towards understanding non-abelian axion inflation

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introduction and overview

case for (thermal) axion inflation[1]

thanks to a shift symmetry, the potential remains "flat" — even in the presence of thermal corrections, evading known obstacles[2]

yet the friction can be large, offerring for a mechanism to remove energy from the inflaton and heat up the standard model plasma

$$V_0(\varphi) \simeq m^2 f_a^2 \left[1 - \cos\left(\frac{\varphi}{f_a}\right) \right]$$

$$f_a \approx 1.25\, m_{\text{pl}} \,, \quad m \approx 1.09 \times 10^{-6}\, m_{\text{pl}}$$

[within 2σ of planck data]

[1] K. Freese, J.A. Frieman and A.V. Olinto, *Natural inflation with pseudo Nambu-Goldstone bosons*, PRL 65 (1990) 3233; ...

[2] J. Yokoyama and A.D. Linde, *Is warm inflation possible?*, hep-ph/9809409

difference between abelian and non-abelian cases

$$\mathcal{L} \supset \frac{1}{2}\partial^{\mu}\varphi\,\partial_{\mu}\varphi - V_0(\varphi) - \frac{\varphi\,\chi}{f_a}\,, \quad \chi \equiv \frac{\alpha\,\epsilon^{\mu\nu\rho\sigma}F^c_{\mu\nu}F^c_{\rho\sigma}}{16\pi}$$

abelian axion inflation displays remarkable tachyonic instability...[3]

$\Rightarrow$ but backreaction effects are large & difficult to control?

in the non-abelian case, gauge fields self-interact strongly...

$\Rightarrow$ perhaps so much so that gauge fields thermalize? (not φ)

$\Rightarrow$ memory of their history is lost (no mode functions)

$\Rightarrow$ life could be simpler again?

[3] M.M. Anber and L. Sorbo, *Naturally inflating on steep potentials through electromagnetic dissipation*, 0908.4089

remarks on thermalization

"proving" thermalization theoretically is notoriously difficult

for heavy ion collisions, $\gg 10^3$ papers, from preheating-type simulations[4] to advanced perturbative computations[5] to ads/cft

now even heavy quarks ($m_c > 5T_{\max}$) suggested to equilibrate[6]

all this is supported by that, empiricially, hydrodynamics works

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[4] e.g. D. Bödeker, K. Rummukainen, *QCD plasma instability and thermalisation at heavy ion collisions*, 0711.1963

[5] e.g. Y. Fu, J. Ghiglieri, S. Iqbal, A. Kurkela, *Thermalization of non-Abelian gauge theories at next-to-leading order*, 2110.01540

[6] e.g. F. Capellino *et al*, *Hydrodynamization of charm quarks ...*, 2312.10125

our philosophy

introduce a temperature-like parameter T, but keep it dynamical, viewing it as a parametrization of phase space distributions

(technically: we assume that the energy released from φ is distributed ergodically within the gauge sector)

if $T \ll H$, where H is the Hubble rate, T plays no practical role for φ [but it serves as a "seed" for the subsequent reheating]

our temperature is "classical", and often much below the gibbons-hawking temperature $H/(2\pi)$ of de sitter spacetime

non-abelian theory has a dimensionful parameter, $\Lambda_{\rm IR}$

interaction between the inflaton and gauge fields:

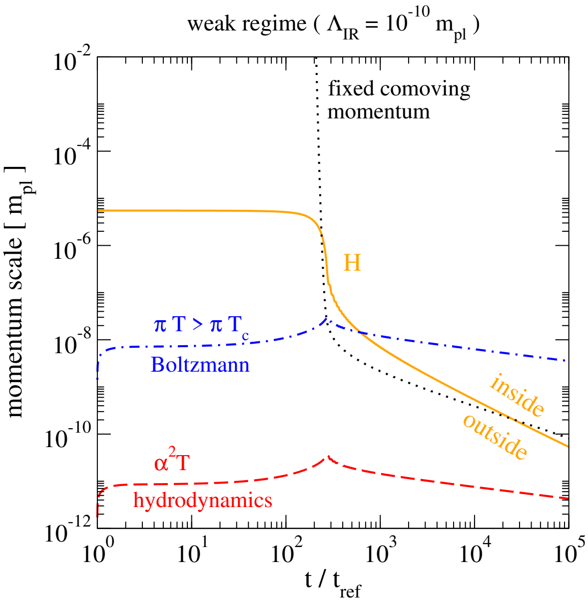
$$\mathcal{L} \supset \frac{1}{2}\partial^\mu\varphi\,\partial_\mu\varphi - V_0(\varphi) - \frac{\varphi\,\chi}{f_a}\,, \quad \chi \equiv \frac{\alpha\,\epsilon^{\mu\nu\rho\sigma}F^c_{\mu\nu}F^c_{\rho\sigma}}{16\pi}$$

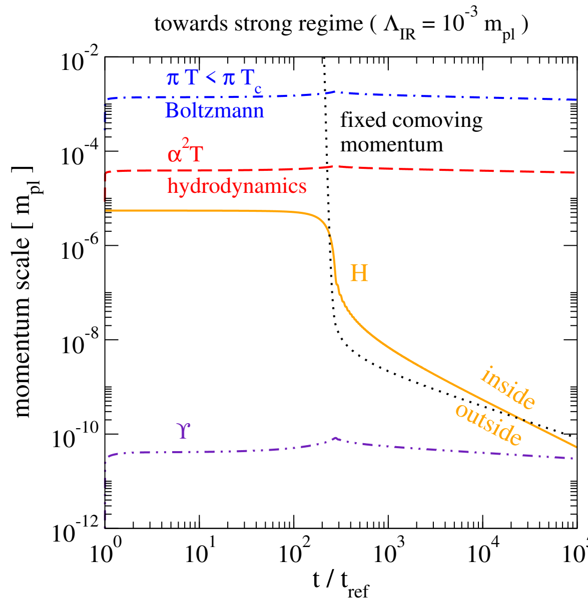
gauge field self-interactions are parametrized by $\alpha = \frac{g^2}{4\pi} \Leftrightarrow \Lambda_{\rm IR}$;
for energy scale $\omega \gg \Lambda_{\rm IR}$ or temperature $2\pi T \gg \Lambda_{\rm IR}$,

$$\alpha \simeq \frac{6\pi}{11N_{\rm c}}\ln^{-1}\left[\frac{\sqrt{\omega^2+(2\pi T)^2}}{\Lambda_{\rm IR}}\right]$$

confinement sets in if $\max\{\omega, 2\pi T\} \ll 2\pi\Lambda_{\rm IR}$

prototypical scenarios[7] for a small and large $\Lambda_{\rm IR}$





deep in the weak regime, T plays no role during inflation

at large $\Lambda_{\rm IR}$, we can have $\pi T \gg m$, but $\Upsilon \ll H$

[7] H. Kolesova, ML, S. Procacci, *Maximal temperature of ... dark sectors*, 2303.17973

basic equations and the friction Υ

equations for the background solution ($\varphi = \bar{\varphi} + \delta\varphi$)

$$\ddot{\bar{\varphi}} + (3H + \Upsilon)\dot{\bar{\varphi}} + \partial_{\varphi} V \simeq 0 \, ,$$

$$\dot{e}_r + 3H\big(e_r + p_r - T\partial_T V\big) - T(\partial_T \dot{V}) \simeq \Upsilon \dot{\bar{\varphi}}^2$$

consistent with overall energy conservation $e + 3H(e + p) = 0$, where $e = e_r + \dot{\bar{\varphi}}^2/2 + V - T\partial_T V$ and $p = p_r + \dot{\bar{\varphi}}^2/2 - V$

the friction Υ transfers energy from $\dot{\bar{\varphi}}$ to “radiation” (e_r, p_r)

dispersive representation of Υ

Υ originates from a coupling between φ and gauge fields

$$\mathcal{L} \supset -\frac{\varphi\,\chi}{f_a}\,, \quad \chi \equiv \frac{\alpha\,\epsilon^{\mu\nu\rho\sigma} F^c_{\mu\nu} F^c_{\rho\sigma}}{16\pi}$$

through linear response theory ($\varphi \leftrightarrow \chi$), the influence of χ on φ can be related[8,9] to the "spectral function" of χ alone,

$$\Upsilon(\omega) = \frac{1}{f_a^2}\frac{\rho(\omega)}{\omega}$$

this incorporates both vacuum decays $\varphi \to gg$ (for $\omega \gg 2\pi T$) and plasma scatterings $\varphi + X \to Y$ (for $\omega \ll 2\pi T$)

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[8] L.D. McLerran, E. Mottola and M.E. Shaposhnikov, *Sphalerons and axion dynamics in high-temperature QCD*, PRD 43 (1991) 2027

[9] ML and S. Procacci, ... *inflation with complete medium response*, 2102.09913

2-point correlation functions

$$\rho(\omega) \equiv \int_{-\infty}^{\infty} \mathrm{d}t \, e^{i\omega t} \int_{\mathbf{x}} \Big\langle \frac{1}{2} \big[\chi(t,\mathbf{x}) \, , \, \chi(0,\mathbf{y}) \big] \Big\rangle_T$$

$$C_{\mathrm{S}}(\omega) \equiv \int_{-\infty}^{\infty} \mathrm{d}t \, e^{i\omega t} \int_{\mathbf{x}} \Big\langle \frac{1}{2} \big\{ \chi(t,\mathbf{x}) \, , \, \chi(0,\mathbf{y}) \big\} \Big\rangle_T$$

the two time orderings are related to each other,

$$C_{\mathrm{S}}(\omega) \overset{|\omega| \ll T}{=} \frac{2T \, \rho(\omega)}{\omega}$$

the symmetric ordering has formally a classical limit,

$$C_{\mathrm{S}}^{(\mathrm{cl})}(\omega) \equiv \lim_{\hbar \to 0} C_{\mathrm{S}}(\omega)$$

therefore $\Upsilon(\omega)$ can be estimated via classical simulations

generate gauge configurations at $t = 0$ with boltzmann weight

$$Z^{(\text{cl})} = \int \mathcal{D}U_i \, \mathcal{D}\mathcal{E}_i \, \delta(G) \exp\left\{ -\frac{1}{g^2 T a} \sum_{\mathbf{x}} \left[\sum_{i,j} \mathrm{Tr}(\mathbb{1} - P_{ij}) + \sum_i \mathrm{Tr}(\mathcal{E}_i^2) \right] \right\}$$

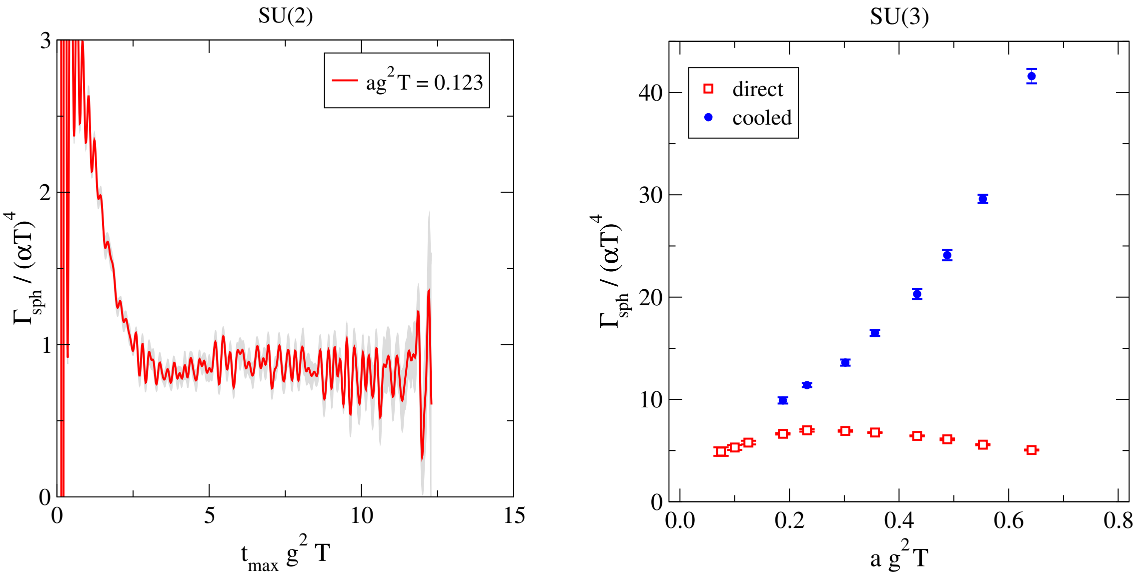
evolve fields to $t > 0$ with equations of motion

$$\begin{aligned} a\,\partial_t U_i(x) &= i\mathcal{E}_i(x) U_i(x) \;, \\ a\,\partial_t \mathcal{E}_i^b(x) &= 2 \sum_{j \neq i} \mathrm{Im}\,\mathrm{Tr}\left\{ T^b \left[P_{ji}(x) + P_{-ji}(x) \right] \right\} \end{aligned}$$

then measure the 2-point correlator of χ, and fourier-transform

example: strong sphaleron rate[10,11]

$$f_a^2\, 2T\Upsilon(0) \;=\; \Gamma_{\mathsf{sph}} \;=\; \lim_{\omega\to 0} C_{\mathsf{S}}(\omega) \;=\; \lim_{t_{\mathsf{max}}\to\infty} \int_{-t_{\mathsf{max}}}^{t_{\mathsf{max}}} \mathrm{d}t\, C_{\mathsf{S}}(t)$$



$\Rightarrow$ the limit $ag^2T \to 0$ is universal (hopefully?)

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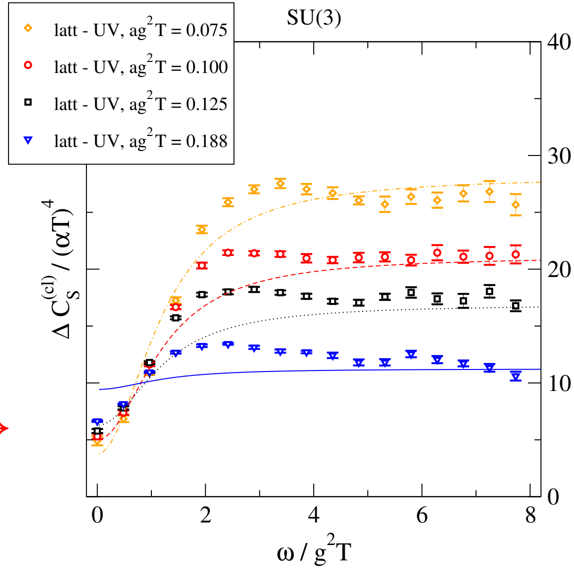
10 "direct method" and extension to $\omega \geq 0$: ML, L. Niemi, S. Procacci, K. Rummukainen, *Shape of the hot topological charge density spectral function*, 2209.13804

11 "cooled": G.D. Moore, M. Tassler, *The sphaleron rate in SU(N) ...*, 1011.1167

the full frequency dependence



sphaleron rate →

⇒ we observe a "transport dip" instead of a "transport peak"

afterwards, lattice needs to be "matched" onto continuum

(i) for IR regime, $\omega < g^2 T$, rescale observables by debye mass squared, to account for interactions between IR and UV modes

(ii) for UV asymptotics, $\omega \sim 1/a$, subtract the lattice result and add the full continuum result, both within perturbation theory

$$C_{\mathsf{S}}\Big|_{\rm cont} \simeq \frac{m^2_{\rm D,latt}}{m^2_{\rm D,cont}} \underbrace{\Big[C_{\mathsf{S}}\Big|_{\rm latt} - C_{\mathsf{S,UV}}\Big|_{\rm latt} \Big]}_{\Delta C_{\mathsf{S}}^{\rm (cl)}} + C_{\mathsf{S,UV}}\Big|_{\rm cont}$$

this leads to a reconstructed continuum expression

$$\Upsilon(\omega) \simeq \frac{d_A \alpha^2}{f_a^2}$$

[$d_A \equiv N_c^2 - 1$, $\kappa \simeq 1.5$, $c_{IR} \simeq 106$, $c_M \simeq 5.1$]

$$\times \left\{ \underbrace{\kappa \, (\alpha N_c T)^3}_{\text{sphaleron rate}} \frac{1 + \frac{\omega^2}{(c_{IR} \alpha^2 N_c^2 T)^2}}{1 + \frac{\omega^2}{(c_M \alpha N_c T)^2}} + \underbrace{\left[1 + 2 n_B \left(\frac{\omega}{2} \right) \right] \frac{\pi \omega^3}{(4\pi)^4}}_{\varphi \to gg} \right\}$$

here n_B is the bose distribution for gauge boson bose enhancement

then need to fix ω — not obvious, since want an equation in t $\Rightarrow$ forward-backward fourier transforms?

in practice, we have considered $0 \leq \omega \leq m$, with $\omega = m$ being the proven choice for reheating

application: gravitational waves

overview

• contrary to common lore, there is a thermal contribution to the tensor spectrum[12] — not flat but with a characteristic f_0^3 shape

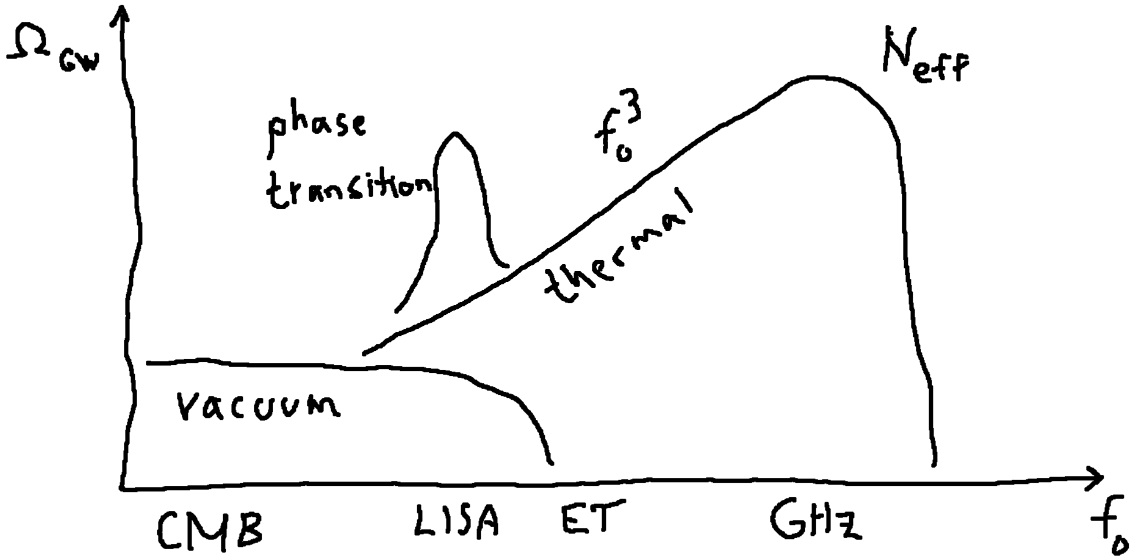
• from the reheating stage, there could be an additional GHz signal, which might be constrained via N_{eff}

• the presence of a non-abelian plasma with $T_{\rm max} > T_{\rm c}$ forces us to think about subsequent (dark sector?) phase transitions

all are "interesting", but none pose strong constraints on the benchmarks considered (i.e. no over-production)

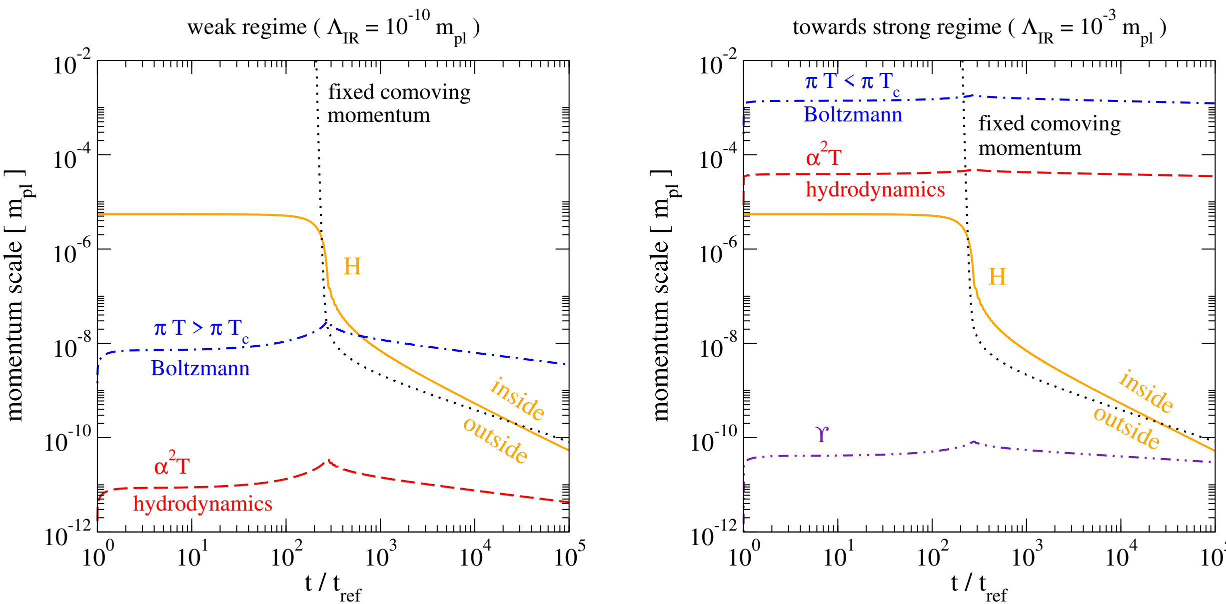
[12] Y. Qiu and L. Sorbo, ... *tensor perturbations in warm inflation*, 2107.09754

sketch



Ω_{GW}
phase
transition
f_0^3
thermal
N_{eff}
vacuum
CMB
LISA
ET
GHz
f_0

reminder: prototypical scenarios



[have this in mind first]

f_0^3 shape: basic ingredients

the non-abelian plasma has non-trivial dissipative coefficients, like the shear viscosity $\eta \sim T^3/\alpha^2$ and the bulk viscosity ζ

the fluctuation-dissipation theorem asserts that dissipation is balanced by hydrodynamic fluctuations, whose autocorrelator is proportional to the same dissipative coefficients[13]

$$\langle\, T^{ij}_{\text{hydro}}(x)\, T^{mn}_{\text{hydro}}(y)\,\rangle = 2T \left[\eta\, (\delta^{im}\delta^{jn} + \delta^{in}\delta^{jm}) + \left(\zeta - \frac{2\eta}{3}\right) \delta^{ij}\delta^{mn} \right] \frac{\delta^{(4)}(x-y)}{\sqrt{-\det g}}$$

such a local white noise leads to a characteristic "hydrodynamic" shape of the gravitational wave spectrum

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[13] E.M. Lifshitz and L.P. Pitaevskii, *Statistical Physics, Part 2*, secs. 88-89; J.I. Kapusta, B. Müller and M. Stephanov, *Relativistic theory of hydrodynamic fluctuations with applications to heavy-ion collisions*, 1112.6405

f_0^3 shape: general result for the tensor power spectrum[14]

$$\mathcal{P}_{\mathrm{T}}(k) = \frac{32\,k^3}{\pi m_{\mathrm{pl}}^2}\left\{ \overbrace{\frac{H^2(1+k^2\tau_e^2)}{2k^3}}^{\text{vacuum part}} + \underbrace{\frac{32\pi}{m_{\mathrm{pl}}^2}\int_{-\infty}^{\tau_e} \mathrm{d}\tau_i\, G_{\mathrm{R}}^2(\tau_e,\tau_i,k)\, T(\tau_i)\,\eta(\tau_i)}_{\text{from thermal fluctuations}} \right\}$$

[τ_e = end of inflation]

[14] P. Klose, ML, S. Procacci, *Gravitational wave background from vacuum and thermal fluctuations during axion-like inflation*, 2210.11710

f_0^3 shape: largest thermal contribution comes from $\sim T_{\max}$

$$\frac{\delta\,\mathcal{P}_{\mathrm{T}}(k)}{\delta\,(T\eta(\tau_i))} = \frac{32^2\,k^3}{m_{\mathrm{pl}}^4}\quad \underbrace{G^2_{\mathrm{R}}(\tau_e,\tau_i,k)}_{\text{constant for } k\ll aH}$$

multiplying $\mathcal{P}_T$ with the post-inflation transfer function[15] yields

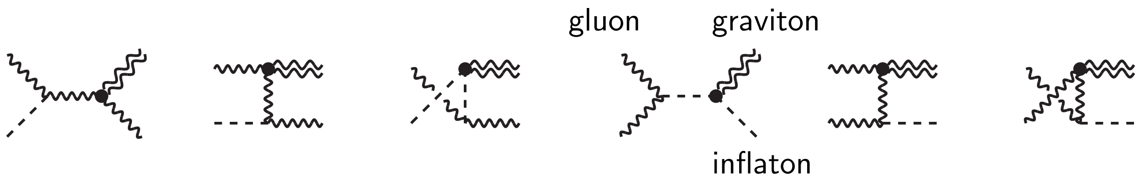
$$\Omega_{\mathrm{GW}}h^2 \supset A\left(\frac{f_0}{\mathsf{Hz}}\right)^3\left(\frac{T\eta}{m_{\mathrm{pl}}^4}\right)_{\mathsf{max}}\,, \qquad (T\eta)_{\mathsf{max}} \sim \frac{T^4_{\max}}{\alpha^2_{\mathsf{min}}}\,,$$

with the estimate $A \sim 10^{-9}$ for $\Lambda_{\mathrm{IR}} \ll m_{\mathrm{pl}} \Rightarrow$ "so and so"

[15] assuming frequencies that re-enter the horizon within the radiation-dominated epoch

N_{eff}: at $\pi T \gg m$, $2 \to 2$ single-graviton (h) production

this could be from SM,[16] BSM,[17] or inflaton processes[18,19]



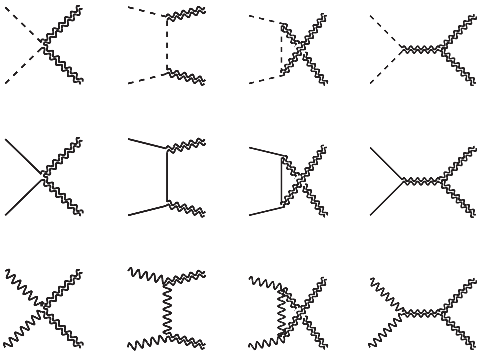
[16] J. Ghiglieri, ML, *Gravitational wave background from Standard Model physics: qualitative features*, 1504.02569; J. Ghiglieri, G. Jackson, ML, Y. Zhu, *Gravitational wave background from Standard Model physics: complete leading order*, 2004.11392

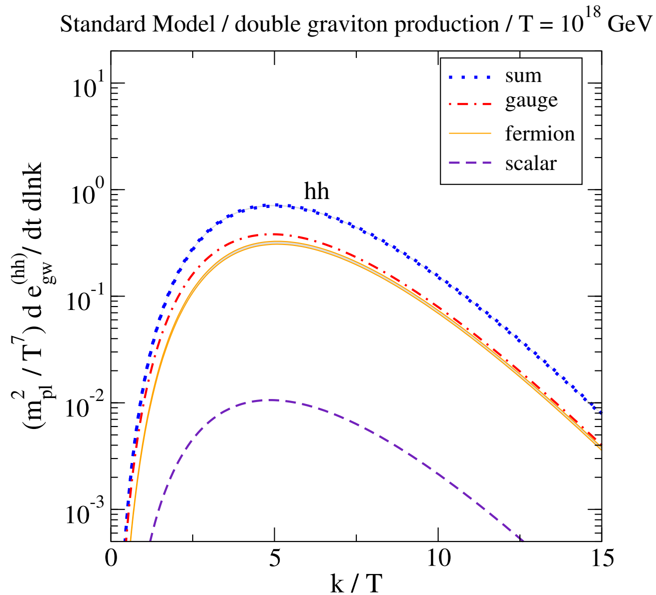
[17] A. Ringwald, J. Schütte-Engel, C. Tamarit, 2011.04731; L. Castells-Tiestos, J. Casalderrey-Solana, 2202.05241; F. Muia, F. Quevedo, A. Schachner, G. Villa, 2303.01548; M. Drewes, Y. Georis, J. Klaric, P. Klose, 2312.13855; ...

[18] P. Klose, ML, S. Procacci, *Gravitational wave background from non-Abelian reheating after axion-like inflation*, 2201.02317

[19] e.g. N. Bernal, S. Cléry, Y. Mambrini, Y. Xu, *Probing Reheating with Graviton Bremsstrahlung*, 2311.12694; A. Tokareva, *Gravitational Waves from Inflaton Decay and Bremsstrahlung*, 2312.16691; ...

$N_{\rm eff}$: double-graviton $(\rm hh)$ rates can be added[20]





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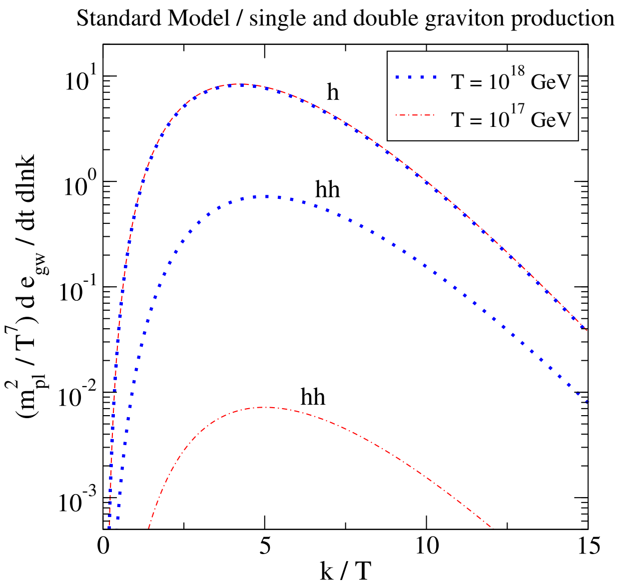
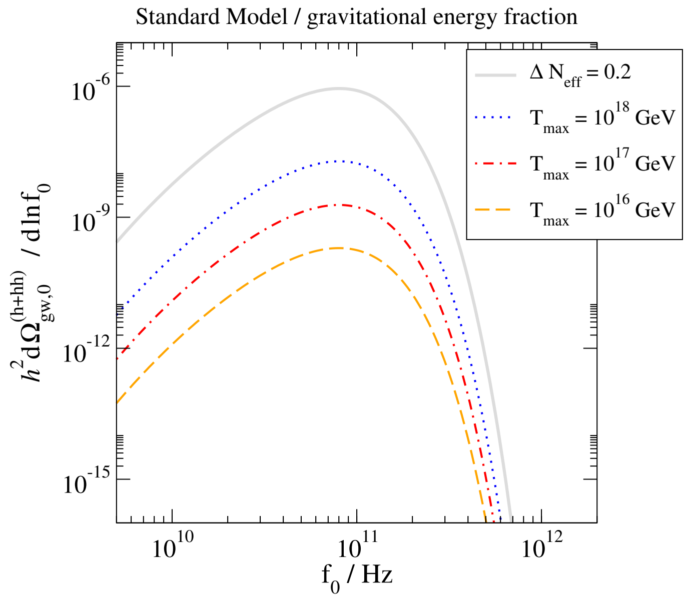
[20] J. Ghiglieri, J. Schütte-Engel, E. Speranza, *Freezing-In Gravitational Waves*, 2211.16513; J. Ghiglieri, ML, J. Schütte-Engel, E. Speranza, *Double-graviton production from Standard Model plasma*, 2401.08766

$N_{\rm eff}$: the signal is observable only at very high $T_{\rm max}$





hh-rate becomes dominant
at $T > 4 \times 10^{18}$ GeV

at $T_{\rm max} < 10^{18}$ GeV,
$\Delta N_{\rm eff} < 0.004$

phase transitions: effect of matter domination

if $T_{\rm max} > T_{\rm c}$, there is a (dark sector?) thermal phase transition

if $\Upsilon \ll H_* \equiv \{$Hubble rate at phase transition point$\}$, inflaton oscillations lead to a matter domination era, which suppresses any inside-horizon gravitational wave signal[21,22]

$$h^2\,\Omega_{\rm gw} \simeq 1.65\times 10^{-5}\,\frac{g_e}{g_s}\left(\frac{100}{g_s}\right)^{1/3}\left(\frac{\Upsilon}{H_*}\right)\left(\frac{\min\{\Upsilon,\Gamma\}}{H_*}\right)^{2/3}\frac{2}{3(1+2w_*)}\,\frac{e_{{\rm gw},*}}{e_{r+\varphi,*}}$$

$[\Upsilon =$ inflaton friction , $\Gamma =$ equilibration rate for dark sector$]$

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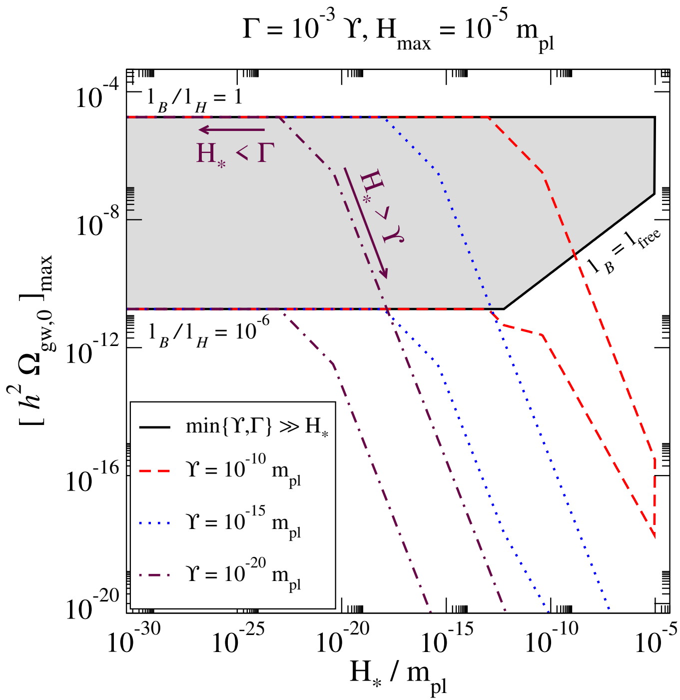
[21] e.g. J. Ellis, M. Lewicki and V. Vaskonen, ... *gravitational waves produced in a strongly supercooled phase transition*, 2007.15586; F. Ertas, F. Kahlhoefer and C. Tasillo, ... *listening to phase transitions in hot dark sectors*, 2109.06208

[22] H. Kolesova, ML, *Update on gravitational wave signals from post-inflationary phase transitions*, 2311.03718

phase transitions: suppression if $\Upsilon \ll H_*$

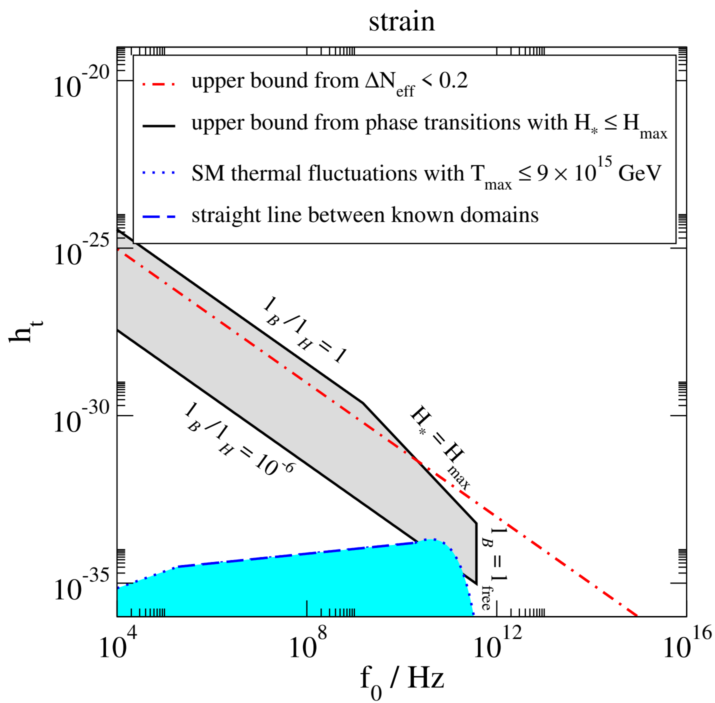


← maximal signal

← suppression

⇒ want large Υ during reheating, or late transition (small H_*)

at the largest f_0, phase transitions merge with scatterings!



if bubble separation ($\ell_{\rm B}$) is as short as the mean free path ($\ell_{\rm free}$), we have just thermal fluctuations

what should be done better?

there are many intriguing features, but...

$\Rightarrow$ could something more be said about the thermalization (separately of the plasma, and of φ)?

$\Rightarrow$ if $T_{\rm max} < T_{\rm c}$ (confinement phase), the important coefficients Υ and η become inaccurate — how to improve on them?

$\Rightarrow$ how much are curvature perturbations modified from cold-inflation predictions when approaching the strong regime?[23]

$\Rightarrow$ we looked at pure gauge; how about the effect of fermions?[24]

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[23] e.g. M. Mirbabayi and A. Gruzinov, *Shapes of non-Gaussianity in warm inflation*, 2205.13227; G. Ballesteros, A. Perez Rodríguez and M. Pierre, *Monomial warm inflation revisited*, 2304.05978; G. Montefalcone, V. Aragam, L. Visinelli and K. Freese, *WarmSPy: a numerical study of cosmological perturbations in warm inflation*, 2306.16190

[24] e.g. K.V. Berghaus, P.W. Graham, D.E. Kaplan, G.D. Moore and S. Rajendran, *Dark energy radiation*, 2012.10549; M. Drewes and S. Zell, *On Sphaleron Heating in the Presence of Fermions*, 2312.13739

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why is the temperature stationary at early times?

suppose that Δe from inflaton compensates for the hubble dilution, so that $\dot{e}_r - T(\dot{\partial_T V}) \simeq 0$

$$\dot{e}_r + 3H\big(e_r + p_r - T\partial_T V\big) - T(\dot{\partial_T V}) \simeq \Upsilon\dot{\varphi}^2$$

$$\overset{e+p\,=\,Ts}{\Longrightarrow} \qquad \underbrace{3T_{\rm stat}s}_{\text{strongly } T\text{-dependent}} \qquad \simeq \qquad \underbrace{\frac{\Upsilon}{H}\frac{(\partial_\varphi V)^2}{(3H+\Upsilon)^2}}_{\text{weakly } T\text{-dependent}}$$

a solution exists and represents a stable fixed point![25]

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[25] including the strong sphaleron rate: K.V. Berghaus, P.W. Graham and D.E. Kaplan, *Minimal warm inflation*, 1910.07525; W. DeRocco, P.W. Graham and S. Kalia, *Warming up cold inflation*, 2107.07517