partial^\sigma\phi\partial_\sigma\phi$$
$$-2\xi\nabla_\mu\nabla_\nu\phi + 2\xi g_{\mu\nu}\phi\Box\phi + \xi G_{\mu\nu}\phi^2 - \frac{1}{2}m^2 g_{\mu\nu}\phi^2.$$

- We can take into account the quantum fluctuations of the field ϕ by considering the expansion of the field around its background (or classical mean field) value ϕ,

$$\phi(\tau,\mathbf{x}) = \phi_b(\tau) + \delta\phi(\tau,\mathbf{x}),$$

$$\boxed{\langle T_{\mu\nu}^{vac}\rangle \equiv -\rho_\Lambda g_{\mu\nu} + \langle T_{\mu\nu}^{\delta\phi}\rangle.}$$

Total
vacuum contribution
(needs renormalization!!)

$$ds^2 = g_{\mu\nu}\,dx^\mu dx^\nu = a^2(\tau)\eta_{\mu\nu}\,dx^\mu dx^\nu$$

$\text{sign}(g_{\mu\nu}) = (-,+,+,+)$

Fluctuations split in Fourier modes:

$$\delta\phi(\tau,\mathbf{x}) = \frac{1}{(2\pi)^{3/2}a}\int d^3k\left[A_{\mathbf{k}}e^{i\mathbf{kx}}h_k(\tau) + A_{\mathbf{k}}^{\dagger}e^{-i\mathbf{kx}}h_k^*(\tau)\right]$$

$$(\Box - m^2 - \xi R)\delta\phi(\tau,x) = 0 \Rightarrow \boxed{h_k'' + \Omega_k^2 h_k = 0,}$$ **(mode equation)**

$$h_k' h_k^* - h_k h_k^{*\prime} = i$$

$$\Omega_k^2 \equiv k^2 + a^2 m^2 + \boxed{a^2(\xi - 1/6)R}$$ **(non-trivial!)**

The solution is $$h_k(\tau) \sim \frac{e^{i\int^{\tau} W_k(\tau_1)d\tau_1}}{\sqrt{W_k(\tau)}},$$

$$W_k^2 = \Omega_k^2 - \frac{1}{2}\frac{W_k''}{W_k} + \frac{3}{4}\left(\frac{W_k'}{W_k}\right)^2$$

In order to solve this equation we should use the **WKB approximation** or **adiabatic regularization**. (slowly varying) Ω_k **!!**

$$W_k = \omega_k^{(0)} + \omega_k^{(2)} + \omega_k^{(4)} \ldots ,$$ (**Adiabatic expansion**)

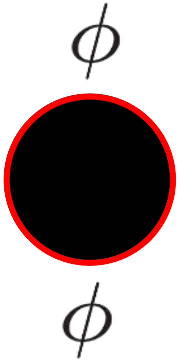
$$\begin{cases} \omega_k^{(2)} = \dfrac{a^2\Delta^2}{2\omega_k} + \dfrac{a^2 R}{2\omega_k}(\xi - 1/6) - \dfrac{\omega_k''}{4\omega_k^2} + \dfrac{3\omega_k'^2}{8\omega_k^3}\,, \\ \omega_k^{(4)} = -\dfrac{1}{2\omega_k}\left(\omega_k^{(2)}\right)^2 + \dfrac{\omega_k^{(2)}\omega_k''}{4\omega_k^3} - \dfrac{\omega_k^{(2)\prime\prime}}{4\omega_k^2} - \dfrac{3\omega_k^{(2)}\omega_k'^2}{4\omega_k^4} + \dfrac{3\omega_k'\omega_k^{(2)\prime}}{4\omega_k^3}\,. \end{cases}$$

$$\begin{cases} \omega_k^{(0)} \equiv \omega_k = \sqrt{k^2 + a^2 M^2}, \\ \omega_k' = a^2\mathcal{H}\dfrac{M^2}{\omega_k}, \quad \omega_k'' = 2a^2\mathcal{H}^2\dfrac{M^2}{\omega_k} + a^2\mathcal{H}'\dfrac{M^2}{\omega_k} - a^4\mathcal{H}^2\dfrac{M^4}{\omega_k^3}\,. \end{cases}$$

The non-appearance of the odd adiabatic orders is justified by means of general covariance.

Explains why only even powers of H:

$$\boxed{\Lambda(H) = c_0 + \cancel{c_1 H} + c_2 H^2 + \cancel{c_3 H^3} + c_4 H^4 + \ldots}$$



one-loop

$T_{00}^{\delta\phi}$ up to 4th adiabatic order:

$$\left\langle T_{00}^{\delta\phi} \right\rangle = \int dkk^2 \Bigg[|h_k'|^2 + (\omega_k^2 + a^2\Delta^2)\,|h_k|^2$$

$$\left(\xi - \frac{1}{6}\right)\left(-6\mathcal{H}^2|h_k|^2 + 6\mathcal{H}(h_k' h_k^* + h_k^{*\prime} h_k)\right)\Bigg]$$

unrenormalized

ZPE

UV-divergent !!

$$\left\langle T_{00}^{\delta\phi} \right\rangle = \frac{1}{8\pi^2 a^2}\int dkk^2 \Bigg[2\omega_k + \frac{a^4M^4\mathcal{H}^2}{4\omega_k^5} - \frac{a^4M^4}{16\omega_k^7}(2\mathcal{H}''\mathcal{H} - \mathcal{H}'^2 + 8\mathcal{H}'\mathcal{H}^2 + 4\mathcal{H}^4)$$

$$+ \frac{7a^6M^6}{8\omega_k^9}(\mathcal{H}'\mathcal{H}^2 + 2\mathcal{H}^4) - \frac{105a^8M^8\mathcal{H}^4}{64\omega_k^{11}}$$

$$+ \left(\xi - \frac{1}{6}\right)\left(-\frac{6\mathcal{H}^2}{\omega_k} - \frac{6a^2M^2\mathcal{H}^2}{\omega_k^3} + \frac{a^2M^2}{2\omega_k^5}(6\mathcal{H}''\mathcal{H} - 3\mathcal{H}'^2 + 12\mathcal{H}'\mathcal{H}^2)\right.$$

$$\left. - \frac{a^4M^4}{8\omega_k^7}(120\mathcal{H}'\mathcal{H}^2 + 210\mathcal{H}^4) + \frac{105a^6M^6\mathcal{H}^4}{4\omega_k^9}\right)$$

$$+ \left(\xi - \frac{1}{6}\right)^2\left(-\frac{1}{4\omega_k^3}(72\mathcal{H}''\mathcal{H} - 36\mathcal{H}'^2 - 108\mathcal{H}^4) + \frac{54a^2M^2}{\omega_k^5}(\mathcal{H}'\mathcal{H}^2 + \mathcal{H}^4)\right)\Bigg]$$

$$+ \frac{1}{8\pi^2a^2}\int dkk^2 \Bigg[\frac{a^2\Delta^2}{\omega_k} - \frac{a^4\Delta^4}{4\omega_k^3} + \frac{a^4\mathcal{H}^2M^2\Delta^2}{2\omega_k^5} - \frac{5}{8}\frac{a^6\mathcal{H}^2M^4\Delta^2}{\omega_k^7}$$

$$+ \left(\xi - \frac{1}{6}\right)\left(-\frac{3a^2\Delta^2\mathcal{H}^2}{\omega_k^3} + \frac{9a^4M^2\Delta^2\mathcal{H}^2}{\omega_k^5}\right)\Bigg] + \ldots,$$

$$\Delta^2 \equiv m^2 - M^2$$

Quartically, quadratically and log divergent contributions for bosons:

$$\langle T_{00}^{\delta\phi}\rangle^{(0-2)}\Big|_{(M=m)} = \frac{1}{4\pi^2 a^2}\int dk k^2 \omega_k(m) - \frac{3\left(\xi-\frac{1}{6}\right)}{4\pi^2 a^2}\int dk k^2\left(\frac{\mathcal{H}^2}{\omega_k(m)} + \frac{a^2 m^2 \mathcal{H}^2}{\omega_k^3(m)}\right)$$

Quartically and logarithmic divergent contributions for spin-1/2 fermions:

$$\langle T_{00}^{\delta\psi}\rangle\Big|_{(M=m)} = -\frac{1}{\pi^2 a^2}\int dk k^2 \omega_k(m) + \frac{1}{8\pi^2}\int_0^\infty dk k^2 \frac{m^2}{\omega_k^3(m)}\mathcal{H}^2$$

For Minkowski spacetime and minimal coupling to gravity:

$$\boxed{\langle T_{00}^{\delta\phi}\rangle^{\rm Mink}\Big|_{(M=m,\xi=0)} = \frac{1}{4\pi^2}\int dk k^2 \omega_k = \int \frac{d^3k}{(2\pi)^3}\left(\frac{1}{2}\hbar\omega_k\right)}$$

- We compute terms up to 4th order because the divergences are only present up to this adiabatic order.
- We define the renormalized ZPE in curved space-time at the scale M as follows:

$$\boxed{\langle T_{00}^{\delta\phi}\rangle_{Ren}(M) \equiv \langle T_{00}^{\delta\phi}\rangle(m) - \langle T_{00}^{\delta\phi}\rangle^{(0-4)}(M)}$$

$$\langle T_{00}^{\delta\phi}\rangle_{\rm Ren}(M) = \frac{a^2}{128\pi^2}\left(-M^4 + 4m^2M^2 - 3m^4 + 2m^4\ln\frac{m^2}{M^2}\right)$$
$$-\left(\xi-\frac{1}{6}\right)\frac{3\mathcal{H}^2}{16\pi^2}\left(m^2 - M^2 - m^2\ln\frac{m^2}{M^2}\right) + \left(\xi-\frac{1}{6}\right)^2\frac{9\left(2\mathcal{H}''\mathcal{H} - \mathcal{H}'^2 - 3\mathcal{H}^4\right)}{16\pi^2a^2}\ln\frac{m^2}{M^2} + \dots$$

$$\Downarrow$$

$$\boxed{\mathcal{M}_{\rm Pl}^2(M)G_{\mu\nu} + \rho_\Lambda(M)g_{\mu\nu} + \alpha(M)\ {}^{(1)}H_{\mu\nu} = \langle T_{\mu\nu}^{\delta\phi}\rangle_{\rm ren}(M)\,.}$$

$$\mathcal{M}_{\rm Pl}^2(M) = \frac{G^{-1}(M)}{8\pi}$$

Off-shell subtraction: $\Longrightarrow$ **Exploring different scales**

$$\boxed{\text{VED} = \rho_\Lambda + \text{ZPE}}$$

$$\left\langle T_{\mu\nu}^{vac} \right\rangle \equiv -\rho_\Lambda g_{\mu\nu} + \left\langle T_{\mu\nu}^{\delta\phi} \right\rangle \Rightarrow \rho_{vac}(M) = \rho_\Lambda(M) + \frac{\left\langle T_{00}^{\delta\phi} \right\rangle_{Ren}(M)}{a^2}$$

$$\Downarrow$$

$$\boxed{\begin{aligned}\rho_{\rm vac}(M) &= \rho_\Lambda(M) + \frac{1}{128\pi^2}\left(-M^4 + 4m^2M^2 - 3m^4 + 2m^4 \ln\frac{m^2}{M^2}\right) \\ &+ \left(\xi - \frac{1}{6}\right)\frac{3\mathcal{H}^2}{16\pi^2 a^2}\left(M^2 - m^2 + m^2\ln\frac{m^2}{M^2}\right) + \left(\xi - \frac{1}{6}\right)^2 \frac{9\left(2\mathcal{H}''\mathcal{H} - \mathcal{H}'^2 - 3\mathcal{H}^4\right)}{16\pi^2 a^4}\ln\frac{m^2}{M^2} + \cdots\end{aligned}}$$

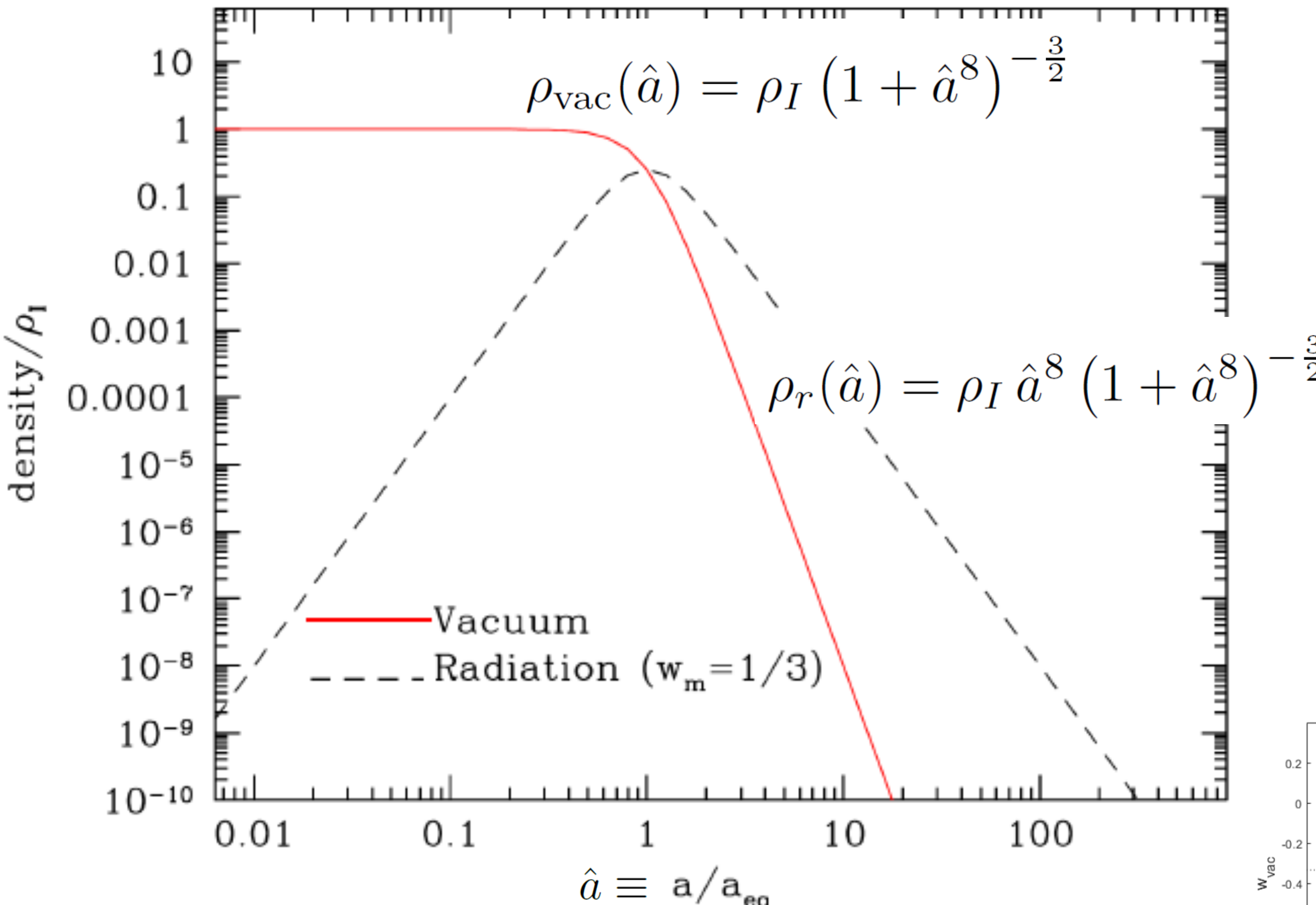
$$\left.\begin{array}{l}\text{in Minkowski space } (H=0) \\ \rho_{\rm vac}(M) \quad \text{must be RG invariant}\end{array}\right\} \quad \beta_{\rho_\Lambda}(M) = M\frac{\partial\rho_\Lambda(M)}{\partial M} = \frac{1}{2(4\pi)^2}(M^2 - m^2)^2$$

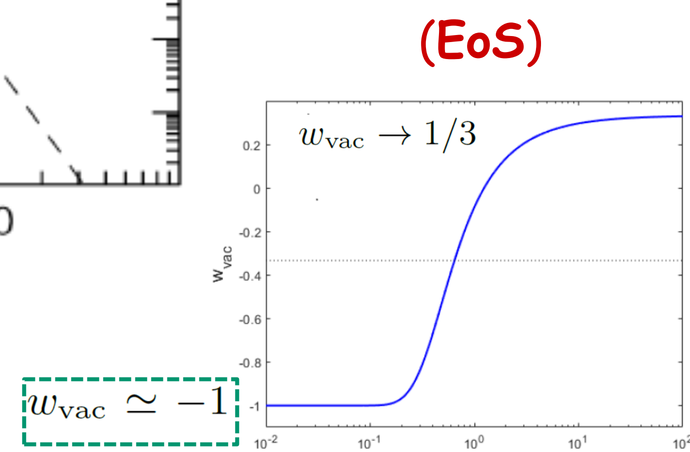
➢ Impact for the early universe: RVM-inflation!

$$\boxed{\Lambda(H) = c_0 + 3\nu H^2 + 3\alpha \frac{H^{n+2}}{H_I^n}}$$

$$\begin{cases} \rho_r(\hat{a}) = \tilde{\rho}_I (1-\nu) \dfrac{\hat{a}^{2n(1-\nu)}}{\left[1+\hat{a}^{2n(1-\nu)}\right]^{\frac{n+2}{n}}} \\ \hat{a} = \dfrac{a}{a_*} \\ \rho_\Lambda(\hat{a}) = \tilde{\rho}_I \dfrac{1+\nu\hat{a}^{2n(1-\nu)}}{\left[1+\hat{a}^{2n(1-\nu)}\right]^{\frac{n+2}{n}}} \,. \end{cases}$$

RVM-inflation and vacuum decay





QUANTUM VACUUM Pressure and EoS

Calculations up to 6th adiabatic order yield:

$$\boxed{P_{\rm vac}(M) = -\rho_{\rm vac}(M)} + \text{ corrections } \;\; !!$$

$$+ f_2(M,\dot{H}) + f_4(M,H,\dot{H},...,\dddot{H}) + f_6(\dot{H},...,\overset{\cdot\cdot\cdot\cdot\cdot}{H})$$

e.g. $$f_2(M,\dot{H}) = \frac{(\xi-\frac{1}{6})}{8\pi^2}\dot{H}\left(m^2 - M^2 - m^2\ln\frac{m^2}{M^2}\right)$$

Equation of State of the QUANTUM VACUUM (EoS)

C. Moreno-Pulido, JSP, arXiv:2207.07111 [gr-qc] (EPJC 2022)

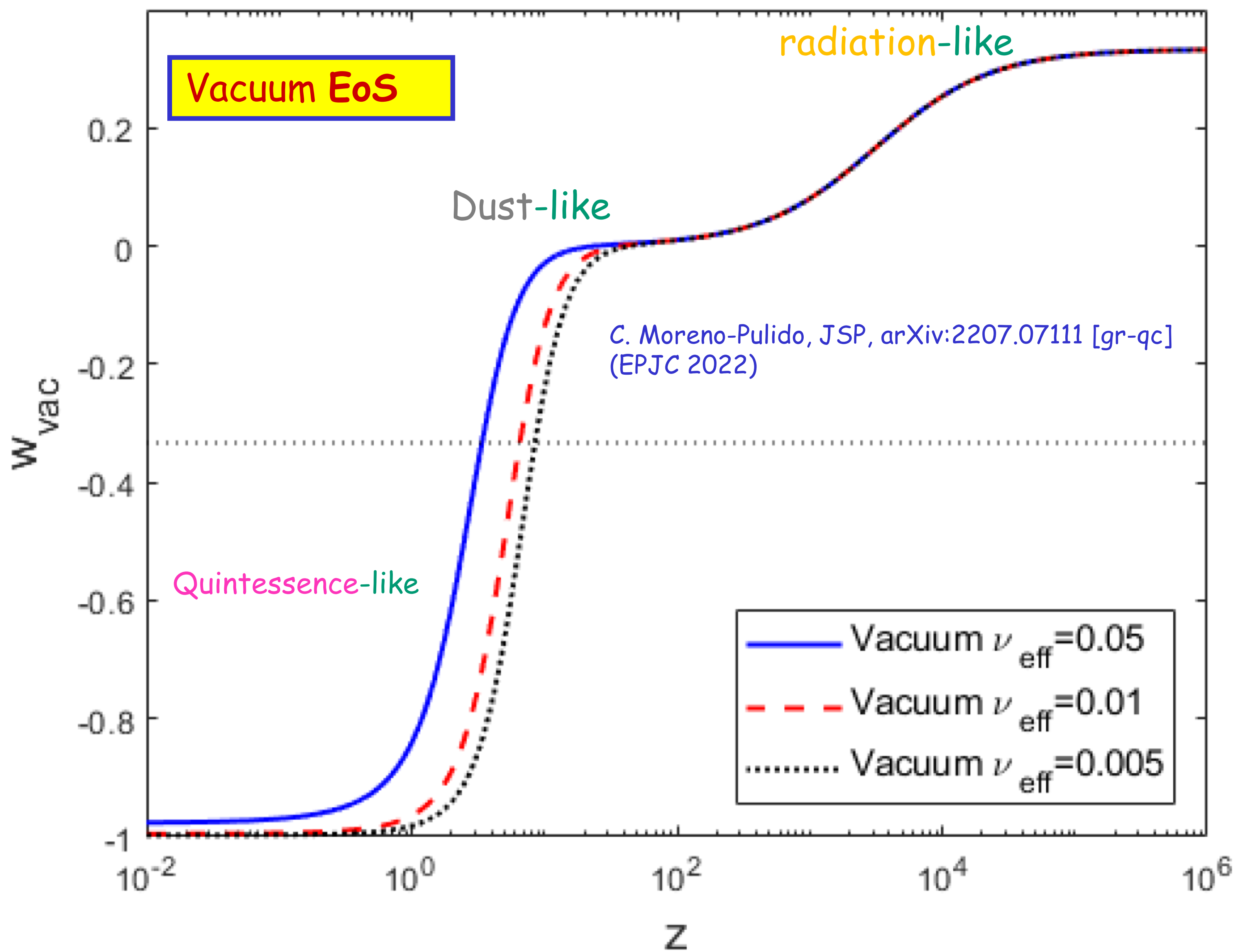
$$w_{\rm vac}(H) \equiv \frac{P_{\rm vac}(H)}{\rho_{\rm vac}(H)} \simeq -1 + \frac{f_2(\dot{H})}{\rho_{\rm vac}(H)}$$

$$\simeq -1 + \left(\xi - \frac{1}{6}\right)\frac{\dot{H}m^2}{8\pi^2\rho_{\rm vac}(H)}\left(1 - \ln\frac{m^2}{H^2}\right)$$

$$= -1 + \frac{\nu_{\rm eff}\left(\Omega_m^0(1+z)^3 + \frac{4}{3}\Omega_r^0(1+z)^4\right)}{\Omega_{\rm vac}^0 + \nu_{\rm eff}\left[-1 + \Omega_m^0(1+z)^3 + \Omega_r^0(1+z)^4 + \Omega_v^0\right]}$$

$$= \begin{cases} \frac{1}{3} \quad \text{for} \quad z \gg z_{\rm eq} \quad \text{with} \quad \Omega_r^0(1+z) \gg \Omega_m^0, & \text{radiation behavior } (\nu_{\rm eff} \neq 0), \\ 0 \quad \text{for} \quad \mathcal{O}(1) < z \ll z_{\rm eq} \quad \text{with} \quad \Omega_m^0 \gg \Omega_r^0(1+z), & \text{dust behavior } (\nu_{\rm eff} \neq 0), \\ \boxed{-1 + \nu_{\rm eff}\frac{\Omega_m^0}{\Omega_{\rm vac}^0}(1+z)^3}^{\,*} \quad \text{for} \quad -1 < z < \mathcal{O}(1)\,, & \text{quintessence behavior } (\nu_{\rm eff} > 0) \end{cases}$$

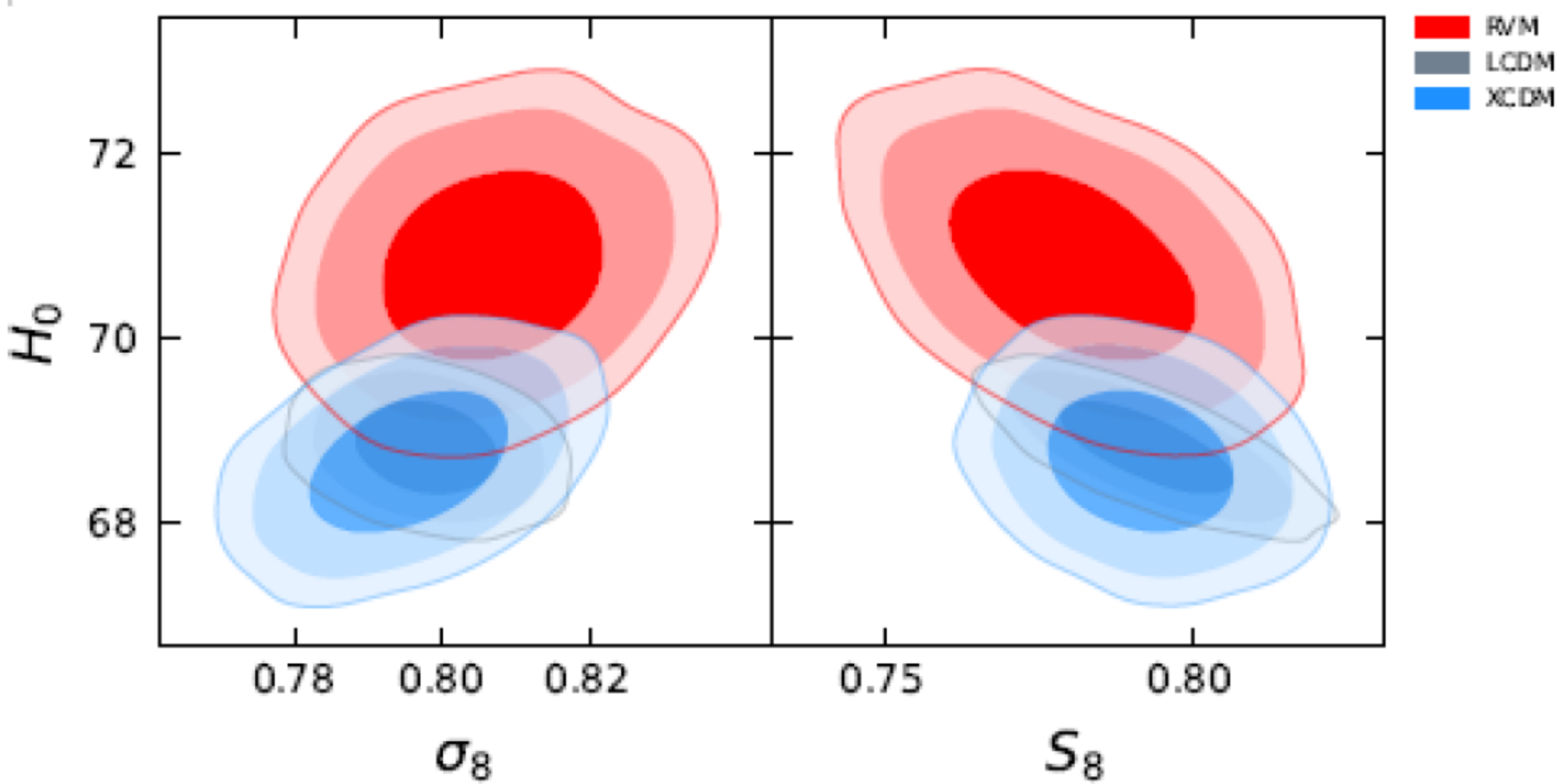
$*$ Same EoS as obtained in lattice QG, (Dai et al., arXiv:2408.08963 [hep-lat])



(Joan Solà, KCL 2024)

JSP,A.Gómez-Valent, J. de Cruz Pérez and C. Moreno-Pulido

arXiv:2304.11157



➢ Composite DE: the LXCDM model

LXCDM ⇒ $\left\{ \begin{array}{l} \bar{\Lambda}\text{CDM} \quad \text{(running } \Lambda) \; ! \\ + \text{ a new component } \mathbf{X} \quad \text{(Phantom Matter)} \end{array} \right.$

Originally introduced long ago: J.Grande,JS, H.Stefancic, gr-qc/0604057 (JCAP)

RVM running

$$\dot{\rho}_\Lambda + \dot{\rho}_X + \alpha_X\,\rho_X\,H = 0\,, \qquad \alpha_X \equiv 3(1+\omega_X)$$

The LXCDM keeps the ``coincidence ratio'' under control:

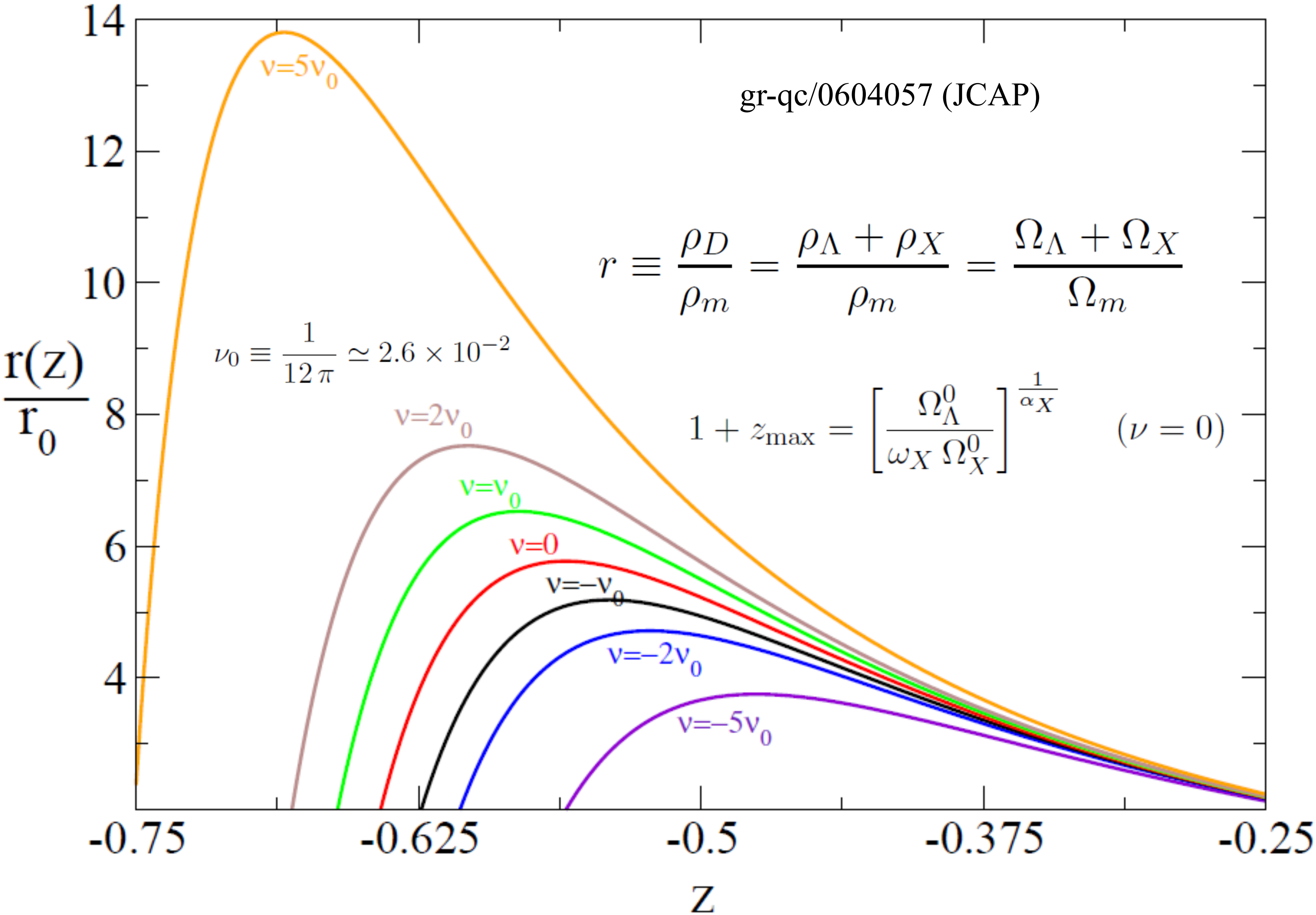
$$r \equiv \frac{\rho_D}{\rho_m} = \frac{\rho_\Lambda + \rho_X}{\rho_m} = \frac{\Omega_\Lambda + \Omega_X}{\Omega_m}$$

The latter remains bounded during the entire cosmic evolution, it may afford a solution to the "cosmic coincidence problem".

In the phantom matter region (see EoS diagram), where

$$\omega_X < -1 \qquad \rho_X < 0$$

the Universe is halted in the future, coincidence problem may be solved and Big Rip avoided despite it being in the phantom domain !



(Joan Solà, KCL 2024)

➢ EoS Diagram: Energy conditions for the cosmic fluids

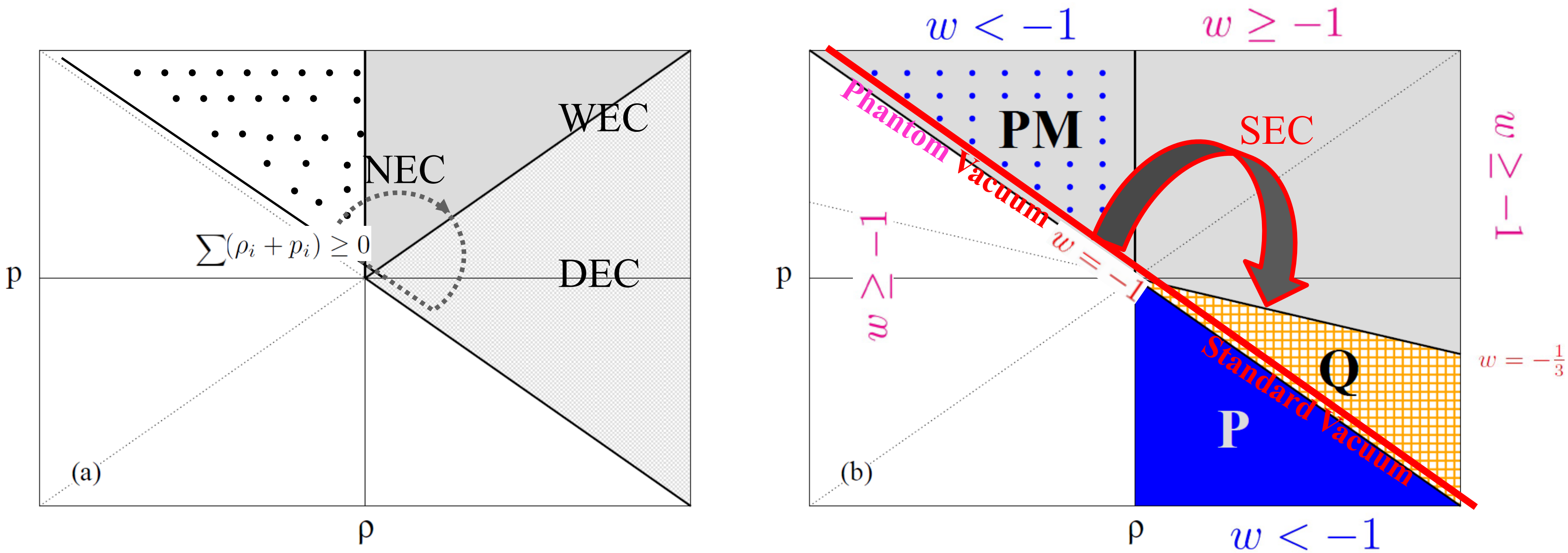


Figure 1: Energy conditions: **(a)** The shaded regions fulfill the Weak Energy Condition (WEC): $\rho \geq 0$ and $\rho + p \geq 0$. The lighter-shaded one also satisfies the Dominant Energy Condition (DEC): $\rho \geq |p|$; **(b)** The region shaded in gray (with or without dots) fulfills the Strong Energy Condition (SEC): $\rho + p \geq 0$ and $\rho + 3p \geq 0$. The quintessence (Q) region ($-1 < \omega_e < -1/3$) is marked cross-hatched, and the *usual* phantom region ($\omega_e \leq -1$ with $\rho > 0$) is indicated by P. The gray-dotted region corresponds to an unusual kind of phantom $\omega_e < -1$ with $\rho < 0$, which we call "Phantom Matter" (PM). Note that PM satisfies the SEC. The cosmon behaves in many cases as PM within the ΛXCDM model.

J. Grande, JS, H. Stefancic LXCDM: gr-qc/0604057

N. Mavromatos and JS arXiv:2105.02659

➢Composite DE: the wXCDM model

``Phantom matter: a challenging solution to the cosmological tensions"

A. Gómez-Valent and J. Solà Peracaula [arXiv:2404.18845] (ApJ 2024, to appear)

Effective formulation of the LXCDM model with two components: X and Y:

The reason why type II RVM with threshold cannot make it for H_0 is because the latter is not forced to increase at low z to match the angular diameter distance to the last scattering surface.

$$\theta_* = \frac{r_d}{D_A(z_*)}$$

$$r_d = \int_{z_d}^{\infty} \frac{c_s(z)\,dz}{H(z)}$$

(sound horizon)

We asume no early DE Changes modifying r_d

$$D_A(z_*) \rightarrow \int_0^{z_*} \frac{dz'}{H(z')}$$

(Ang. Diam. distance)

Assume, instead, compensating changes between X and Y.

Net effect

H_0 higger !!

θ_*: Measured by Planck with 0.01% precision

(Joan Solà, KCL 2024)

Double DE Strategy

Consider a LXCDM variant (wXCDM) with two components, X and Y, aimed at increasing H_0:

We explore a **twofold** wCDM-type of **DE** regime with components X and Y **below** the last scattering surface:

- i) X has (w_X, ρ_X) and acts first during the expansion: $z > z_t$
  Stands for the **PM**-like regime $(w_X \lesssim -1, \rho_X < 0)$

transition redshift z_t

- ii) Y has (w_Y, ρ_Y) and acts second during the expansion: $z < z_t$
  Represents the **quintessence**-like regime $(w_Y \gtrsim -1, \rho_Y > 0)$

Cosmological Data

wXCDM
free parameters
(z_t, w_X, w_Y)

→ Full Planck 2018 CMB polarization, temperature and lensing likelihoods
→ Pantheon+ compilation of SNIa
→ Cosmic chronometers
→ 2D BAO
→ Redshift-space distortions (LSS)

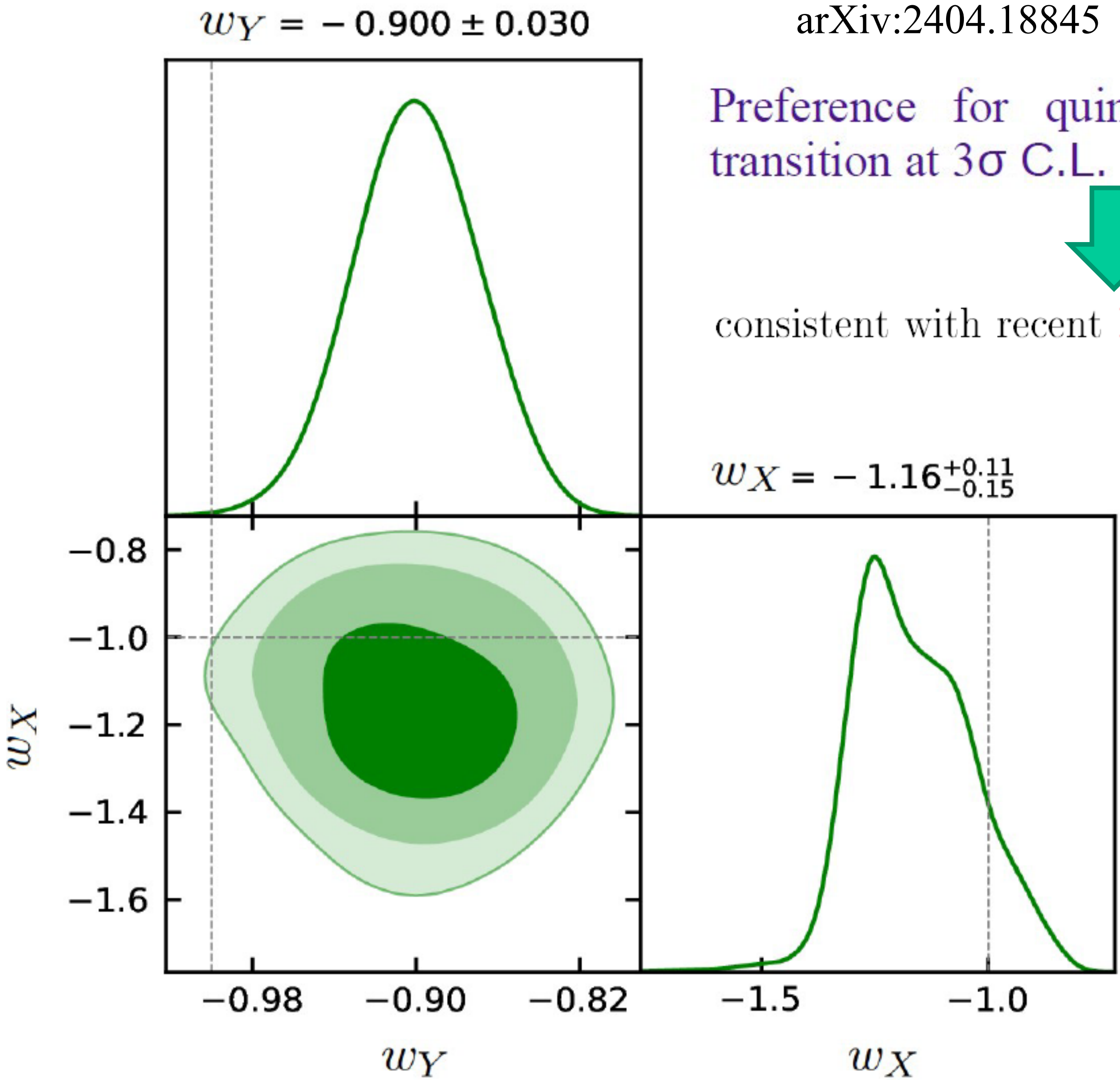
Note: **3D BAO** also tested, but BAO issue is currently in quarantine...

(Joan Solà, KCL 2024)

arXiv:2404.18845 (ApJ 2024)

Preference for quintessence after the transition at 3σ C.L.

consistent with recent **DESI 2024** results (arXiv:2404.03002)



For $(w_X, w_Y) = (-1, -1)$

Λ_sCDM

(Akarsu et al, arXiv:2307.10899)

(Joan Solà, KCL 2024)

Why wXCDM can cure the tensions?

- **H_0-tension**

Because $\Omega_X < 0$, a higher value of H in the quintessence stage is enforced to preserve the angular diameter distance D_A to the last scattering surface, which is essentially fixed from the very precise measurement of θ_*:

$$D_A \sim \int_0^{z_*} \frac{dz'}{H(z')} = \underbrace{\int_0^{z_t} \frac{dz'}{H(z')}}_{\substack{(\textbf{quintessence}\text{ part})\\ (\Omega_Y > 0,\ w_Y \gtrsim -1)}} + \underbrace{\int_{z_t}^{z_*} \frac{dz'}{H(z')}}_{\substack{(\textbf{phantom matter}\text{ part})\\ (\Omega_X < 0,\ w_X \lesssim -1)}}$$

- **Growth (σ_8, S_8)-tension**

i) Equation for density contrast of X (PM) in the $w\mathbf{X}$CDM model reads

$$\delta_m'' + \frac{3}{2a}\underbrace{\left(1 - \Omega_X(a) w_X\right)}_{\text{Friction term decreases}} \delta_m' - \frac{3}{2a^2}\underbrace{\left(1 - \Omega_X(a)\right)}_{\text{Poisson term increases}}\delta_m = 0 \qquad \text{(arXiv:2404.18845)}$$

⇒ structure formation is **enhanced thanks to PM !** **and...**

(Joan Solà, KCL 2024)

PM approach could provide an explanation for the recent **JWST data**

ii) Equation for density contrast of Y (quintessence-like field) in the wXCDM:

$$\delta_m'' + \frac{3}{2a}\left(1 - \Omega_Y(a) w_Y\right)\delta_m' - \frac{3}{2a^2}\left(1 - \Omega_Y(a)\right)\delta_m = 0$$

Friction term increases   Poisson term decreases

$$(\Omega_Y > 0,\ w_Y \gtrsim -1)$$

helps solving the growth problem !!

structure formation is hindered under quintessence-like regime

consistent with recent **DESI 2024** results (arXiv:2404.03002) !!

(Joan Solà, KCL 2024)

Parameter	ΛCDM	wXCDM	Λ_sCDM
ω_b	0.02281 ± 0.00014 (0.02278)	0.02241 ± 0.00013 (0.02260)	$0.02236^{+0.00016}_{-0.00018}$ (0.02232)
$\omega_{\rm dm}$	0.1153 ± 0.0009 (0.1148)	0.1199 ± 0.0010 (0.1196)	$0.1205^{+0.0015}_{-0.0016}$ (0.1216)
$\ln(10^{10}A_s)$	$3.066^{+0.016}_{-0.018}$ (3.080)	3.037 ± 0.014 (3.034)	$3.036^{+0.015}_{-0.016}$ (3.030)
τ	$0.069^{+0.008}_{-0.010}$ (0.076)	0.051 ± 0.008 (0.048)	$0.050^{+0.008}_{-0.009}$ (0.046)
n_s	0.978 ± 0.004 (0.981)	0.967 ± 0.004 (0.969)	$0.966^{+0.004}_{-0.005}$ (0.961)
H_0 [km/s/Mpc]	$69.82^{+0.41}_{-0.44}$ (70.05)	$72.75^{+0.57}_{-0.71}$ (72.36)	$72.24^{+0.99}_{-0.75}$ (73.82)
z_t	–	$1.46^{+0.02}_{-0.01}$ (1.47)	$1.61^{+0.22}_{-0.18}$ (1.47)
w_X	–	$-1.16^{+0.13}_{-0.16}$ (-1.16)	–
w_Y	–	-0.90 ± 0.03 (-0.88)	–
Ω_m^0	0.283 ± 0.005 (0.280)	0.269 ± 0.005 (0.272)	0.267 ± 0.005 (0.264)
M	$-19.372^{+0.011}_{-0.012}$ (-19.362)	$-19.273^{+0.015}_{-0.016}$ (-19.282)	$-19.278^{+0.026}_{-0.020}$ (-19.261)
σ_{12}	0.780 ± 0.007 (0.884)	0.776 ± 0.007 (0.772)	$0.782^{+0.007}_{-0.006}$ (0.784)
$\chi^2_{\rm min}$	4166.76	4107.62	4120.04
ΔDIC	–	57.94	40.16
ΔAIC	–	53.14	44.72

The `fundamental' physics behind the wXCDM

Phenomenologically motivated in the old LXCDM model gr-qc/0604057 [gr-qc],
but theoretically bolstered in: **"Stringy RVM"**

N. Mavromatos and JSP (arXiv: 2105.02659 [hep-th] and 2012.07971 [hep-ph]) (EPJ)

and Nick's talk **!!**

In a nutshell….

The path towards the de Sitter epoch involves a transition…

⇓

➢ from phantom vacuum to true vacuum (dS)

Early Universe "Stringy RVM"

In the early universe, before and during inflation, it is assumed that only fields from the gravitational multiplet of the string exist, which implies that the relevant bosonic part of the effective action pertinent to the dynamics of the inflationary period is given by

$$S_B^{\rm eff} = \int d^4x\sqrt{-g}\Big[-\frac{1}{2\kappa^2}\,R + \frac{1}{2}\,\partial_\mu b\,\partial^\mu b + \sqrt{\frac{2}{3}}\,\frac{\alpha'}{96\,\kappa}\,b(x)\,R_{\mu\nu\rho\sigma}\,\widetilde{R}^{\mu\nu\rho\sigma} + \dots\Big]\,.$$

$$\alpha' = M_s^{-2} \qquad \kappa = \sqrt{8\pi G} = M_{\rm Pl}^{-1} \qquad M_{\rm Pl} \neq M_s \quad \text{in general}$$

It involves the usual Hilbert-Einstein term and the Kalb-Ramond axion field, $b(x)$, which is coupled to the **gravitational Chern-Simons topological density** through the string tension α'. Such topological term when averaged over the de Sitter spacetime produces an effective contribution to the vacuum energy density of the form $\sim H^4$.

During inflation $R_{\mu\nu\rho\sigma}\,\widetilde{R}^{\mu\nu\rho\sigma}$ $\Rightarrow$ triggers H^4 contributions to ρ_Λ !

$$\sim\ H^4_{\rm inflation}$$

!!

Detailed review:

(N.E. Mavromatos and JSP)

arXiv:2012.07971 (EPJ-ST 2021)

See talk by Nick Mavromatos !

N. Mavromatos & JSP
arXiv:2105.02659 [hep-th]
(EPJP 2021)

$$\begin{cases} \rho_{\rm total} = \rho^b + \rho^{\rm gCS} + \rho^{\rm condensate} \\ p_{\rm total} = p^b + p^{\rm gCS} + p^{\rm condensate} \end{cases}$$

$$p^b = +\rho^b \qquad p^{\rm condensate} = -\rho^{\rm condensate} \qquad p^{\rm gCS} = \frac{1}{3}\,\rho^{\rm gCS}$$

$$\rho^b = -\frac{2}{3}\,\rho^{\rm gCS} > 0. \qquad \rho_{\rm Cotton\ tensor} < 0.$$

$$p^b + p^{\rm gCS} = \rho^b + \frac{1}{3}\,\rho^{\rm gCS} = -\frac{1}{3}\,\rho^{\rm gCS} = -(\rho^b + \rho^{\rm gCS}) > 0$$

phantom vacuum

Transitory state !!

But adding the gCS condensates…

$$\boxed{p_{\rm total} = -\rho_{\rm total} < 0 \qquad \rho_{\rm total} = \rho^b + \rho^{\rm gCS} + \rho^{\rm condensate} > 0}$$

Final de Sitter phase

Possible explanation of the JWST data **!!**

True Vacuum **!!**

(Joan Solà, KCL 2024)

Summarized conclusions

- **Dynamical DE**: natural proposal for an **expanding Universe**
- The **RVM** based on a **running Λ** term in interaction with matter or **G** is theoretically **well motivated**
- **Running vacuum models** seem to describe **better** the observations SNIa+BAO+$H(z)$+LSS+CMB **than the ΛCDM**
- Provide a **consistent solution** to the main **tensions**
- **Composite DE** with **with running vacuum** further improve a **possible solution** to the **cosmological tensions**
- These ideas may signal a **micro and macro connection** **between** the **LSS** of the Universe and the **quantum phenomena** in the **microcosmos**

(Joan Solà, KCL 2024)