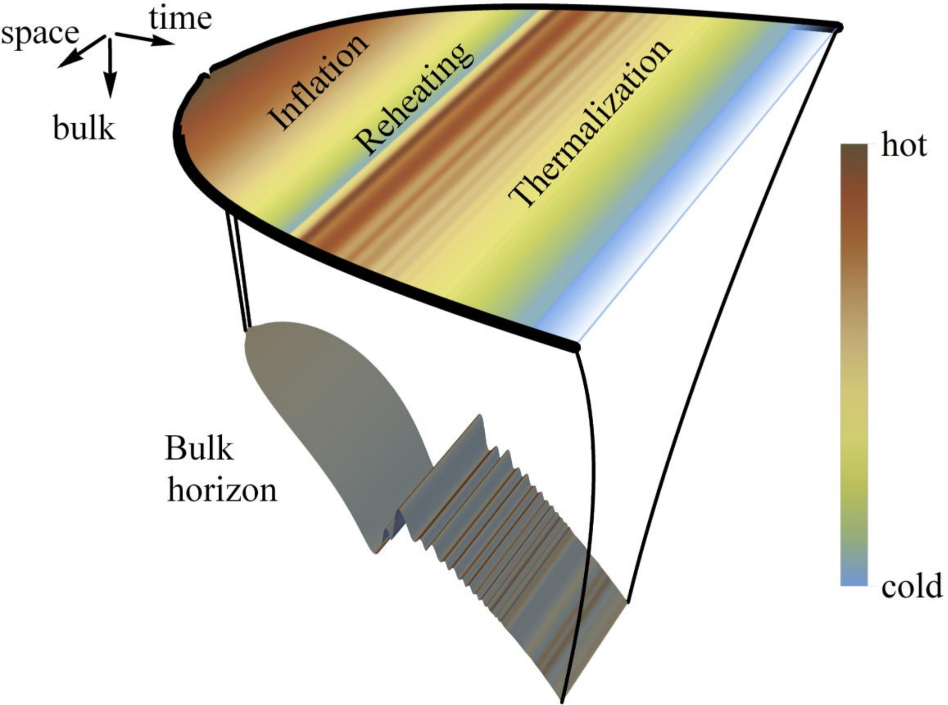
A DYNAMICAL INFLATON COUPLED TO STRONGLY INTERACTING MATTER



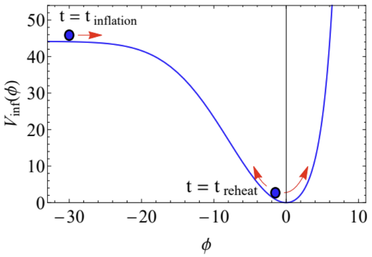
HOLOGRAPHIC QFTS ON CURVED BACKGROUNDS

With Christian Ecker, Elias Kiritsis, Jorge Casalderrey-Solana and David Mateos

Reference: 2011.08194 (JHEP), 2109.10355 (JHEP) and 2302.06618 (PRL)

Wilke van der Schee

Particle Production and Thermal Effects in Inflation
King's College (virtual), 5 February 2024



CONCLUSION

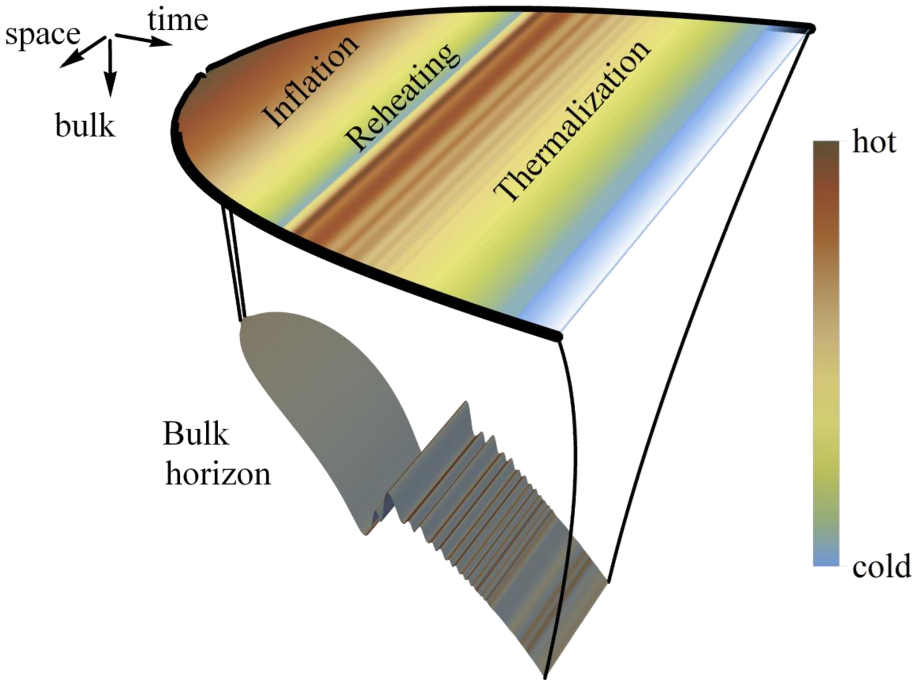
Strongly coupled QFT as a mechanism for thermalization

- Proof-of-principle: sQFT plus inflaton with dynamical gravity
  - Gravity is semi-classical; no quantum gravity
- Inflaton sources QFT: natural reheating when rolling down
- Boundary metric sources bulk metric
  - Bulk thermalises `as fast as possible' (time ~ 1/temperature)

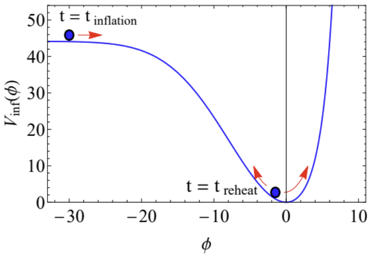
Some crucial differences

- We start at high temperature; never `empty' de Sitter
  - Similar to warm inflation?
  - Technical problem to get enough e-foldings?
- QFT similar to N=4 SYM, link with SM not obvious?

Boundary: FLRW_{3+1} universe



Bulk: anti-de-Sitter_{4+1} with FLRW slices



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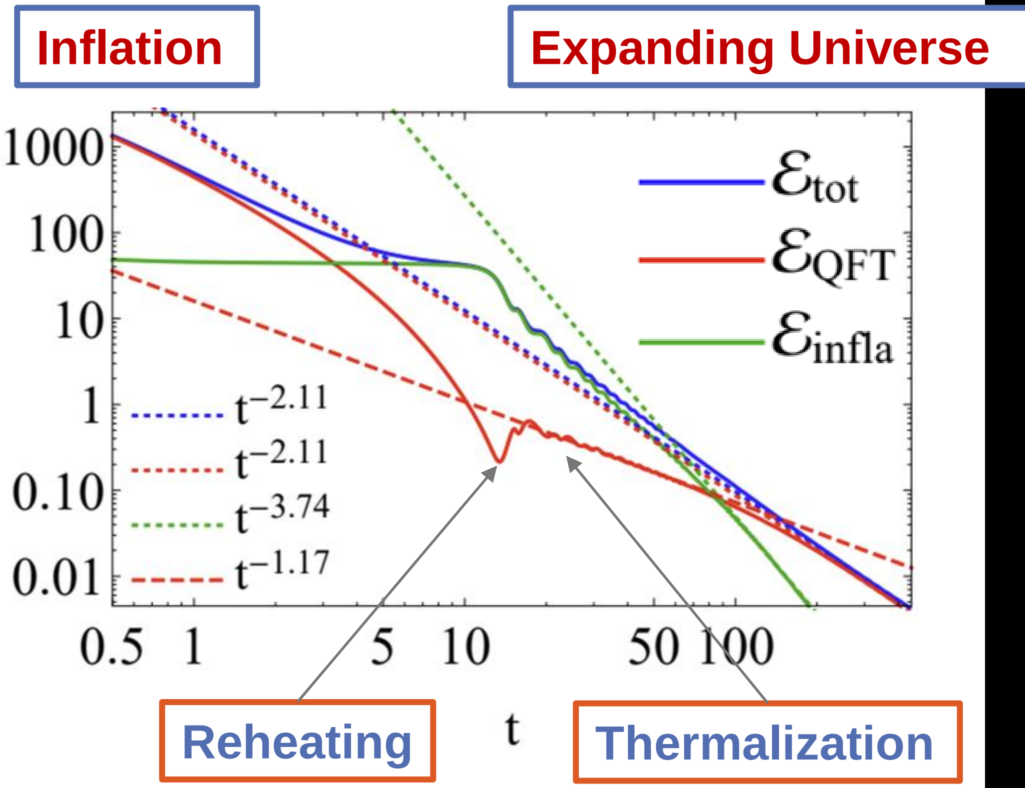

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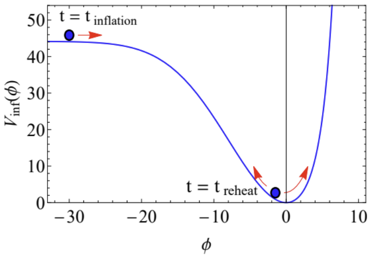

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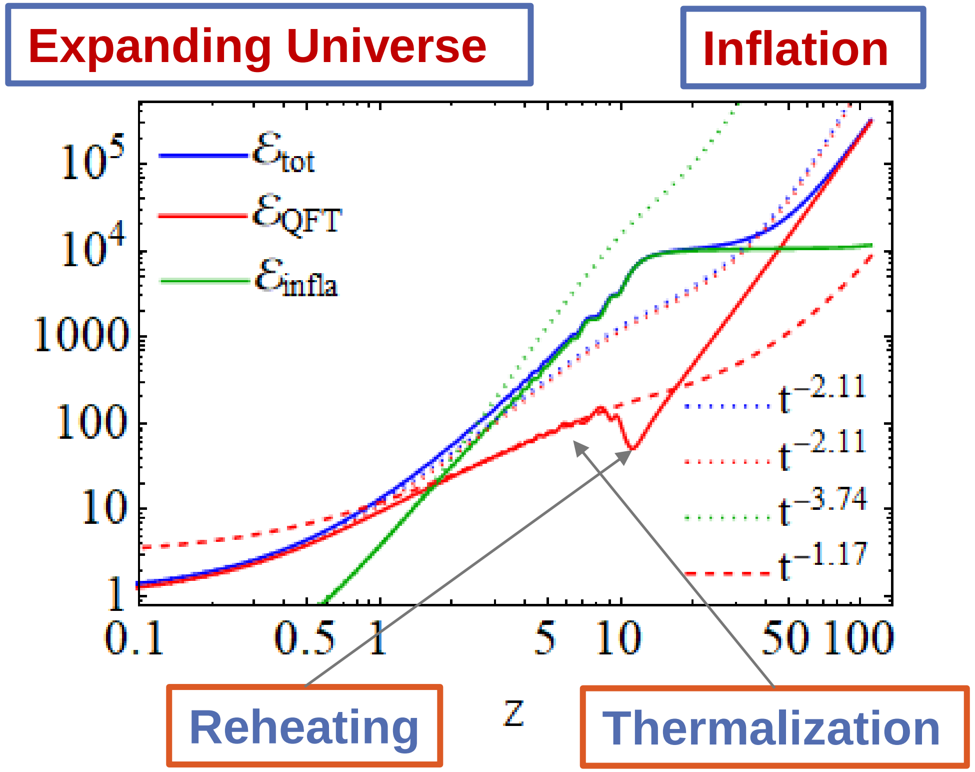


INFLATION, PREHEATING AND REHEATING

$$S = \int d^4x\sqrt{-g}\left[-\frac{m_{\rm Pl}^2}{2}R + \frac{1}{2}\nabla_\mu\phi\nabla^\mu\phi - V(\phi)\right] + S_{\rm matter}$$

$$\ddot{\bar\phi} + 3H\dot{\bar\phi} + \partial_{\bar\phi}V(\bar\phi) = 0$$

How to get a working universe

- Inflation/cooling + reheating/thermalisation
- The inflaton energy needs to be transferred to Standard Model

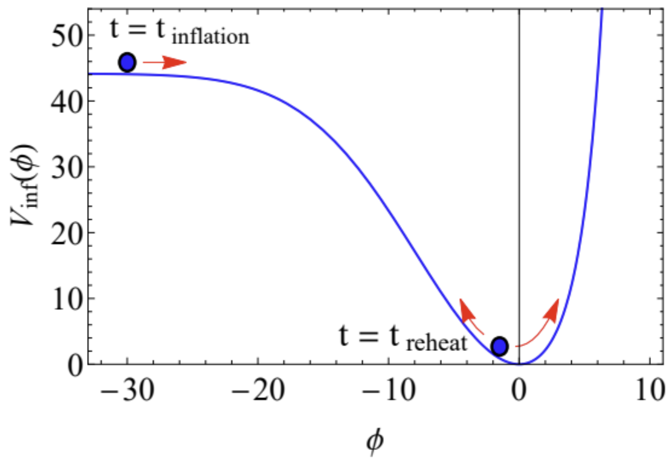
Perturbative Reheating

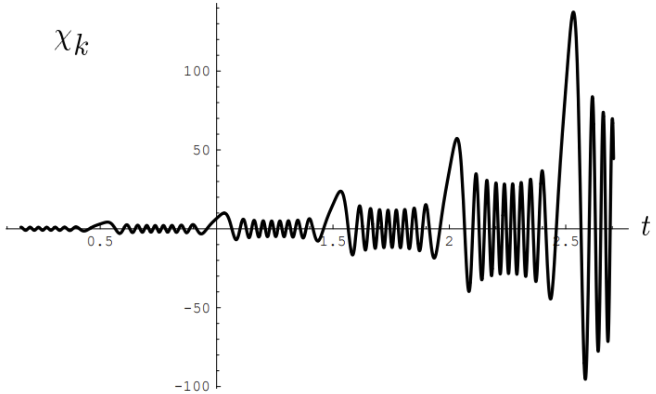
- Inflaton decays at an (approximately) constant rate (faster than H)
- Decaying particles thermalise → big bang

Preheating

- Bose enhancement leads to resonances: exponential transfer of energy
- Bose enhanced states thermalise/reheat → big bang

Inflaton potential





Lev Kofman, Andrei Linde and Alexei A. Starobinsky, Towards the Theory of Reheating After Inflation (1997)

SHORT INTRO ON HOLOGRAPHY

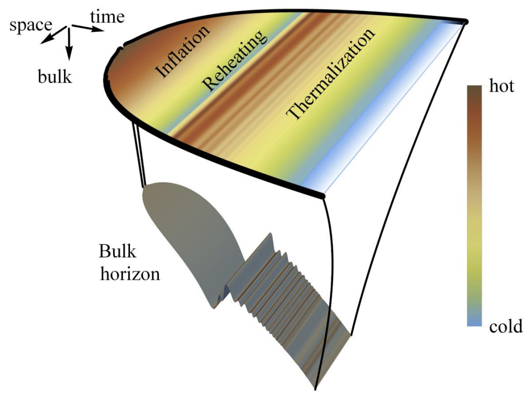
Exact equivalence between string theory and quantum field theory

$$\mathcal{Z}_{bulk}\left[\phi(\vec{x},z)|_{z=0}=\phi_0(\vec{x})\right]=\langle e^{\int d^4x\phi_0(\vec{x})\mathcal{O}(\vec{x})}\rangle_{\text{Field Theory}}$$

$$\langle\mathcal{O}\rangle=-i\frac{\delta Z_{\text{bulk}}[\phi_{(0)}]}{\delta\phi_{(0)}}\overset{N\to\infty}{=}\frac{\delta S[\phi_{(0)}]}{\delta\phi_{(0)}}$$

Dictionary:

- Source operator is a boundary condition for the bulk field
- Expectation value as a near-boundary derivative



- Scalar operator ←→ scalar field
- Stress tensor ←→ metric
    - Boundary metric as a source; *not necessarily dynamic*
- Simplifies in large N + strong coupling limit

Juan Maldacena, The large-N limit of superconformal field theories and supergravity (1997)

INFLATION, REHEATING AND HOLOGRAPHY

$$S = \int d^4x\sqrt{-g}\left[-\frac{m_{\rm Pl}^2}{2}R + \frac{1}{2}\nabla_\mu\phi\nabla^\mu\phi - V(\phi)\right] + S_{\rm matter}$$

$$\ddot{\phi}_0 = -U\phi_0\langle\mathcal{O}\rangle - 3H\dot{\phi}_0 - V'(\phi_0)$$

Holographic set-up

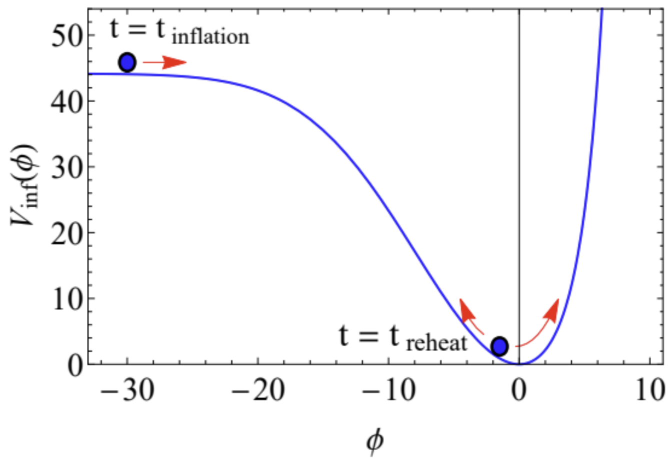
- ‘standard’ EH + inflaton + coupling to scalar:

$$S = S_{\rm EH+inf} + S_{\rm hol} + S_{\rm int}$$

$$S_{\rm int} = \int \mathrm{d}^4x\sqrt{-\gamma}\,U(\phi)\,\mathcal{O}_{\rm QFT}$$

- Inflaton acts as a source for the scalar operator

Inflaton potential

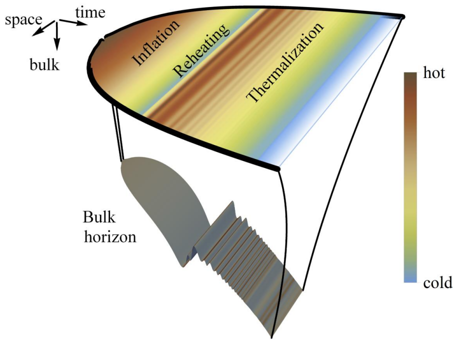


Two technical complications:

- Sources are now time-dependent numerical functions
- holographic renormalisation becomes more complicated

Trial and error:

- Initial inflaton position, inflaton velocity, coupling strength, shape potential
- Balance between numerical stability, runtime and making a ‘realistic’ universe



NON-CONFORMAL MODEL ON DE SITTER$_4$

De Sitter is conformally flat: almost trivial for CFT

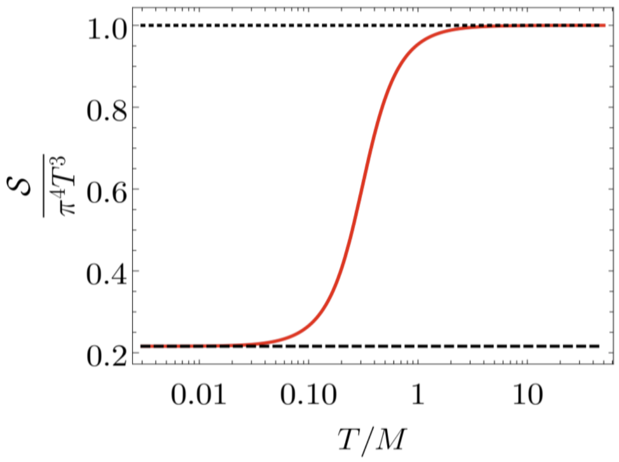
- Break scale invariance by V(Φ) with source M (will be inflaton)

$$S = \frac{2}{8\pi G}\int_{\mathcal{M}} d^5x\sqrt{-g}\left(\frac{1}{4}R[g] - \frac{1}{2}(\partial\phi)^2 - V(\phi)\right)$$

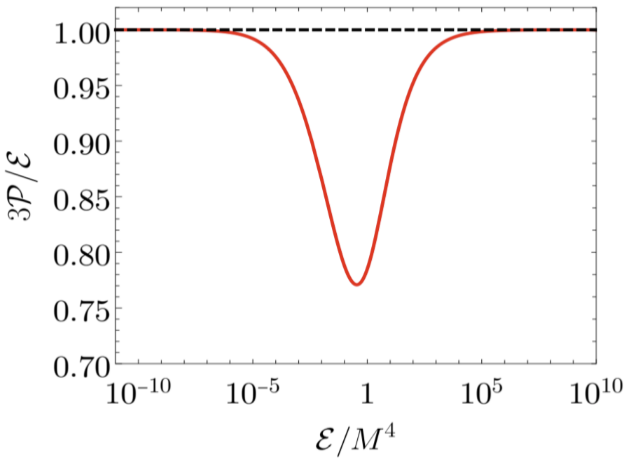
$$L^2 V(\phi) = -3 - \frac{3}{2}\phi^2 - \frac{1}{3}\phi^4 + \left(\frac{1}{3\phi_M^2} + \frac{1}{2\phi_M^4}\right)\phi^6 - \frac{1}{12\phi_M^4}\phi^8$$

- Leads to non-trivial EOS and bulk viscosity (no shear considered):

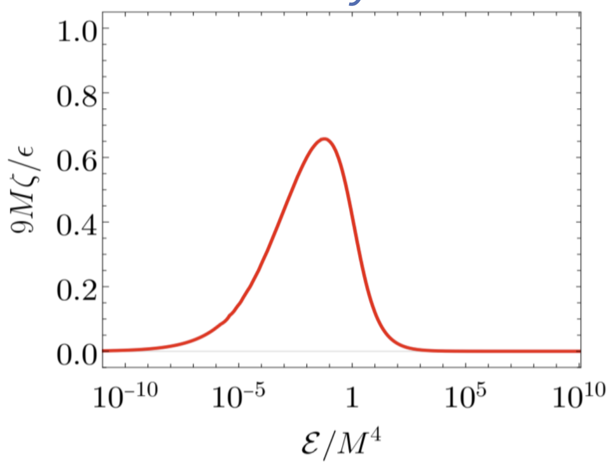
Entropy density



Trace anomaly



Bulk viscosity



Jorge Casalderrey-Solana, Christian Ecker, David Mateos and WS, Strong-coupling dynamics and entanglement in de Sitter space (2020)

$$ds_{\mathrm{b}}^2 = -\mathrm{d}t^2 + S_0(t)^2 \mathrm{d}\vec{x}^2$$

A DYNAMICAL BOUNDARY

Study dynamics including semi-classical gravity:

$$R_{\mu\nu} - \frac{1}{2} R\, g_{\mu\nu} = 8\pi G_{N,4} \langle T_{\mu\nu} \rangle$$

$$\ddot{\phi}_0 = -U\phi_0 \langle \mathcal{O} \rangle - 3H\dot{\phi}_0 - V'(\phi_0)$$

- Stress-tensor includes possible cosmological constant
- NB: renormalisation counterterms are now physical
- We treat the boundary Newton constant as a (small) parameter

Dynamics of scale factor $S_0(t)$ is a consequence of Friedmann equations

How to initialise the dynamics

- Start with thermal Minkowski profile and small boundary $G_{N,4}$

Jewel Ghosh, Elias Kiritsis, Francesco Nitti and Lukas Witkowski, Back-reaction in de Sitter QFTs: holography, gravitational DBI action and f(R) gravity (2020)
Paul Chesler and Abraham Loeb, Holographic duality and mode stability of de Sitter space in semiclassical gravity (2020)
Christian Ecker, WS, David Mateos and Jorge Casalderrey-Solana , Holographic evolution with dynamical boundary gravity (2021)

$$ds_{\mathrm{b}}^2 = -\mathrm{d}t^2 + S_0(t)^2 \mathrm{d}\vec{x}^2$$

INTERMEZZO: EMPTY DE SITTER

Two ways to study strongly coupled QFT on empty de Sitter:

- ‘Standard’: metric is non-normalisable mode, so just use $S_0(t) = e^{Ht}$
- Natural end-point of dynamical gravity with positive cosmological constant

Expectations:

- De Sitter has a Hawking temperature $T = \frac{H}{2\pi}$
- Expect an entropy proportional to the cosmological horizon (?)
- Is this related to the event or apparent horizon?
  - A priori not: horizon gives a volume law entropy
  - More robust: entanglement entropy, through bulk extremal surfaces

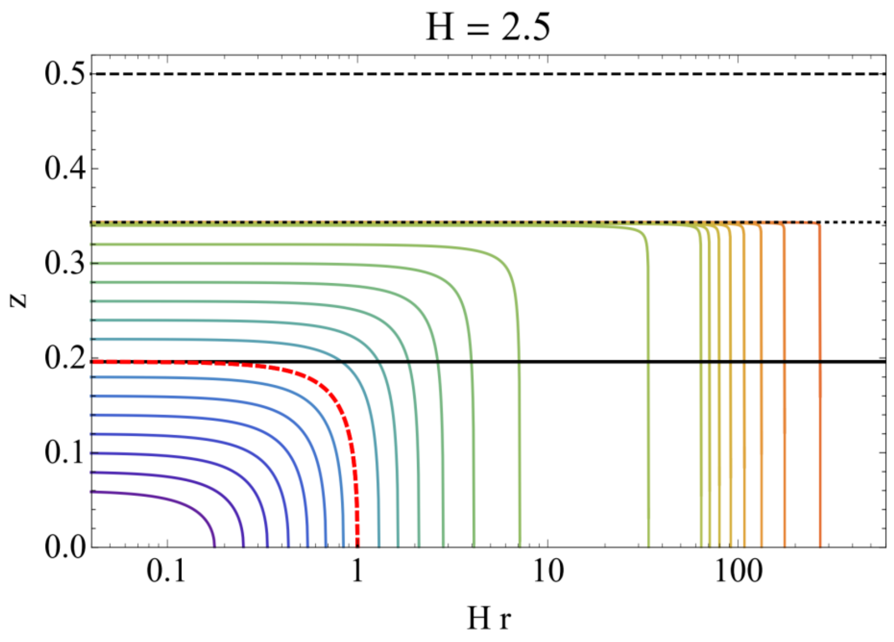
Jorge Casalderrey, Christian Ecker, David Mateos and WS, Strong-coupling dynamics and entanglement in de Sitter space (2020)

$$ds_b^2 = -dt^2 + S_0(t)^2 d\vec{x}^2 \,. \quad S_0(t) = e^{Ht}$$

BULK GEOMETRY WITH dS BOUNDARY

Event, apparent and 'entanglement' horizons for bulk dual to empty de Sitter

- The apparent horizon is much beyond the event horizon
- Also shown: extremal surfaces
- `Temperature' of AH is negative; curiosity? (from surface gravity of bulk horizon)
- Only `superobservers' can see behind event horizon (if and only if (!))

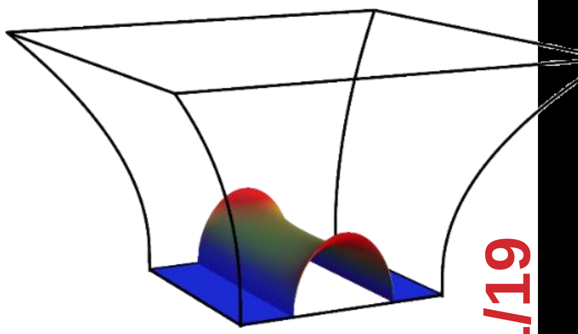


Apparent horizon: $T = -\frac{H}{2\pi}$

Entanglement horizon: $T = 0$

Event horizon: $T = \frac{H}{2\pi}$

Boundary



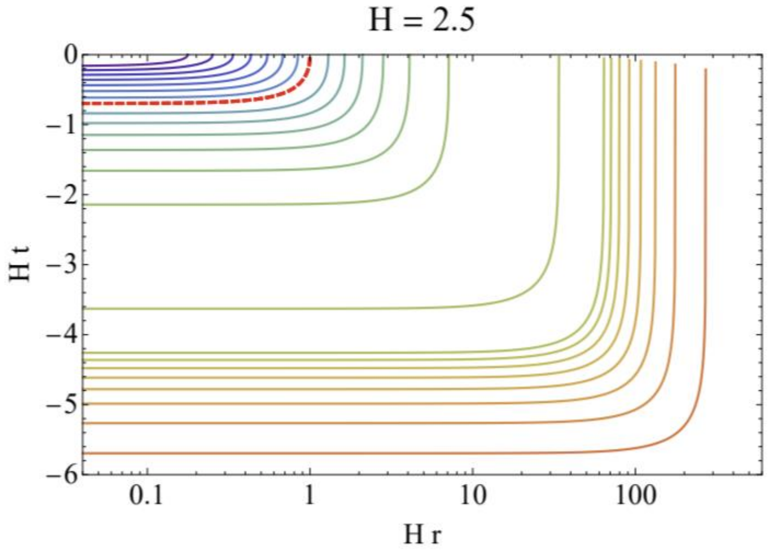
Cosmological horizon

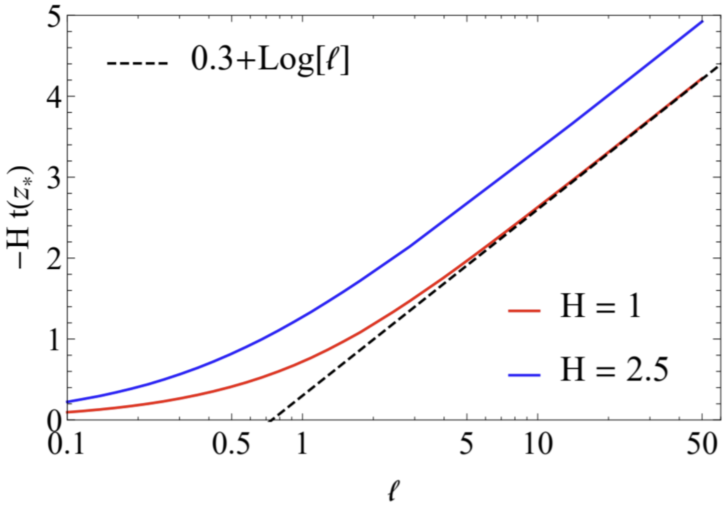
Jorge Casalderrey, Christian Ecker, David Mateos and WS, Strong-coupling dynamics and entanglement in de Sitter space (2020)

ENTANGLEMENT IN DE SITTER

Extremal surfaces go backward in time

- Time at the deepest point grows exactly as log(l) for large l
- Implies that `entanglement horizon' contribution has a **constant instead of volume law** contribution
- Standard `area law' divergence still applies





Cosmological horizon

A DYNAMICAL BOUNDARY – CONSISTENT INITIAL CONDITIONS

$$ds_{\rm b}^2 = -\mathrm{d}t^2 + S_0(t)^2 \mathrm{d}\vec{x}^2$$

Near-boundary expansion scalar (and metric):

$$\Phi = \frac{\phi(t)}{r} + \frac{\phi_2(t)}{r^3} + \frac{1}{r}\sum_{n\geq 3}\frac{\phi_n(t)}{r^n} + \frac{1}{r}\sum_{n\geq 2}\frac{\psi_n(t)\log r}{r^n} + \cdots$$

- Crucial subtlety: logarithmic terms ψ_n solely determined by sources $S_0(t)$
- Consistent IC not much of a problem with known source (ψ_n known analytically)

Dynamics of scale factor *$S_0(t)$* is now consequence of Friedmann equations

Sufficiently smooth solution requires knowledge of sufficient # of log's

- Log's depend on time derivatives of $S_0(t)$
- Solution: extract $\partial_t^4 S_0(t)$ from near-boundary expansion of scalar (extract ϕ_n)
- Use time derivates to treat first few logs analytically

Final subtlety: VEV and energy density depend on *$S_0''(t)$ and $\phi''(t)$*

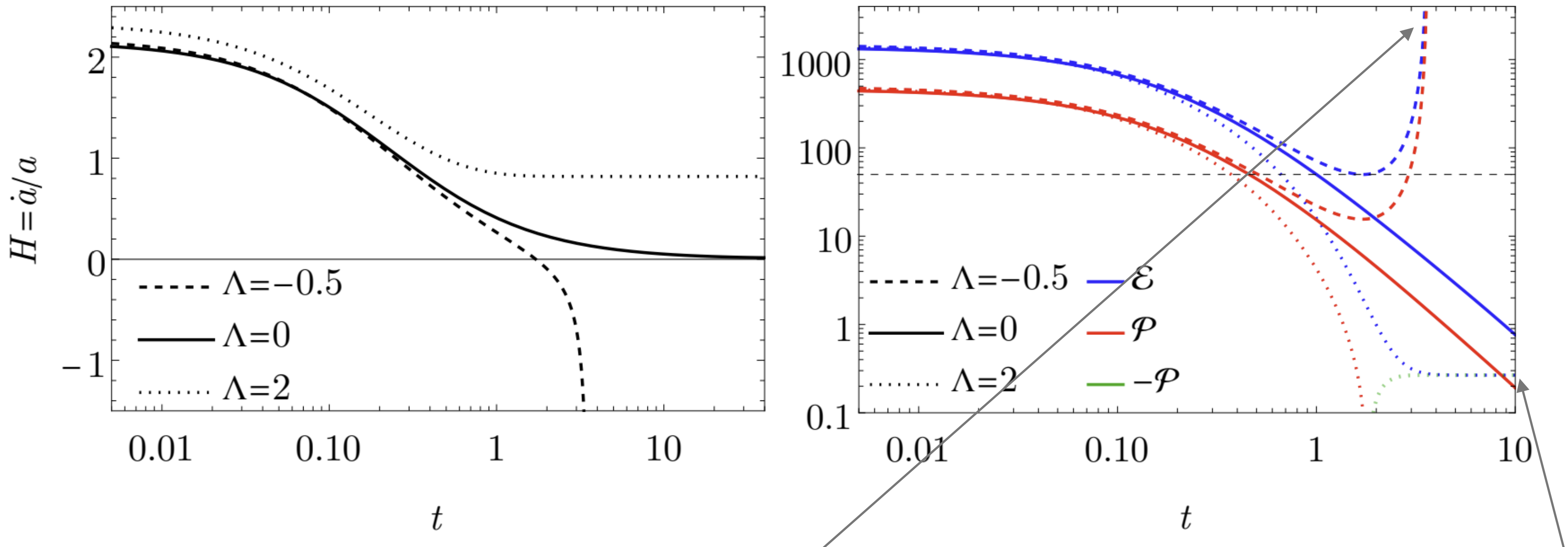
- Solve resulting 6th order polynomial equation numerically

Sebastian de Haro, Kostas Skenderis and Sergey N. Solodukhin, Holographic Reconstruction of Spacetime and Renormalization in the AdS/CFT Correspondence (2001)

$$ds_{\mathrm{b}}^2 = -\mathrm{d}t^2 + S_0(t)^2 \mathrm{d}\vec{x}^2$$

BOUNDARY GRAVITY ONLY

Stress-energy tensor for zero, positive and negative Λ



Positive Λ settles down to de Sitter state as studied before, includes (small) Casimir energy

Now also asymptotically flat + Big Crunch geometries

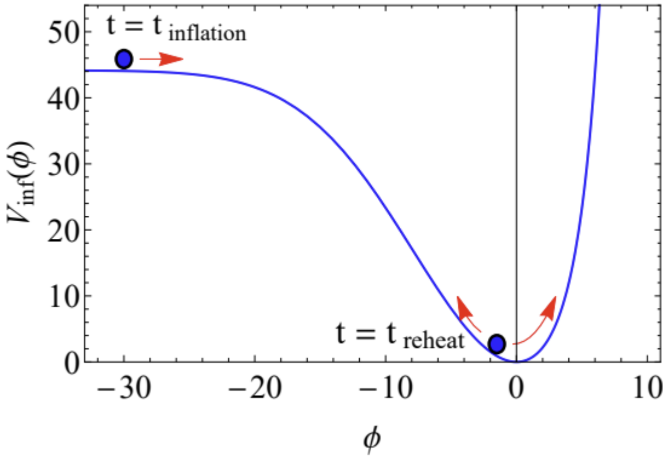
Christian Ecker, WS, David Mateos and Jorge Casalderrey-Solana, Holographic evolution with dynamical boundary gravity (2021)

BOUNDARY GRAVITY WITH AN INFLATON

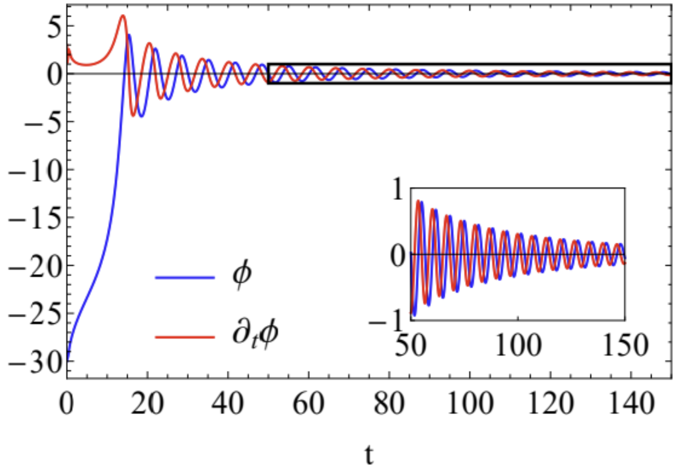
Inflaton rolls down and undergoes damped oscillation

Rolls down relatively fast (helps numerical stability)

Inflaton potential



Inflaton dynamics



INFLATION AND REHEATING

Initially dominated by QFT energy

- Quickly dilutes due to expansion

Then dominated by inflaton

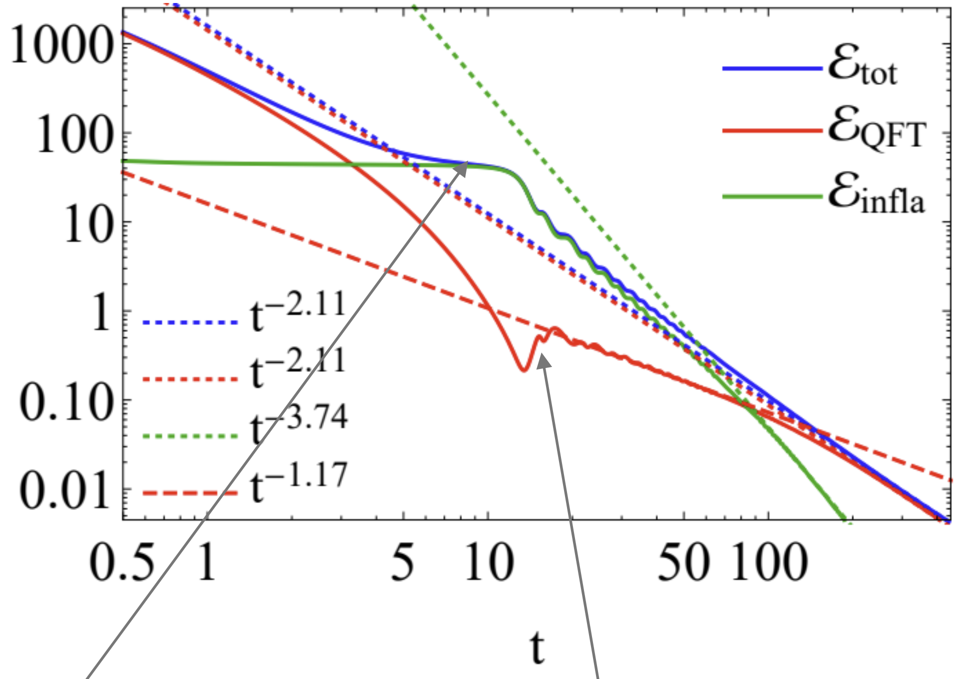
- Inflation, while slowly rolling down

Then inflaton reaches bottom: reheating

- Inflaton loses energy to QFT

Finally universe dominated by QFT in thermal equilibrium

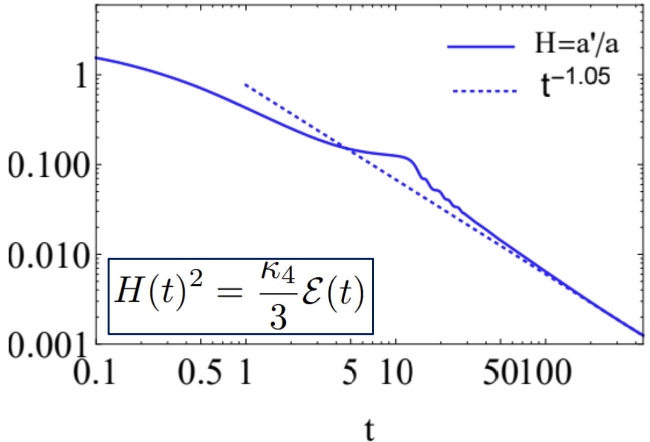
Energy densities (log-log)



Inflating phase (t ~ 5 till 12)

Reheating (around t=13)

Hubble rate



TEMPERATURE FROM BULK AND QFT

Three ways to determine temperature:

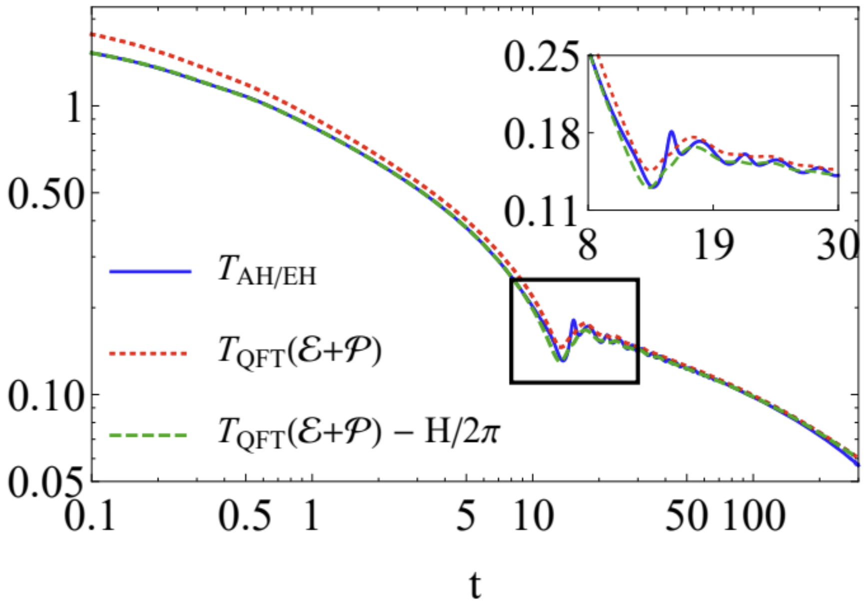
- From energy density and EOS
- From apparent horizon
- From event horizon (agrees to apparent horizon)

Always `close to equilibrium'

- Somewhat curious Casimir shift by de Sitter temperature
- Also present in analytical solution of conformal case

$$\epsilon = \frac{3}{8}\pi^2 N^2 T^4 + \frac{3N^2(\dot{a})^4}{32\pi^2 a^4}$$

Temperature (log-log)



Alex Buchel, Michal P. Heller and Jorge Noronha, Entropy Production, Hydrodynamics, and Resurgence in the Primordial Quark-Gluon Plasma from Holography (2016)

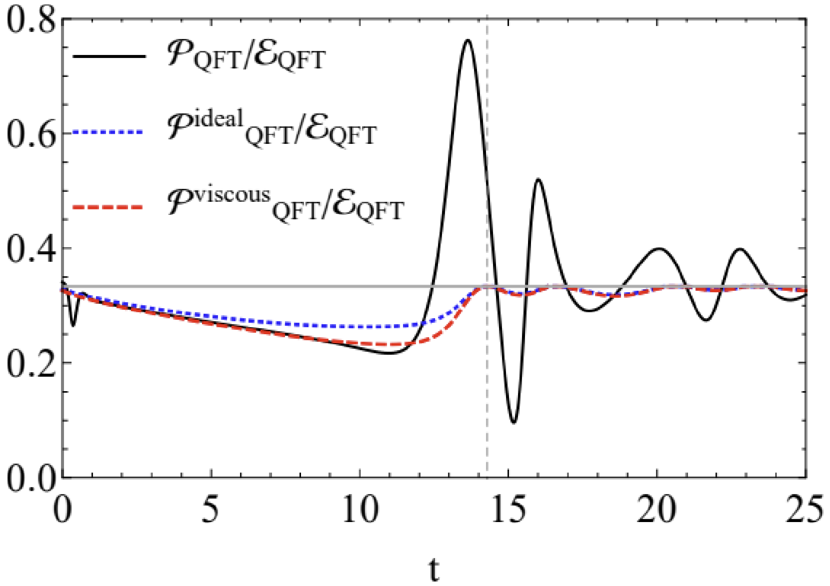
HYDRODYNAMICS

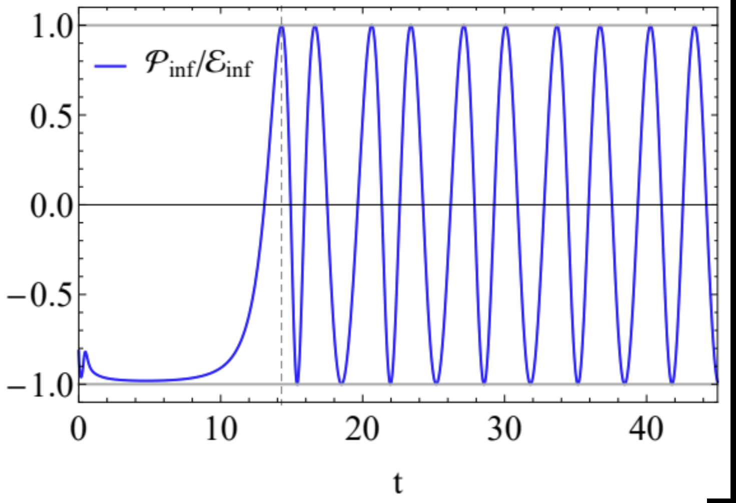
$$\langle \hat{T}^{ij} \rangle = \epsilon\, u^i u^j + p_{\rm eq}(\epsilon)\Delta^{ij} - \eta(\epsilon)\,\sigma^{ij} - \zeta(\epsilon)\Delta^{ij}\overline{\nabla}^k u_k + O(\overline{\nabla}^2)\,,$$

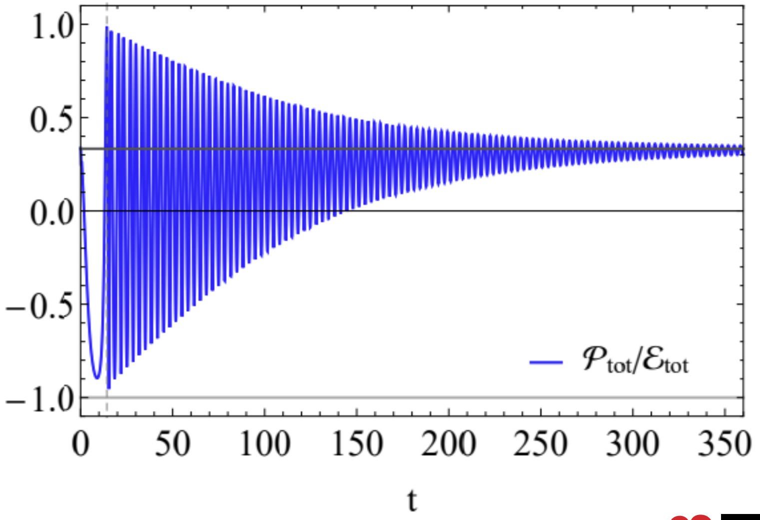
$$\sigma^{ij} = \Delta^{ik}\Delta^{jl}(\overline{\nabla}_k u_l + \overline{\nabla}_l u_k) - \frac{2}{3}\Delta^{ij}\overline{\nabla}_k u^k\,,$$

Hydrodynamic constitutive relation (ideal+viscous):

$$\Delta\mathcal{P}^{\rm hydro}(t) \equiv \frac{N^2}{2\pi^2}\left\{\Delta p_{\rm eq}(\Delta\mathcal{E}(t)) - 3H\zeta(\Delta\mathcal{E}(t))\right\} + O(H^2)$$

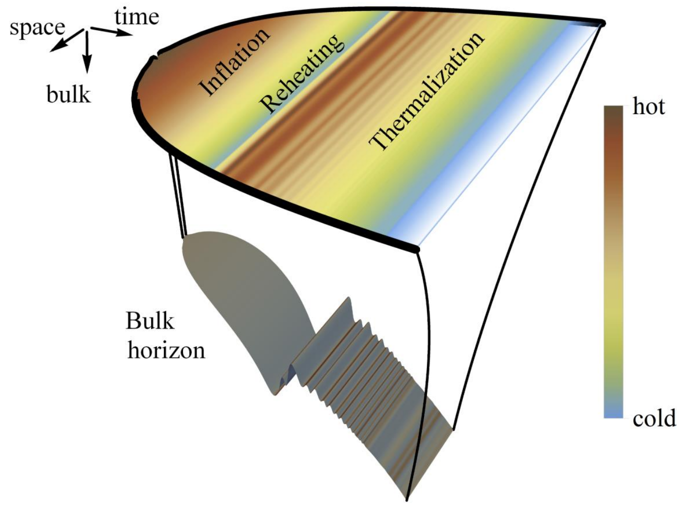






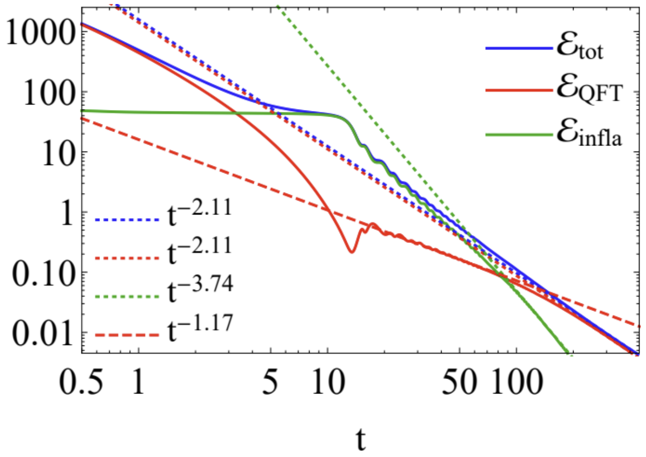
- Quick initial hydrodynamisation (t~1)
- **Viscous** hydrodynamics during inflation (t ~ 5 – 13)
- Far-from-equilibrium during reheating (t~ 13 – 20)

DISCUSSION



Dynamical gravity and inflaton on the boundary

- Technical progress: how to deal with the logs?
- Several interesting evolution: flat space, Big Crunch
- Interesting holographic temperatures and entropies in de Sitter



Inflation and reheating

- First principles computation of strongly coupled reheating
- Strongly coupled physics can thermalise after preheating
- Currently not a `realistic universe'

Future

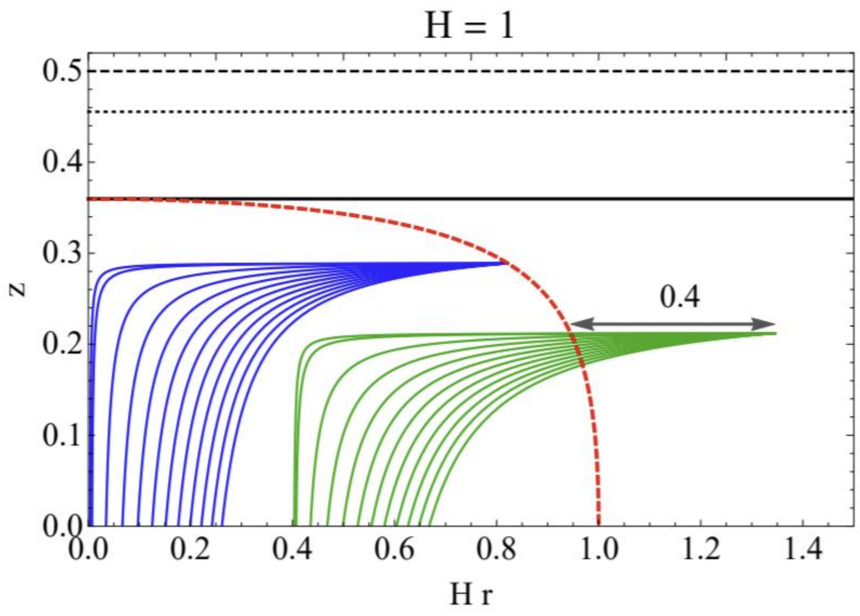
- Realistic universe? Many e-foldings based on hydro?
- Do we understand entropy of de Sitter?

BACK-UP

COSMOLOGICAL HORIZON

Extremal surface dual to cosmological horizon:

- Separates points in the bulk from which light can reach the (boundary) origin
- *Boundary cosmological horizon → full bulk cosmological horizon*



Jorge Casalderrey, Christian Ecker, David Mateos and WS, Strong-coupling dynamics and entanglement in de Sitter space (2020)

$$ds_{\mathrm{b}}^2 = -\mathrm{d}t^2 + S_0(t)^2 \mathrm{d}\vec{x}^2$$

SOME NUMERICAL DETAILS

Three bulk fields depending on time t and bulk direction r :

$$\mathrm{d}s^2_{\mathrm{bulk}} = -A(r,t)\mathrm{d}t^2 + 2\mathrm{d}r\mathrm{d}t + S(r,t)^2\mathrm{d}\vec{x}^2, \quad \Phi = \Phi(r,t)$$

Characteristic formulation leads to nested set of linear ODEs (spectral elements):

$$S'' = -\frac{2}{3}S\left(\Phi'\right)^2,$$

$$\dot{S}' = -\frac{2\dot{S}S'}{S} - \frac{2SV}{3},$$

$$\dot{\Phi}' = \frac{V'}{2} - \frac{3\dot{S}\Phi'}{2S} - \frac{3S'\dot{\Phi}}{2S},$$

$$A'' = \frac{12\dot{S}S'}{S^2} + \frac{4V}{3} - 4\dot{\Phi}\Phi'$$

Boundary 4D Einstein-inflaton equations:

$$H(t)^2 = \frac{\kappa_4}{3}\mathcal{E}(t),$$

$$\frac{a''(t)}{a(t)} = -\frac{1}{2}\left(\kappa_4\mathcal{P}(t) + H(t)^2\right),$$

$$\phi''(t) = \partial_\phi U(\phi(t))\,\mathcal{O}_{\mathrm{QFT}}(t) - 3H(t)\phi'(t) - \partial_\phi V_{\mathrm{inf}}(\phi(t))$$

S and Φ require boundary conditions: scale factor a(t) and inflaton ϕ(t)

- Normally sources are specified by hand; here: 4D Einstein-Inflaton equations
- Boundary conditions implemented through near-boundary expansion
- Subtract first few logs analytically (for stability in spectral elements)
- A requires initial boundary condition (energy density), then determined by SE-conservation

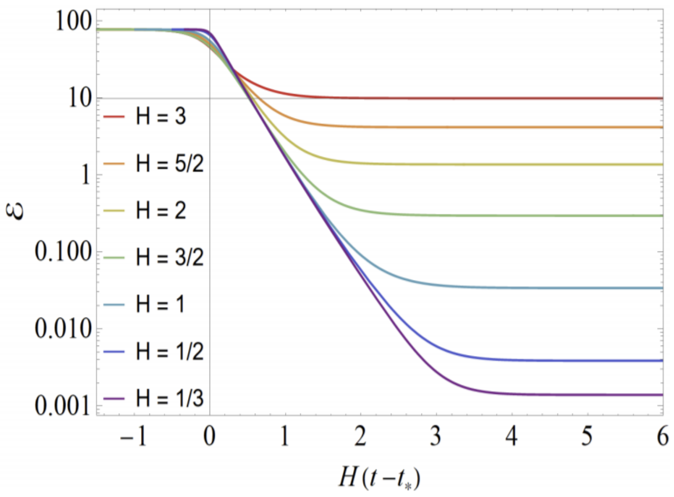
See also www.wilkevanderschee.nl for the full numerical code.

TIME EVOLUTION OF THE PROTOCOL

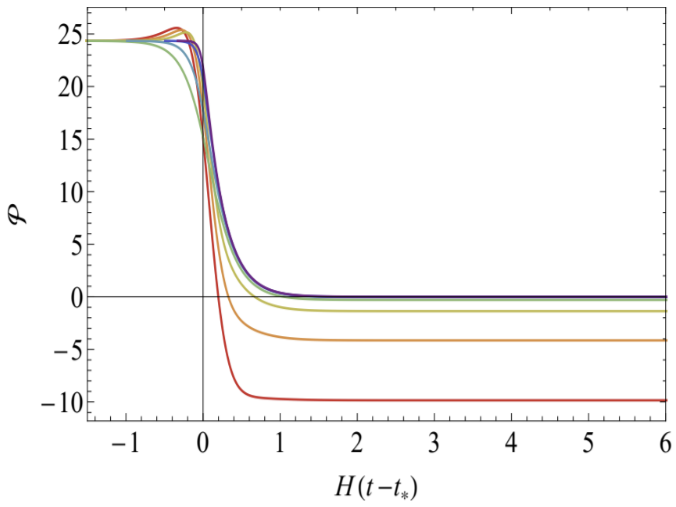
Evolution of stress tensor for different Hubble constants

- Energy density decreases towards vacuum energy (VE) (afterwards renormalised to zero)
- Pressure decreases, changes sign and becomes –VE
- Enthalpy is scheme independent, decays due to expansion

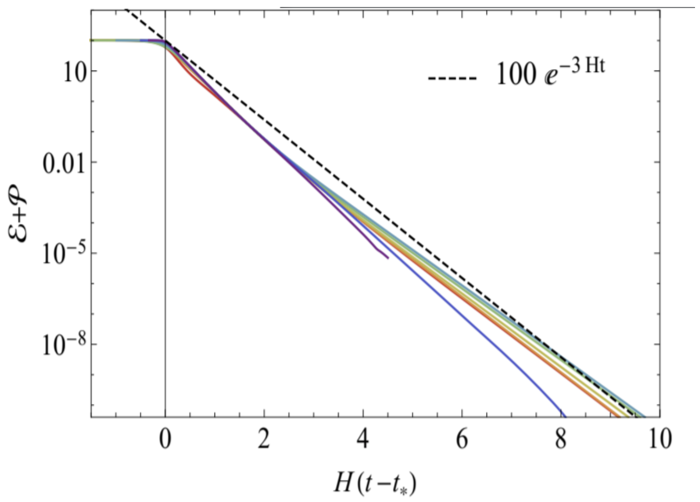
Energy density



Pressure

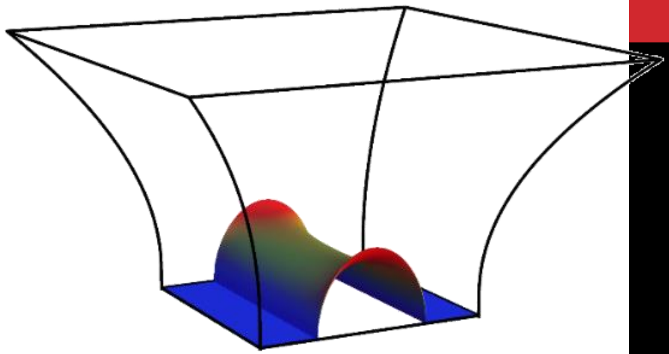


Enthalpy



$$ds^2 = -A(r,t)\mathrm{d}t^2 + 2\mathrm{d}r\mathrm{d}t + S(r,t)^2\mathrm{d}\vec{x}^2$$

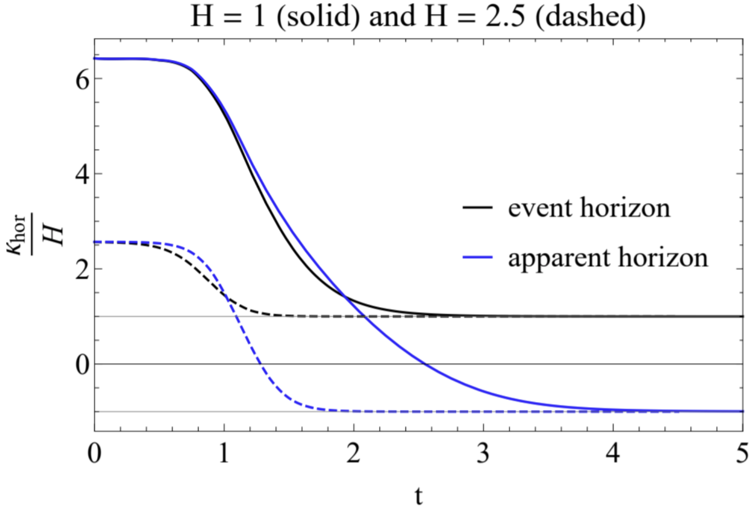
BLACK HOLE THERMODYNAMICS



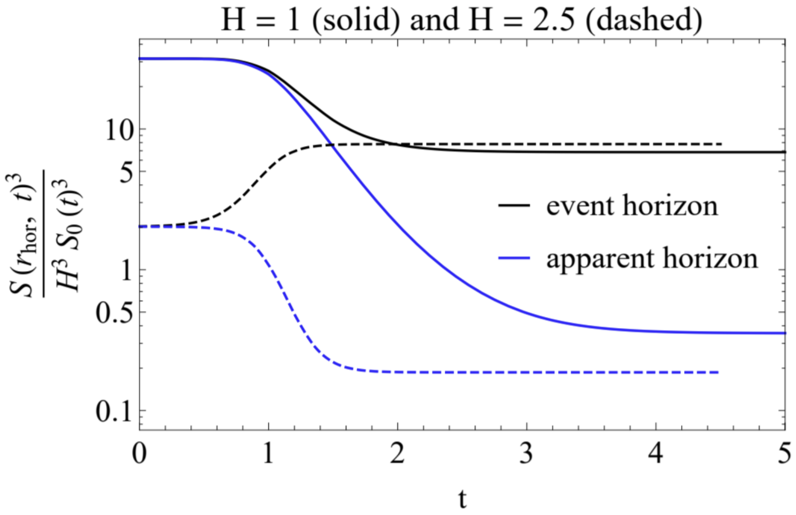
Keep track of bulk event and apparent horizons (EF coordinates)

- Dynamical setting: horizons not coincide at late times:
- Surface gravities can be shown analytically: $\kappa_{\mathrm{EH}} = -\kappa_{\mathrm{AH}} = H$
  EH confirms Hawking's temperature in de Sitter
- Area density apparent horizon vanishes for conformal theory

Surface gravity



Area densities



Willy Fischler, Sandipan Kundu and Juan Pedraza, Entanglement and out-of-equilibrium dynamics in holographic models of de Sitter QFTs (2013)
Alex Buchel, Entanglement entropy of N = 2* de Sitter vacuum (2019)

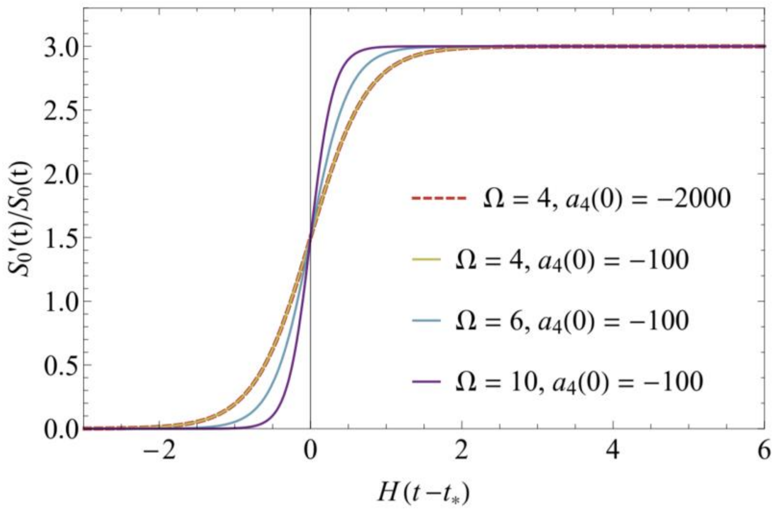
HOW WE SET UP A STATE

Non-trivial boundary metric: $ds_b^2 = -\mathrm{d}t^2 + S_0(t)^2 \mathrm{d}\vec{x}^2 \,. \quad S_0(t) = e^{Ht}$

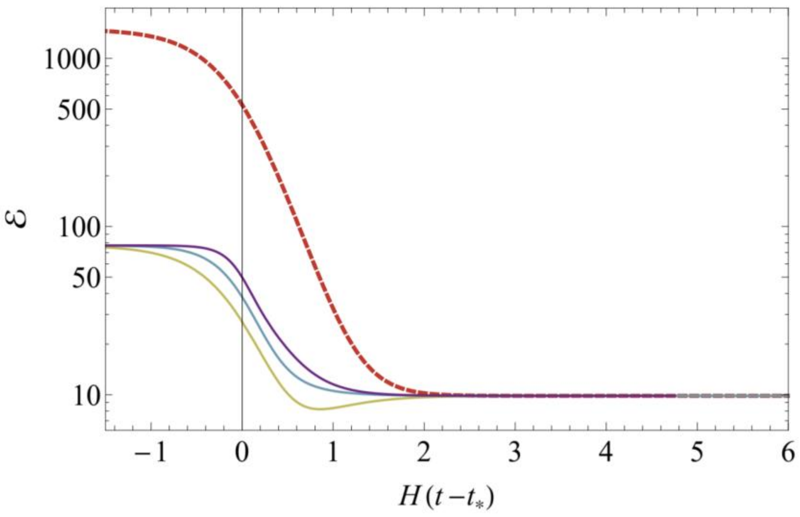
Start with thermal (high-temperature) state in flat space

- Quench system by suitable fast tanh to constant Hubble parameter
- Energy density decreases towards final `vacuum energy' (VE)
- Final (Bunch-Davis)-VE is ambiguous → chose scheme with zero VE

Hubble parameter



Energy density



THE APPROACH TOWARDS HYDRODYNAMICS

$$ds_{\mathrm{b}}^2 = -\mathrm{d}t^2 + S_0(t)^2 \mathrm{d}\vec{x}^2 \,. \quad S_0(t) = e^{Ht}$$

Comparing with the hydrodynamic constituent relations:

$$T_{\perp}^{\mu\nu} = P(\varepsilon)\Delta^{\mu\nu} - \eta(\varepsilon)\sigma^{\mu\nu} - \zeta(\varepsilon)\Delta^{\mu\nu}(\nabla\cdot u)\,,$$
$$\Delta^{\mu\nu} = g^{\mu\nu} + u^{\mu}u^{\nu}$$

Symmetric set-up: only non-trivial part is bulk viscosity:

$$\Delta\mathcal{P}^{\mathrm{hydro}}(t) \equiv \Delta p_{\mathrm{eq}}(\Delta\mathcal{E}(t)) - 3H\zeta(\Delta\mathcal{E}(t)) + O(H^2)\,,$$

A subtlety: EOS and viscosity computed in flat space;
what is the energy density in de Sitter space?

We decided to fix renormalisation freedom so that late time solution has zero energy density

(in any case: ambiguity is order H^2)
(also, note that scheme depends on H for our choice)

HOLOGRAPHIC RENORMALISATION

$$ds_{\mathrm{b}}^2 = -\mathrm{d}t^2 + S_0(t)^2 \mathrm{d}\vec{x}^2 \,. \quad S_0(t) = e^{Ht}$$

$$ds^2 = -A(r,t)\mathrm{d}t^2 + 2\mathrm{d}r\mathrm{d}t + S(r,t)^2 \mathrm{d}\vec{x}^2 \,, \quad \phi = \phi(r,t) \,,$$

Action needs (scheme-dependent) counter-terms:

$$S_{\mathrm{ct}} = \frac{1}{8\pi G} \int \mathrm{d}^4 x \sqrt{-\gamma} \left[\left(-\frac{1}{8} R[\gamma] - \frac{3}{2} - \frac{1}{2}\phi^2 \right) + \frac{1}{2} (\log \rho)\, \mathcal{A} + \left(\alpha \mathcal{A} + \beta \phi^4 \right) \right],$$

$$\mathcal{A} = \mathcal{A}_g + \mathcal{A}_\phi \,, \qquad \mathcal{A}_g = \frac{1}{16}\left(R^{ij}R_{ij} - \frac{1}{3}R^2\right), \qquad \mathcal{A}_\phi = -\frac{\phi^2}{12} R$$

Leads to an ambiguity in the stress-tensor:

$$\begin{aligned}\mathcal{E}(t) = &-\frac{3a_{(4)}(t)}{4} - M\bar{\phi}_{(2)}(t) + \frac{3S_0'(t)^4}{16S_0(t)^4} + M^2\left(\xi(t)^2 + \frac{S_0'(t)^2}{8S_0(t)^2} + \frac{2S_0''(t)}{3S_0(t)}\right) \\ &- M^2\alpha\frac{S_0'(t)^2}{2S_0(t)^2} - M^4\left(\beta - \frac{7}{36}\right),\end{aligned}$$

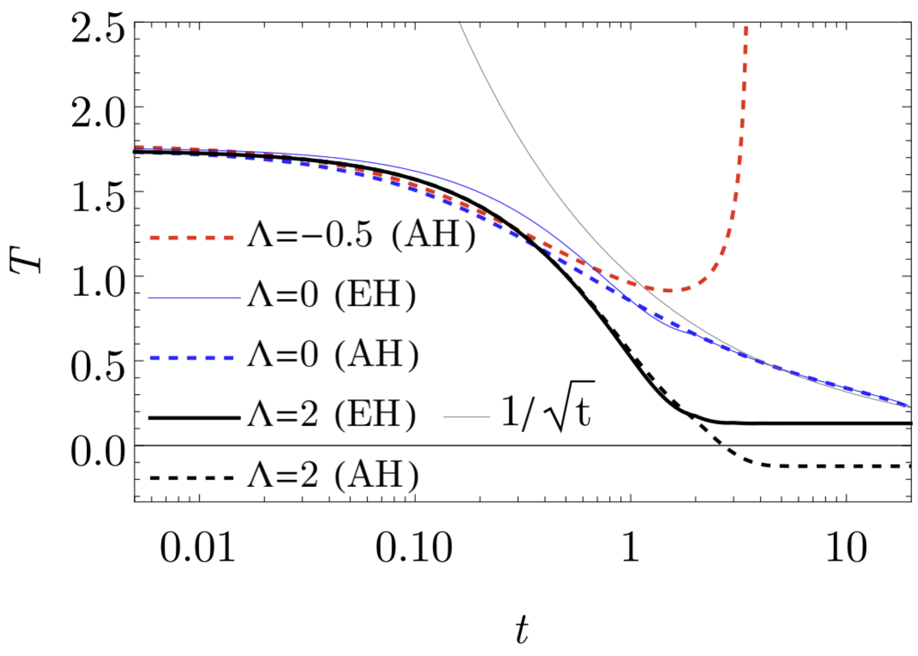
$$\begin{aligned}\mathcal{P}(t) = &-\frac{a_{(4)}(t)}{4} + \frac{1}{3}M\bar{\phi}_{(2)}(t) + \frac{S_0'(t)^2\left(S_0'(t)^2 - 4S_0(t)S_0''(t)\right)}{16S_0(t)^4} \\ &- \frac{M^2}{3}\left(\xi(t)^2 + \frac{S_0'(t)^2}{8S_0(t)^2} + \frac{13S_0''(t)}{12S_0(t)}\right) + M^2\alpha\left(\frac{S_0'(t)^2}{6S_0(t)^2} + \frac{S_0''(t)}{3S_0(t)}\right) + M^4\left(\beta - \frac{5}{108}\right)\end{aligned}$$

α and β encode scheme dependencies (cosmological constant);
fixed such that late time solution has vanishing energy

Ambiguities come in at order H^2

BOUNDARY GRAVITY - TEMPERATURES

Temperatures extracted from surface gravity event and apparent horizons



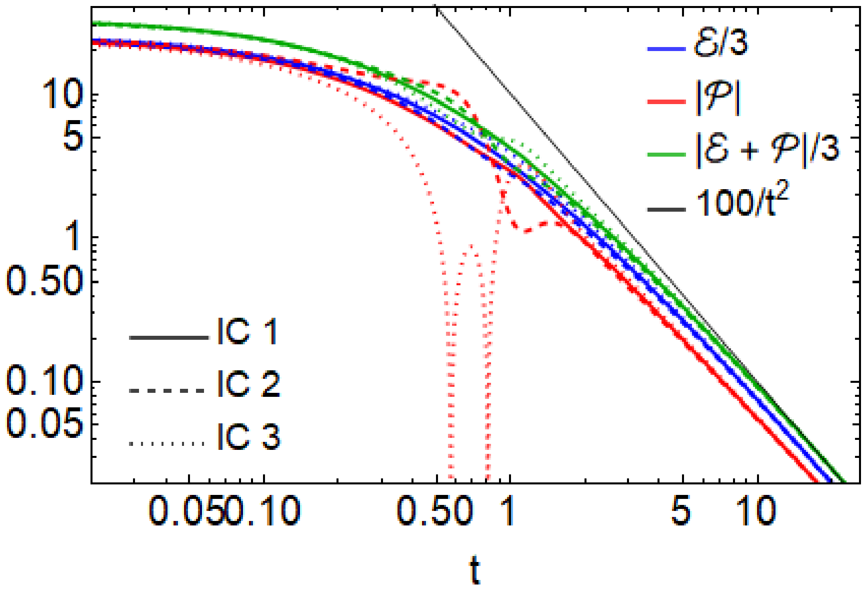
De Sitter case again settles down with negative AH 'temperature'

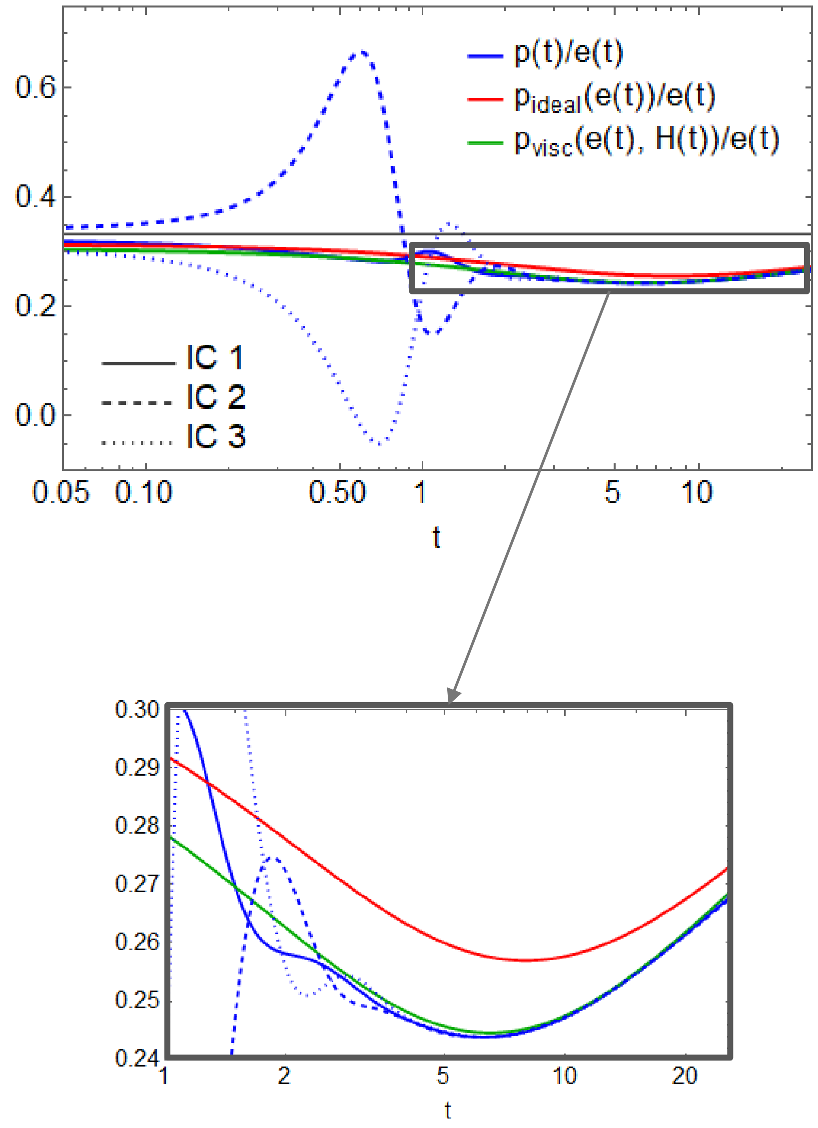
For the big crunch geometry the EH location is not obvious (no future timelike infinity)

HYDRODYNAMISATION AND BOUNDARY GRAVITY

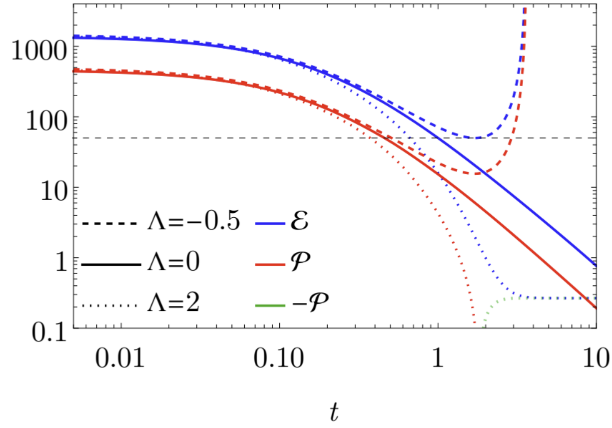
Three different initial conditions for Λ = 0:

Stress-tensor, and hydrodynamisation:
all hydrodynamise within a time of ~1/T





HYDRODYNAMISATION AND BOUNDARY GRAVITY



AdS case: fast hydrodynamisation, hydro all the way till big crunch

dS case: hydrodynamisation, but falls out of equilibrium.
Casimir energy important.