Applications of Field Theory to Hermitian and Non-Hermitian Systems

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Book of Abstracts

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Resurgence and Non-perturbative Physics

Resurgent asymptotics unifies perturbative and non-perturbative expansions into a single object, a trans-series, thereby revealing a surprisingly deep network of relations between perturbative and non-perturbative physics. This is a general mathematical formalism with roots in work of Stokes, Dingle and Ecalle, and which has many applications in physics. I will describe the basic ideas underlying resurgence, and illustrate with examples from non-linear differential equations, quantum mechanical spectral problems, and partition functions in matrix models and quantum field theory.

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Exact WKB analysis for PT symmetric quantum mechanics: Study of the Ai-Bender-Sarkar conjecture

We consider exact WKB analysis to a calPT symmetric quantum mechanics defined by the potential, $V(x) = \omega^2 x^2 + gx^2(ix)^{\varepsilon=2}$ with $\omega \in \mathbb{R}_{\geq 0}, g \in \mathbb{R}_{> 0}$. We in particular aim to verify a conjecture proposed by Ai-Bender-Sarkar (ABS), that pertains to a relation between D-dimensional calPT-symmetric theories and analytic continuation (AC) of Hermitian theories concerning the energy spectrum or Euclidean partition function. For the purpose, we construct energy quantization conditions by exact WKB analysis and write down their transseries solution by solving the conditions. By performing alien calculus to the energy solutions, we verify validity of the ABS conjecture and seek a possibility of its alternative form by Borel resummation theory if it is violated. Our results claim that the validity of the ABS conjecture drastically changes depending on whether $\omega>0$ or $\omega=0$: If $\omega>0$, then the ABS conjecture is violated when exceeding the semi-classical level of the first non-perturbative order, but its alternative form is constructable by Borel resummation theory. If $\omega=0$, then, these energies are independent solutions, and no alternative form of the ABS conjecture can be reformulated by Borel resummation theory.

References

- * W.-Y. Ai, C. M. Bender, and S. Sarkar, "PT-symmetric - $g\phi 4$ theory," Phys. Rev. D 106 no. 12, (2022) 125016, arXiv:2209.07897 [hep-th].
- * S. Kamata, "Exact WKB analysis for PT-symmetric quantum mechanics: Study of the Ai-Bender-Sarkar conjecture," Phys. Rev. D 109 no. 8, (2024) 085023, arXiv:2401.00574 [hep-th].

3

Complex phases in quantum mechanics

Schroedinger's equation is a local differential equation and boundary conditions are required to determine the solution uniquely. Depending on the choice of boundary conditions, a given Hamiltonian may describe several different physically observable phases, each exhibiting its own characteristic global symmetry.

Harnessing the power of reflectionless scattering modes in atomic and molecular systems

We demonstrate that reflectionless scattering mode (RSM) theory can be used to reveal the existence of long-sought-after quantum mechanical parity-time reversal (PT) symmetry behavior in standard ultracold-atom scattering experiments. In addition, we demonstrate the applicability of RSM theory to ultracold chemical reactions, a discovery which opens the door to the development of powerful quantum technologies.

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Large N insights for the PT-symmetric Higgs at finite temperature

In the Standard Model of Physics, the Higgs field is in a broken phase at low temperature, and in a symmetric phase at high temperature, with a second order transition in between. In this talk, I consider a systematic expansion in N»1 scalar field components to access the non-perturbative properties of the PT-symmetric field theory at low and high temperature. The results are qualitatively different from the Standard Model and may have implications for our understanding of how baryons have formed in our universe.

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The Structure of Perturbative Renormalization Group Functions

RG functions are used in QFT to evolve theories between energy scales and are frequently used in both phenomenology and theory. As we push to ever higher loop-order beta functions in generic gauge-Yukawa-quartic theories, we have observed and utilized an underlying structure to obtain results at 4-3-2 loop order: the beta functions satisfy what is known as the Weyl Consistency Conditions. Going to these high orders in perturbation theory, we also found that the usual MS-bar definition of the beta functions is ambiguous and can even exhibit poles in the dimensional regulator. I will clarify how this is linked to flavor symmetries and how we can recover unambiguous, finite functions.

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Resurgence and Modularity: a large-N story

Breakdown of the Meissner effect at the zero exceptional point in non-Hermitian two-band BCS model

The spontaneous symmetry breaking of a continuous symmetry in complex field theory at the exceptional point of the parameter space is known to exhibit interesting phenomena, such as the breakdown of a Higgs mechanism. In this work, we derive the complex Ginzburg-Landau model from a non-Hermitian two-band BCS model via path integral and investigate its spontaneous symmetry breaking. We find that analog to the Higgs mechanism, the Meissner effect of the complex Ginzburg-Landau model also breaks down at the exceptional point while the gap parameters stay finite.

9

High-density QCD: a paradigm for PT symmetry in field theory

The physics of QCD at nonzero density and temperature is rich, accessible via theory and experiment, and important for several areas of physics. The path integral is complex at nonzero density, and difficult to evaluate: there is a sign problem which reflects an underlying PT symmetry. We show that novel new phases associated with PT symmetry breaking, including liquid-like phases and inhomogeneous phases, are likely in finite-density QCD and related models. A diverse set of analytic and computational methods give a consistent, if incomplete, picture of possible phase structure and other features. We propose experimental signatures which may give a direct indication of PT symmetry breaking at the next generation of heavy-ion experiments.

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Gradient Properties of RG Flows

General properties of the renormalisation group (RG) are of immense theoretical interest, as they have implications for the evolution of physical systems from high to low energies. In a perturbative setting, RG flows are determined by a vector field, the beta function, that can be computed in a loop expansion. In this talk, we will discuss the gradient property of the RG up to six loops in multi-scalar models in d=4 and d=4- ϵ dimensions. After elucidating a variety of subtleties, we will derive and discuss highly nontrivial constraints that need to be satisfied for the RG flow to be gradient.

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Integrability in Non-Hermitian Physics and Field Theories

This talk will review recent developments linking integrable models, soliton theory, and field theories. Through several examples, it will illustrate how the influence of non-Hermitian physics and PT symmetry leads to new theories, integrable models, and soliton solutions. These advances have interesting applications across various fields, including classical field theories, many-particle models, and black hole physics.

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New physics from pseudo-Hermiticity

Physical frameworks are often at their most interesting when we stress-test their underlying assumptions. This is no less true of quantum field theory, and no less true of relaxing the constraint of Hermiticity of the Hamiltonian. I will review progress in pseudo-Hermitian quantum field theory and argue that non-Hermiticity represents an exciting paradigm in which to attack long-standing problems in modern physics.

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Complex Metastable condensates in Chern-Simons Quantum Gravity

In the context of string-inspired Chern Simons (CS) gravity, involving axion-like fields interacting with gravitational CS anomalous terms, I discuss the formation of condensates of the latter due to quantum graviton modes of chiral gravitational wave type, that could exist in the primordial universe. The condensates are complex, with their imaginary parts being associated with instabilities that determine the lifetime of the respective vacuum. I will argue that such condensates can lead to linear axion monodromy potentials, which in turn imply the entrance of the Universe into an inflationary phase, the duration of which is determined by the magnitude of the imaginary part of the gravitational CS condensate. I discuss some elementary phenomenology of such cosmologies and show agreement with the data.

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Running vacuum approach to renormalizing the vacuum energy in cosmological spacetime and the cosmological constant problem

In the last few years the idea that the vacuum energy density (VED) is a running' quantity with the cosmological evolution has become phenomenologically advantageous, as it helps in alleviating the current cosmological tensions afflicting the Λ CDM. The theoretical studies backing up this approach go under the name of "running vacuum model" (RVM). Using this framework, based on quantum field theory (QFT) in curved spacetime, one can show that the properly renormalized VED in FLRW spacetime is free from fine-tuning troubles since the vacuum dynamics proves to be a power series

of the Hubble rate H and of its time derivatives. The calculation is performed using an off-shell version of the adiabatic renormalization procedure, which leads to a smooth cosmic evolution of the VED with H, ρ_{-} vac(H). As a result the "cosmological constant" Λ appears here as the nearly sustained value of $8\pi G(H)\rho_{-}$ vac(H) around (any) given epoch H, where G(H) is the gravitational coupling at the corresponding epoch, which runs very mildly (logarithmically) with H. The VED evolution between points H and H_0 of the cosmic expansion history reads $\delta\rho_{-}$ vac(H)~\nu_eff m_Pl^2(H^2-H_0^2) (where |\nu_eff|\ll1) and m_Pl is the Planck mass. The effective coefficient \nu_eff receives contributions from all the quantized matter fields and can be explicitly computed in QFT. Remarkably, there are also higher powers of H which can trigger inflation in the early universe within a new inflationary paradigm calledRVM-inflation', which does not make use of (ad hoc) inflaton fields. Finally, the equation of state (EoS) of the running vacuum also receives quantum corrections from bosons and fermion fields, shifting its value from -1. The striking consequence is that the EoS of the quantum vacuum may nowadays appear as quintessence, which is consistent with the recent DESI results.

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PT phases and Dark Energy

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We consider a string inspired effective axion anomalously coupled to Abelian gauge fields and gravity via Chern-Simons couplings. By considering the renormalisation group flows in the flat space limit it is observed that a Hermitian parity symmetric phase of the theory can flow into a non-Hermitian parity-time symmetric phase.

This behavior has implications for Chern-Simons gravity. The repulsive nature of gravity, usually attributed to the existence of a positive cosmological constant, is reinterpreted at large scales as a flow from a Hermitian (attractive) gravitational theory, to a

cPT-symmetric (repulsive) gravity in the infrared. The discussion here is presented in the context of a Chern-Simons gravitational theory but it may be valid more generally in gravity with torsion. The validity of such a scenario in realistic theories might alleviate the need for de Sitter phases in the current epoch of the cosmological evolution, thus avoiding their associated conceptual and technical complications.

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Nucleation seeds and vacuum bubbles

In nature, most examples of bubbe nucleation occur around nucleation seeds. Clouds are an example, were particles of dust are needed to start off the process of making rain. The theory of nucleation around seeds in phase transitions or in false vacuum decay is a rich one, with interesting outcomes. The theory has applications to laboratory vacuum decay experiments, stimulated decay from collisions or black holes, and may shed some light on environmental questions such as cloud formation.

Dynamics of thermal bubble nucleation

First-order phase transitions proceed via bubble nucleation. This is true regardless of whether the transition is happening in your kettle or on a cosmological scale in the very early universe. The rate of bubble nucleation determines many gross features of cosmological phase transitions, including the peak frequency of the resulting gravitational wave signal. At high temperatures, the rate factorises into a product of statistical and dynamical parts, the latter of which is much less well understood. I will discuss recent works suggesting that our best estimates for the dynamical part are far off the mark, and avenues of progress.

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Building a Quantum Black Hole Simulator

Horizons can occur in a wide range of physical situations, many of which we can construct in the lab, leading to the field of Analog Gravity. Most gravity simulators observe features, like super-radiance, that are analysed as a continuum effect in gravity, whereas many interesting "beyond GR" features theorise about the impact of quantised aspects of the black hole. In this talk, I will discuss recent experimental work on a liquid helium giant vortex that naturally has quantisation, and how we hope to build a quantised analog black hole that can start to explore "black hole" phenomena in a much broader context.

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Towards the Asymptotically Safe Landscape

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A consistent theory of quantum gravity needs to be able to incorporate the matter degrees of freedom present in our Universe, for example, the matter fields and symmetries of the Standard Model. In asymptotically safe quantum gravity, the key questions are for which matter content the interacting ultraviolet fixed point exists, and which low-energy matter interactions are connected to the ultraviolet fixed point. I will give a comprehensive overview of the status of these questions, in particular, highlighting recent results towards the ultraviolet completion of the Standard Model with asymptotically safe gravity and implications for dark matter models.

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CP conservation in the strong interactions

There is no empirical evidence for CP conservation in the strong interactions. As there generally is a renormalizable, parity-odd coupling between the field strength and its dual, this requires an explanation from theory.

I will therefore first review what interactions are present when constructing an effective theory for hadrons from QCD. But I will also point out, that from such considerations alone, it cannot be decided whether the effective interactions (that, e.g., give mass to eta-prime) are misaligned (CP violation) or aligned (no CP violation) with the quark mass phase.

To see whether or not there is a material effect of the parity-odd operator in QCD requires therefore an understanding of how field configurations from different topological sectors contribute to the path integral or, in canonical quantization, whether topology implies different ground states that are in general not parity eigenstates. To that end, I will review the pertinent homeomorphisms between the SU(2) subgroups of the strong interactions and the boundaries of spacetime or spatial hypersurfaces.

As for the Euclidean path integral approach, I will note that pure gauge configurations on the boundary only follow when the latter is placed at infinity. Picard-Lefschetz theory then implies that steepest-descent integration contours cover all field configurations within a topological sector that one can find in the infinite spacetime volume. Consequently, the limit of infinite spacetime volume must be taken before summing over sectors, and it turns out that parity violation then vanishes. Commuting these limits, as tacitly done in standard approaches, corresponds to a singular deformation of the original Cauchy contour, falsely suggesting parity-violating results.

Regarding canonical quantization, I will note that the usually considered theta-vacua are not properly normalizable, which is at odds with the probability interpretation from the axioms of quantum mechanics. The root of this problem is the summation over gauge-redundant configurations in the orthonormality relations among theta-vacua. Imposing that (in temporal gauge) the wave functionals and Hilbert-space operators are well-defined when the inner product covers each physical field configuration one time and one time only, I recover that the consistent states satisfy Gauß' law and are moreover eigenstates of parity.

References: 2001.07152 [hep-th] 2403.00747 [hep-th] 2404.16026 [hep-ph]

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Taming large logarithms and the scale invariant instanton

Computing the false vacuum decay rate for the classically scale invariant scalar theory is particularly relevant for assessing the stability of the Standard Model. This proves particularly challenging as the classical scale invariance introduces an additional non-normisable zero-mode that cannot be treated through standard methods. We show how the Green's function method can be employed to compute the effect of quantum fluctuations, and how it can be improved via renormalisation group to keep under control the large logarithms that haunt the effective action. Finally, a self-consistent evaluation procedure allows us to identify the optimal renormalisation scale that minimises the large logarithms.

QFTs in the early universe

Some seemingly well-behaved quantum field theories in Minkowski space may develop pathological behaviors in certain gravitational backgrounds. This seems to be especially true of QFTs of higher spin. For example, the early universe may be used as a probe such theories. Perhaps there is a swampland of Minkowskian QFTs that cannot be lifted to certain cosmological backgrounds.

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Gauge Fields in the Early Universe and their remnants in the Sky

Gauge fields and fermions are the building blocks of particle physics models. I will summarize some progress in understanding their production and contribution to the physics of cosmic inflation. In particular, I will focus on the case of non-Abelian gauge fields in axion inflation and discuss their rich phenomenology as well as their observable signatures on CMB and GW background. In these models Parity and CP are spontaneously broken in inflation which makes them natural settings for matter asymmetry. Finally, I will present a possible realization of this setup based on embedding axion-inflation in Left-Right symmetric extensions of the SM. In this model, a pure quantum effect, i.e. the chiral anomaly of the SU(2)_R gauge field, provides a common origin for baryogenesis and Right-handed neutrino production.

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Ultraviolet finite resummation of perturbative quantum gravity

If the metric is chosen to depend exponentially on the conformal factor, and if one works in a gauge where the conformal factor has the wrong sign propagator, perturbative quantum gravity corrections can be partially resummed into a series of terms each of which is ultraviolet finite. These new terms however are not perturbative in some small parameter, and are not individually BRST invariant, or background diffeomorphism invariant. With appropriate parametrisation, the finiteness property holds true also for a full phenomenologically relevant theory of quantum gravity coupled to (beyond the standard model) matter fields, provided massive tadpole corrections are set to zero by a trivial renormalisation.

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Bosonic dark-matter: How to mix Particle Physics, Cosmology and Cold Atoms

We present from first principles, under the Schwinger-Keldysh path integral formalism, equations for bosonic, non-relativistic and self-interacting dark matter which can include both a condensed, low momentum "fuzzy" component and one with higher momenta that may be approximated as a collection of particles. The equations can describe both CDM and Fuzzy Dark Matter in a unified way. We show that self-interaction could play an important role in the dynamics of the two components, for example in the initial generation and growth of the condensate through the presence of stochastic noises and dissipative terms.

We also present the linear regime of this mixed model and we show how the existence of these two components and the interaction between the condensate and particles could bypass Lyman alpha forest bounds for the typical Fuzzy Dark Matter.

Why quantum gravity made me fall in love with domain walls

Domain walls are a defect that arises when a vacuum manifold is discontinuous. They are often regarded as a problem - literally the "domain wall problem" - but if you can get rid of them, they could be an interesting source of gravitational waves. If the domain walls result from a breaking a global symmetry, the most common way of doing so always struck me as contrived - having an unnaturally small bias term. Quantum gravity is expected to violate all global symmetries - but the process is generally a non-perturbative process like an instanton/wormhole. This means the effective scale of explicit global symmetry breaking is many orders of magnitude above the Planck scale. This makes gravitational waves from domain walls natural. Moreover, if dark matter is protected by a global symmetry which is violated by the same mechanism, one can acquire an independent measurement of a qualitative feature of quantum gravity. Finally, the domain walls themselves can catalyze primordial black hole production, making quantum gravity the indirect source of dark matter.

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Finite-volume effects in QFT and preventing singularities in gravity

Finite-volume effects in QFT allow the violation of the Null Energy Condition, without modified gravity or exotic matter. After reviewing few known features arising from the Casimir effect, I will explain that tunnelling between degenerate vacua also induces NEC violation. I will then discuss resulting mechanisms which can help avoid singularities in gravity.

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Axions, String Theory, and the Swampland

Axions provide one of the most promising routes for bridging the gap between string theory and experiment. In this talk, we will focus on several observed features of the string landscape (e.g. the absence of global symmetries and the Weak Gravity Conjecture) and their implications for axion physics (e.g. the axion quality problem and axion inflation). I will also introduce the notion of an "axion bitower," which can be understood as a tower of towers of particles coupled to an axion field. We will see that such bitowers are ubiquitous in string theory, and they may have important observational effects.

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Spatially oscillating correlations in strongly-interacting four-fermion model with generalized PT-symmetry

In this talk, I present the phase diagram of a (2+1)-dimensional four-fermion model at finite temperature and chemical potential, which is invariant under a chiral symmetry transformation as well as a generalized PT-symmetry transformation. Besides the ordinary phase with chiral symmetry breaking, a regime is observed where mesonic two-point correlation functions feature spatial oscillations, but are still exponentially damped. The role of the P-symmetry and the T-symmetry is played by charge conjugation and complex conjugation, respectively. This generalized PT-symmetry is also present in QCD at non-vanishing baryon chemical potential, for which the observed regime, recently termed a quantum pion liquid, could be a relevant scenario. The oscillatory behavior is generated by mixing between scalar and vector condensates. Moreover, I find that inhomogeneous condensates

are disfavored against homogeneous ones, akin to previous findings. If possible in the time available, I will also present evidence that the quantum pion liquid (without translational symmstry breaking) is favored over an inhomogeneous phase (with translational symmetry breaking) when including bosonic quantum fluctuations in lattice Monte Carlo simulations.

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New Hermitian and Non-Hermitian Toda field theories and Calogero models from infinite symmetries

Many integrable theories can be formulated universally in terms of Lie algebraic root systems. Examples are conformal field theories that can be expressed in terms of the simple roots of finite Lie algebras, massive field theories that can be written in terms of simple roots of the affine Kac-Moody algebras and Calogero (Moser-Sutherland) models that require the entire root system of the finite Lie algebras in their formulation. Here we discuss extensions to similar systems based on hyperbolic and Lorentzian Kac-Moody algebras. We discuss various properties of these models, including their integrability and invariance with regard to infinite Weyl groups of affine, hyperbolic and Lorentzian type. Some of these model are Hermitian whereas others are Non-Hermitian.

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Taming large logarithms and the scale invariant instanton

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Computing the false vacuum decay rate for the classically scale invariant scalar theory is particularly relevant for assessing the stability of the Standard Model.

This proves particularly challenging as the classical scale invariance introduces an additional non normalisable zero-mode that cannot be treated through standard methods.

We show how the Green's function method can be employed to compute the effect of quantum fluctuations, and how it can be improved via renormalisation group to keep under control the large logarithms that haunt the effective action.

Finally, a self-consistent evaluation procedure allows us to identify the optimal renormalisation scale that minimises the large logarithms.

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Perturbative and non-perturbative aspects of flavour oscillations

I will review recent work on the quantization for mixed fields, with particular reference to the case of neutrinos. I will argue on the physical relevance of flavor states, which are associated to a condensate structure of the flavor vacuum, induced by a Bogoliubov transformation. Phenomenological consequences of this result will be discussed.

Based on the analogy of oscillating neutrinos with unstable particles, I will show that finite time perturbation theory (approximately) reproduces the oscillation formula obtained by means of the non-perturbative flavor Fock space approach. Finally, will report recent results on the quantum correlations associated to neutrino oscillations, which make them a possible tool for quantum information tasks.

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Breakdown of the Meissner effect at the zero exceptional point in non-Hermitian two-band BCS model

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The spontaneous symmetry breaking of a continuous symmetry in complex field theory at the exceptional point of the parameter space is known to exhibit interesting phenomena, such as the breakdown of a Higgs mechanism. In this work, we derive the complex Ginzburg-Landau model from a non-Hermitian two-band BCS model via path integral and investigate its spontaneous symmetry breaking. We find that analog to the Higgs mechanism, the Meissner effect of the complex Ginzburg-Landau model also breaks down at the exceptional point while the gap parameters stay finite.

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Exploring the landscape of fermionic theories at large N

Theories of self-interacting fermions play an important role in particle and condensed matter physics, covering effective descriptions of the strong nuclear force, the critical behaviour of Dirac materials such as graphene, and more. In this talk, I will discuss functional RG flows for fermionic systems in the large-N limit. I provide conditions under which these flows become exact, and exactly solvable, and provide the most general form of their quantum effective actions. I exemplify the method for fermionic theories with scalar, pseudo-scalar, vector, axial-vector, and derivative interactions in various dimensions. Results include phenomena such as chiral symmetry breaking and dynamical generation of fermion mass, interacting fixed points, universal scaling dimensions of operators, conformal manifolds with exactly marginal interactions, the spontaneous breaking of scale symmetry, and the appearance of a massless dilaton. Exact dualities with bosonised versions of theories are also discussed.

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