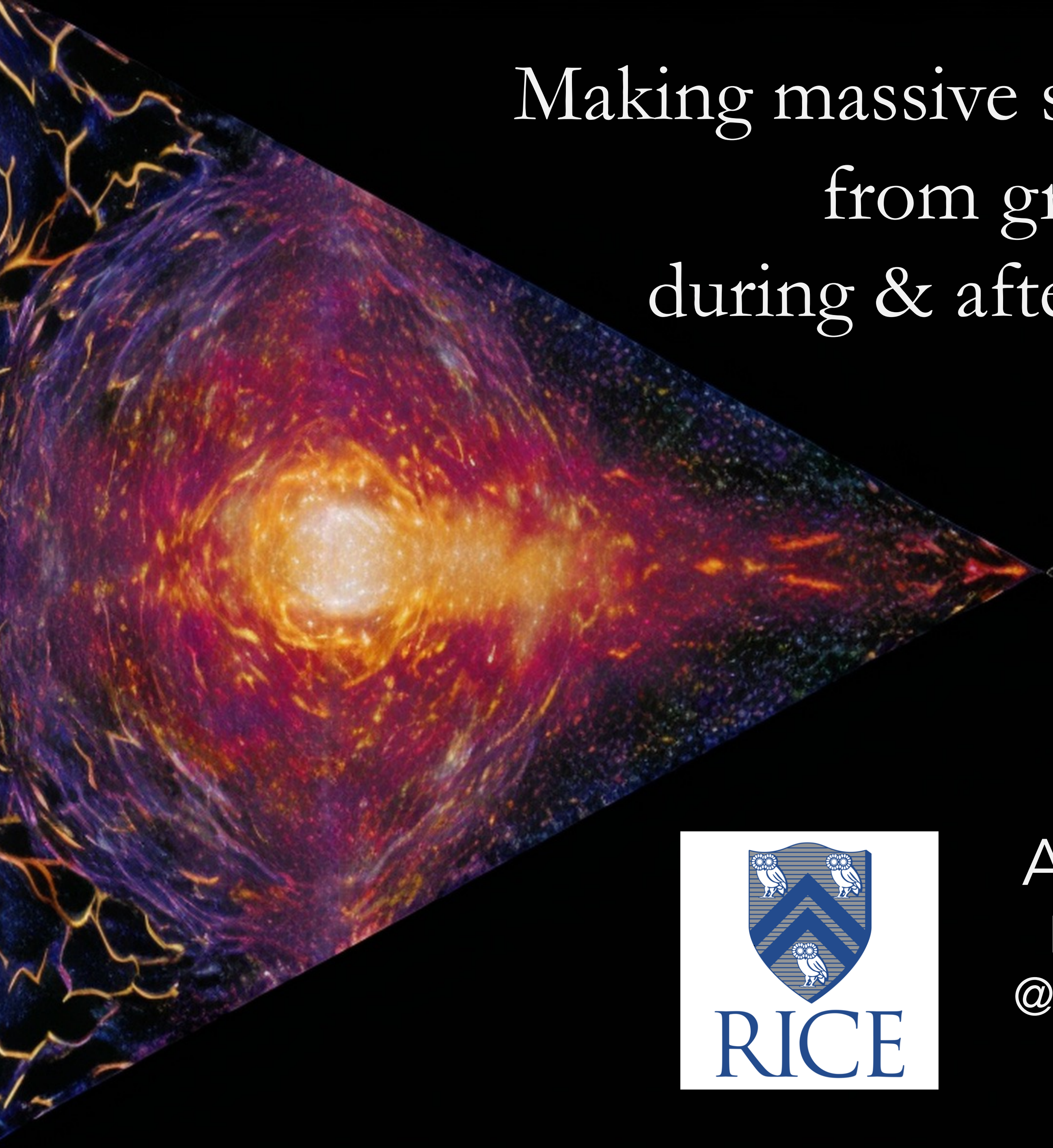
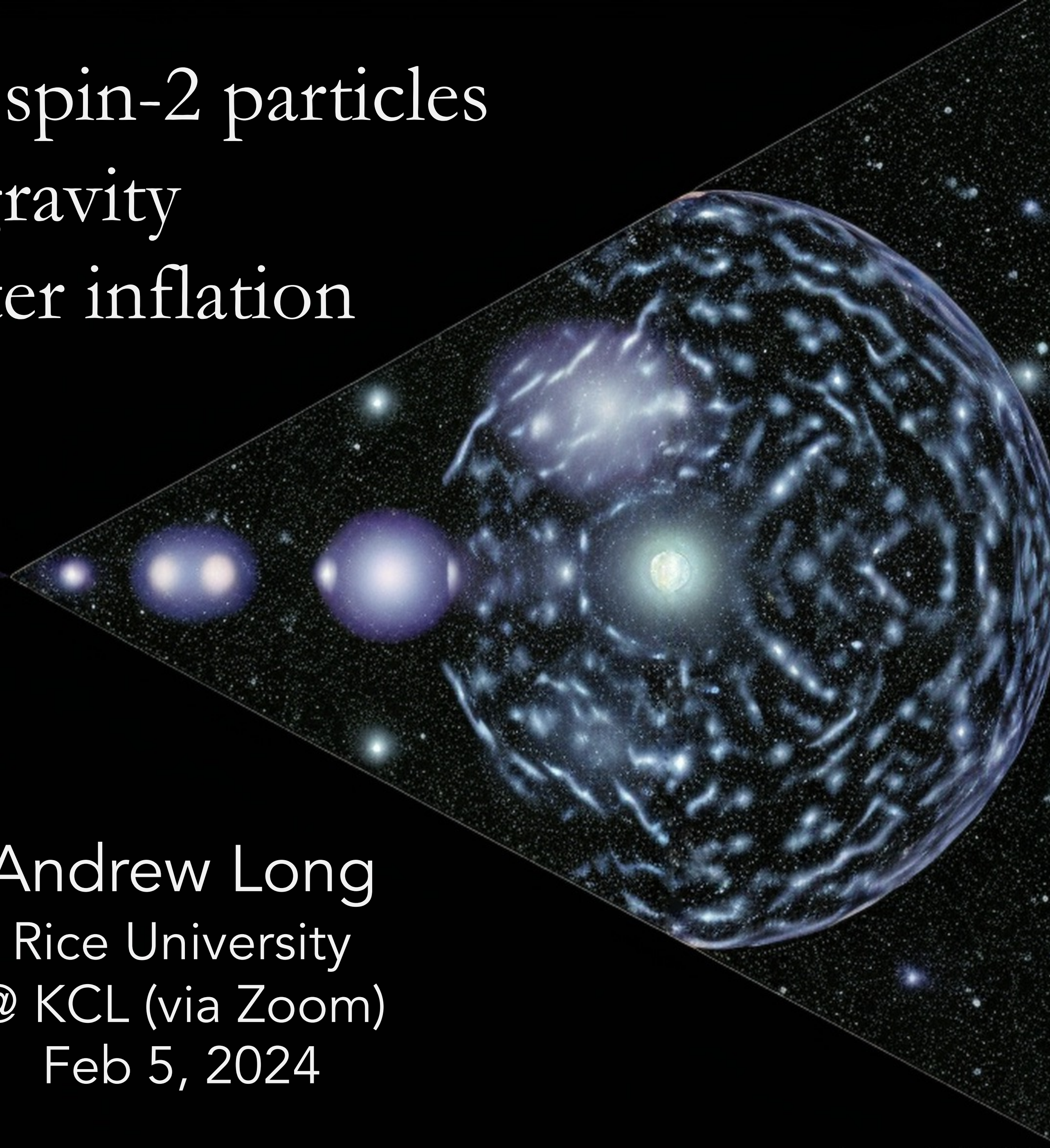




Making massive spin-2 particles from gravity during & after inflation

Andrew Long
Rice University

@ KCL (via Zoom)
Feb 5, 2024

Image credit: Chris Stabb

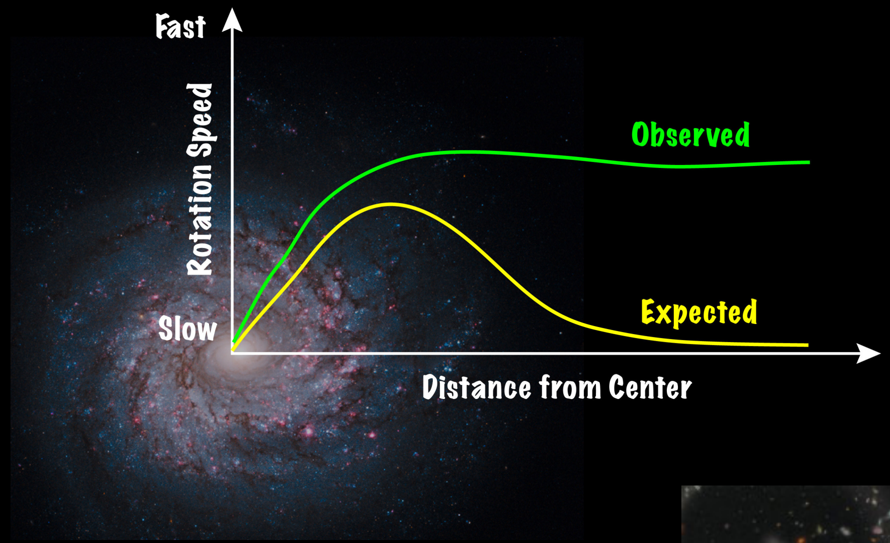
motivation

making dark matter from gravity

dark matter pulls on things

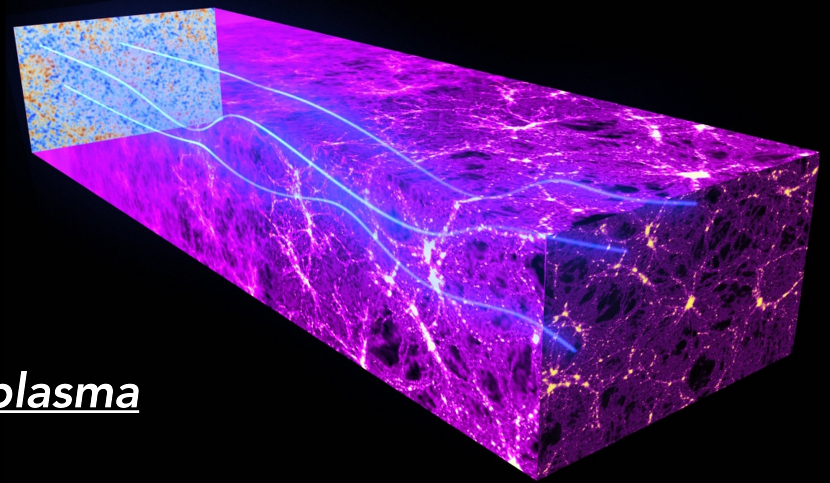
Dark matter pulls on stars in galaxies

(galactic rotation curves)



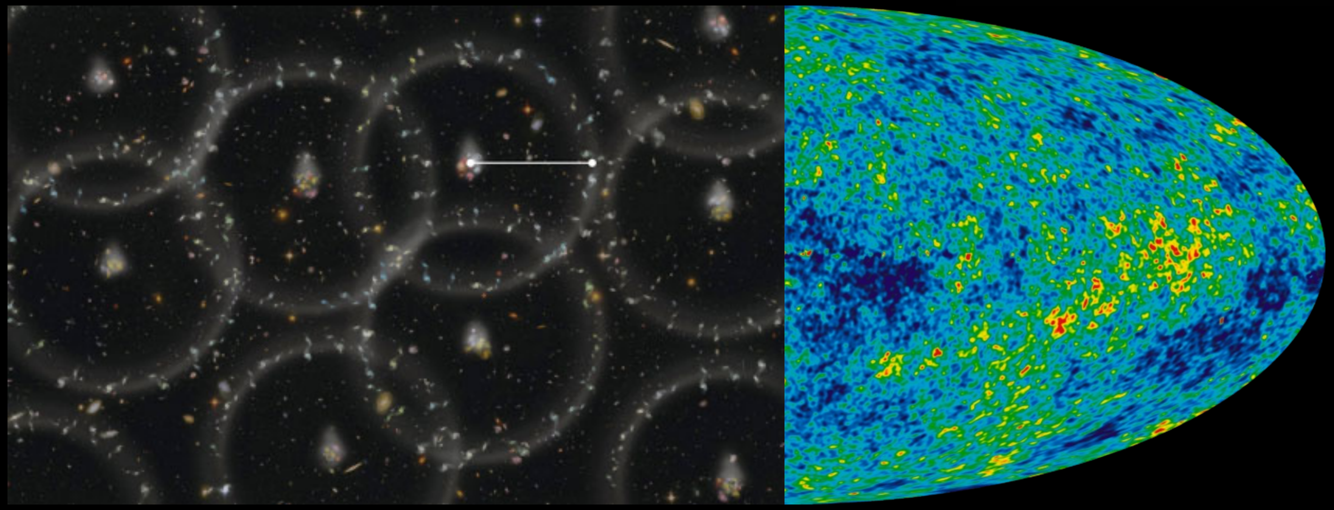
Dark matter pulls on light

(gravitational lensing)

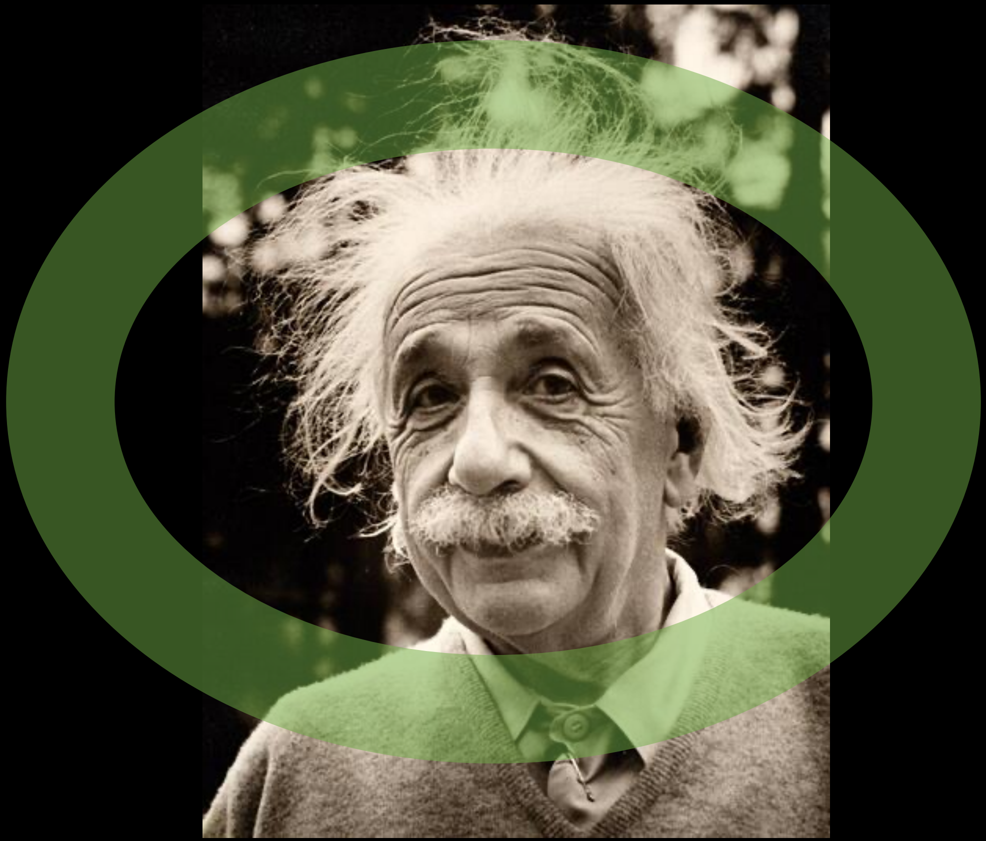


Dark matter pulled on e^-p^+ plasma

(CMB & large scale structure)



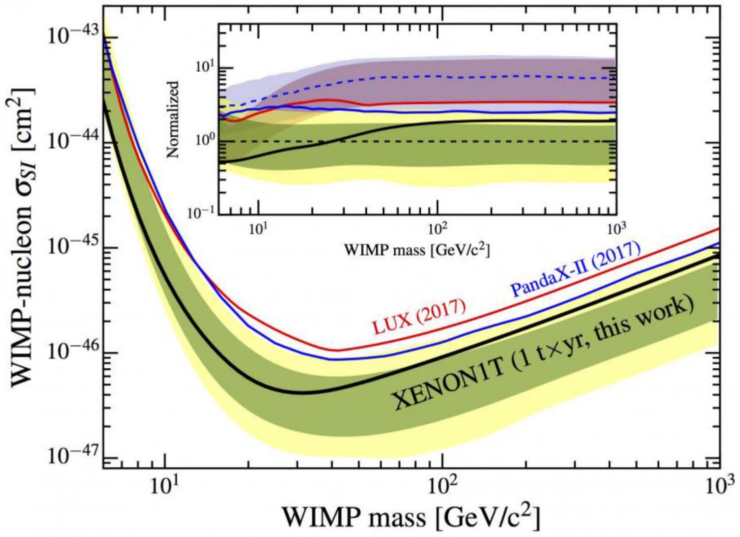
don’t need a dark force



no evidence (yet) of dark matter bumping into things

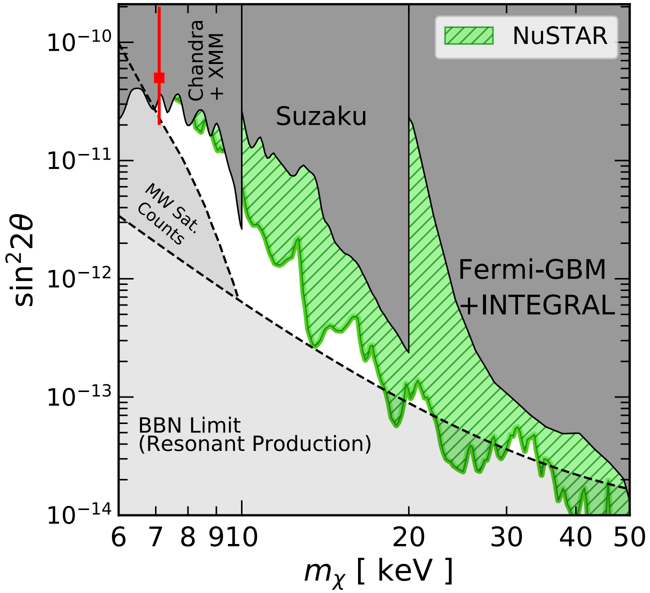
No dark matter bumping into things

(direct detection; 1805.12562)



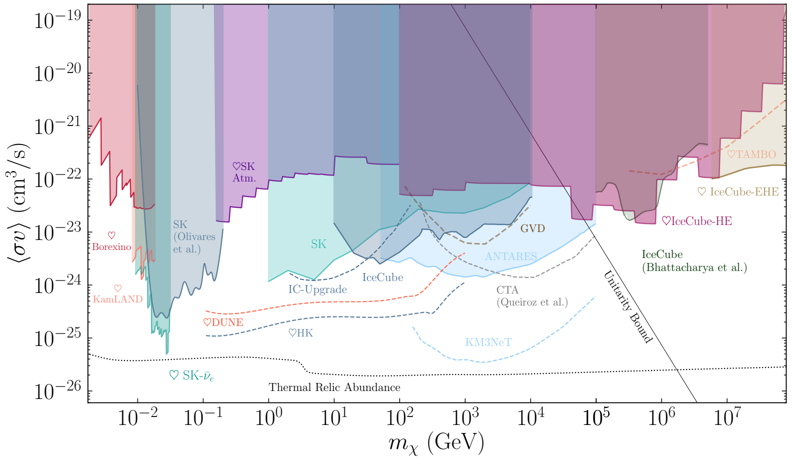
No dark matter decaying into things

(X-ray emission; 1908.09037)



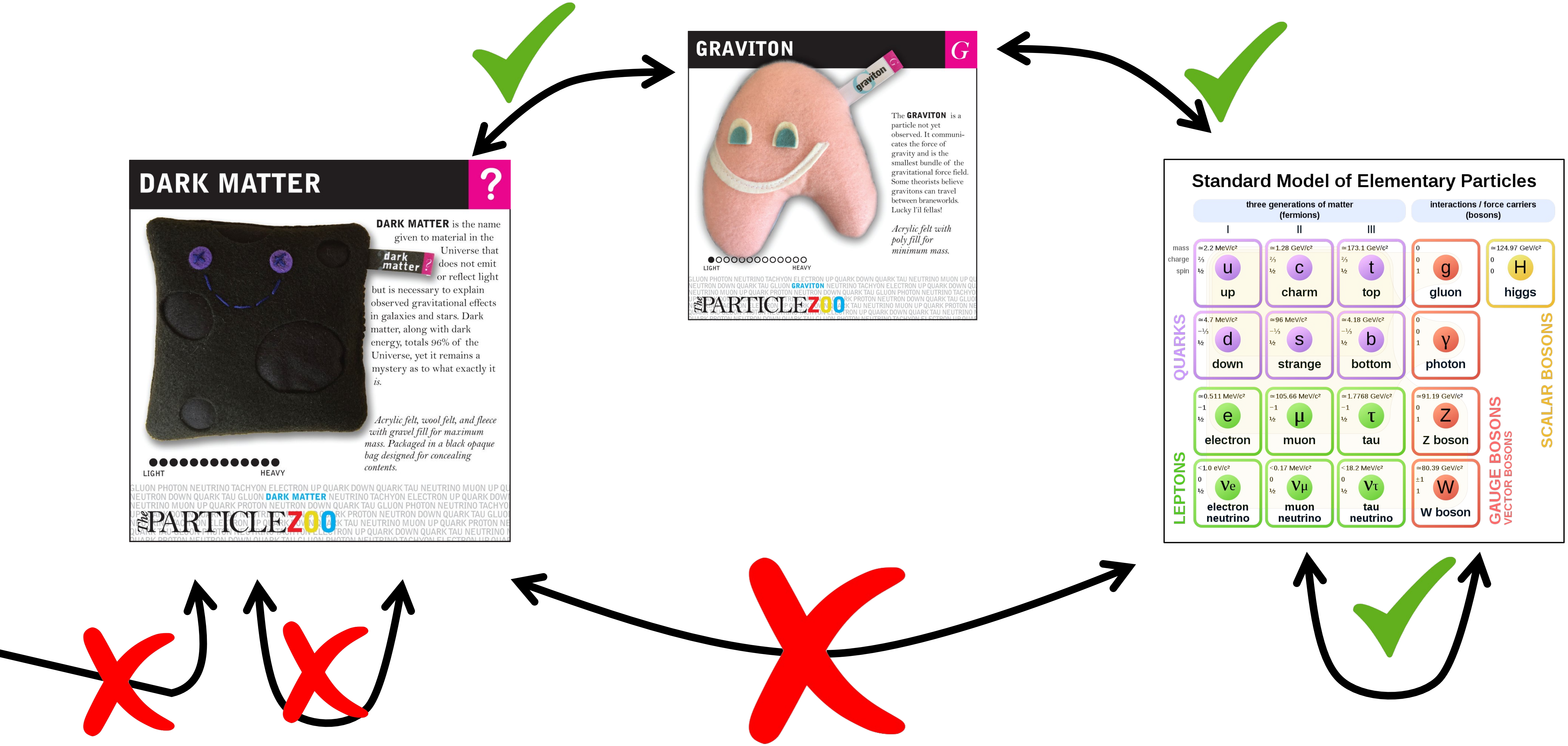
No dark matter bumping into itself

(annihilation to ν's; 1912.09486)



(notwithstanding hints of new physics, there's no overwhelming evidence)

the hypothesis:



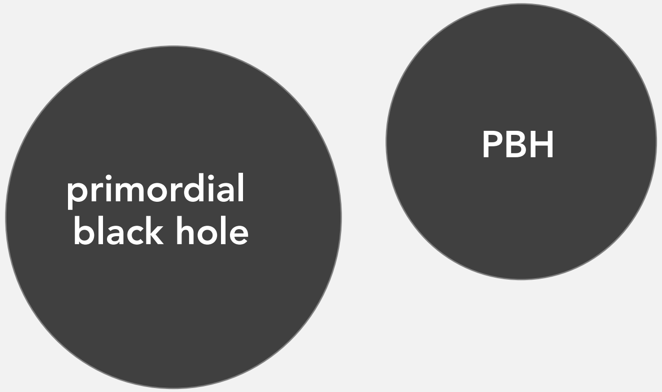
the problem:

where did all the dark matter come from?

(how do we use gravity to make dark matter?)

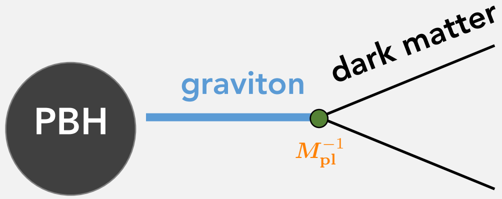
ideas …

the DM is a collection of primordial black holes



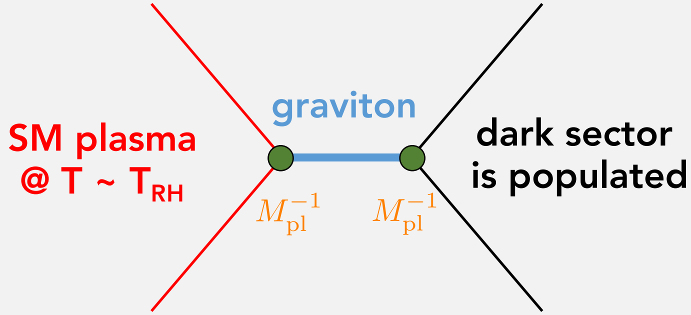
[many authors, esp. after LIGO BBH merger (2016)]

the DM is produced from PBH evaporation



[Hooper, Krnjaic, & McDermott (2019)]

the DM is a produced from thermal freeze-in

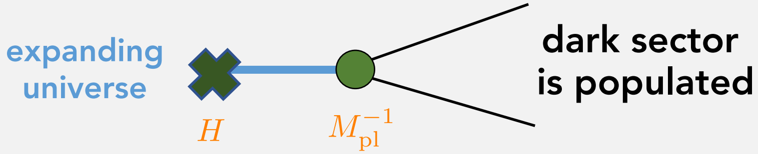


[Garny, Sandora, & Sloth (2015)], [Garcia, Kaneta, Mambrini, Olver, Verner (2021)], [Clery, Mambrini, Olive, Sherkin, Verner (2022)]

[see talk by Gabriele Montefalcone]

this talk:

the DM is produced from cosmological expansion during (or at the end) of inflation



[Kuzmin & Tkachev (1999); Chung, Kolb, & Riotto (1999)]

What do I want to tell you about?

(1) CGPP as inflationary quantum fluctuations

(2) CGPP as graviton-mediated scattering

(3) A theory of massive spin-2 during inflation

(4) CGPP of massive spin-2 particles

(5) Summary & discussion

(1) CGPP as inflationary quantum fluctuations

Establishing the framework: scalar spectator

covariant action

$$S = \int \mathrm{d}^4x \, \sqrt{-g} \left[\tfrac{1}{2} g^{\mu\nu} \partial_\mu \chi \partial_\nu \chi - \tfrac{1}{2} m_\chi^2 \chi^2 \right]$$

+

FLRW spacetime

$$(\mathrm{d}s)^2 = a(\eta)^2 \left[(\mathrm{d}\eta)^2 - |\mathrm{d}\boldsymbol{x}|^2 \right]$$

$R(\eta) = -6a''/a^3$

Fourier decomposition

$$\chi(\eta, \boldsymbol{x}) = \frac{1}{a(\eta)} \int \frac{\mathrm{d}^3\boldsymbol{k}}{(2\pi)^3} \, a_{\boldsymbol{k}} \, \chi_k(\eta) \, \mathrm{e}^{\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{x}} + \mathrm{c.c.}$$

equations of motion

$$\chi_k''(\eta) + \omega_k^2(\eta) \, \chi_k(\eta) = 0$$

a harmonic oscillator with time-dependent frequency

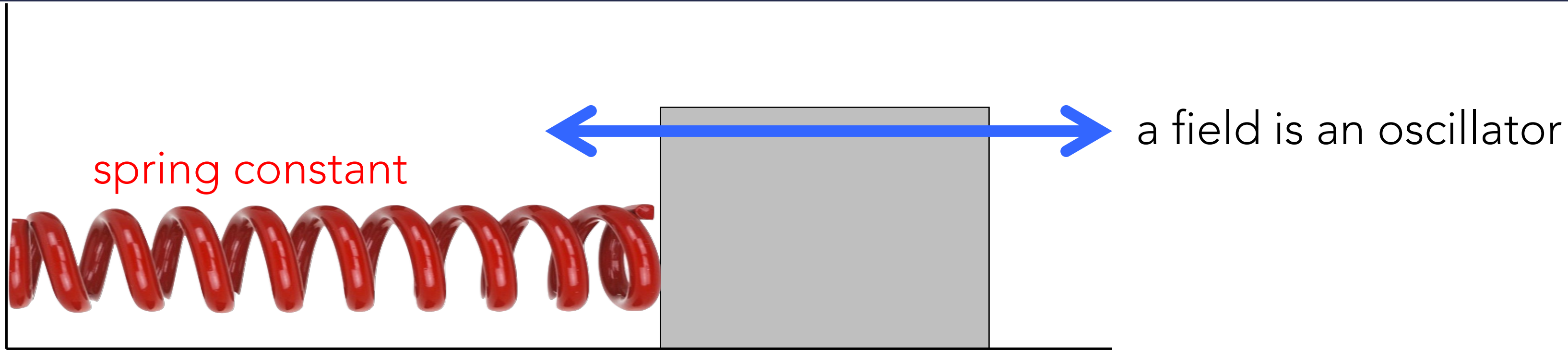
$$\omega_k^2(\eta) = k^2 + a(\eta)^2 m_\chi^2 + \tfrac{1}{6} a(\eta)^2 R(\eta)$$

initial conditions

Bunch-Davies vacuum

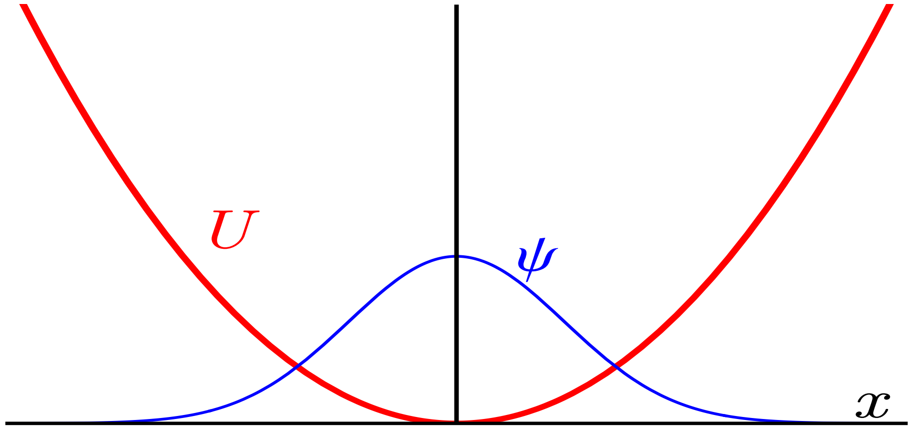
$$\chi_k(\eta) \sim \mathrm{e}^{-\mathrm{i}k\eta} / \sqrt{2k} \quad (\text{as } \eta \to -\infty)$$

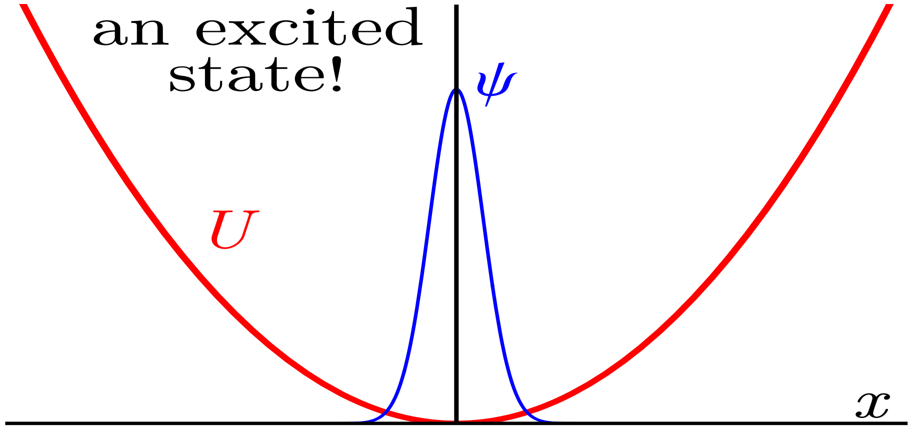
An analogy with 1D quantum mechanics



Spring constant is varied slowly (adiabatically)

Spring constant is varied quickly (*non*-adiabatically)





Various interesting observables

w.r.t. BD vacuum

correlation functions $\langle \chi(\eta, \boldsymbol{x}) \, \chi(\eta, \boldsymbol{y}) \rangle$

power spectra $\Delta_\chi^2(k) = \dfrac{k^3}{2\pi^2} P_\chi(k)$

average energy density $\bar{\rho}(\eta) = \langle \tfrac{1}{2}\chi'(\eta, \boldsymbol{x})^2 + \tfrac{1}{2}\boldsymbol{\nabla}\chi(\eta, \boldsymbol{x})^2 + \tfrac{1}{2}m_\chi^2 \chi(\eta, \boldsymbol{x})^2 \rangle$

density contrast $\delta(\eta, \boldsymbol{x}) = \rho(\eta, \boldsymbol{x}) \, / \, \bar{\rho}(\eta, \boldsymbol{x}) - 1$

comoving number density

$$a^3 n = \int \mathrm{d} \ln k \, \frac{k^3}{2\pi^2} |\beta_k|^2$$

$$|\beta_k|^2 = \frac{\omega_k}{2} |\chi_k|^2 + \frac{1}{2\omega_k} |\partial_\eta \chi_k|^2 - \frac{1}{2}$$

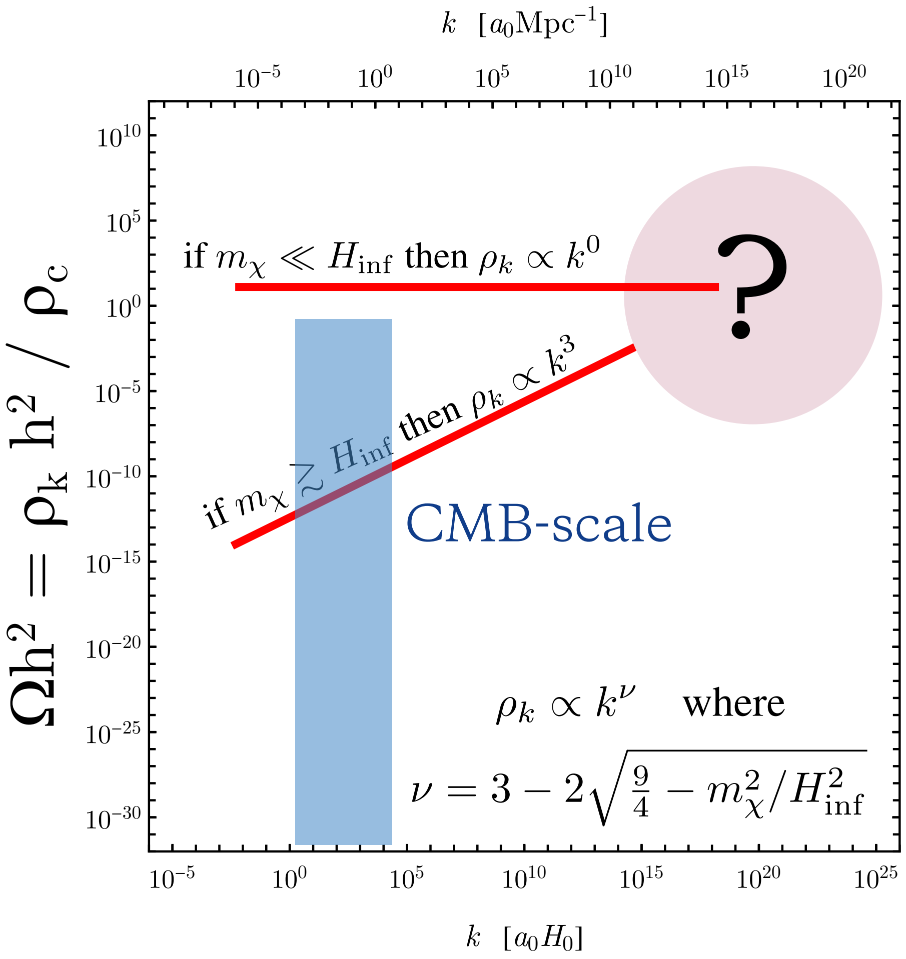
assume $a^3 n$ is constant after production (i.e. no decay, annihilation, production)

at late times when the field is non-relativistic we have $\bar{\rho} \approx m_\chi n$

see 2312.09042 for more info

Intuition for light vs. heavy

for fields that are
light during inflation
…
scale-invariant
energy spectrum
…
for CMB observables,
you don't care about
the high-k modes



for fields that are
heavy during inflation
…
most energy is carried
by the high-k modes
…
to calculate their
spectrum, you need to
model the end of
inflation & reheating
…
often necessary to rely
on numerical methods

(2) CGPP as inflationary graviton-mediated scattering

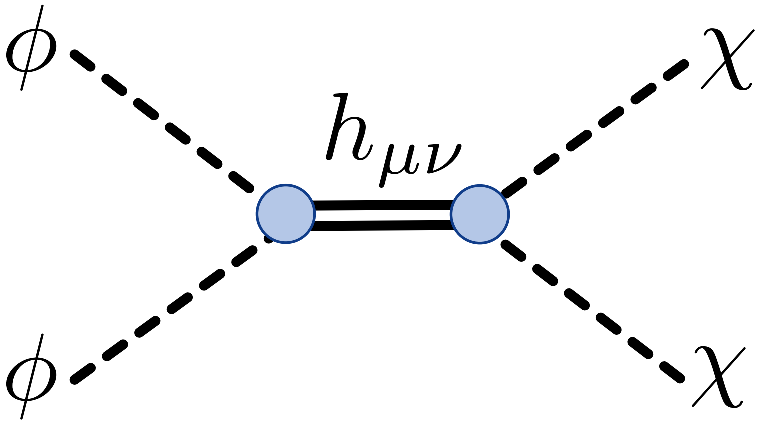
Gravity-mediated inflaton annihilation

Ema, Nakayama, & Tang (2015)
Chung, Kolb, & AL (2018)

After inflation, inflaton-field oscillations correspond to a condensate of zero-momentum spin-0 inflaton particles

$H_e < m_\chi \ll m_\phi$

s-channel graviton



rate estimates

$$\sigma_{\phi\phi\to\chi\chi} \sim m_\phi^2 / M_{\rm pl}^4$$

$$n_\phi \sim m_\phi \phi_e^2 \, (a/a_e)^{-3}$$

$$\Gamma_{\phi\phi\to\chi\chi} \approx \frac{3/8}{16\pi} \frac{m_\phi^3 \phi_e^2}{M_{\rm pl}^4} \left(\frac{a}{a_e}\right)^{-3}$$

$k_\chi \approx a m_\phi$ at production

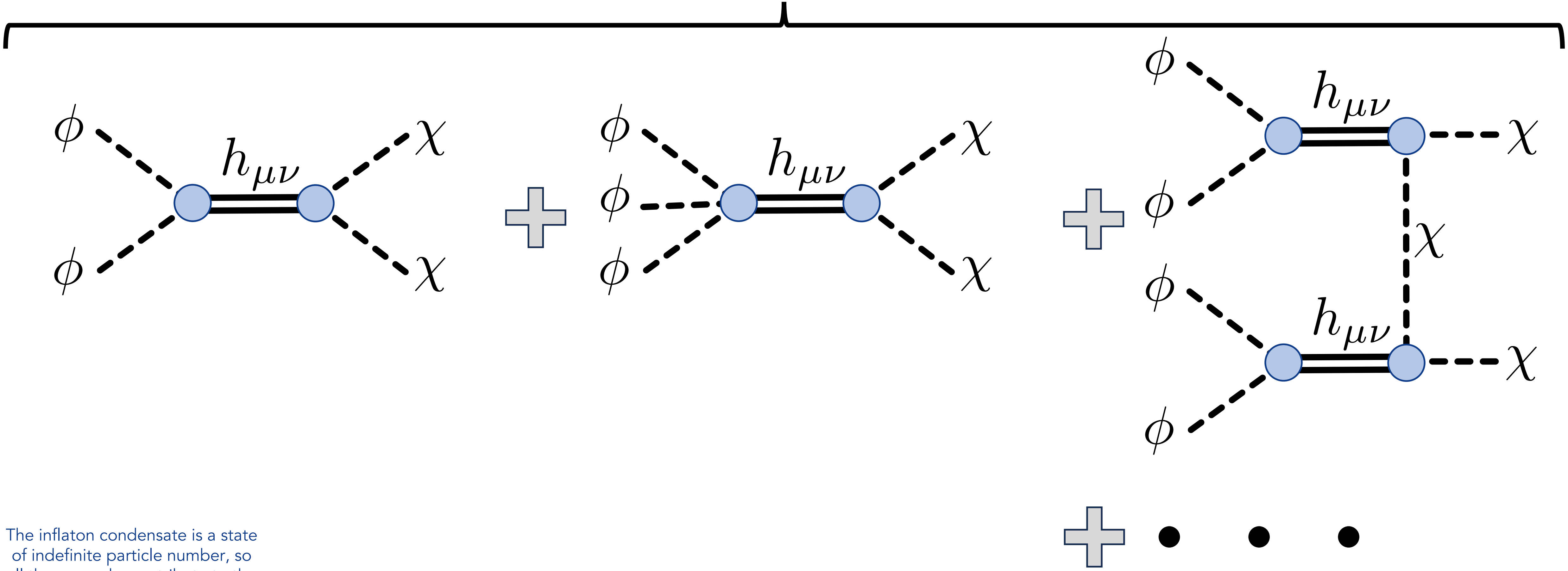
χ-spectrum: high-momentum power-law tail	$a^3 n_k \propto k^{-3/2}$ $a_e m_\phi < k < a_{\rm RH} m_\phi$

$$\phi\phi \to \chi\chi \quad \Rightarrow \quad n_k \propto k^{-3/2}$$
$$\phi\phi\phi \to \chi\chi \quad \Rightarrow \quad n_k \propto k^{-9/2}$$
$$n\phi \to \chi\chi \quad \Rightarrow \quad n_k \propto k^{-3(2n-3)/2}$$

There's interference too!

Basso, Chung, Kolb, AL [2209.01713]

ϕ-condensate $\rightarrow \chi\chi$



The inflaton condensate is a state of indefinite particle number, so all these graphs contribute to the same amplitude

Analytical analysis

Bogolubov coefficient

$$n_k \propto |\beta_k|^2$$

$$\beta_k \approx \int_{-\infty}^{\infty} \mathrm{d}t \, \frac{\dot{\omega}_k}{2\omega_k} \, e^{-2i\int^t \mathrm{d}t' \, \omega_k / a}$$

Resonant contributions

$$n m_\phi \approx 2\sqrt{k^2/a^2 + m_\chi^2}$$

$$\beta_k \approx \beta_k^{(1\to2)} + \beta_k^{(2\to2)} + \beta_k^{(3\to2)} + \cdots$$

$$\beta_k^{(n\to2)} = \mathcal{A}_k^{(n\to2)} \, e^{i\Phi_k^{(n\to2)}}$$

$$\Delta\Phi_k^{(n\to2)} = \Phi_k^{(n\to2)} - \Phi_{k,\mathrm{leading}}^{(n\to2)}$$

Lowest-order terms

$$\mathcal{A}_k^{(1\to2)} = -\kappa_1^{-15/4} \, 3\alpha_3 \sqrt{\frac{-\frac{i}{2}\pi}{\frac{1}{4} - r_\chi^2}} \, r_\chi^2 \left(1 + \mathcal{O}(\kappa_1^{-3})\right) \,, \tag{4.3a}$$

$$\mathcal{A}_k^{(2\to2)} = \kappa_2^{-9/4} \frac{3}{16} \sqrt{\frac{-i\pi}{1 - r_\chi^2}} \, r_\chi^2 \left(1 + \frac{x_0 + x_1 r_\chi^2 + x_2 r_\chi^4 - 416 r_\chi^6 + 384 r_\chi^8}{1024(1 - r_\chi^2)^2} \kappa_2^{-3} + \mathcal{O}(\kappa_2^{-6})\right) \,, \tag{4.3b}$$

$$\mathcal{A}_k^{(3\to2)} = \kappa_3^{-15/4} \frac{\alpha_3}{9} \sqrt{\frac{-\frac{3}{2}i\pi}{\frac{9}{4} - r_\chi^2}} \, r_\chi^2 \left(1 + \mathcal{O}(\kappa_3^{-3})\right) \,, \tag{4.3c}$$

$$\mathcal{A}_k^{(4\to2)} = \kappa_4^{-21/4} \frac{3\left(-21 + 68\alpha_3^2 + 24\alpha_4 + 12 r_\chi^2\right)}{4096} \sqrt{\frac{-2i\pi}{4 - r_\chi^2}} \, r_\chi^2 \left(1 + \mathcal{O}(\kappa_4^{-3})\right) \,, \tag{4.3d}$$

$$\Delta\Phi_k^{(1\to2)} = \kappa_1^{-3/2} \left(\frac{y_0^{(1)} + y_1^{(1)} r_\chi^2 - 1280 r_\chi^4}{480\left(1 - 4r_\chi^2\right)} + z^{(1)} + \mathcal{O}(\kappa_1^{-3})\right) \,, \tag{4.4a}$$

$$\Delta\Phi_k^{(2\to2)} = \kappa_2^{-3/2} \left(\frac{y_0^{(2)} + y_1^{(2)} r_\chi^2 - 80 r_\chi^4}{960\left(1 - r_\chi^2\right)} + z^{(2)} + \mathcal{O}(\kappa_2^{-3})\right) \,, \tag{4.4b}$$

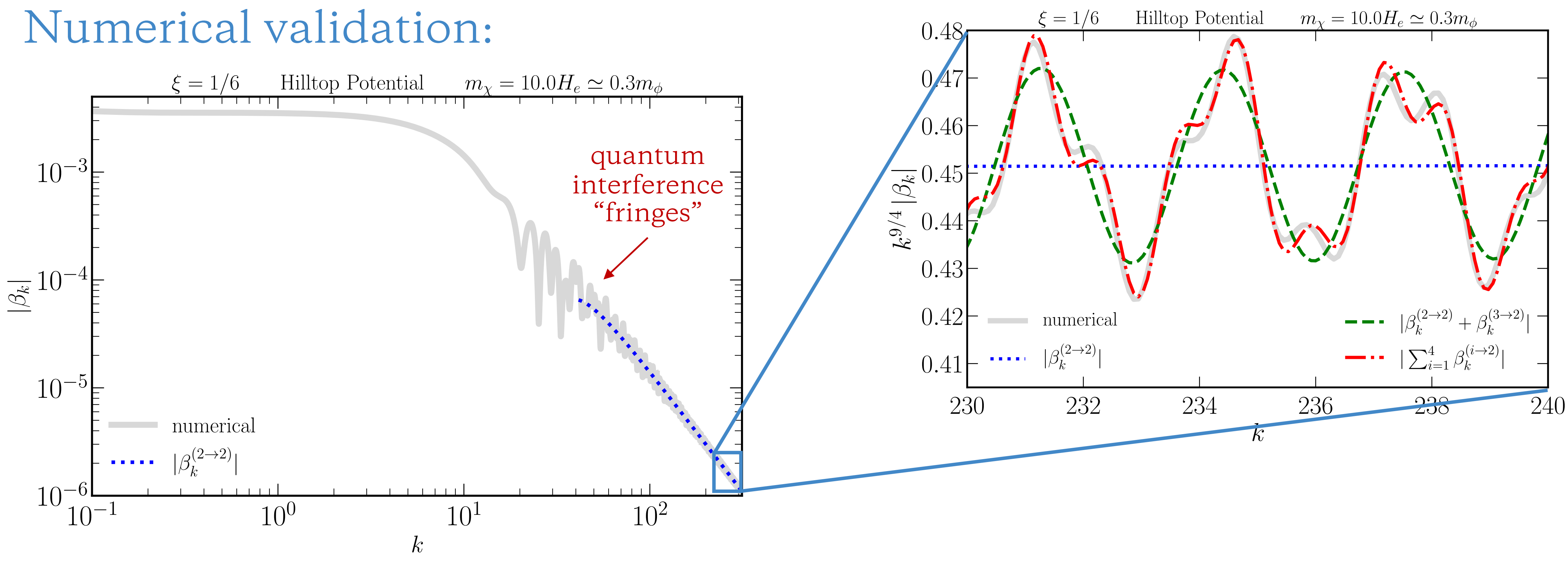
$$\Delta\Phi_k^{(3\to2)} = \kappa_3^{-3/2} \left(\frac{y_0^{(3)} + y_1^{(3)} r_\chi^2 - 1280 r_\chi^4}{12960\left(9 - 4r_\chi^2\right)} + z^{(3)} + \mathcal{O}(\kappa_3^{-3})\right) \,, \tag{4.4c}$$

$$\Delta\Phi_k^{(4\to2)} = \kappa_4^{-3/2} \left(\frac{y_0^{(4)} + y_1^{(4)} r_\chi^2 + y_2^{(4)} r_\chi^4 + 2588 r_\chi^6}{960\left(4 - r_\chi^2\right)\left(-21 + 68\alpha_3^2 + 24\alpha_4 + 12 r_\chi^2\right)} + z^{(4)} + \mathcal{O}(\kappa_4^{-3})\right) \,, \tag{4.4d}$$

Quantum interference fringes

Basso, Chung, Kolb, AL [2209.01713]

Numerical validation:

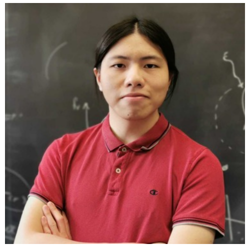


quick example

scalar CGPP in
alpha-attractor inflation

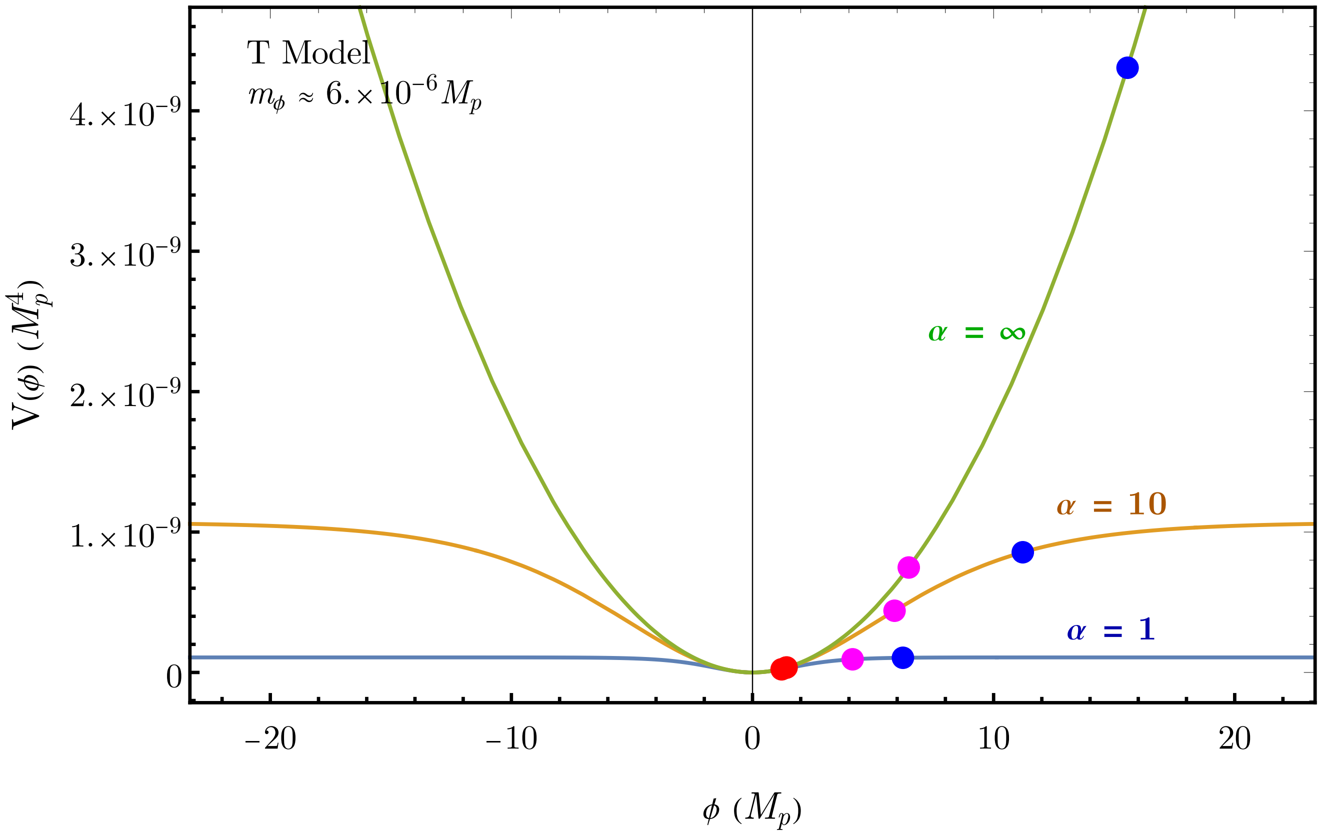
Example: alpha attractor inflation

[Ling & Long (2101.11621)]



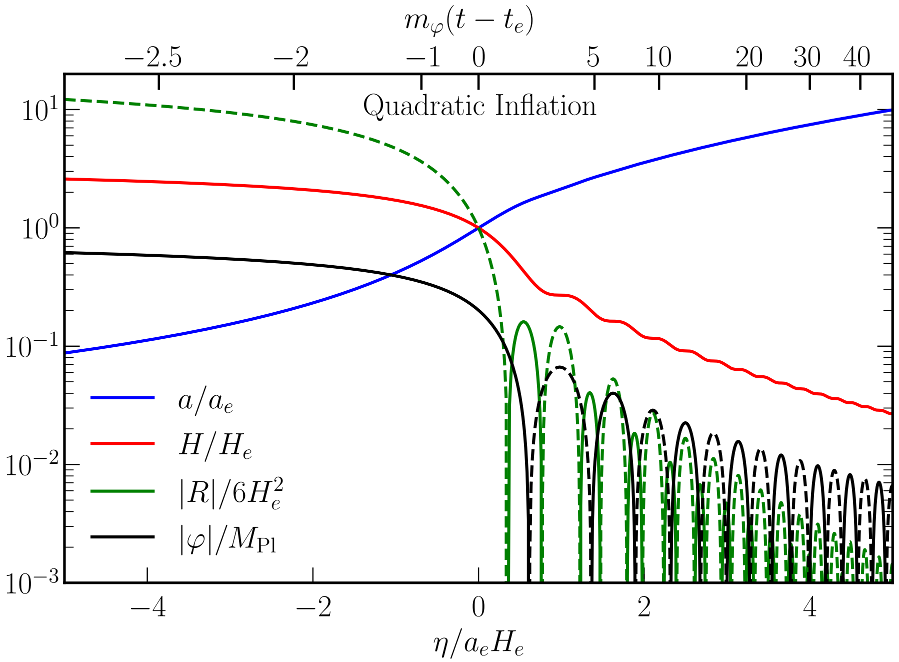
T-model alpha attractor

$$V_T(\phi) = \alpha \mu^2 M_p^2 \tanh^2 \frac{\phi}{\sqrt{6\alpha} M_p}$$



$$\Rightarrow \begin{cases} \phi(t) \\ a(t) \end{cases}$$

FRW background

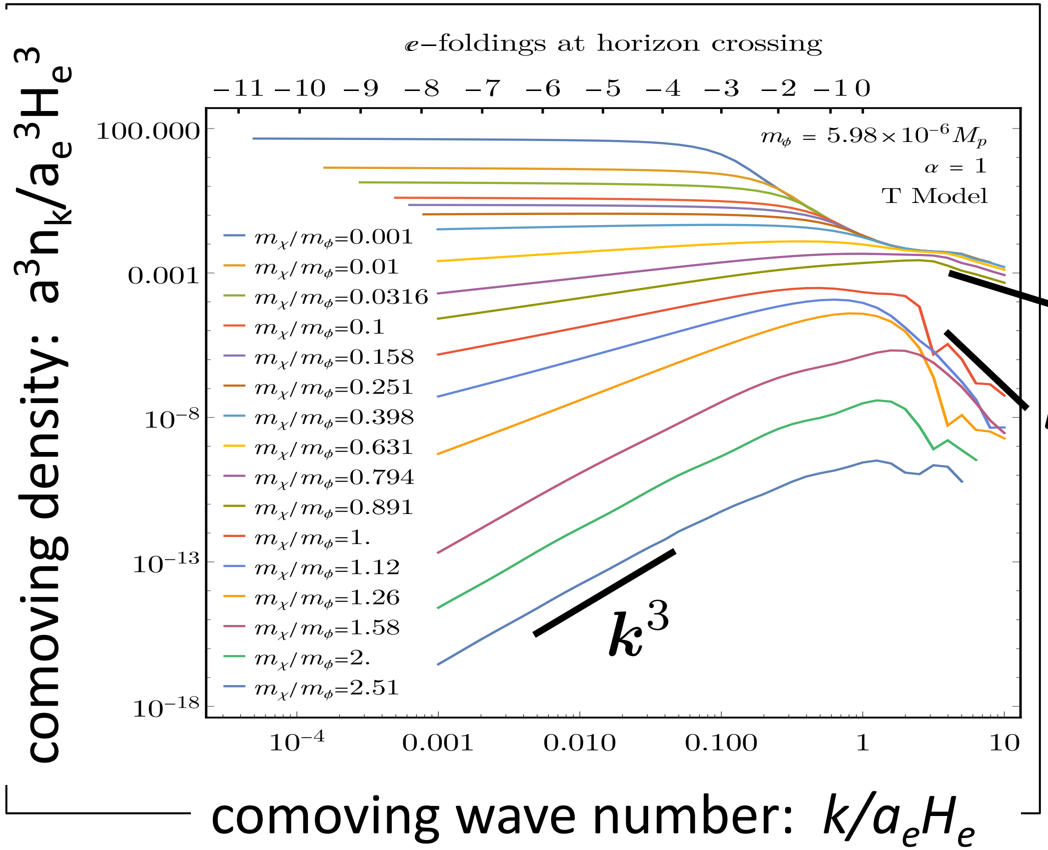


Numerical results

[Ling & AL (2101.11621)]

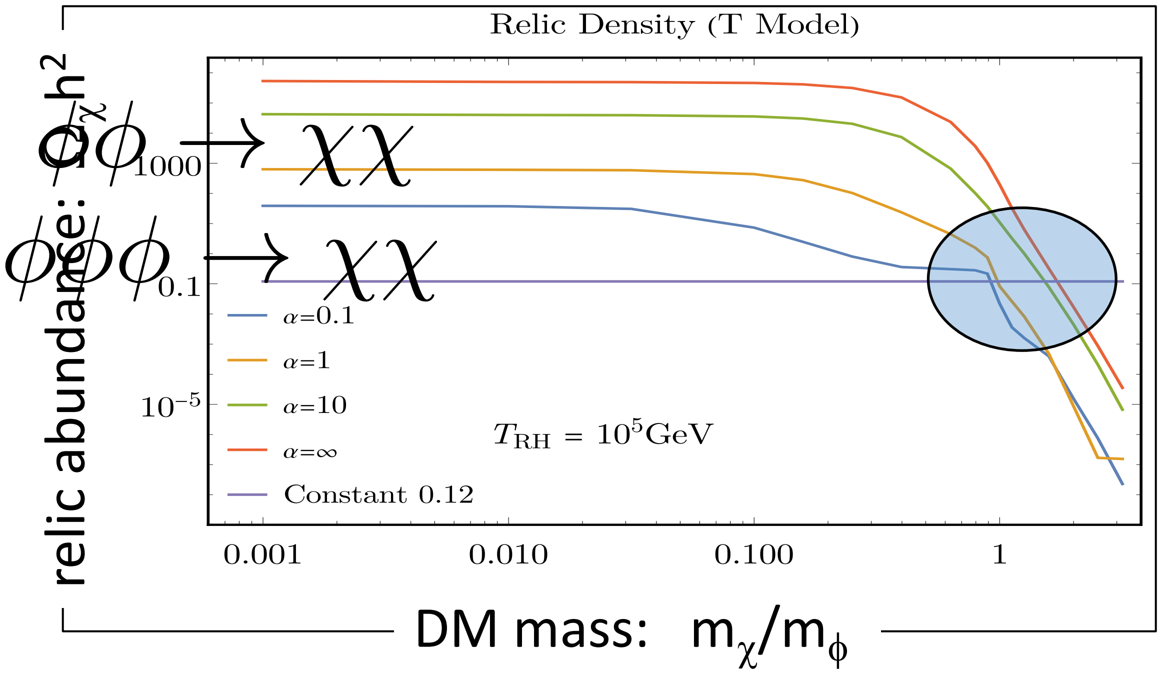
$n_k \propto k^{\nu}$ with $\nu = 3 - 2\left[\frac{9}{4} - \frac{m^2}{H_{\rm inf}^2}\right]^{1/2}$

comoving number density



relic abundance

$$\Omega_\chi h^2 \simeq (0.114)\left(\frac{m_\chi}{10^{10}\ \text{GeV}}\right)\left(\frac{H_e}{10^{10}\ \text{GeV}}\right)\left(\frac{T_{\rm RH}}{10^8\ \text{GeV}}\right)\left(\frac{a^3 n}{a_e^3 H_e^3}\right)$$

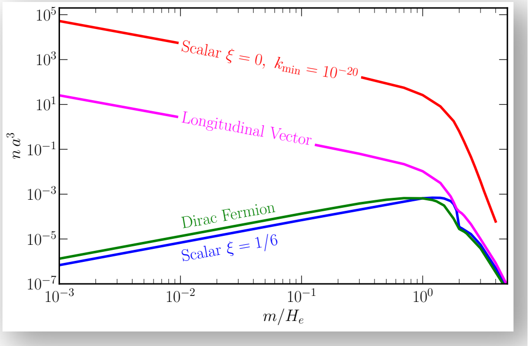


CGPP can account for all the dark matter

what about particles with spin?
CGPP occurs for them too!

Studies of CGPP for particles w/ spin

$$S = \int \mathrm{d}^4x \sqrt{-g}\, \mathscr{L}$$



spin-0 (scalar field)

Chung, Kolb, & Riotto (1998)
Kuzmin & Tkachev (1998)
Herring, Boyanovsky, & Zentner (2020)
Brandenberger, Kamali, & Ramos (2023)

$$\mathscr{L} = \frac{1}{2} g^{\mu\nu} \partial_\mu \varphi \partial_\nu \varphi - \frac{1}{2} m^2 \varphi^2 + \frac{1}{2} \xi \varphi^2 R$$

spin-1/2 (spinor field)

Kuzmin & Tkachev (1998)
Chung, Everett, Yoo, & Zhou (2011)
Hashiba, Ling, & AL (2206.14204)

$$\mathscr{L} = \frac{i}{2} \bar{\Psi} \underline{\gamma}^\mu (\nabla_\mu \Psi) - \frac{1}{2} m \bar{\Psi} \Psi + \mathrm{h.c.}$$

spin-1 (vector field)

Dimopoulos (2006) – not for DM; Graham, Mardon, & Rajendran (2016); Ahmed, Grzadkowski, & Socha (2020); Kolb & AL (2009.03828); Cembranos et al (2023)

[see talk by Jose Cembranos]

$$\mathscr{L} = -\frac{1}{4} g^{\mu\alpha} g^{\nu\beta} F_{\mu\nu} F_{\alpha\beta} + \frac{1}{2} m^2 g^{\mu\nu} A_\mu A_\nu - \frac{1}{2} \xi_1 R g^{\mu\nu} A_\mu A_\nu - \frac{1}{2} \xi_2 R^{\mu\nu} A_\mu A_\nu$$

spin-3/2 (vector-spinor field)

Kallosh, Kofman, Linde, & Van Proeyen (1999); Giudice, Riotto, & Tkachev (1999); Lemoine (1999); Kolb, AL, & McDonough (2102.10113); Kaneta, Ke, Mambrini, Olive, Verner (2023)

$$\mathscr{L} = \frac{i}{4} \bar{\Psi}_\mu (\underline{\gamma}^\mu \underline{\gamma}^\rho \underline{\gamma}^\sigma - \underline{\gamma}^\sigma \underline{\gamma}^\rho \underline{\gamma}^\mu)(\nabla_\rho \Psi_\sigma) + \frac{1}{2} m \bar{\Psi}_\mu \underline{\gamma}^\mu \underline{\gamma}^\sigma \Psi_\sigma + \mathrm{h.c.}$$

spin-2 (tensor field)

Alexander, Jenks, McDonough (2020)
Kolb, Ling, AL, & Rosen (2302.04390)

$$\mathscr{L} = \tfrac{1}{2} \nabla h_{\mu\nu} \nabla h^{\mu\nu} - \tfrac{1}{2} m^2 h_{\mu\nu} h^{\mu\nu} + \cdots$$

larger reps (Kalb-Ramond)

Capanelli, Jenks, Kolb, McDonough (2023)

(3) A theory of massive spin-2 during inflation

General relativity

Covariant action for metric field $g_{\mu\nu}$

$$S[g_{\mu\nu}] = \int \mathrm{d}^4x \sqrt{-g} \left[\tfrac{1}{2} M_P^2 \, R[g] \right]$$

Linearize around Minkowski spacetime

$$g_{\mu\nu} = \eta_{\mu\nu} + \frac{2}{M_P} h_{\mu\nu}$$

$$S[h_{\mu\nu}] = \int \mathrm{d}^4x \left[-\tfrac{1}{2}\partial_\lambda h_{\mu\nu}\partial^\lambda h^{\mu\nu} + \partial_\mu h^{\nu\lambda}\partial_\nu h^\mu{}_\lambda - \partial_\mu h^{\mu\nu}\partial_\nu h + \tfrac{1}{2}\partial_\mu h \partial^\mu h + O(h^3) \right]$$

$$(\, h = \eta^{\mu\nu} h_{\mu\nu} \,)$$

Counting degrees of freedom

$$h_{\mu\nu} \sim 16_{\text{components}} - 6_{\text{symmetric}} - 4_{\text{gauge}} - 4_{\text{constraint}} = 2_{\text{dof}}$$

$$(\, \delta h_{\mu\nu} = \partial_\mu \xi_\nu + \partial_\nu \xi_\mu \,) \quad (\text{transverse \& traceless})$$

➔ these are the two polarization modes of the massless graviton (x & + or h = +2, -2)

Adding a mass

Fierz & Pauli (1939)
see also the nice reviews by
Hinterbichler 1105.3735
and de Rham 1401.4173

Try to add mass terms

$$\delta S[h_{\mu\nu}] = \int \mathrm{d}^4x \left[-\tfrac{1}{2} m_1^2 h_{\mu\nu} h^{\mu\nu} - \tfrac{1}{2} m_2^2 h^\mu_\mu h^\nu_\nu \right]$$

A poor choice of these mass parameters leads to a theory with a ghost (in addition to a massive spin-2)

$$m^2_{\mathrm{ghost}} = -\frac{1}{2} \frac{m_1^2 + 4m_2^2}{m_1^2 + m_2^2}$$

(Boulware-Deser ghost)

A clever choice of parameters avoids the ghost and yields a healthy theory of massive spin-2 field

$$S_{\mathrm{FP}}[h_{\mu\nu}] = \int \mathrm{d}^4x \left[-\tfrac{1}{2}\partial_\lambda h_{\mu\nu}\partial^\lambda h^{\mu\nu} + \partial_\mu h^{\nu\lambda}\partial_\nu h^\mu_{\ \lambda} - \partial_\mu h^{\mu\nu}\partial_\nu h + \tfrac{1}{2}\partial_\mu h \partial^\mu h - \tfrac{1}{2} m^2 \left(h_{\mu\nu}h^{\mu\nu} - h^2 \right) \right]$$

(Fierz-Pauli action)

$$h_{\mu\nu} \sim 16_{\mathrm{components}} - 6_{\mathrm{symmetric}} - 1_{\mathrm{gauge}} - 4_{\mathrm{constraint}} = 5_{\mathrm{dof}}$$

➔ the five polarization modes of a massive graviton (helicity = -2, -1, 0, +1, +2)

Going to FRW background – failed attempt

(Fierz-Pauli action)

$$S_{\rm FP}[h_{\mu\nu}] = \int \mathrm{d}^4x \left[-\tfrac{1}{2}\partial_\lambda h_{\mu\nu}\partial^\lambda h^{\mu\nu} + \partial_\mu h^{\nu\lambda}\partial_\nu h^\mu_{\ \lambda} - \partial_\mu h^{\mu\nu}\partial_\nu h + \tfrac{1}{2}\partial_\mu h \partial^\mu h - \tfrac{1}{2}m^2\left(h_{\mu\nu}h^{\mu\nu} - h^2\right)\right]$$

try promoting Minkowski derivatives to FRW covariant derivatives

$$\nabla_\lambda h_{\mu\nu} = \partial_\lambda h_{\mu\nu} - \Gamma^\rho_{\lambda\mu} h_{\rho\nu} - \Gamma^\rho_{\lambda\nu} h_{\mu\rho}$$

$$S[h_{\mu\nu}] = \int \mathrm{d}^4x \left[-\tfrac{1}{2}\nabla_\lambda h_{\mu\nu}\nabla^\lambda h^{\mu\nu} + \nabla_\mu h^{\nu\lambda}\nabla_\nu h^\mu_{\ \lambda} - \nabla_\mu h^{\mu\nu}\nabla_\nu h + \tfrac{1}{2}\nabla_\mu h \nabla^\mu h - \tfrac{1}{2}m^2\left(h_{\mu\nu}h^{\mu\nu} - h^2\right)\right]$$

This procedure would re-introduce the Boulware-Deser ghost. Going to an FRW bkg without also introducing the matter sector is a violation of gauge symmetry. $(\delta h_{\mu\nu} = \partial_\mu \xi_\nu + \partial_\nu \xi_\mu)$

Successful attempt

Let's add a matter sector

$$S[g_{\mu\nu}] = \int \mathrm{d}^4x \sqrt{-g} \left[\tfrac{1}{2} M_P^2 R[g] - \tfrac{1}{2} g^{\mu\nu} \nabla_\mu \phi \nabla_\nu \phi - V(\phi) \right]$$

Linearize around an FRW background

$$g_{\mu\nu} = \bar{g}_{\mu\nu}^{(\mathrm{FRW})} + \frac{2}{M_P} u_{\mu\nu} \quad \text{and} \quad \phi = \bar{\phi}^{(\mathrm{FRW})} + \varphi_u$$

Resulting quadratic action

$$\begin{aligned}
\mathcal{L}^{(2)}_{\text{massless}} &= \mathcal{L}^{(2)}_{uu} + \mathcal{L}^{(2)}_{u\,\varphi_u} + \mathcal{L}^{(2)}_{\varphi_u\varphi_u} \\
\mathcal{L}^{(2)}_{uu} &= -\tfrac{1}{2}\nabla_\lambda u_{\mu\nu}\nabla^\lambda u^{\mu\nu} + \nabla_\mu u^{\nu\lambda}\nabla_\nu u^\mu{}_\lambda - \nabla_\mu u^{\mu\nu}\nabla_\nu u + \tfrac{1}{2}\nabla_\mu u \nabla^\mu u \\
&\quad + \left(\bar{R}_{\mu\nu} - \tfrac{1}{M_P^2}\nabla_\mu\bar{\phi}\nabla_\nu\bar{\phi}\right)\left(u^{\mu\lambda}u_\lambda^{\ \nu} - \tfrac{1}{2}u^{\mu\nu}u\right)\,, \\
\mathcal{L}^{(2)}_{u\,\varphi_u} &= \tfrac{1}{M_P}\left[\left(\nabla_\mu\bar{\phi}\nabla_\nu\varphi_u + \nabla_\nu\bar{\phi}\nabla_\mu\varphi_u\right)\left(u^{\mu\nu} - \tfrac{1}{2}\bar{g}^{\mu\nu}u\right) - V'(\bar{\phi})\varphi_u u\right]\,, \\
\mathcal{L}^{(2)}_{\varphi_u\varphi_u} &= -\tfrac{1}{2}\nabla_\mu\varphi_u\nabla^\mu\varphi_u - \tfrac{1}{2}V''(\bar{\phi})\varphi_u^2\,.
\end{aligned}$$

(massless spin-2 graviton + inflaton perturbation)

$$\begin{aligned}
\mathcal{L}^{(2)}_{\text{massive}} &= \mathcal{L}^{(2)}_{vv} + \mathcal{L}^{(2)}_{v\,\varphi_v} + \mathcal{L}^{(2)}_{\varphi_v\varphi_v} \\
\mathcal{L}^{(2)}_{vv} &= -\tfrac{1}{2}\nabla_\lambda v_{\mu\nu}\nabla^\lambda v^{\mu\nu} + \nabla_\mu v^{\nu\lambda}\nabla_\nu v^\mu{}_\lambda - \nabla_\mu v^{\mu\nu}\nabla_\nu v + \tfrac{1}{2}\nabla_\mu v \nabla^\mu v \\
&\quad + \left(\bar{R}_{\mu\nu} - \tfrac{1}{M_P^2}\nabla_\mu\bar{\phi}\nabla_\nu\bar{\phi}\right)\left(v^{\mu\lambda}v_\lambda^{\ \nu} - \tfrac{1}{2}v^{\mu\nu}v\right) \\
&\quad - \tfrac{1}{2}m^2\left(v^{\mu\nu}v_{\mu\nu} - v^2\right)\,, \\
\mathcal{L}^{(2)}_{v\,\varphi_v} &= \tfrac{1}{M_P}\left[\left(\nabla_\mu\bar{\phi}\nabla_\nu\varphi_v + \nabla_\nu\bar{\phi}\nabla_\mu\varphi_v\right)\left(v^{\mu\nu} - \tfrac{1}{2}\bar{g}^{\mu\nu}v\right) - V'(\bar{\phi})\varphi_v v\right]\,, \\
\mathcal{L}^{(2)}_{\varphi_v\varphi_v} &= -\tfrac{1}{2}\nabla_\mu\varphi_v\nabla^\mu\varphi_v - \tfrac{1}{2}V''(\bar{\phi})\varphi_v^2\,.
\end{aligned}$$

(massive spin-2 + inflaton perturbation)

Another approach: ghost-free bigravity

Hassan & Rosen (2012)

Field content: two metrics & two scalars

$$g_{\mu\nu} \, , \quad f_{\mu\nu} \, , \quad \phi_g \, , \quad \phi_f$$

A theory of bigravity with a minimal coupling to matter

$$S = \int \mathrm{d}^4 x \left[\tfrac{1}{2} M_g^2 \sqrt{-g}\, R[g] \; + \; \tfrac{1}{2} M_f^2 \sqrt{-f}\, R[f] \right.$$ (metric kinetic terms)

$$- \, m^2 M_*^2 \sqrt{-g}\, V(\mathbb{X};\, \beta_n)$$ (metric interactions)

$$\left. + \sqrt{-g}\, \mathcal{L}_g(g, \phi_g) \; + \; \sqrt{-f}\, \mathcal{L}_f(f, \phi_f) \right]$$ (coupling to matter)

Matter-sector Lagrangians

$$\mathcal{L}_g(g, \phi_g) = -\tfrac{1}{2} g^{\mu\nu} \nabla_\mu \phi_g \nabla_\nu \phi_g - V_g(\phi_g)$$

$$\mathcal{L}_f(f, \phi_f) = -\tfrac{1}{2} f^{\mu\nu} \nabla_\mu \phi_f \nabla_\nu \phi_f - V_f(\phi_f)$$

$$\left(\begin{array}{c} M_*^{-2} = M_g^{-2} + M_f^{-2} \\ M_P^2 = M_g^2 + M_f^2 \end{array} \right)$$

(4) CGPP of massive spin-2 particles

Separate out the 5 different polarization modes

Perform a scalar-vector-tensor (SVT) decomposition

$$v_{\mu\nu}(\eta, \boldsymbol{x}) \sim \text{massive spin-2}$$
$$\sim (\text{helicity } \lambda = \pm 2) \oplus (\text{helicity } \lambda = \pm 1) \oplus (\text{helicity } \lambda = 0)$$

Tensor sector

$$\chi''_{k,\lambda}(\eta) + \omega_k^2(\eta)\, \chi_{k,\lambda}(\eta) = 0 \qquad \text{for } \lambda = \pm 2$$

$$\omega_k^2(\eta) = k^2 + a^2 m^2 + \tfrac{1}{6} a^2 R$$

Vector sector

$$\chi''_{k,\lambda}(\eta) + \omega_k^2(\eta)\, \chi_{k,\lambda}(\eta) = 0 \qquad \text{for } \lambda = \pm 1$$

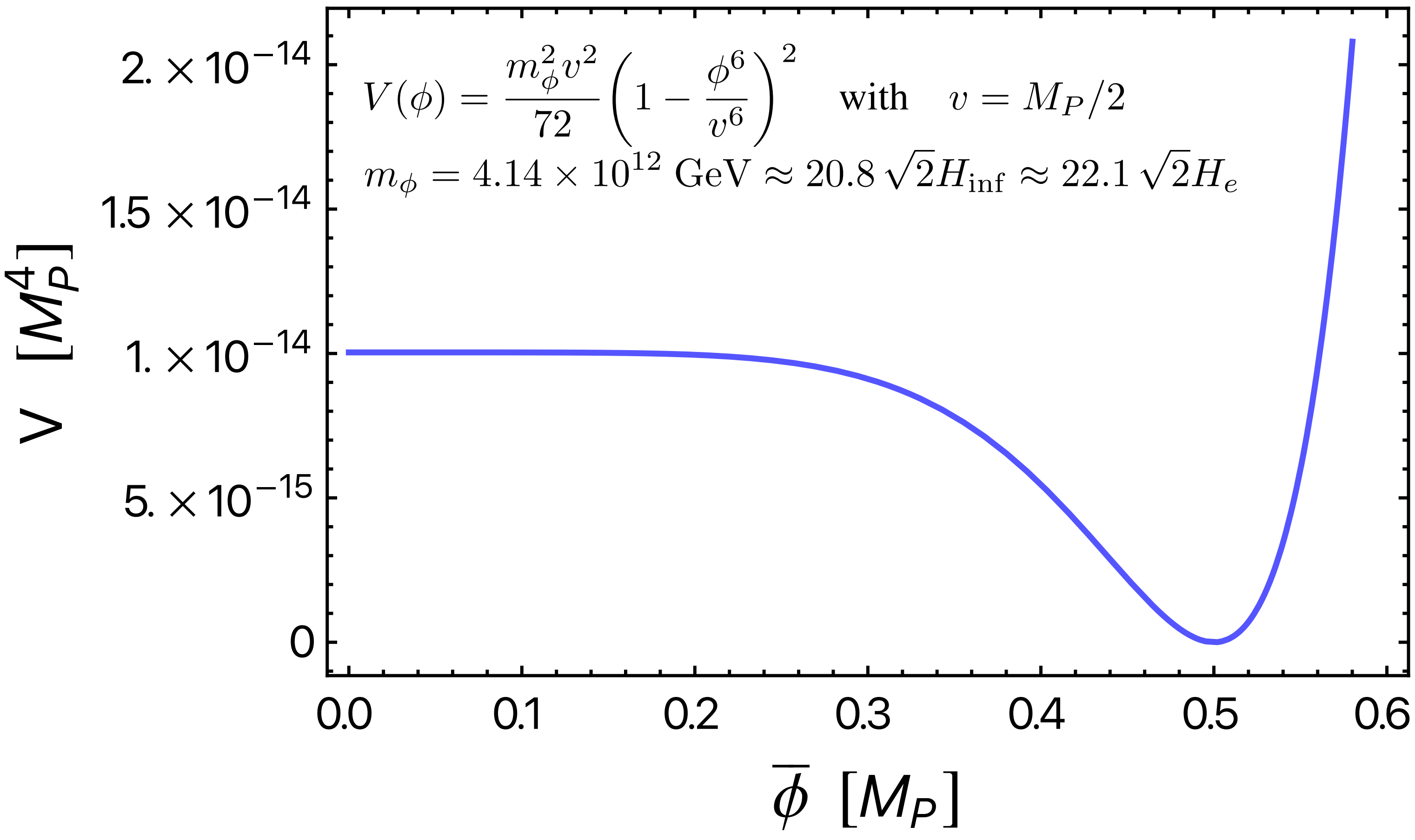
$$\omega_k^2(\eta) = k^2 + a^2 m^2 - f''/f \quad \text{where} \quad f = a^2/\sqrt{k^2 + a^2 m^2}$$

Scalar sector – it's complicated!

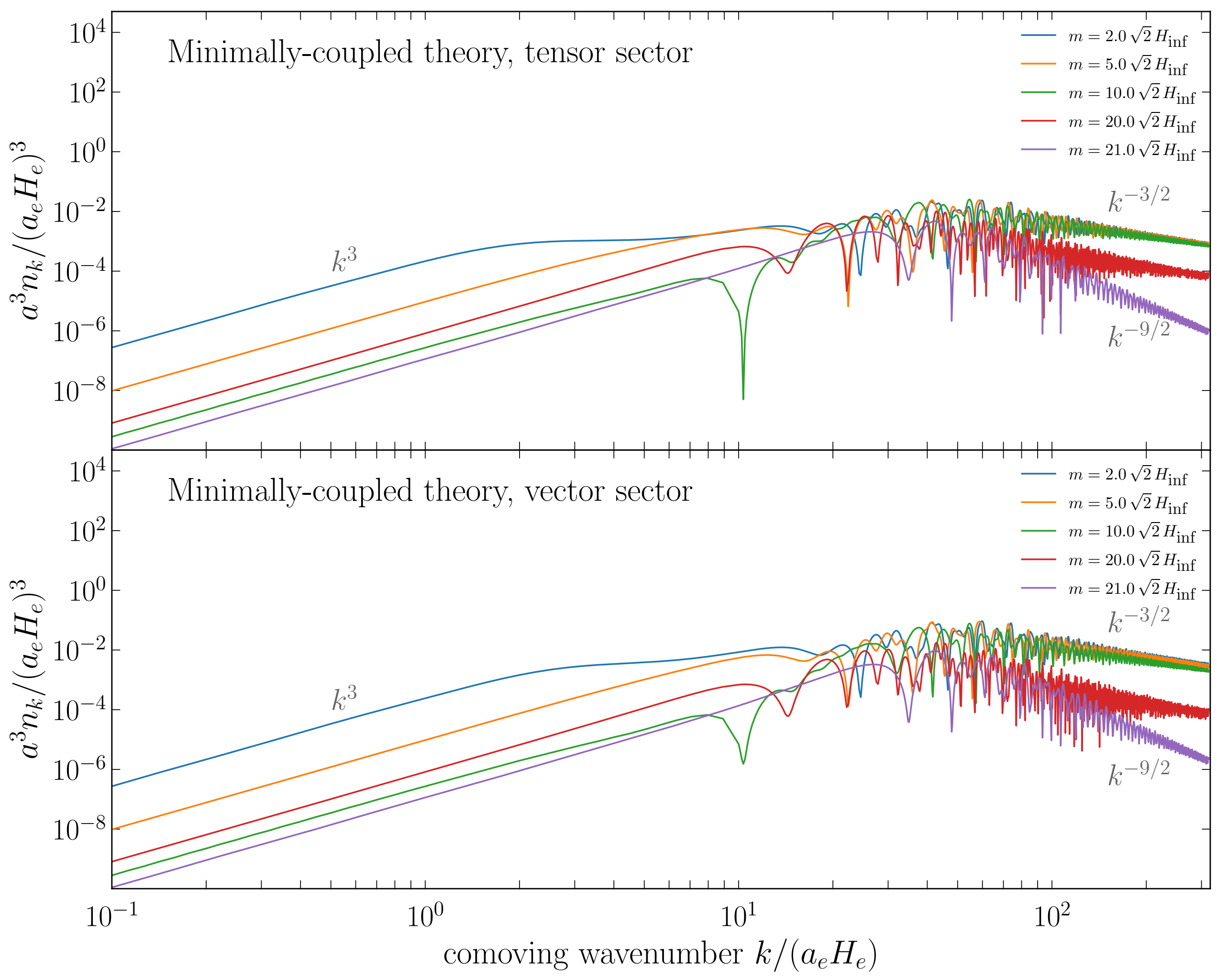
$$L_{S,\boldsymbol{k}} = K_\Pi\, |\tilde{\Pi}'|^2 + M_\Pi\, |\tilde{\Pi}|^2 + K_\mathcal{B}\, |\tilde{\mathcal{B}}'|^2 + M_\mathcal{B}\, |\tilde{\mathcal{B}}|^2 + \lambda_1\, \tilde{\Pi}^* \tilde{\mathcal{B}}' + \lambda_0\, \tilde{\Pi}^* \tilde{\mathcal{B}}$$

Hilltop inflation

a hilltop model of inflation



CGPP for tensor & vector sectors

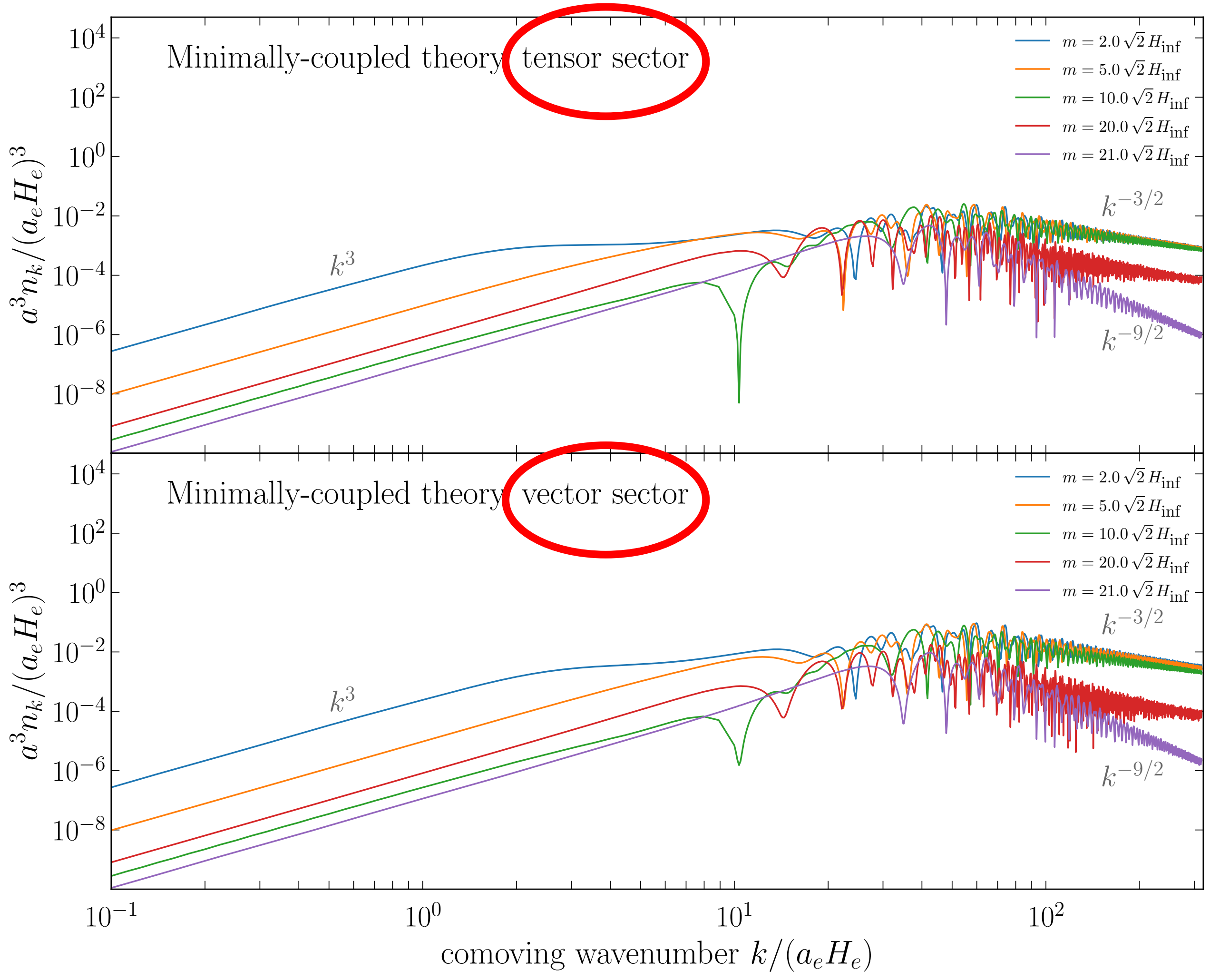


Notable features:

1. Similar results for tensors & vectors
2. Low-k power law ~ k^3
3. High-k power law ~ $k^{-3/2}$ or $k^{-9/2}$
4. Wiggles!

see 2302.04390

CGPP for tensor & vector sectors



Notable features:

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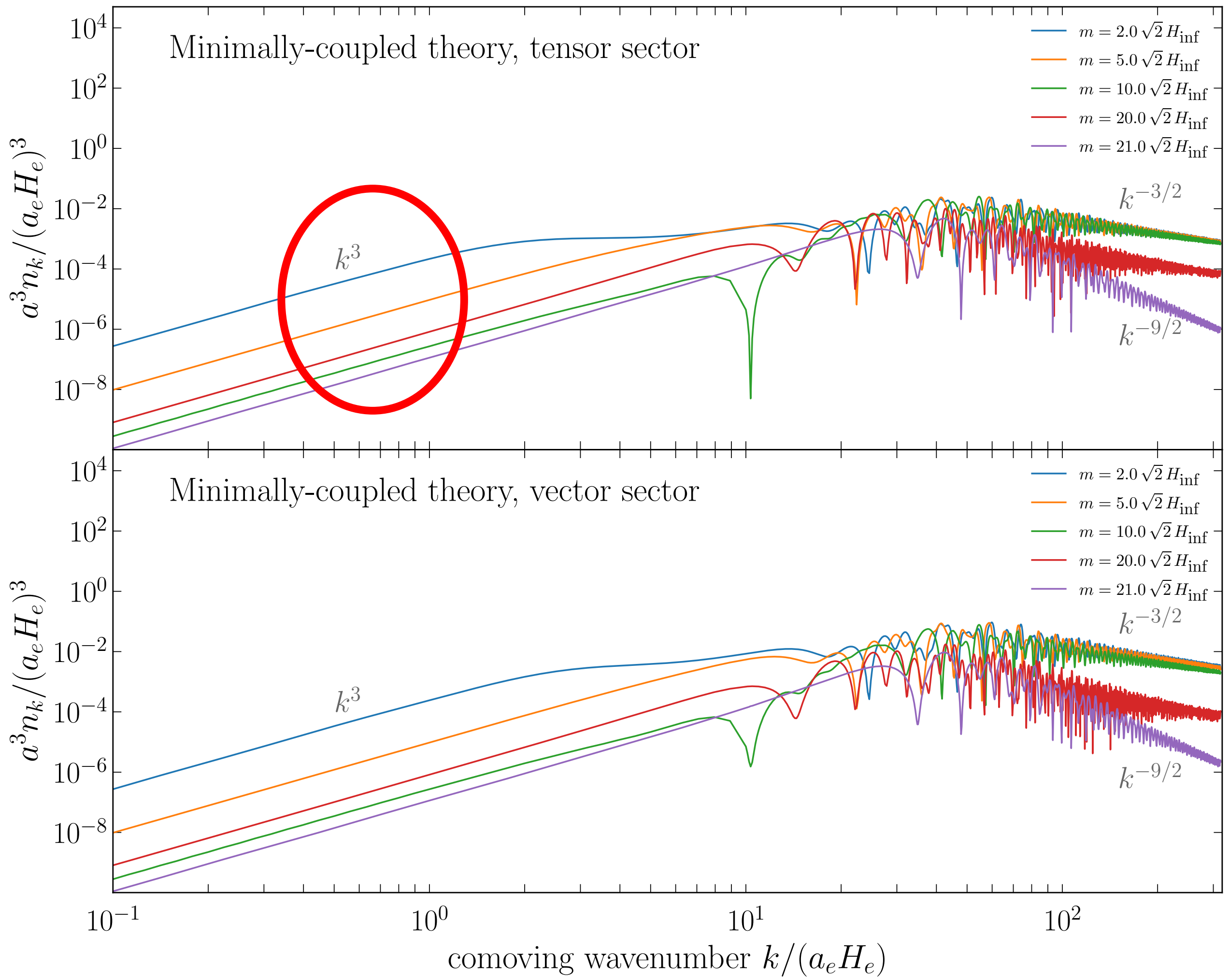
tensor sector: $\omega_k^2(\eta) = k^2 + a^2 m^2 + \frac{1}{6} a^2 R$

vector sector: $\omega_k^2(\eta) = k^2 + a^2 m^2 - f''/f$

equal for nonrelativistic modes

see 2302.04390

CGPP for tensor & vector sectors



Notable features:

1. Similar results for tensors & vectors
2. Low-k power law ~ k^3
3. High-k power law ~ $k^{-3/2}$ or $k^{-9/2}$
4. Wiggles!

$$n_k \propto k^\nu \quad \text{with} \quad \nu = 3 - 2\left[\frac{9}{4} - \frac{m^2}{H_{\rm inf}^2}\right]^{1/2}$$

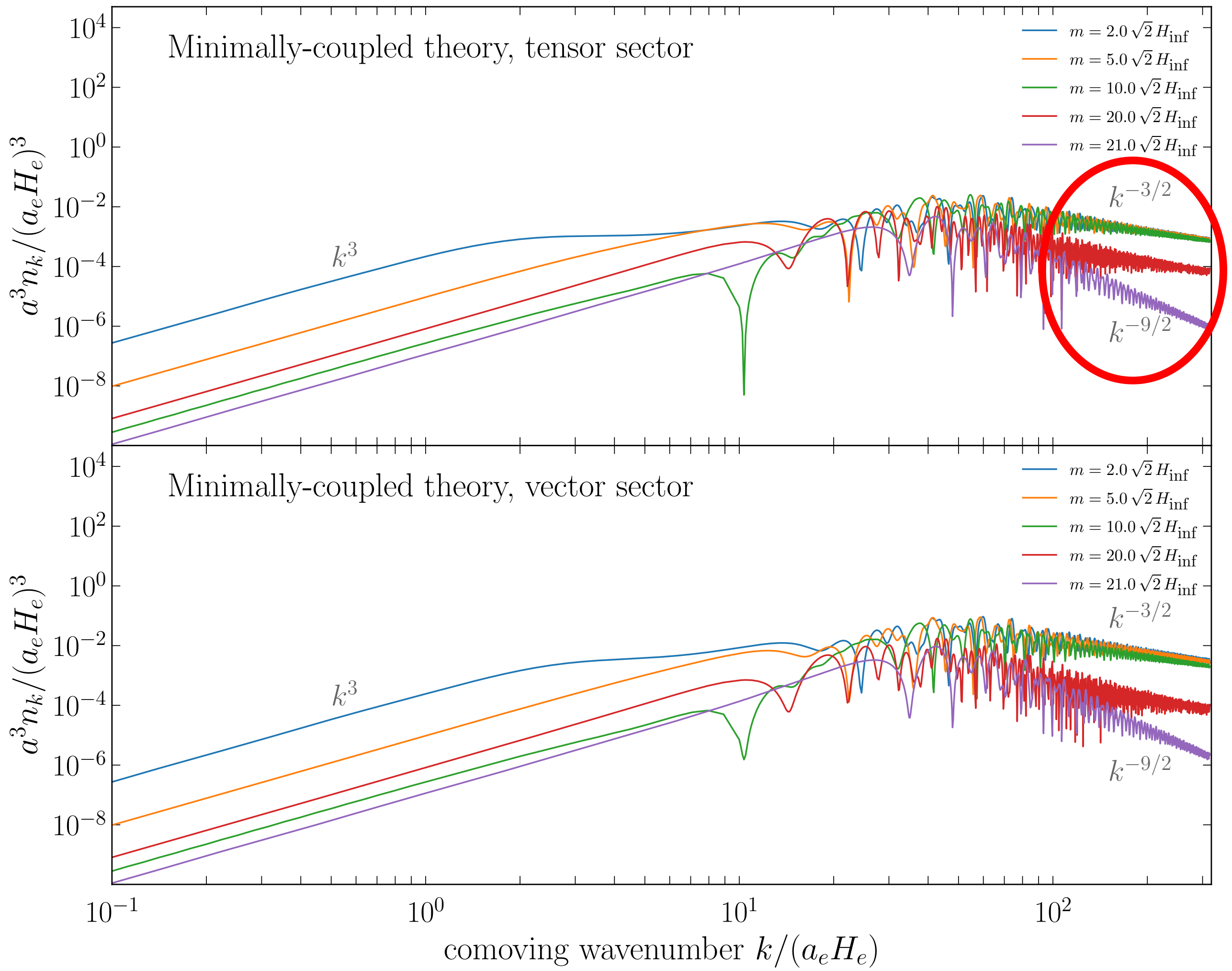
$$\mathrm{Re}[\nu] = 3 \quad \text{for } m > \frac{3}{2} H_{\rm inf}$$

low-k modes have familiar dS solution

see 2302.04390

CGPP for tensor & vector sectors

[Ema, Nakayama, & Tang (2018)]
[Chung, Kolb, AL (2018)]



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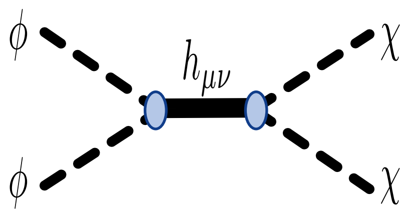


inflaton annihilation

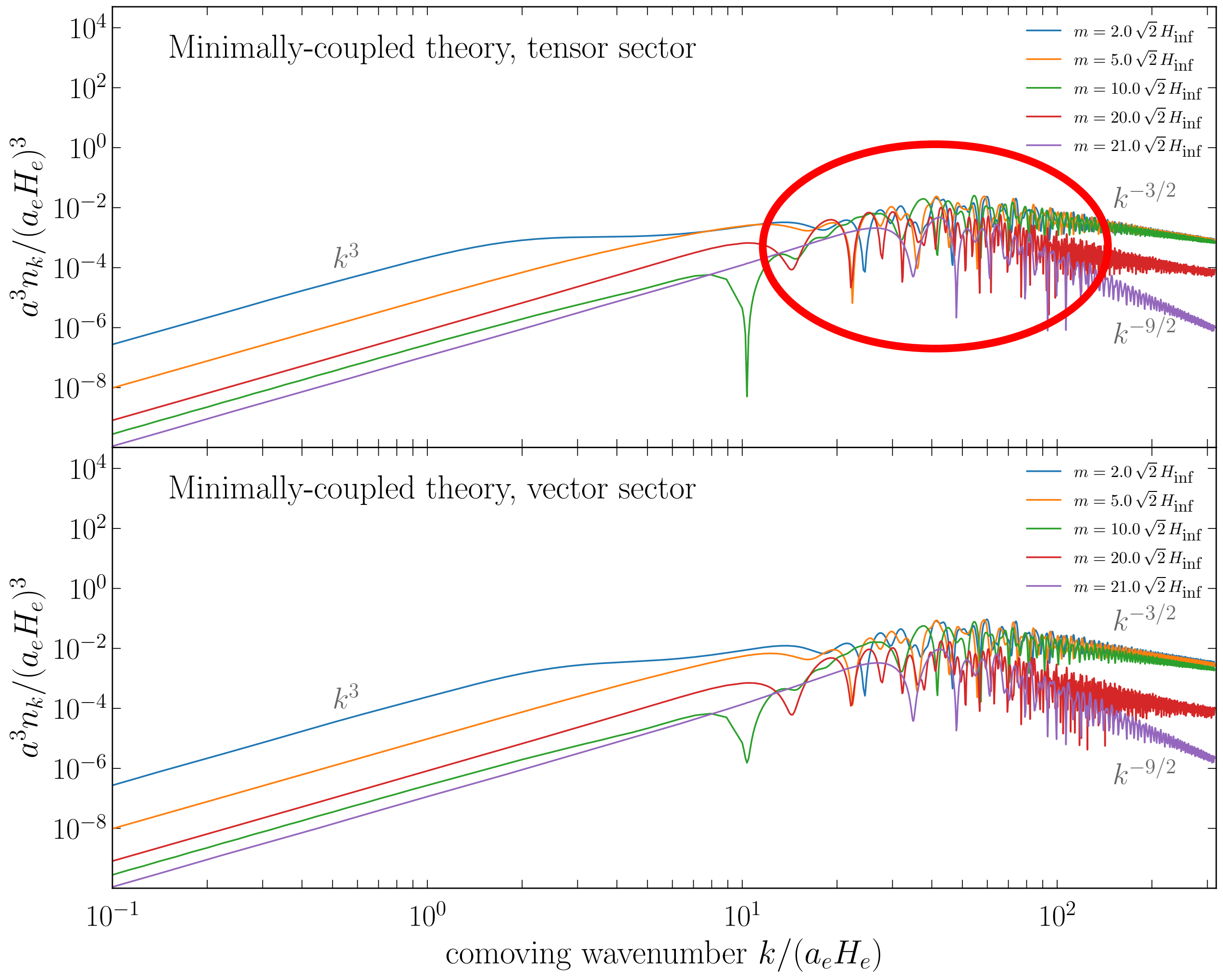
$$\Gamma_{\phi\phi\to\chi\chi} \simeq \frac{C}{16\pi}\frac{\Phi^2}{M_P^4}m_\phi^4$$

$$\phi\phi \to \chi\chi \quad \Rightarrow \quad n_k \propto k^{-3/2}$$
$$\phi\phi\phi \to \chi\chi \quad \Rightarrow \quad n_k \propto k^{-9/2}$$
$$n\phi \to \chi\chi \quad \Rightarrow \quad n_k \propto k^{-3(2n-3)/2}$$

see 2302.04390

CGPP for tensor & vector sectors

[Basso, Chung, Kolb, AL (2022)]



Notable features:

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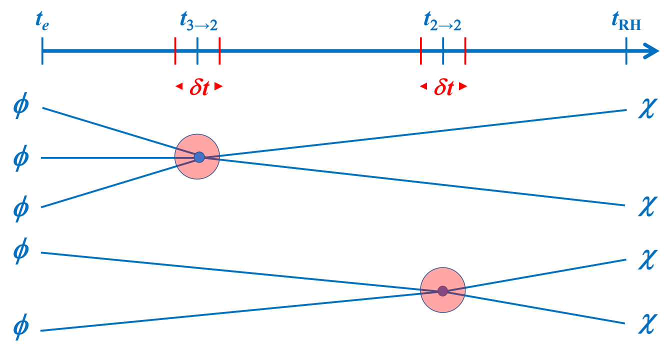
2. Low-k power law ~ k^3
3. High-k power law ~ $k^{-3/2}$ or $k^{-9/2}$
4. Wiggles!

interference between annihilation channels



What about the scalar sector?

(longitudinal polarization: $\lambda = 0$)

Scalar sector

Scalar metric perturbations mix with scalar inflaton perturbation

$$L_S = \text{a messy function of } A,\ B,\ E,\ F, \text{ and } \varphi_v$$

After imposing constraints (and a LOT of algebra) there are only two propagating degrees of freedom

$$L_{S,\boldsymbol{k}} = K_\Pi\,|\tilde{\Pi}'|^2 + M_\Pi\,|\tilde{\Pi}|^2 + K_{\mathcal{B}}\,|\tilde{\mathcal{B}}'|^2 + M_{\mathcal{B}}\,|\tilde{\mathcal{B}}|^2 + \lambda_1\,\tilde{\Pi}^*\tilde{\mathcal{B}}' + \lambda_0\,\tilde{\Pi}^*\tilde{\mathcal{B}}$$

$$K_\varphi = \frac{a^2}{2}\frac{H^2k^4 + 3a^2(m^2 - m_H^2)H^2k^2 + \frac{9}{4}a^4m^2(m^2 - m_H^2)H^2}{H^2k^4 + 3a^2(m^2 - m_H^2)H^2k^2 + \frac{3}{8}a^4m^2(6m^2H^2 - 4H^2m_H^2 - m_H^4)} \tag{3.17a}$$

$$M_\varphi = \frac{a^2}{2}\frac{c_{10}k^{10} + c_8k^8 + c_6k^6 + c_4k^4 + c_2k^2 + c_0}{\left[H^2k^4 + 3a^2(m^2 - m_H^2)H^2k^2 + \frac{3}{8}a^4m^2(6m^2H^2 - 4H^2m_H^2 - m_H^4)\right]^2} \tag{3.17b}$$

$$\begin{aligned}
c_{10} &= H^4 \\
c_8 &= \tfrac{1}{2}a^2H^2\Big[\big(12m^2H^2 + 8H^4 - 14H^2m_H^2 - m_H^4\big) + 4\tfrac{HV'(\bar\phi)\bar\phi'}{aM_P^2} + 2H^2V''(\bar\phi)\Big] \\
c_6 &= \tfrac{3}{8}a^4H^2\Big[\big(36m^4H^2 + 72m^2H^4 - 82m^2H^2m_H^2 - 64H^4m_H^2 \\
&\qquad - 7m^2m_H^4 + 40H^2m_H^4 + 8m_H^6\big) \\
&\qquad + 8\big(3m^2 - 4m_H^2\big)\tfrac{HV'(\bar\phi)\bar\phi'}{aM_P^2} \\
&\qquad + 16\big(m^2 - m_H^2\big)H^2V''(\bar\phi)\Big] \\
c_4 &= \tfrac{3}{8}a^6\Big[4H^2\big(9m^6H^2 + 36m^4H^4 + 16m^2H^6 - 30m^4H^2m_H^2 - 76m^2H^4m_H^2 \\
&\qquad - 3m^4m_H^4 + 31m^2H^2m_H^4 + 24H^4m_H^4 + 6m^2m_H^6 - 6H^2m_H^6 - 3m_H^8\big) \\
&\qquad - 4m^2H^2\big(H^2 - m_H^2\big)\tfrac{V'(\bar\phi)^2}{M_P^2} \\
&\qquad + \big(36m^4H^2 + 8m^2H^4 - 94m^2H^2m_H^2 + m^2m_H^4 + 48H^2m_H^4\big)\tfrac{HV'(\bar\phi)\bar\phi'}{aM_P^2} \\
&\qquad + \big(36m^4H^2 - 58m^2H^2m_H^2 - m^2m_H^4 + 24H^2m_H^4\big)H^2V''(\bar\phi)\Big] \\
c_2 &= \tfrac{9}{32}a^8m^2\Big[H^2\big(18m^6H^2 + 120m^4H^4 + 128m^2H^6 - 78m^4H^2m_H^2 - 384m^2H^4m_H^2 \\
&\qquad - 9m^4m_H^4 + 132m^2H^2m_H^4 + 128H^4m_H^4 + 23m^2m_H^6 - 32H^2m_H^6 - 16m_H^8\big) \\
&\qquad - 8H^2\big(2m^2H^2 - 2m^2m_H^2 + m_H^4\big)\tfrac{V'(\bar\phi)^2}{M_P^2} \\
&\qquad + 4\big(6m^4H^2 - 22m^2H^2m_H^2 + m^2m_H^4 + 14H^2m_H^4\big)\tfrac{HV'(\bar\phi)\bar\phi'}{aM_P^2} \\
&\qquad + 4\big(m^2 - m_H^2\big)\big(12m^2H^2 - 10H^2m_H^2 - m_H^4\big)H^2V''(\bar\phi)\Big] \\
c_0 &= \tfrac{27}{32}a^{10}m^4\Big[-2H^2\big(2m^2H^2 - 2m^2m_H^2 + m_H^4\big)\tfrac{V'(\bar\phi)^2}{M_P^2} \\
&\qquad - m^2\big(2H^2 - m_H^2\big)\big(4H^2 + m_H^2\big)\tfrac{HV'(\bar\phi)\bar\phi'}{aM_P^2} \\
&\qquad + \big(m^2 - m_H^2\big)\big(6m^2H^2 - 4H^2m_H^2 - m_H^4\big)H^2V''(\bar\phi)\Big]
\end{aligned}$$

$$K_B = \frac{a^6m^2}{8}\frac{\big(8m^2H^2 - 6H^2m_H^2 - m^2m_H^2\big)k^4}{H^2k^4 + 3a^2(m^2 - m_H^2)H^2k^2 + \frac{3}{8}a^4m^2(6m^2H^2 - 4H^2m_H^2 - m_H^4)} \tag{3.17c}$$

$$M_B = \frac{a^6m^2}{8}\frac{c_{10}k^{10} + c_8k^8 + c_6k^6 + c_4k^4}{\left[H^2k^4 + 3a^2(m^2 - m_H^2)H^2k^2 + \frac{3}{8}a^4m^2(6m^2H^2 - 4H^2m_H^2 - m_H^4)\right]^2} \tag{3.17d}$$

$$\begin{aligned}
c_{10} &= H^2\big(8m^2H^2 - 8H^4 - 2H^2m_H^2 - m^2m_H^2\big) \\
c_8 &= a^2H^2\Big[\big(30m^4H^2 + 32m^2H^4 - 96H^6 - 3m^4m_H^2 - 56m^2H^2m_H^2 \\
&\qquad + 48H^4m_H^2 + 5m^2m_H^4 + 6H^2m_H^4\big) \\
&\qquad + \big(4m^2 - 24H^2\big)\tfrac{HV'(\bar\phi)\bar\phi'}{aM_P^2}\Big] \\
c_6 &= \tfrac{3}{8}a^4m^2\Big[\big(96m^4H^4 + 144m^2H^6 - 6m^4H^2m_H^2 - 252m^2H^4m_H^2 - 192H^6m_H^2 \\
&\qquad + 8m^2H^2m_H^4 + 200H^4m_H^4 - 10H^2m_H^6 - m^2m_H^6\big) \\
&\qquad + \big(8m^2m_H^2 - 16H^2m_H^2\big)\tfrac{HV'(\bar\phi)\bar\phi'}{aM_P^2}\Big] \\
c_4 &= \tfrac{3}{8}a^6m^4\Big[\big(36m^4H^4 - 48m^2H^6 + 64H^8 - 12m^2H^4m_H^2 - 32H^6m_H^2 \\
&\qquad - 12m^2H^2m_H^4 + 4H^4m_H^4 + 12H^2m_H^6 - 3m^2m_H^6 + 2m_H^8\big) \\
&\qquad - \big(24m^2H^2 - 16H^4 - 12m^2m_H^2 - 8H^2m_H^2 + 8m_H^4\big)\tfrac{HV'(\bar\phi)\bar\phi'}{aM_P^2}\Big]
\end{aligned}$$

$$L_2 = \frac{a^3m^2\bar\phi'}{2M_PH}\frac{H^2k^4 + \frac{3}{2}a^2(m^2 - m_H^2)H^2k^2}{H^2k^4 + 3a^2(m^2 - m_H^2)H^2k^2 + \frac{3}{8}a^4m^2(6m^2H^2 - 4H^2m_H^2 - m_H^4)} \tag{3.17e}$$

$$L_1 = -\frac{a^4m^2\bar\phi'}{M_P}\frac{\big(H^2 - \frac{1}{4}m_H^2 - \frac{1}{2}\frac{aHV'(\bar\phi)}{\bar\phi'}\big)k^4 - \frac{3}{2}a^2(m^2 - m_H^2)\big(H^2 + \frac{1}{4}m_H^2 + \frac{1}{2}\frac{aHV'(\bar\phi)}{\bar\phi'}\big)k^2}{H^2k^4 + 3a^2(m^2 - m_H^2)H^2k^2 + \frac{3}{8}a^4m^2(6m^2H^2 - 4H^2m_H^2 - m_H^4)} \tag{3.17f}$$

$$L_0 = \frac{a^3m^2\bar\phi'}{2M_PH}\frac{c_{10}k^{10} + c_8k^8 + c_6k^6 + c_4k^4 + c_2k^2}{\left[H^2k^4 + 3a^2(m^2 - m_H^2)H^2k^2 + \frac{3}{8}a^4m^2(6m^2H^2 - 4H^2m_H^2 - m_H^4)\right]^2} \tag{3.17g}$$

$$\begin{aligned}
c_{10} &= H^4 \qquad\qquad (3.17\text{h}) \\
c_8 &= \tfrac{1}{2}a^2H^4\Big[\big(9m^2 + 12H^2 - 13m_H^2\big) - 4\tfrac{aHV'(\bar\phi)}{\bar\phi'}\Big] \\
c_6 &= \tfrac{3}{8}a^4H^2\Big[\big(18m^4H^2 + 32m^2H^4 + 64H^6 - 48m^2H^2m_H^2 - 64H^4m_H^2 \\
&\qquad + m^2m_H^4 + 28H^2m_H^4\big) \\
&\qquad + 8\big(-4m^2H^2 + 4H^4 + m^2m_H^2\big)\tfrac{aHV'(\bar\phi)}{\bar\phi'}\Big] \\
c_4 &= \tfrac{3}{16}a^6m^2H^2\Big[\big(18m^4H^2 - 24m^2H^4 + 256H^6 - 54m^2H^2m_H^2 - 160H^4m_H^2 \\
&\qquad + 9m^2m_H^4 + 60H^2m_H^4 - 7m_H^6\big) \\
&\qquad + 4\big(-30m^2H^2 + 32H^4 + 12m^2m_H^2 + 4H^2m_H^2 - 7m_H^4\big)\tfrac{aHV'(\bar\phi)}{\bar\phi'}\Big] \\
c_2 &= \tfrac{9}{16}a^8m^4H^2\big(2H^2 - m_H^2\big)\Big[-\big(4H^2 + m_H^2\big)\big(3m^2 - 4H^2 - m_H^2\big) \\
&\qquad + 4\big(-3m^2 + 2H^2 + 2m_H^2\big)\tfrac{aHV'(\bar\phi)}{\bar\phi'}\Big]
\end{aligned}$$

Scalar sector

Scalar metric perturbations mix with scalar inflaton perturbation

$$L_S = \text{a messy function of } A,\ B,\ E,\ F, \text{ and } \varphi_v$$

After imposing constraints (and a LOT of algebra) there are only two propagating degrees of freedom

$$L_{S,\boldsymbol{k}} = K_\Pi\,|\tilde{\Pi}'|^2 + M_\Pi\,|\tilde{\Pi}|^2 + K_\mathcal{B}\,|\tilde{\mathcal{B}}'|^2 + M_\mathcal{B}\,|\tilde{\mathcal{B}}|^2 + \lambda_1\,\tilde{\Pi}^*\tilde{\mathcal{B}}' + \lambda_0\,\tilde{\Pi}^*\tilde{\mathcal{B}}$$

The second kinetic term coefficient is

$$K_\mathcal{B} = \frac{3a^6m^2(m^2 - m_H^2)}{4k^4 + 12a^2(m^2 - m_H^2)k^2 + 9a^4m^2(m^2 - m_H^2)}$$

and where we've defined: $m_H^2(\eta) = 2H(\eta)^2\left[1 - \epsilon(\eta)\right]$ where $\epsilon(\eta) = -\dot{H}/H^2$

Beware of ghosts

Higuchi (1986)
see also: Fasiello & Tolley (2013)

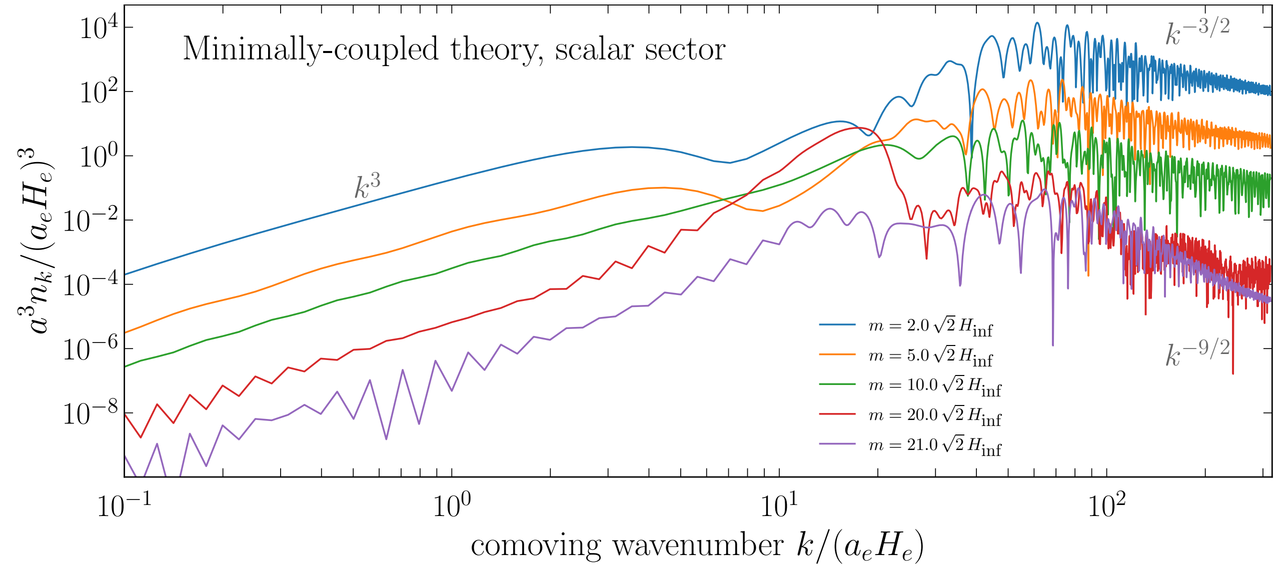
A wrong-sign kinetic term leads to dangerous ghosts!

For massive spin-2 particles in FRW spacetime, ghost avoidance requires:

$$\boxed{m^2 > m_H^2(\eta)} = 2H(\eta)^2\left[1-\epsilon(\eta)\right] \quad \text{where} \quad \epsilon(\eta) = -\dot{H}/H^2$$

- ➔ Generalizes the Higuchi bound (for dS) to FRW spacetime
- ➔ After inflation $\varepsilon > 1$ and any positive m^2 is ghost-free
- ➔ Implications for ultra-light spin-2 dark matter (e.g., time-dep mass)
- ➔ Implications for Kaluza-Klein (compact extra dimensions)
- ➔ Our numerical analysis focuses on $m^2 > 2\,H_{inf}^2$ to avoid the ghost

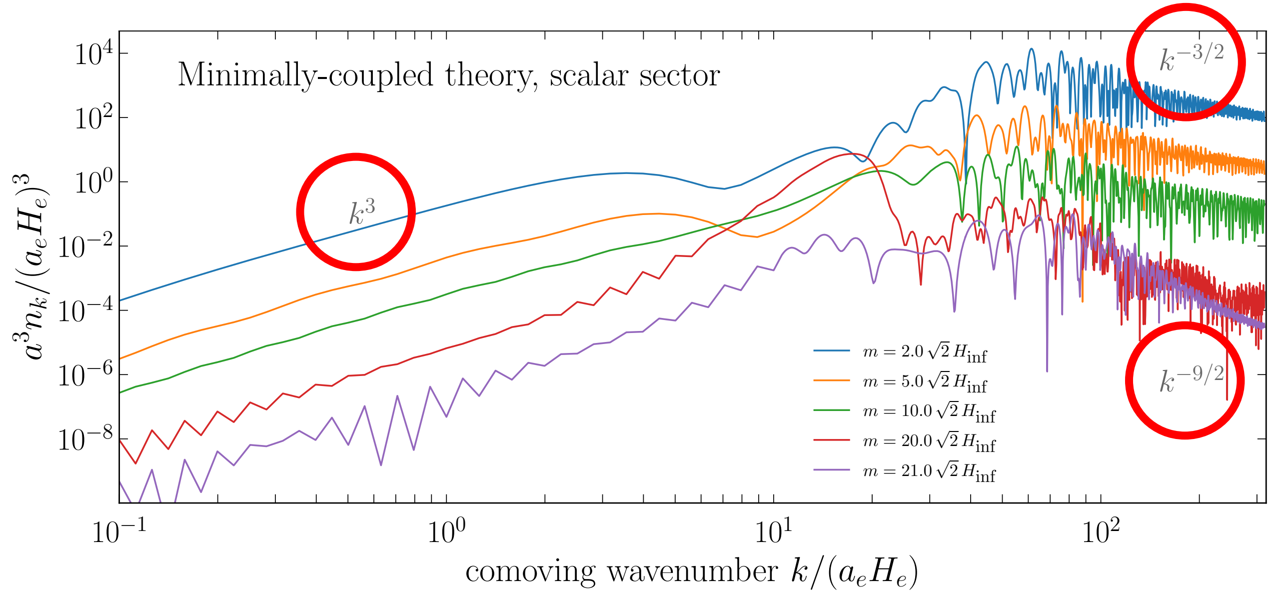
Scalar sector - spectra



Notable features:

1. Same power laws & wiggles as T/V
2. Lowering mass raises amplitude

Scalar sector - spectra



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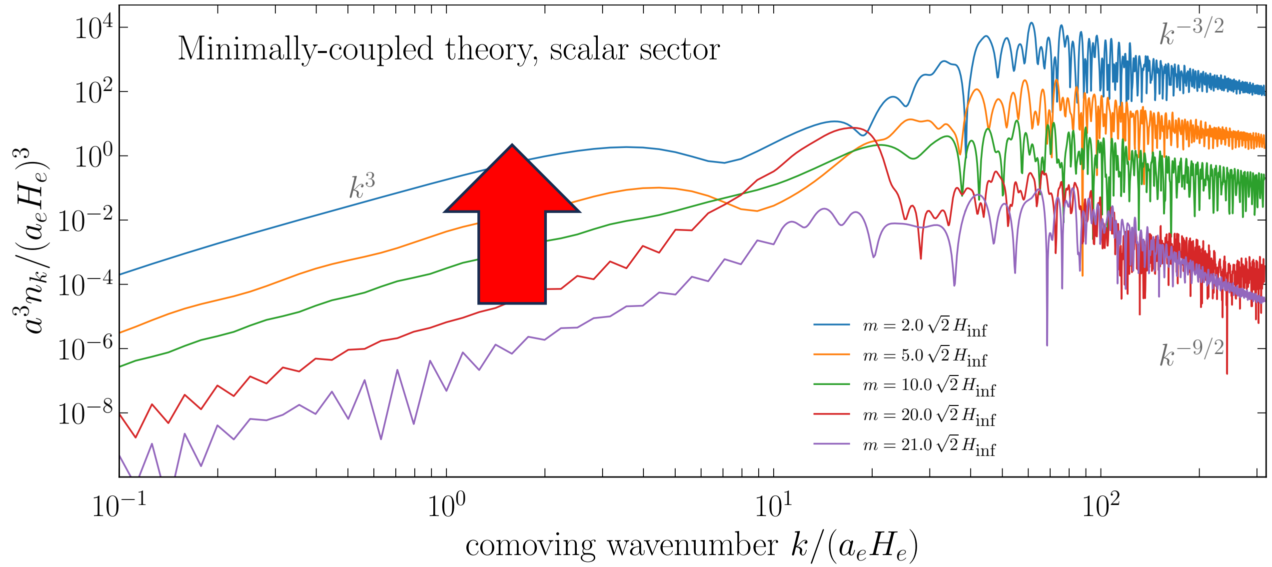
Scalar sector - spectra



Notable features:

1. Same power laws & wiggles as T/V
2. Lowering mass raises amplitude

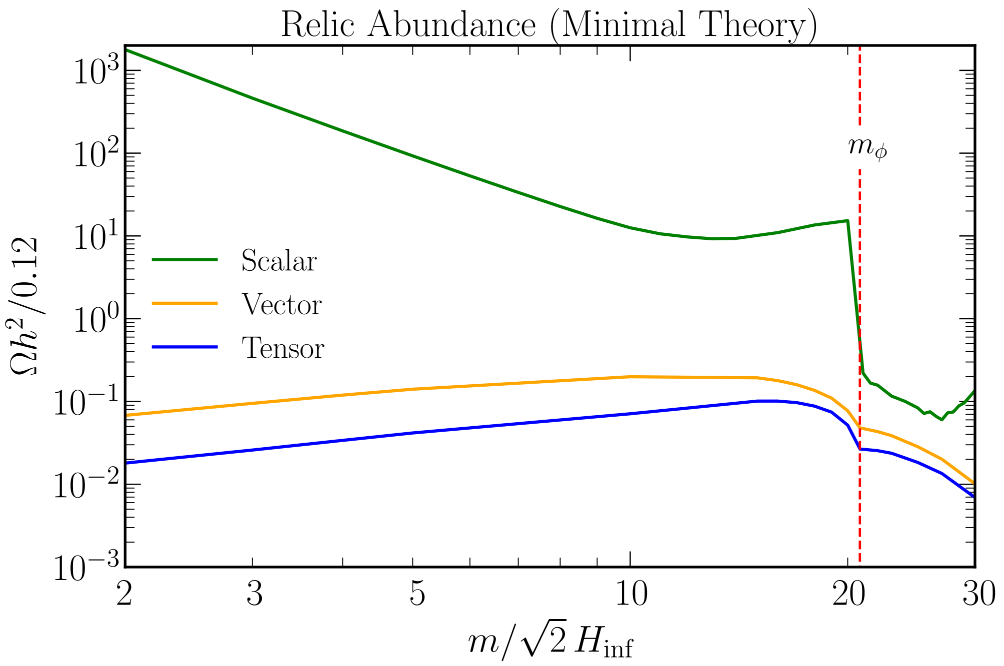
Implications for spin-2 dark matter

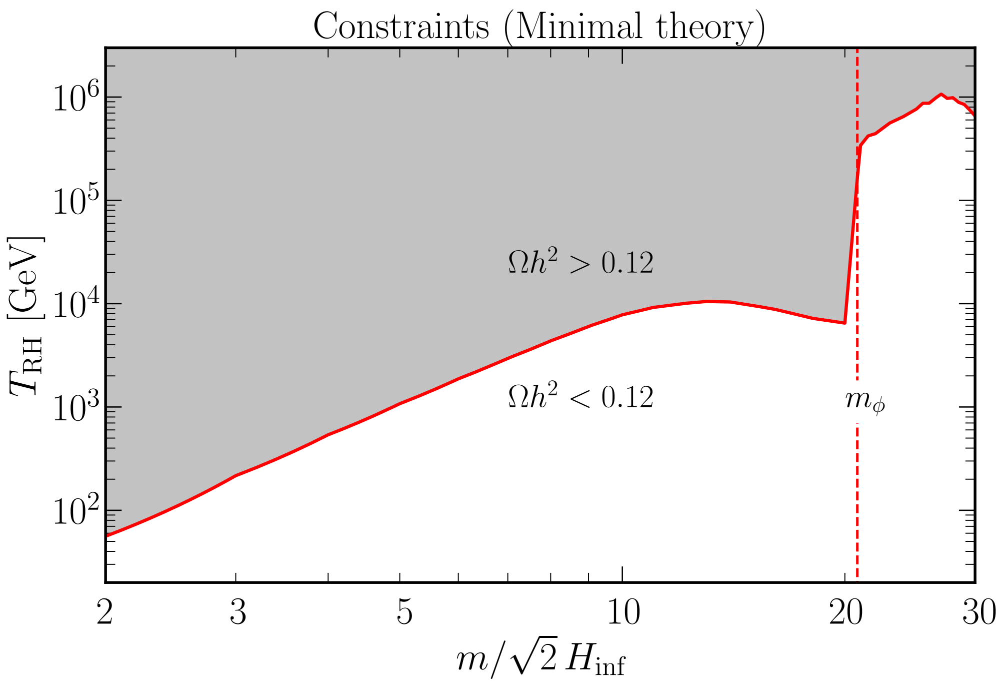
see also: Babichev et. al. (2016)

Assume: massive spin-2 particles are cosmologically long-lived

Relic abundance

$$\Omega h^2 \approx (0.114) \left(\frac{m}{10^{10}\ \mathrm{GeV}}\right) \left(\frac{H_e}{10^{10}\ \mathrm{GeV}}\right) \left(\frac{T_{\mathrm{RH}}}{10^{8}\ \mathrm{GeV}}\right) \left(\frac{a^3 n}{a_e^3 H_e^3}\right)$$





(5) Summary & discussion

Summary

Questions: If dark matter only interacts gravitationally, how was it produced? What if it has spin-2?

Things I talked about:

- review: CGPP = quantum excitations of a spectator field during/after inflation
- review: CGPP = graviton-mediated inflaton annihilation after inflation
- We showed: interference effects lead to fringes ("wiggles") in the CGPP spectrum
- We developed: a theory of massive spin-2 particles on an FRW background (bigravity)
- We calculated: predicted spectrum & relic abundance of massive spin-2 particles
  - CGPP of massive spin-2 particles can account for all the dark matter
- As a by-product: we derived an FRW-generalization of the Higuchi bound

Things to talk about:

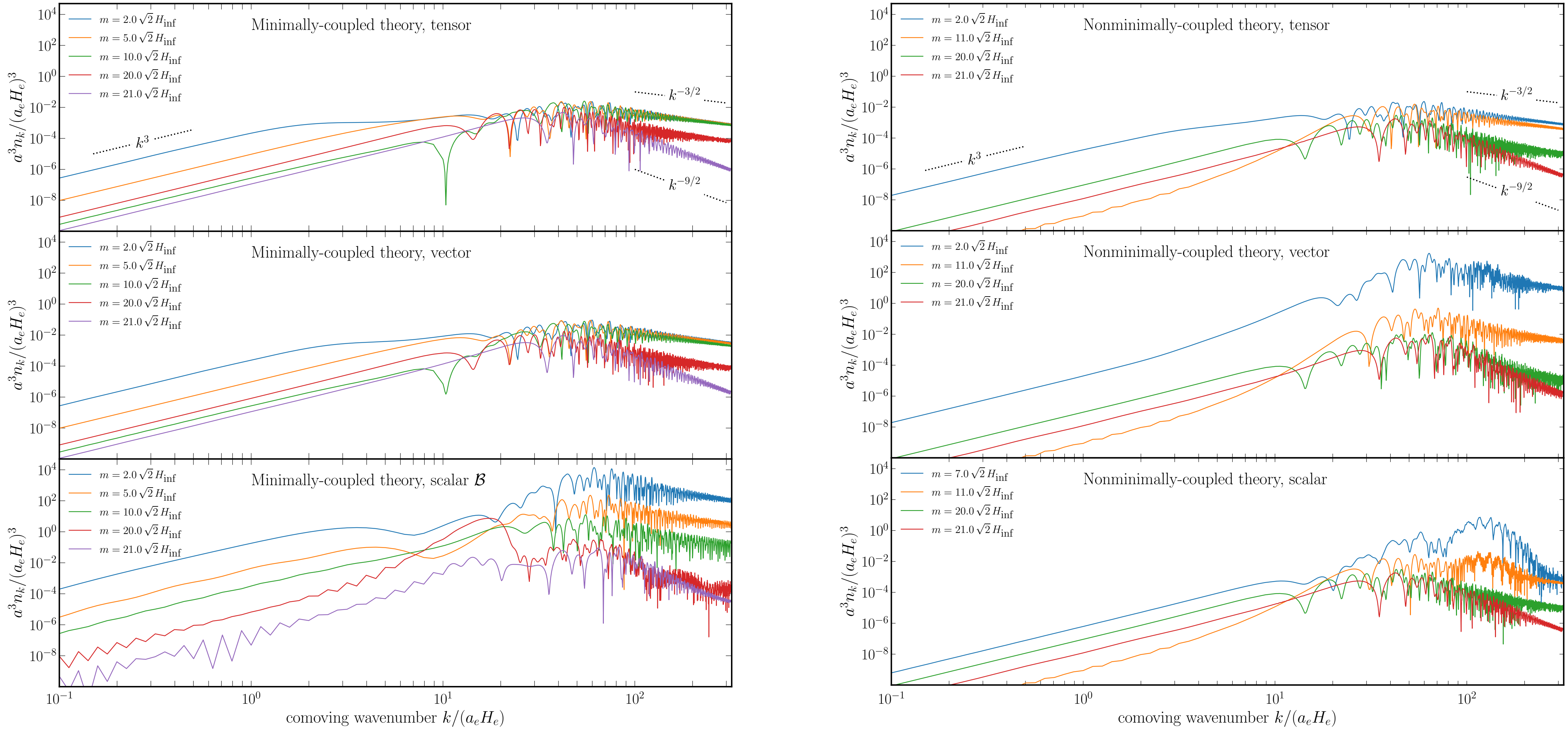
- CGPP for massive spin-2 particles in Kaluza-Klein, hadronic resonances, Regge trajectories?
- The dynamics of a system that approaches (crosses?) the FRW Higuchi bound?
- Stabilizing massive spin-2 dark matter?
- Observational probes: isocurvature, free streaming, non-Gaussianity, CMB hot spots

backup slides

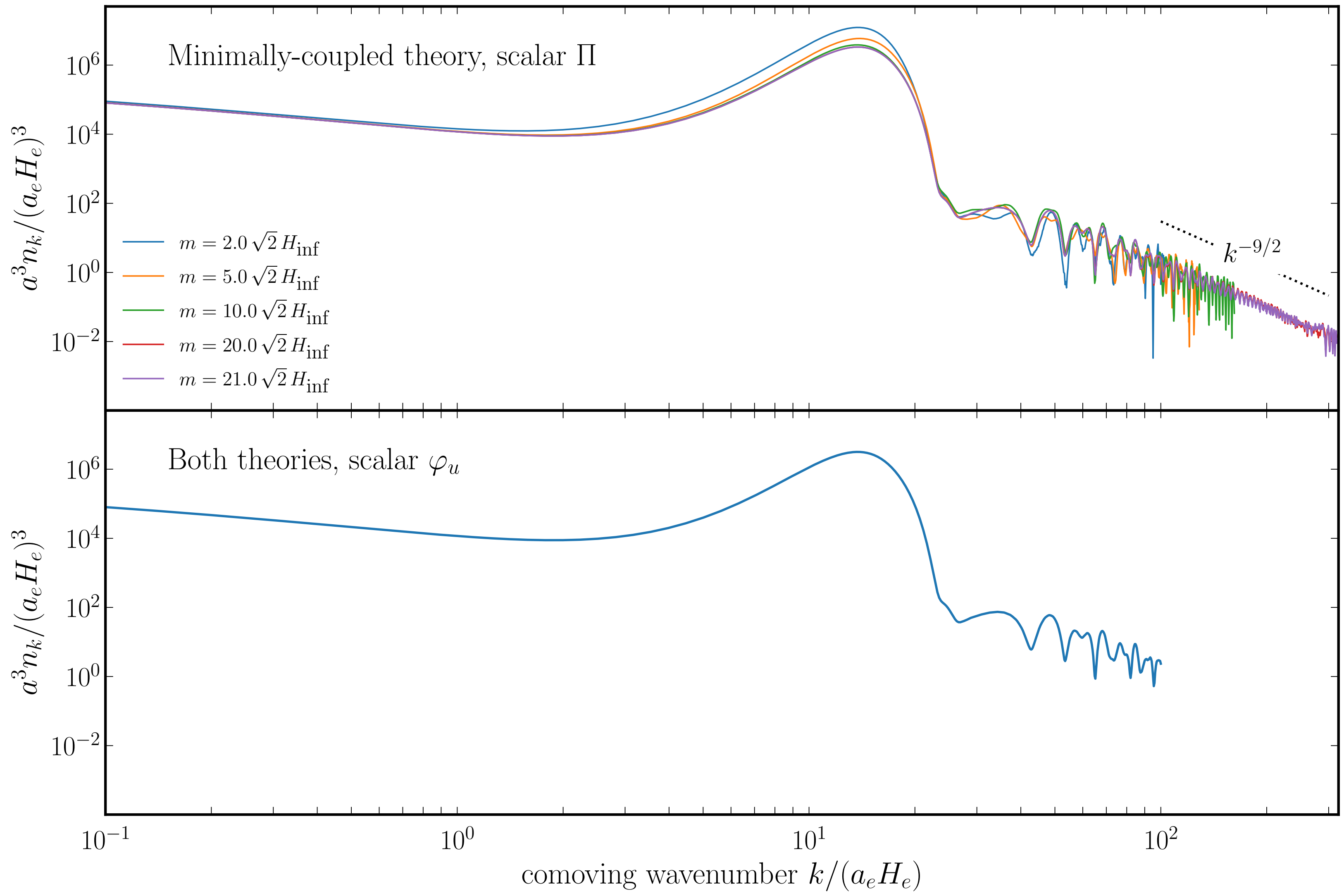
spin-2 CGPP

extra plots

Numerical results – spectra



Scalar sector - spectra



- We also calculate spectra for the inflaton-like scalar perturbations. This is just the usual quasi-scale invariant spectrum of curvature perturbations.

ghost-free bigravity

details

Ghost-free bigravity

Hassan & Rosen (2012)

Field content: two metrics & two scalars

$$g_{\mu\nu} \; , \quad f_{\mu\nu} \; , \quad \phi_g \; , \quad \phi_f$$

A theory of bigravity with a minimal coupling to matter

$$S = \int \mathrm{d}^4 x \Big[\tfrac{1}{2} M_g^2 \sqrt{-g}\, R[g] \; + \; \tfrac{1}{2} M_f^2 \sqrt{-f}\, R[f] \quad \text{(metric kinetic terms)}$$

$$- m^2 M_*^2 \sqrt{-g}\, V(\mathbb{X};\, \beta_n) \quad \text{(metric interactions)}$$

$$+ \sqrt{-g}\, \mathcal{L}_g(g, \phi_g) \; + \; \sqrt{-f}\, \mathcal{L}_f(f, \phi_f) \Big] \quad \text{(coupling to matter)}$$

Matter-sector Lagrangians

$$\mathcal{L}_g(g, \phi_g) = -\tfrac{1}{2} g^{\mu\nu} \nabla_\mu \phi_g \nabla_\nu \phi_g - V_g(\phi_g)$$

$$\mathcal{L}_f(f, \phi_f) = -\tfrac{1}{2} f^{\mu\nu} \nabla_\mu \phi_f \nabla_\nu \phi_f - V_f(\phi_f)$$

$$\left(\begin{matrix} M_*^{-2} = M_g^{-2} + M_f^{-2} \\ M_P^2 = M_g^2 + M_f^2 \end{matrix} \right)$$

Proportional background & mirroring conditions

Backgrounds plus perturbations:

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + \tfrac{2}{M_g} h_{\mu\nu}\,, \quad f_{\mu\nu} = \bar{f}_{\mu\nu} + \tfrac{2}{M_f} k_{\mu\nu}\,, \quad \phi_g = \bar{\phi}_g + \varphi_g\,, \quad \text{and} \quad \phi_f = \bar{\phi}_f + \varphi_f$$

We seek solutions of the background equations of motion with

$$\bar{g}_{\mu\nu} = \bar{f}_{\mu\nu} = \text{FRW} \qquad \text{and} \qquad \frac{1}{M_g}\,\bar{\phi}_g = \frac{1}{M_f}\,\bar{\phi}_f \equiv \frac{1}{M_P}\,\bar{\phi}$$

The existence of such solutions places a constraint on the models:

$$\frac{1}{M_g^2}\,V_g(\bar{\phi}_g) = \frac{1}{M_f^2}\,V_f(\bar{\phi}_f) \equiv \frac{1}{M_P^2}\,V(\bar{\phi})$$

(mirroring condition)

Then the backgrounds obey the usual equations of motion (EOM) for an inflationary cosmology:

bkg. metric EOM: $\bar{R}_{\mu\nu} - \tfrac{1}{2}\bar{g}_{\mu\nu}\bar{R} + \Lambda \bar{g}_{\mu\nu} = \frac{1}{M_P^2}\,\bar{T}_{\mu\nu}$

bkg. inflaton EOM: $\Box\bar{\phi} - V'(\bar{\phi}) = 0$

Perturbations

Change variables

$$\frac{1}{M_*}\, u_{\mu\nu} = \frac{1}{M_f} h_{\mu\nu} + \frac{1}{M_g}\, k_{\mu\nu} \;, \qquad \frac{1}{M_*}\, v_{\mu\nu} = \frac{1}{M_g}\, h_{\mu\nu} - \frac{1}{M_f}\, k_{\mu\nu}$$

$$\frac{1}{M_*}\, \varphi_u = \frac{1}{M_f}\, \varphi_g + \frac{1}{M_g}\, \varphi_f \;, \qquad \frac{1}{M_*}\, \varphi_v = \frac{1}{M_g}\, \varphi_g - \frac{1}{M_f}\, \varphi_f$$

Quadratic action

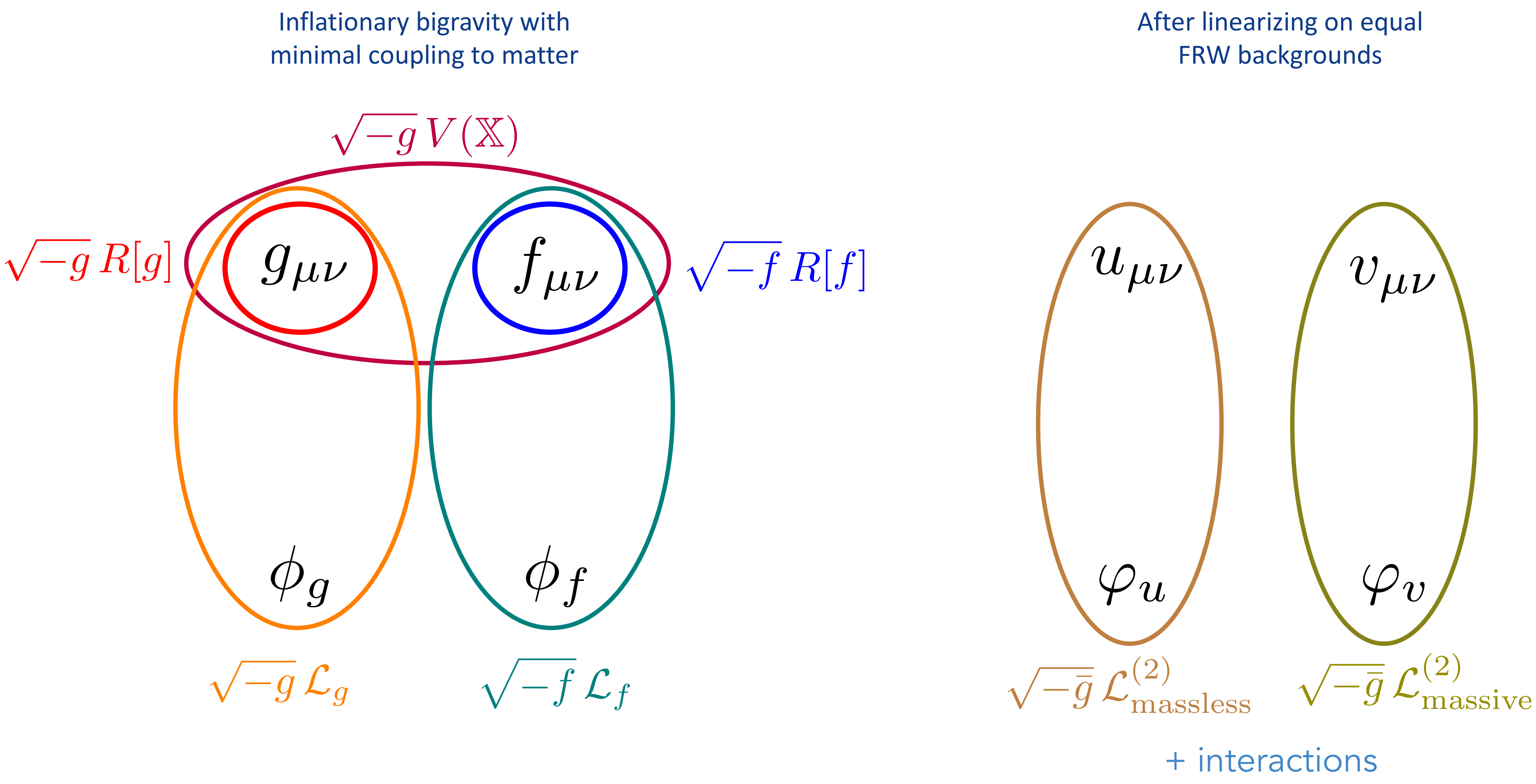
$$S = \int \mathrm{d}^4 x \, \sqrt{-\bar{g}} \, \left(\mathcal{L}^{(2)}_{\text{massless}} + \mathcal{L}^{(2)}_{\text{massive}} + \textbf{interactions} \right)$$

$$\begin{aligned}
\mathcal{L}^{(2)}_{\text{massless}} &= \mathcal{L}^{(2)}_{uu} + \mathcal{L}^{(2)}_{u\,\varphi_u} + \mathcal{L}^{(2)}_{\varphi_u \varphi_u} \\
\mathcal{L}^{(2)}_{uu} &= -\tfrac{1}{2}\nabla_\lambda u_{\mu\nu}\nabla^\lambda u^{\mu\nu} + \nabla_\mu u^{\nu\lambda}\nabla_\nu u^\mu{}_\lambda - \nabla_\mu u^{\mu\nu}\nabla_\nu u + \tfrac{1}{2}\nabla_\mu u \nabla^\mu u \\
&\quad + \left(\bar{R}_{\mu\nu} - \tfrac{1}{M_P^2}\nabla_\mu\bar{\phi}\nabla_\nu\bar{\phi}\right)\left(u^{\mu\lambda}u_\lambda^{\ \nu} - \tfrac{1}{2}u^{\mu\nu}u\right) , \\
\mathcal{L}^{(2)}_{u\,\varphi_u} &= \tfrac{1}{M_P}\left[\left(\nabla_\mu\bar{\phi}\nabla_\nu\varphi_u + \nabla_\nu\bar{\phi}\nabla_\mu\varphi_u\right)\left(u^{\mu\nu} - \tfrac{1}{2}\bar{g}^{\mu\nu}u\right) - V'(\bar{\phi})\varphi_u u\right] , \\
\mathcal{L}^{(2)}_{\varphi_u\varphi_u} &= -\tfrac{1}{2}\nabla_\mu\varphi_u\nabla^\mu\varphi_u - \tfrac{1}{2}V''(\bar{\phi})\varphi_u^2 \, .
\end{aligned}$$

(massless spin-2 graviton + inflaton perturbation)

$$\begin{aligned}
\mathcal{L}^{(2)}_{\text{massive}} &= \mathcal{L}^{(2)}_{vv} + \mathcal{L}^{(2)}_{v\,\varphi_v} + \mathcal{L}^{(2)}_{\varphi_v \varphi_v} \\
\mathcal{L}^{(2)}_{vv} &= -\tfrac{1}{2}\nabla_\lambda v_{\mu\nu}\nabla^\lambda v^{\mu\nu} + \nabla_\mu v^{\nu\lambda}\nabla_\nu v^\mu{}_\lambda - \nabla_\mu v^{\mu\nu}\nabla_\nu v + \tfrac{1}{2}\nabla_\mu v \nabla^\mu v \\
&\quad + \left(\bar{R}_{\mu\nu} - \tfrac{1}{M_P^2}\nabla_\mu\bar{\phi}\nabla_\nu\bar{\phi}\right)\left(v^{\mu\lambda}v_\lambda^{\ \nu} - \tfrac{1}{2}v^{\mu\nu}v\right) \\
&\quad - \tfrac{1}{2}m^2\left(v^{\mu\nu}v_{\mu\nu} - v^2\right) , \\
\mathcal{L}^{(2)}_{v\,\varphi_v} &= \tfrac{1}{M_P}\left[\left(\nabla_\mu\bar{\phi}\nabla_\nu\varphi_v + \nabla_\nu\bar{\phi}\nabla_\mu\varphi_v\right)\left(v^{\mu\nu} - \tfrac{1}{2}\bar{g}^{\mu\nu}v\right) - V'(\bar{\phi})\varphi_v v\right] , \\
\mathcal{L}^{(2)}_{\varphi_v\varphi_v} &= -\tfrac{1}{2}\nabla_\mu\varphi_v\nabla^\mu\varphi_v - \tfrac{1}{2}V''(\bar{\phi})\varphi_v^2 \, .
\end{aligned}$$

(massive spin-2 + inflaton perturbation)



Inflationary bigravity with
minimal coupling to matter
$\sqrt{-g}\,V(\mathbb{X})$
$\sqrt{-g}\,R[g]$
$g_{\mu\nu}$
$f_{\mu\nu}$
$\sqrt{-f}\,R[f]$
ϕ_g
ϕ_f
$\sqrt{-g}\,\mathcal{L}_g$
$\sqrt{-f}\,\mathcal{L}_f$
After linearizing on equal
FRW backgrounds
$u_{\mu\nu}$
$v_{\mu\nu}$
φ_u
φ_v
$\sqrt{-\bar{g}}\,\mathcal{L}^{(2)}_{\text{massless}}$
$\sqrt{-\bar{g}}\,\mathcal{L}^{(2)}_{\text{massive}}$
+ interactions