# Lattice QCD and BSM: an update

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9/04/2025

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# Outline

## Introduction to the Lattice approach

## Updates from QCD

- $g - 2$
- Scattering & Decays

## Updates from BSM

- Composite Higgs
- Early Universe Phase transitions

# Introduction

## recipe for a lattice calculation

**Ingredients...**

> A euclidean 4-dimensional space-time lattice with spacing $a$ and volume $V$,

> Discretized degrees of freedom $\bar{\psi}(x)$, $\psi(x)$, $U_\mu(x)$,

> An action $S_E = S_g[U] + S_f[\psi, \bar{\psi}, U]$

**...and the recipe**

> for any observable $O(\psi, \bar{\psi}, U)$

$$\langle O \rangle = \frac{1}{\mathcal{Z}} \int \mathcal{D}\psi \mathcal{D}\bar{\psi} \mathcal{D}U \, O[\psi, \, \bar{\psi}, \, U] \, e^{-S_E[\psi, \bar{\psi}, U]} \, ,$$

with

$$\mathcal{Z} = \int \mathcal{D}\psi \mathcal{D}\bar{\psi} \mathcal{D}U \; e^{-S_E[\psi, \bar{\psi}, U]}$$

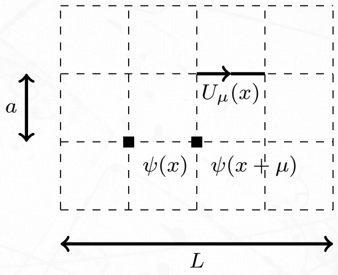


Figure: A two dimensional projection of the hypercubic lattice.

**FAQ:**

> How do we define $U_\mu(x)$?

$$U_\mu(x) = \exp i \int_x^{x+\hat{\mu}} \mathrm{d}x \; A_\mu(x) \; , \quad A_\mu(x) = A_\mu^a T^a$$

> How are $S_g[U]$ and $S_f[\psi, \bar{\psi}, U]$ chosen?
> Every choice has pros and cons.
> as $a \to 0$, they must tend to the well-known continuum actions.

> **Wilson gauge action**,

$$S_g[U] = \beta \sum_{x,\mu<\nu} \left(1 - \frac{1}{N}\text{Re Tr}\, \mathcal{P}_{\mu\nu}(x)\right) \ ,$$

where $\beta = 2N/g_0^2$, $g_0$ bare coupling at scale $1/a$,

$$\mathcal{P}_{\mu\nu}(x) = U_\mu(x)U_\nu(x+\hat{\mu})U_\mu^\dagger(x+\hat{\nu})U_\nu^\dagger(x)$$

> **Wilson-Dirac fermion action**,

$$S_f[\psi,\bar{\psi},U] = \sum_x \bar{\psi}(x)(D+m_0)\psi(y)$$

where $m_0$ is the quark mass parameter, and

$$D = (4/a + m_0)\delta_{yx} - \frac{1}{2a}\sum_\mu \{(1-\gamma_\mu)U_\mu(x)\delta_{y,x+\mu} + (1+\gamma_\mu)U_\mu(x-\mu)^\dagger\delta_{y,x-\mu}\}$$

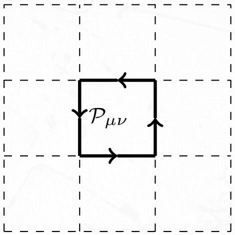


Figure: The elementary plaquette.

**Note:**

> Gauge invariance is safe.
> $4/a$ term: doubling problem.
> $\bar{\psi}(x)$, $\psi(x)$ Grassmann anti-commuting numbers

# Introduction

## Some more details...

**In theory...**

$$\mathcal{Z} = \int \mathcal{D}U \det\left(D + m_0\right)^{1/2} e^{-S_g[U]} \ ,$$

**In pactice**

- Amenable to Monte Carlo simulations.
- Set $a$ using physical observables.
- Extrapolate to $a \to 0$ using

$$O(a) = O(0) + a^n \delta_n O$$

where $\mathcal{O}(0)$ continuum limit.

- Similarly for the infinite volume limit $1/V \to 0$

**Limitations**

- No local chiral fermions in 4d.
- Topological Freezing, metastabilities...
- Sign problem(s): $\mu$, $\theta$,...
- Computational power.

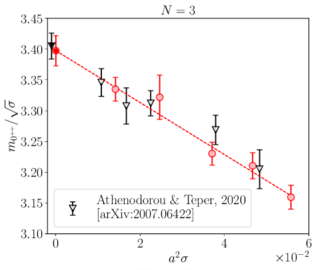


Figure: An example of continuum limit extrapolation, from (Bonanno et al. 2024).

# Typical Computation

**Typical observable**

$$C(t_1 - t_2) = \langle O_1(t_1) O_2(t_2) \rangle - \langle O_1 \rangle \langle O_2 \rangle$$

$O_1(t_1)$ and $O_2(t_2)$ are appropriate operators.

**Example: ground-state energy**

$$C(t) = \sum_n |c_n|^2 e^{-E_n t} \xrightarrow{t \to \infty} |c_0|^2 e^{-E_0 t}$$

**Effective mass**

$$m_{\text{eff}}(t) = -\frac{1}{t} \log \frac{C(t+1)}{C(t)}$$

**Main difficulty**

$$\frac{\text{signal}}{\text{noise}} \propto \exp -E_0 t$$

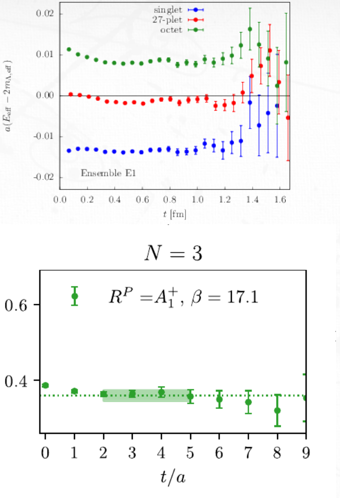


Figure: Top: The H dibaryon multiplet masses, taken from (Francis et al. 2019). Bottom: The $Sp(6)$ glueball mass, taken from (Bennett, Holligan, et al. 2021).

# Take home messages

**Lattice is...**

...first-principle

...non-perturbative

...not a simplification

...amenable to numerical simulations.

...sistematically improvable.

**However...**

...some systems are still beyond reach,

...massive computational needs: sustainability?

# QCD

**QCD**

- Very succesful theory of the strong interaction with

$$\beta(\alpha_s) = -\left(\frac{\alpha_s}{4\pi}\right)^2 (11 - \frac{2}{3}n_f)$$

**Non-perturbative at low energy**

- Confinement: flux-tubes, static potential, glueballs,...

$$V(R) = -\frac{4}{3}\frac{\alpha_s}{R} + \sigma R + \text{const.}$$

- Chiral symmetry breaking: $\pi$ as a pseudo-Nambu-Goldstone boson, EFTs

$$m_\pi^2 \propto (m_u + m_d)$$

**Lattice is the only first-principle method to access this regime**

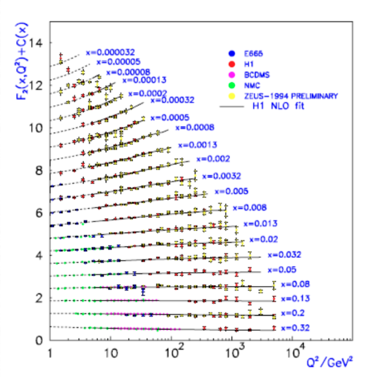


Figure: A compilation of DIS scaling results from (Schlippe)

# Gallery

## Hadron spectrum and Deconfinement

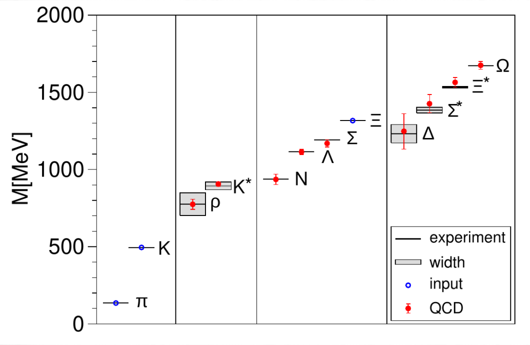


Figure: Determination of the light hadron spectrum, from (Durr et al. 2008).

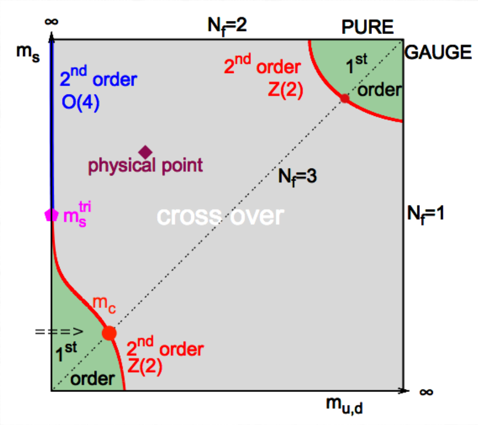


Figure: The so-called Columbia Plot for the deconfinement phase transition, from (Forcrand and D'Elia 2017).

# Scattering on the Lattice

**Resonances**

- > Most of the known hadrons.
- > Quark model: Exotics, Glueballs,...
- > In experiments, asymptotic states

**On the Lattice**

- > Euclidean time
- > Stationary states in a finite box,
- > Discrete spectrum.

**Lüscher's method**: Infinite-volume scattering phase shifts can be calculated finite-volume spectrum and quantization condition

$$\det\left[F^{-1}(E,\mathbf{P},L) - \mathcal{M}(E)\right] = 0$$

see (Luscher 1986a,b).

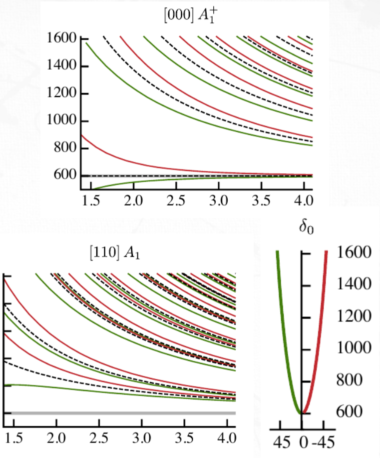


Figure: Weakly attactive (green), weakly repulsive (red) and non-interacting(dashed) spectra, as a function of size, taken from (Briceno, Dudek, and Young 2018).

**Motivation**

- $>$ Diffiult to accomodate in the quark model
- $>$ Unitarized $\chi$-PT suggests two poles.

**The Lattice setup:**

- $>$ $a = 0.0633(4)(6)$ fm, $L = 64a$
- $>$ $m_\pi \sim 200$ MeV, $m_K \sim 487$ MeV,
- $>$ Improved gauge and fermion actions.

**Results:**

- $>$ This analysis suggests the presence of two poles.
- $>$ One is a virtual bound state.
- $>$ One is a resonance.

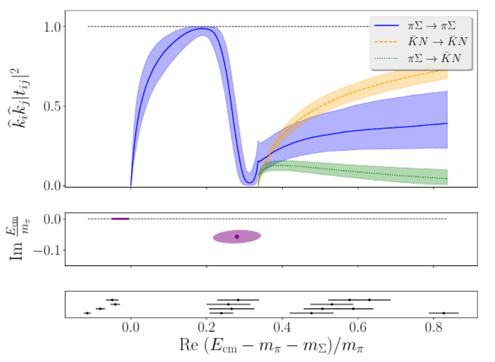


Figure: The isospin $I = 0$ and strangeness $S = -1$ coupled channel $\pi\Sigma - KN^-$ transition amplitudes, and pole position, from (Bulava et al. 2024).

# Muon $g-2$

- **Electron** $g-2$ among the most precise physical observables known to mankind, strong sector practically irrelevant.

$$a_l = \frac{g_l - 2}{2} = \frac{\alpha}{2} + \mathcal{O}(\alpha^2)$$

- Lepton $l$ potentially sensitive to new physics at scale $\Lambda_{\text{NP}}$,

$$a_l - a_l^{\text{SM}} \propto \frac{m_l^2}{\Lambda_{\text{NP}}^2}$$

and $m_\mu / m_e \sim 200$.

**Largest contribution to error:**

- Hadroniv Vacuum polarization (HVP)
- Hadronic Light by Light scattering (HLbL)

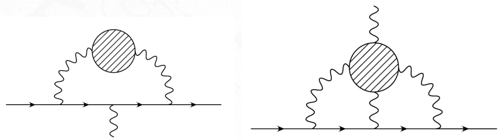

Figure: LOHVP and LOHLBL contributions. The blob represents all connected intermediate haronic states. Taken from (Blum, Hayakawa, and Izubuchi 2012)

**Why Lattice QCD?**

- First-principles, non-perturbative approach.
- Avoids model-dependent assumptions in data-driven methods.
- Essential to match future experimental precision

# Hadron contributions to $g-2$

- **Hadronic Vacuum Polarization (HVP)**: main contibution to uncertainty

$$a_\mu^{HVP,LO} = \left(\frac{\alpha}{\pi}\right)^2 \int_0^\infty \mathrm{d}Q^2 f(Q^2)\Pi(Q^2)$$

where

$$\Pi_{\mu\nu}(Q) = \int d^4x e^{iQx} \langle j_\mu^{em}(x) j_\nu^{em}(0)\rangle$$
$$= (Q_\mu Q_\nu - Q^2)\Pi(Q^2)$$

and $j_\mu^{em}(x) = \sum_f q_f \bar{\psi}_f \gamma_\mu \psi$

- **On the Lattice..**
  - Evaluate current 2-points function.
- **..or Data driven method,**

$$a_\mu^{HVP,LO} = \frac{1}{4\pi^3} \int_{s_0}^\infty \mathrm{d}s \sigma_{had,\gamma}(s) K(s)$$

see Figure.

- **Hadronic Light-by-Light (HLbL)**: smaller contribution to uncertainty

$$\Pi^{\mu\nu\lambda\sigma}(q_1, q_2, q_3) = -i \int \mathrm{d}^4x \mathrm{d}^4y \mathrm{d}^4z$$
$$e^{iq_1x + iq_2y + iq_3z} \langle j_\mu(x) j_\nu(y) j_\lambda(z) j_\sigma(0)\rangle$$

- **only Lattice**
  - Direct: evaluate current 4-points function, several methods.
  - Dispersive analysis.

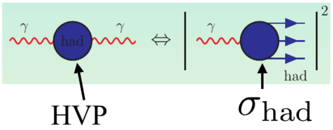


Figure: Calculation of HVP with data driven method, taken from (Davis, 2024)

# Updated BMW'23 results

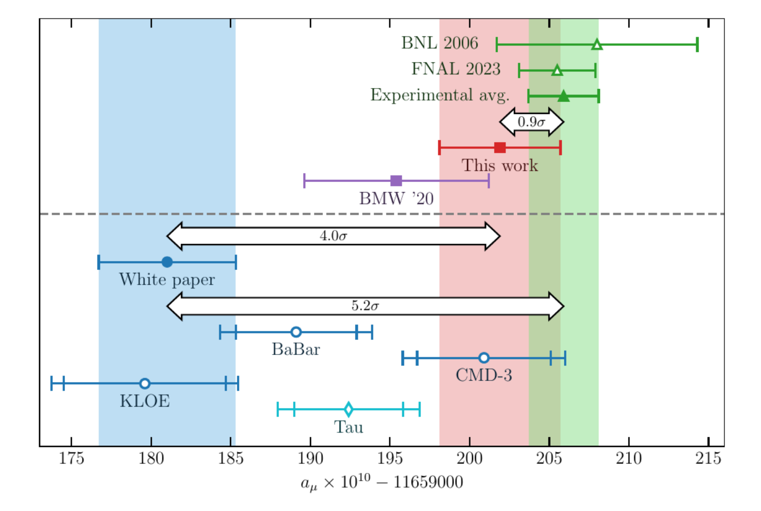


Figure: Blue: data driven methods, Red: Lattice, Green: Experiments. Taken from (Boccaletti et al. 2024).

# BSM

**Shortcomings of the SM..**

> Nature of Dark Matter/Energy?
> Baryogenesys?
> Hierarchy Problem, origins of EW sector?
> No Quantum Gravity!

# BSM

## How can the Lattice be useful?

**Shortcomings of the SM..**

> Nature of Dark Matter/Energy?

> Baryogenesys?

> Hierarchy Problem, origins of EW sector?

> No Quantum Gravity!

**...and how to address them**

> New Dark Sectors, WIMP, SIMP,

> Phase transitions in the early universe,

> Supersymmetry,

> Composite Higgs(CH), Technicolor(TC),...

> Grand Unification and extra dimensions,

> Strings?

# BSM

## How can the Lattice be useful?

**Shortcomings of the SM..**

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- Composite Higgs(CH), Technicolor(TC),...
- Grand Unification and extra dimensions,
- Strings?

**The Lattice can...**

- ...help identify deviations from SM:
  - muon $g-2$,
  - Hadron spectrum,
  - CKM matrix,
  - Rare decays
  - ...
- ...test BSM scenarios
  - TC and CH Models,
  - SIMP Dark Matter
  - Axions
  - ...

# Composite Higgs

# Naturalness problem

**The Higgs boson is...**

- ...what sparks EW symmetry breaking,
- ...a fundamental scalar,
- ...phenomenologically extremely succesfull...

**However it is...**

> trivial?
> Not an explanation for EW.
- ...too light? Indeed,

$$m_H^2 = \delta_{\rm SM} m_H^2 + \delta_{\rm BSM} m_H^2, \quad \delta_{\rm SM} m_H^2 \propto \Lambda_{\rm SM}^2$$

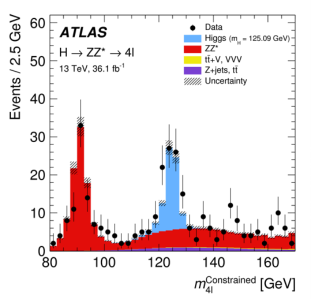


Figure: $H \to ZZ \to 4l$, @13 TeV, data from the LHC.

**If $\Lambda_{\rm SM} \sim M_{\rm GUT}$**

> SM highly generic at $\Lambda_{\rm SM}$
> Then $\dfrac{\delta_{\rm SM} m_H^2}{m_H^2} \sim 10^{-24}$

**If $\Lambda_{\rm SM} \sim 1$ TeV**

> SM highly constrained at $\Lambda_{\rm SM}$.
> New physics very close?

**No new physics at the TeV scale, so far**

# Composite Higgs Models

**Various scenarios...**

> Supersymmetry ?

> Walking Technicolor?

**Composite Higgs**

> Vacuum Misalignement
$\longrightarrow$ **Higgs composite & nearly massless!**

> Partial Compositeness
$\longrightarrow$ **The physical top quark is a superposition of SM+Composite states.**

**On the Lattice**

> Simulate the UV theory.

> “Measure” observables at low energy.

> However: many intereting things accessible.

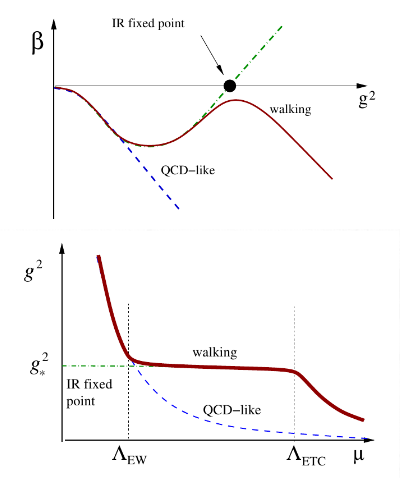


Figure: Sketch of different behaviours for the running coupling, taken from (Bennett, Hong, et al. 2024).

# Conformal window location

For $SU(N)$

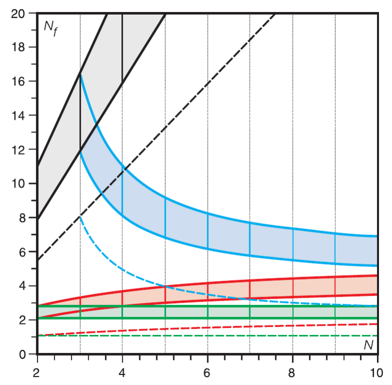


**Figure:** Top: Phase diagram of $SU(N)$ theories in the $(N, N_f)$ plane, taken from (Dietrich and Sannino 2007).

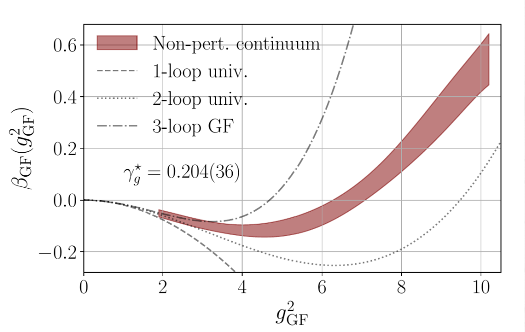


**Figure:** Bottom: Non-perturbative prediction for the beta function at $N_f = 12$, taken from (Hasenfratz and Peterson 2024).

# $Sp(4)$ Partial Compositeness

**Why $Sp(4)$?**

- Landscape of "good" UV completions that can accomodate PC charted. Ferretti and Karateev 2014
- $Sp(4)$ minimal model.

**So far**

- Glueball and Meson spectrum for $Sp(2N_c)$.
- $N_{\text{fund}} = 2$ light mesons spectrum and decay constants.
- $N_{\text{as}} = 3$ conformal window?
- Quenched partial compositeness?

**Limitations**

- $\rho$ still stable.
- Chimera baryons still quenched.

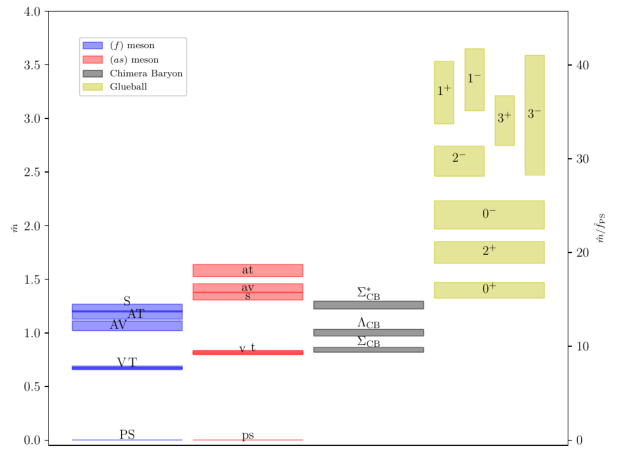


Figure: Multirepresentation quenched spectrum $Sp(4)$, taken from (Bennett, Hong, et al. 2024).

# Dark Sectors

# Lattice and GWs

**Baryogenesys**

- Sacharov conditions.
- SM alone insufficient: CP violation small, EW transition cross-over.

**Dark Matter**

- Many independent clues: structure, lensing, cosmological data,...but no direct detection.
- Cold DM models in tension with small scale structure.

**Novel Strongly Interacting sectors**

- Phase transitions of the first order: $SU(N > 2)$ $\longrightarrow$ GW signature!
- Self Interacting DM (SIDM) scenarios address some issues of $\Lambda CDM$.

**On the Lattice**

- Calculation of transition temperature, and Latent heat.
- Scattering cross sections
- Spectrums
- Axions!

# $Sp(4)$ deconfinement

**Motivation**

- (stochastic) GW signature.
- Non-perturbative calculation needed.

**Setting**

- $Sp(4)$ quenched
- LLR algorithm to access density of states

**Results & Comments**

- Obtained Transition temperature & Latent heat
- Scaling with volume difficult.

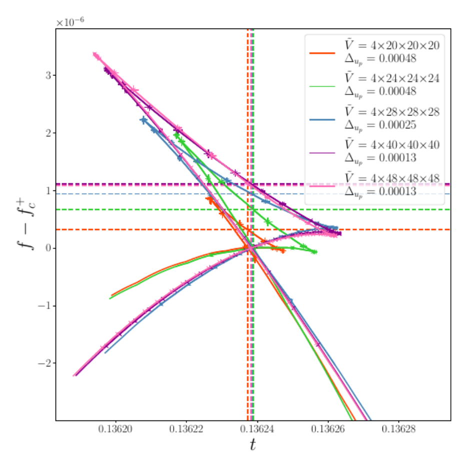


Figure: Behaviour of the free energy of $Sp(4)$ quenched theory around first-order phase transition, taken from (Bennett, Lucini, et al. 2024).

# Take home messages

- Lattice first-principle, non-perturbative, systematically improvable approach.
- Useful for both qualitative and quantitative investigations.
- Lattice has entered its “Young adult” era: strong, healthy and with many exciting possibilities and challenges ahead.

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# Scattering and Decays

## Light and strange vector resonances

**The Lattice setup:**

- $u$, $d$ and also $s$ quarks.
- Domain-Wall fermions, a single lattice spacing.
- What is computed: spectrum at finite volume and at various momenta.
- Value of lat. spacing $a$ set from $\Omega$ baryon mass.
- Sources of systematic error: fits ranges, discretization, masses of $\pi$ and $K$...

**Results:**

$$\rho(770)\begin{cases} M_\rho = 796(5)(50)\text{MeV} \\ \Gamma_\rho = 192(10)(31)\text{MeV} \end{cases}$$

$$K*(892)\begin{cases} M_{K^*} = 893(2)(54)\text{MeV} \\ \Gamma_{K*} = 51(2)(11)\text{MeV} \end{cases}$$

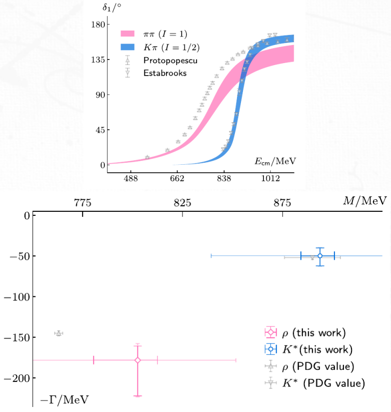


Figure: Figure and results taken from (Boyle et al. 2025a,b).

# Scattering of dark pions in $Sp(4)$

**Motivations**

- Scattering information needed.
- Also important for CH.

**Setting**

- Dirac-Wilson action & HMC algorithm.
- Effective Range Expansion

$$p^{2l+1}\cos\delta_l = -\frac{1}{a_l^{2l+1}} + \frac{p^2}{2r_l^{2l+1}} + \mathcal{O}(p^4)$$

and

$$\sigma_0(p) = \frac{4\pi a_0^2}{|2 - a_0 r_0 p^2 + 2ipa_0|^2}$$

where $a_0$ scattering length and $r_0$ effective range.

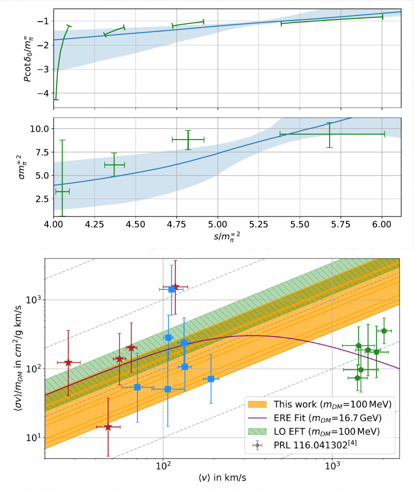


Figure: Top: phase shifts as a function of energy. Bottom: Cross-section as a function of average speed. Taken from (Dengler, Maas, and Zierler 2024).

# Phase transitions in early universe

**Motivation?**

- (stochastic) GW signature.
- Non-perturbative calculation needed.

**Setting**

- Transition is strong at $N(colors) = 8$, hence it allows for superheating & supercooling
- Formulated an EFT and Simulated *that*.
- Used smeared order parameter.

**Results & Comments**

- Possible to resolve critical bubble with appropriate methods.
- Larger volumes are necessary.

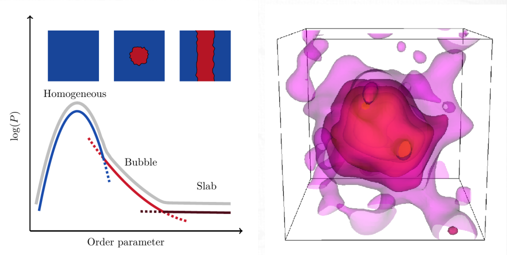


Figure: Left: Free energy behaviour as a function of the order parameter (Hällfors and Rummukainen 2025). Right: Spatial distribution of values of order parameter in two dimensions, taken from (Seppä, Rummukainen, and Weir 2025).

# Charting $Sp(N_c)$ theories

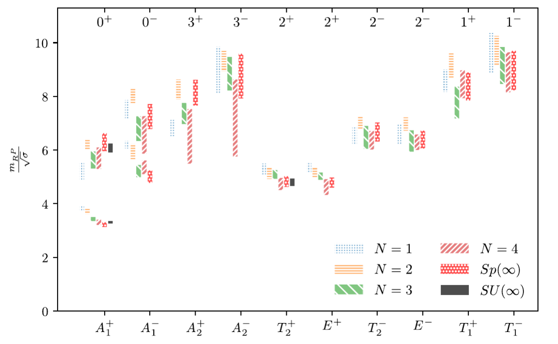


Figure: Quenched glueball spectrum, taken from (Bennett, Holligan, et al. 2021).

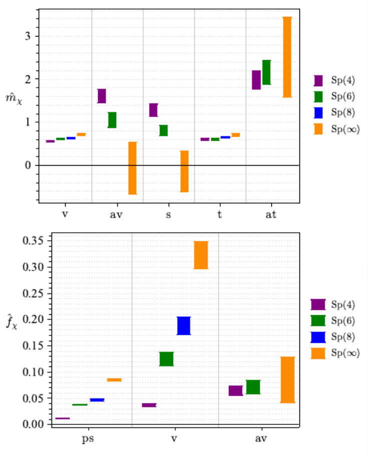


Figure: Quenched meson spectrum, taken from (Bennett, Hong, et al. 2024).