Measuring Atmospheric Neutrino Mixing Without Unitarity

Rory Ramsden
King's College London

IOP Joint APP and HEPP 2025, Cambridge – 8th April

King's College London
UKRI Science and Technology Facilities Council
Super K

Lepton Mixing With Unitarity

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

Lepton mixing matrix mixes the mass states into the flavour states as is the case for quark mixing.

Our current measurements of neutrino oscillations assume unitarity → **6 parameter** problem:

3 angles θ_{ij}

1 complex phase δ

2 mass squared splittings Δm_{ij}^2

$$U_{PMNS} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} + \text{Unitarity}$$

$$|U_{\alpha 1}^{3\nu}|^2 + |U_{\alpha 2}^{3\nu}|^2 + |U_{\alpha 3}^{3\nu}|^2 = 1, \quad \alpha = e, \mu, \tau,$$
$$|U_{ei}^{3\nu}|^2 + |U_{\mu i}^{3\nu}|^2 + |U_{\tau i}^{3\nu}|^2 = 1, \quad i = 1, 2, 3,$$
$$U_{\alpha 1}^{3\nu} U_{\beta 1}^{3\nu,*} + U_{\alpha 2}^{3\nu} U_{\beta 2}^{3\nu,*} + U_{\alpha 3}^{3\nu} U_{\beta 3}^{3\nu,*} = 0, \quad \alpha, \beta = e, \mu, \tau, \quad \alpha \neq \beta,$$
$$U_{ei}^{3\nu} U_{ej}^{3\nu,*} + U_{\mu i}^{3\nu} U_{\mu j}^{3\nu,*} + U_{\tau i}^{3\nu} U_{\tau j}^{3\nu,*} = 0, \quad i, j = 1, 2, 3, \quad i \neq j.$$

Unitarity conditions on rows and columns preserve neutrino oscillation probabilities.

$$= \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13} e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13} e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Lepton Mixing Without Unitarity

$$U_{LMM,3\times3} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu1} & U_{\mu2} & U_{\mu3} \\ U_{\tau1} & U_{\tau2} & U_{\tau3} \end{pmatrix} + \text{~~Unitarity~~}$$

$$= \begin{pmatrix} |U_{e1}| & |U_{e2}| & |U_{e3}| \\ |U_{\mu1}|e^{i\delta_{\mu1}} & |U_{\mu2}|e^{i\delta_{\mu2}} & |U_{\mu3}| \\ |U_{\tau1}|e^{i\delta_{\tau1}} & |U_{\tau2}|e^{i\delta_{\tau2}} & |U_{\tau3}| \end{pmatrix}$$

Without unitarity → **15 parameter** problem:
9 magnitudes $|U_{\alpha i}|$
4 complex phase $\delta_{\alpha i}$
2 mass squared splittings Δm_{ij}^2

We can measure the matrix elements directly as a **model-independent** test for unitarity as is done in quark sector.

If this matrix is non-unitary it could hint that the 3ν mixing matrix is a sub-matrix of a larger lepton mixing matrix → sterile sector.

$$U_{\text{PMNS}}^{\text{Extended}} = \begin{pmatrix} \overbrace{\begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu1} & U_{\mu2} & U_{\mu3} \\ U_{\tau1} & U_{\tau2} & U_{\tau3} \end{pmatrix}}^{U_{\text{PMNS}}^{3\text{x}3}} & \begin{matrix} \cdots & U_{en} \\ \cdots & U_{\mu n} \\ \cdots & U_{\tau n} \end{matrix} \\ \begin{matrix} \vdots & \vdots & \vdots \\ U_{s_n1} & U_{s_n2} & U_{s_n3} \end{matrix} & \begin{matrix} \ddots & \vdots \\ \cdots & U_{s_nn} \end{matrix} \end{pmatrix}$$

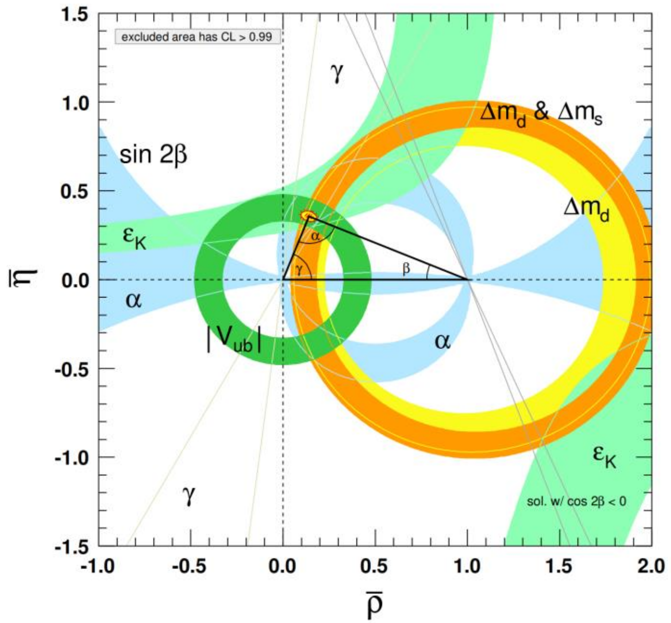
Current Measurements

NuFit JHEP 09 (2020) 178 [arXiv:2007.14792]

Unitarity Assumed

NuFIT 5.3 (2024)

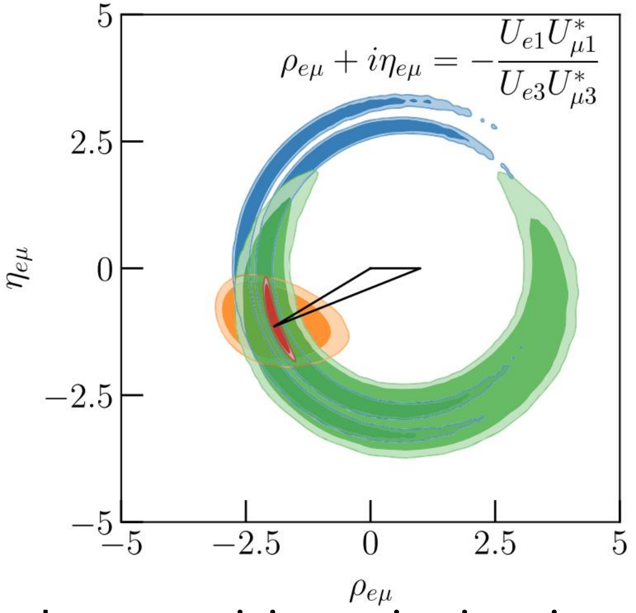
$$|U|^{\text{w/o SK-atm}}_{3\sigma} = \begin{pmatrix} 0.801 \to 0.842 & 0.518 \to 0.580 & 0.142 \to 0.155 \\ 0.236 \to 0.507 & 0.458 \to 0.691 & 0.630 \to 0.779 \\ 0.264 \to 0.527 & 0.471 \to 0.700 & 0.610 \to 0.762 \end{pmatrix}$$



Quark mixing unitarity triangle

P.A. Zyla *et al.* (Particle Data Group), Prog. Theor. Exp. Phys. 2020, 083C01 (2020).

Unitarity Assumed

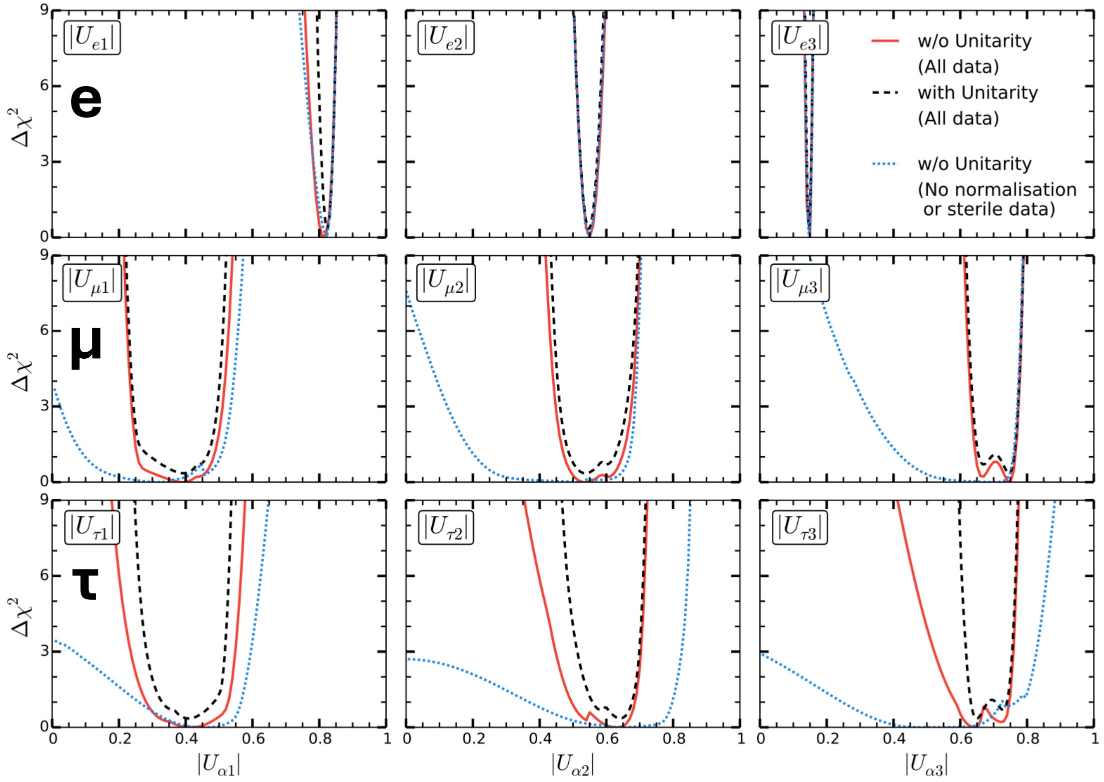


Lepton mixing unitarity triangle

S.A.R. Ellis, K.J. Kelly, S.W. Li (2020). Phys. Rev. D **102**, 115027

S. Parke, M. Ross-Lonergan arXiv:1508.05095v1 [hep-ph]

Unitarity Not Assumed



Electron row much better constrained than muon and tau rows.
Atmospheric neutrinos have lots of muon neutrino oscillations so can we exploit that?

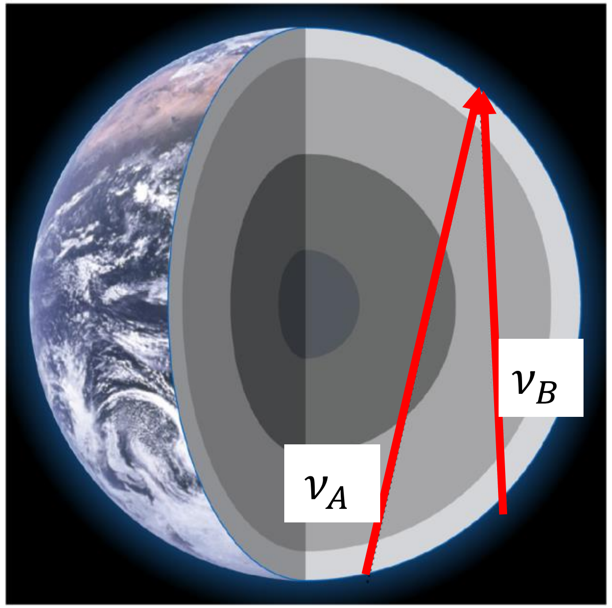
Atmospheric Neutrino Oscillation Probability

$$P_{\alpha\to\beta} = \delta_{\alpha\beta} - 4\sum_{i>j}\mathrm{Re}\left(U^*_{\alpha i}U_{\beta i}U_{\alpha j}U^*_{\beta j}\right)\sin^2\left(\Delta m^2_{ij}\frac{L}{4E}\right) \pm 2\sum_{i>j}\mathrm{Im}\left(U^*_{\alpha i}U_{\beta i}U_{\alpha j}U^*_{\beta j}\right)\sin^2\left(\Delta m^2_{ij}\frac{L}{2E}\right)$$

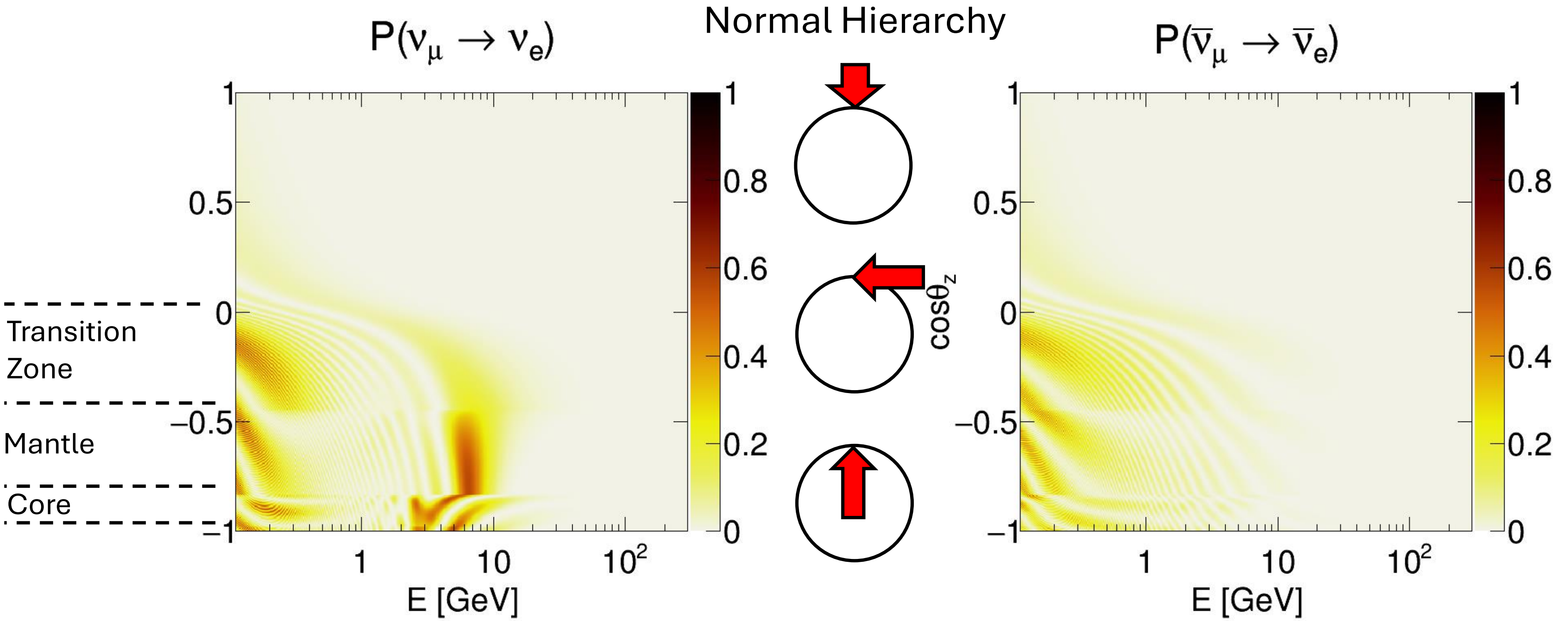
Eg

$$P_{\mu\leftrightarrow e} \approx \sin^2\theta_{23}\sin^2 2\theta_{13}\sin^2\left(\Delta m^2_{23}\frac{L}{4E}\right)$$

Super-K Collaboration (2024). Phys. Rev. D **109**, 072014



Super-K Collaboration (2018). Phys. Rev. D **97**, 072001

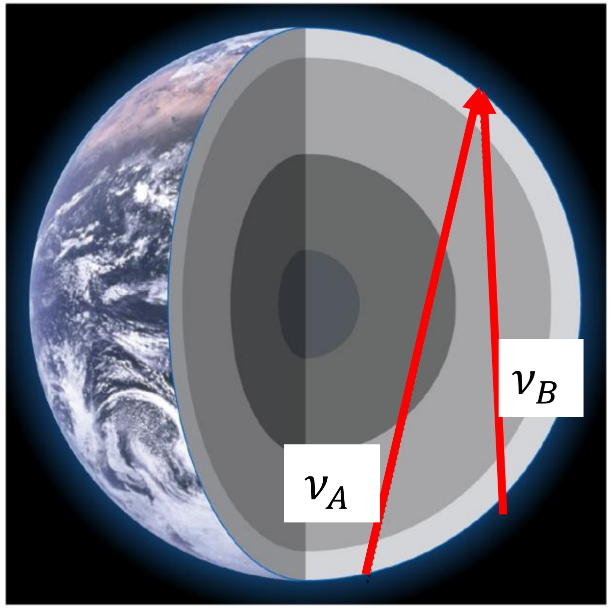


Modification to Atmospheric Oscillations

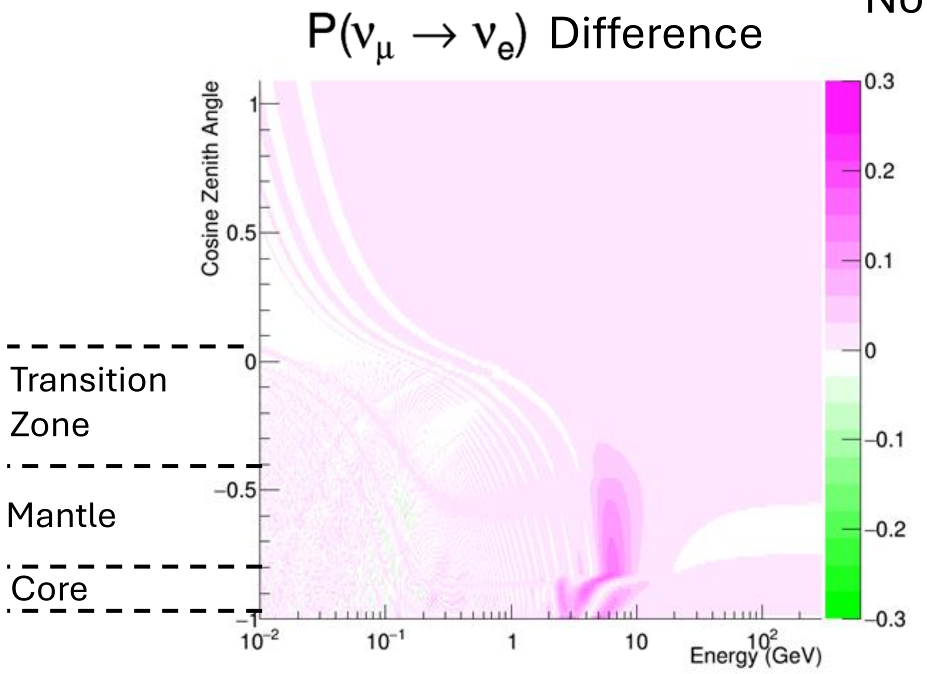
Since the $U_{\alpha i}$ no longer necessarily obey the unitarity conditions → get a 'zero-distance' effect: L, E independent term

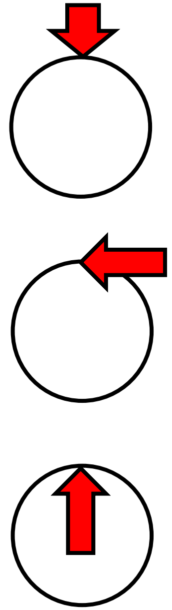
$$P_{\alpha\to\beta} = \left|\sum_i U^*_{\alpha i} U_{\beta i}\right|^2 - 4\sum_{i>j} \mathrm{Re}\left(U^*_{\alpha i} U_{\beta i} U_{\alpha j} U^*_{\beta j}\right)\sin^2\left(\Delta m^2_{ij}\frac{L}{4E}\right) \pm 2\sum_{i>j} \mathrm{Im}\left(U^*_{\alpha i} U_{\beta i} U_{\alpha j} U^*_{\beta j}\right)\sin^2\left(\Delta m^2_{ij}\frac{L}{2E}\right)$$

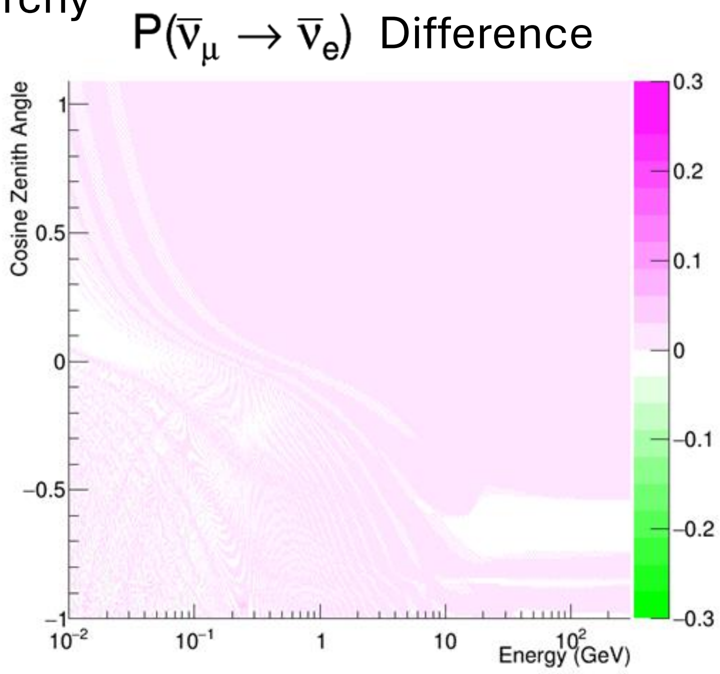
Compare $P(\text{Unitary}) - P(\text{Non-Unitary}) = \text{eg } P(|U_{\mu 3}| = 0.663) - P(|U_{\mu 3}| = 0.653)$



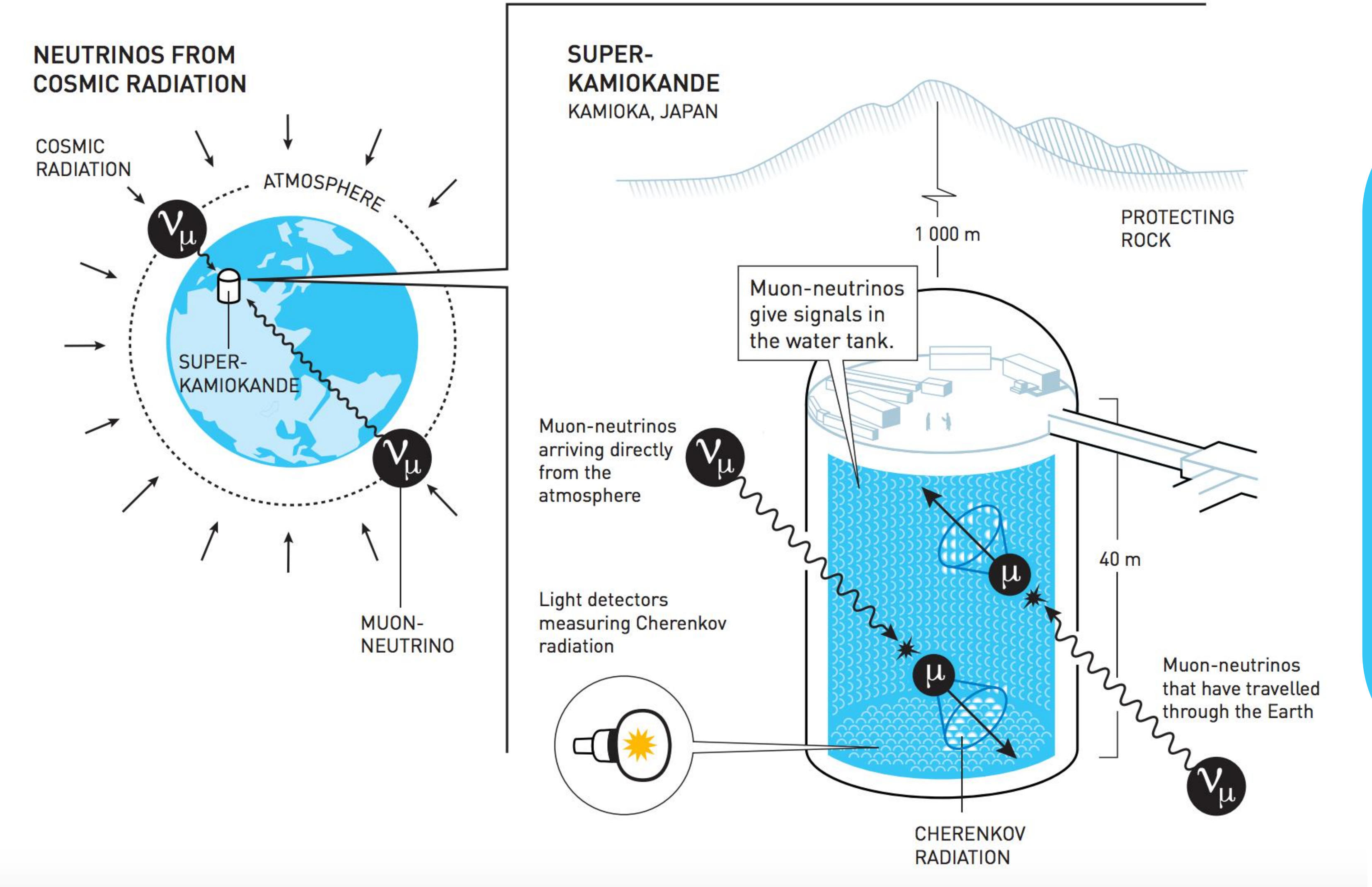
Normal Hierarchy







Super-Kamiokande

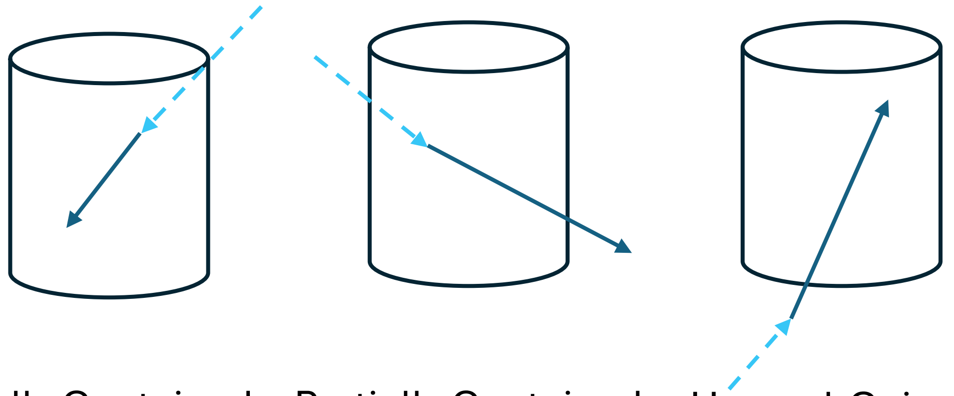


nobelprize.org

- 50 kiloton water-Cherenkov detector in Gifu, Japan
- 22.5 kiloton fiducial mass
- 11000 20” inner detector photosensors
- 1800 8” photosensors in optically separated outer detector for veto of cosmic μ
- 7076 days live-time between 1996-2022 (pure water phase 1996-2020, Gd loaded 2020-present)

Super-Kamiokande Atmospheric Sample

Three main classifications of atmospheric neutrino events

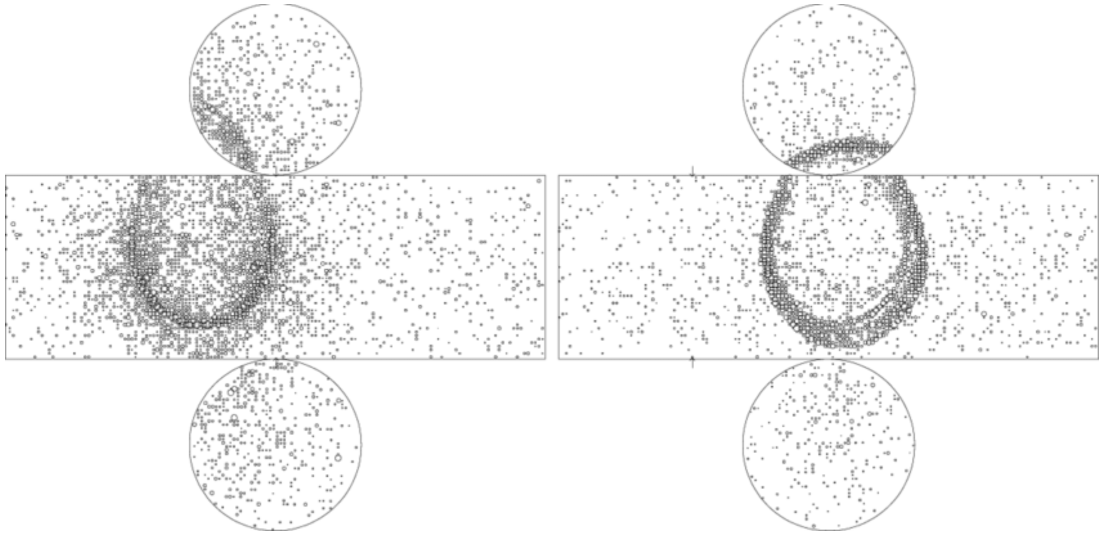


Fully Contained
~10 / day

Partially Contained
~0.6 / day

Upward-Going μ
~1.5 / day

Particle identification

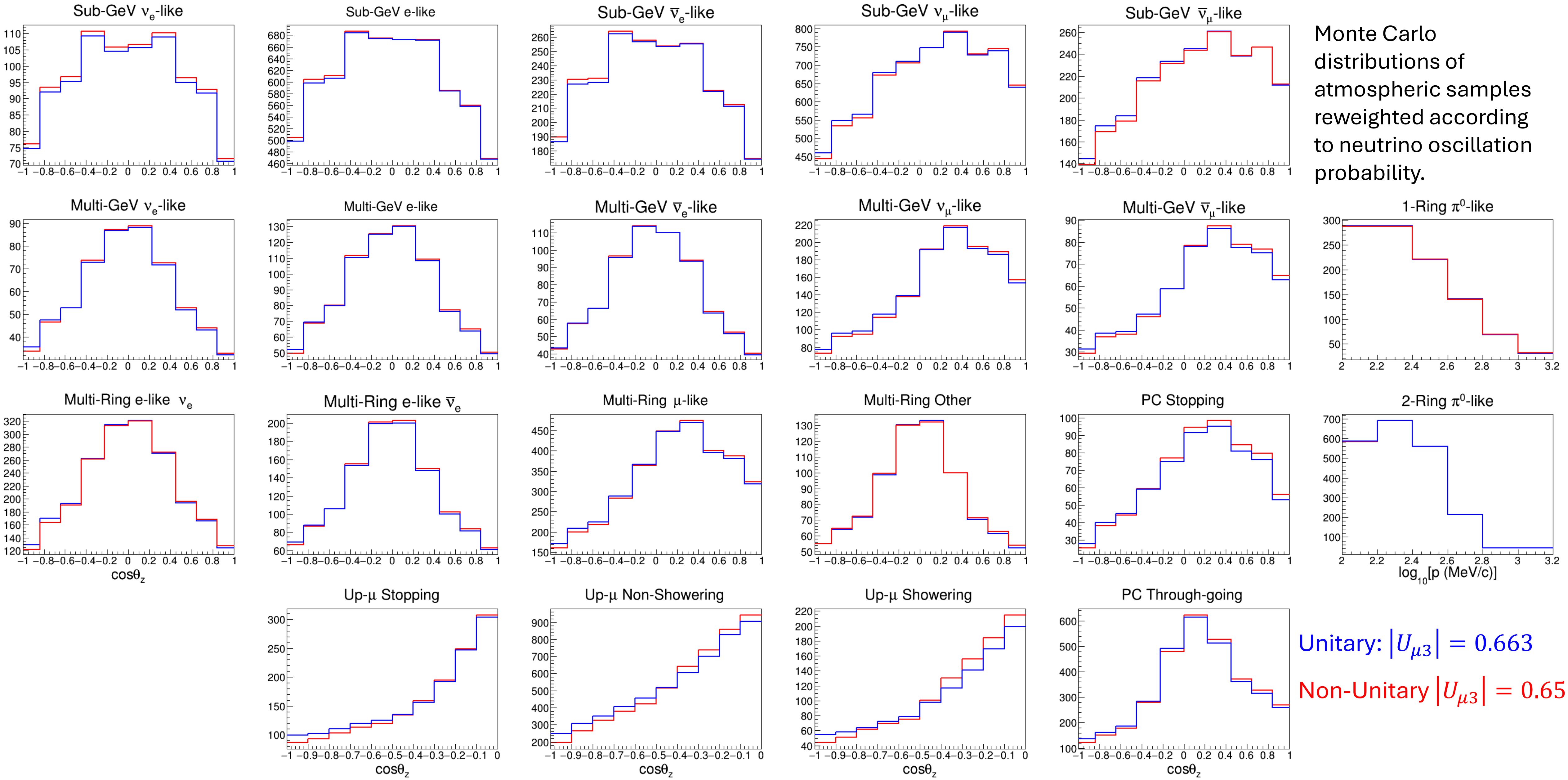


e-like

μ-like

Kajita, Takaaki. (2004). Atmospheric Neutrinos. New Journal of Physics. 6. 194. 10.1088/1367-2630/6/1/194.

Cosine Zenith and Momentum Distributions

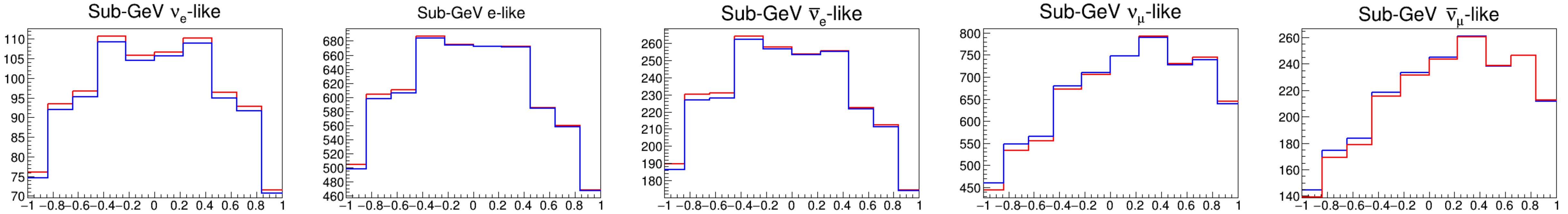


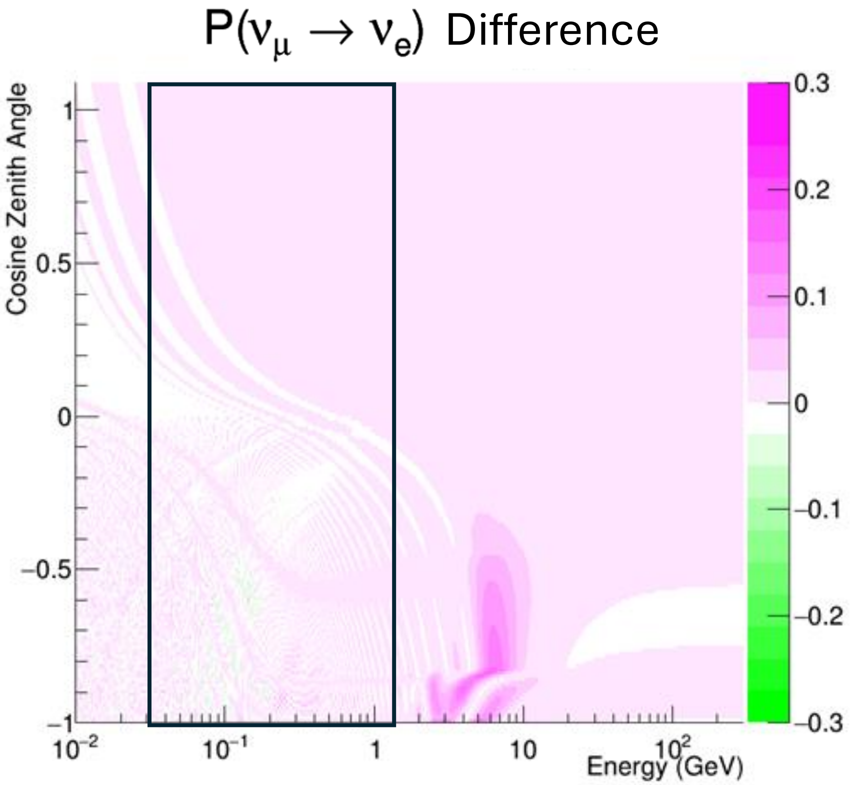
Monte Carlo distributions of atmospheric samples reweighted according to neutrino oscillation probability.

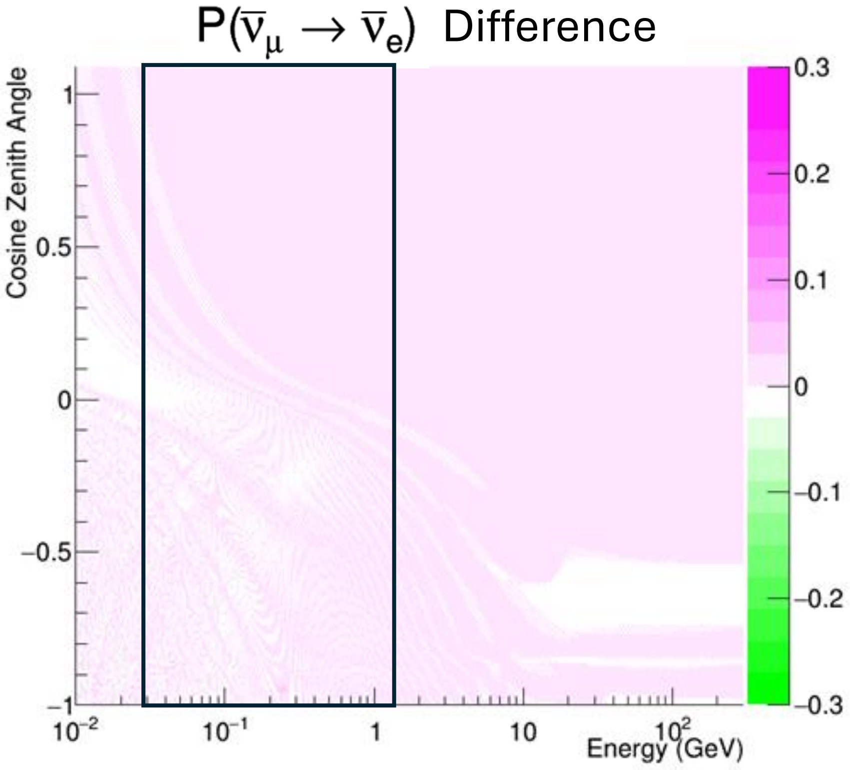
Unitary: $|U_{\mu 3}| = 0.663$

Non-Unitary $|U_{\mu 3}| = 0.653$

Lower Energy Neutrinos (< 1.33 GeV)





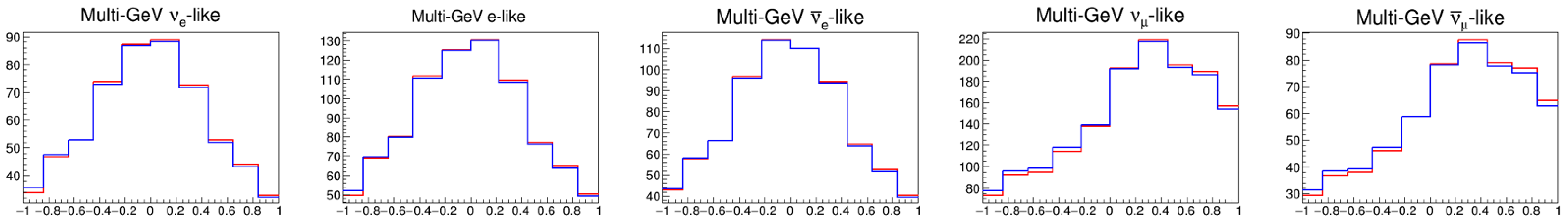


More ν_μ are oscillating to ν_e so more ν_e and less ν_μ

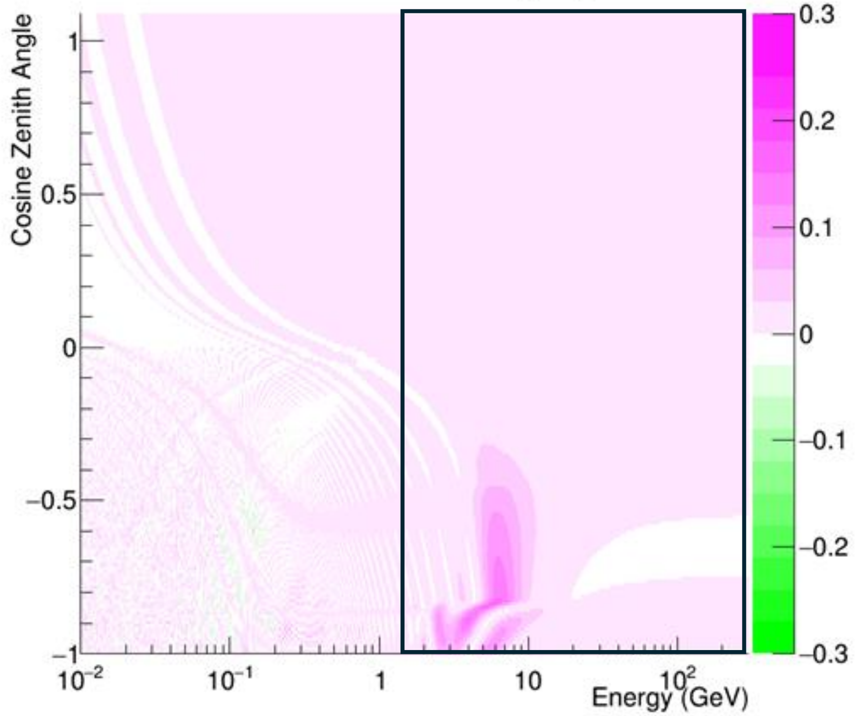
Unitary: $|U_{\mu 3}| = 0.663$

Non-Unitary $|U_{\mu 3}| = 0.653$

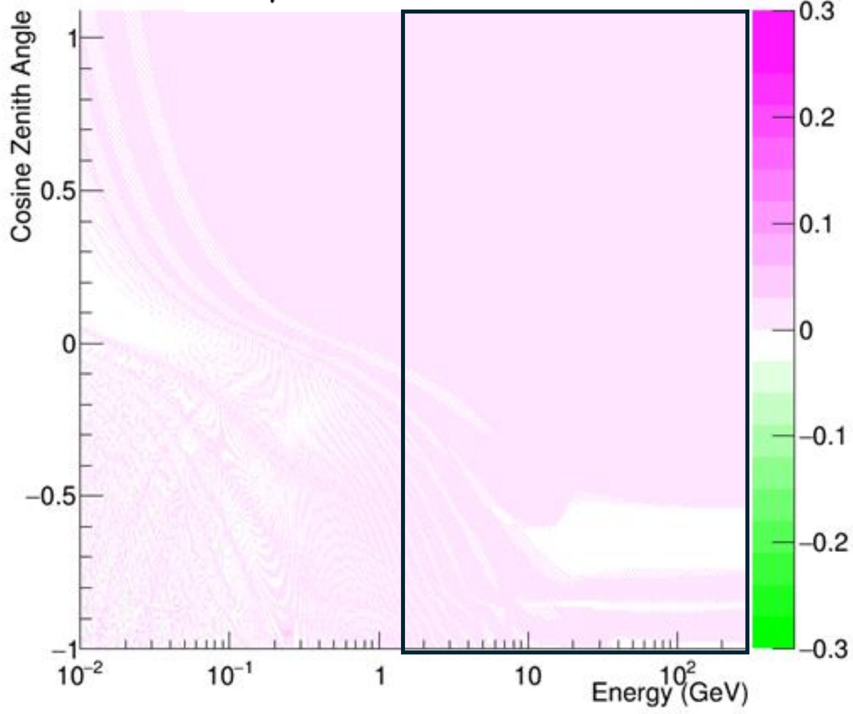
Higher Energy Neutrinos (> 1.33 GeV)



$P(\nu_\mu \to \nu_e)$ Difference



$P(\overline{\nu}_\mu \to \overline{\nu}_e)$ Difference

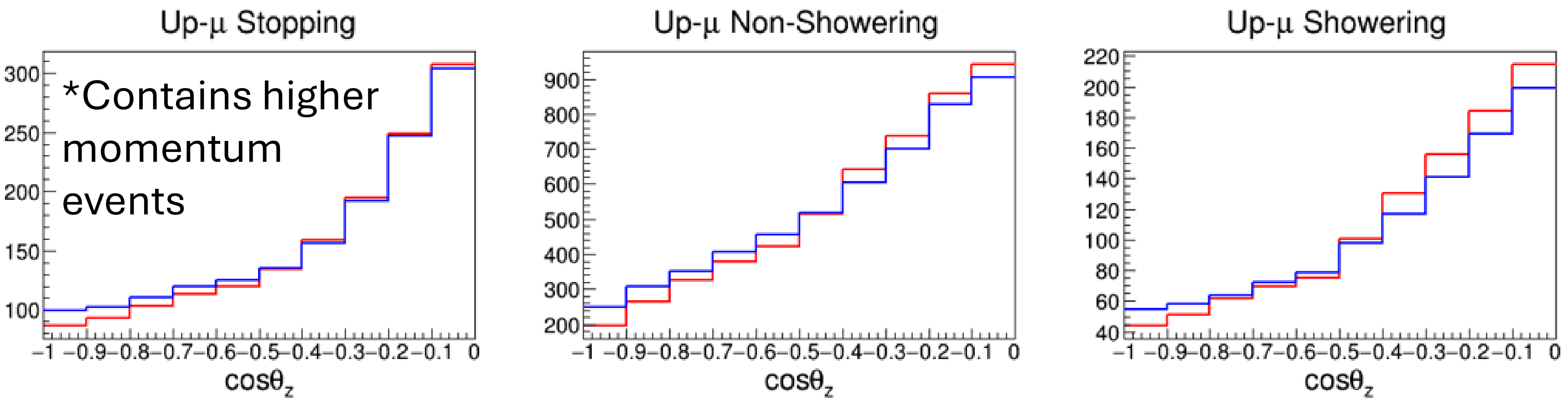


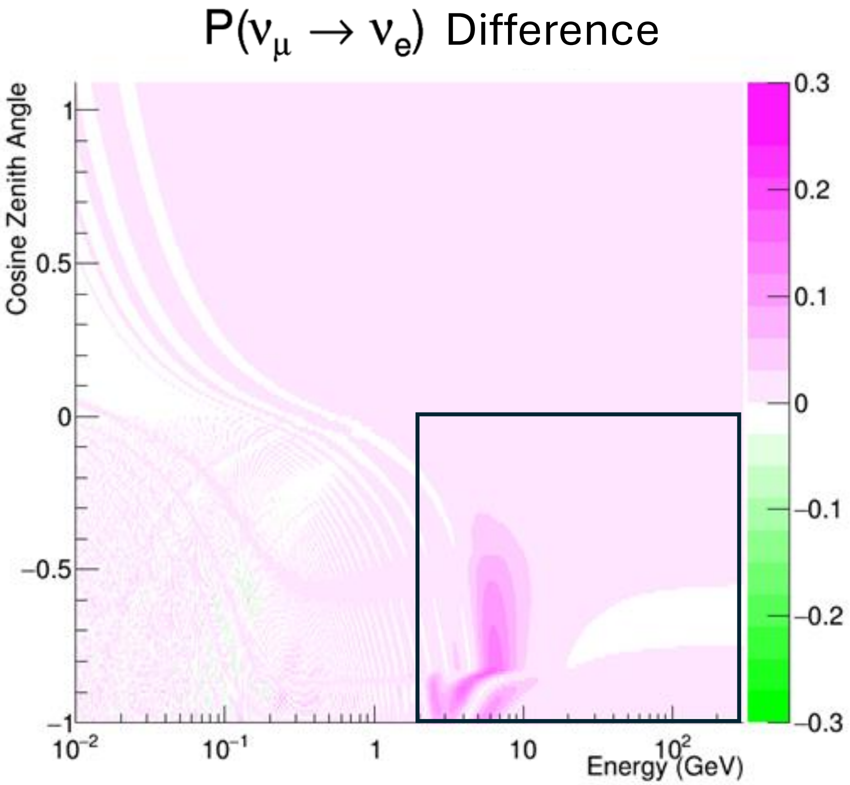
Above horizon less ν_μ are oscillating to ν_e
Below horizon it depends on energy

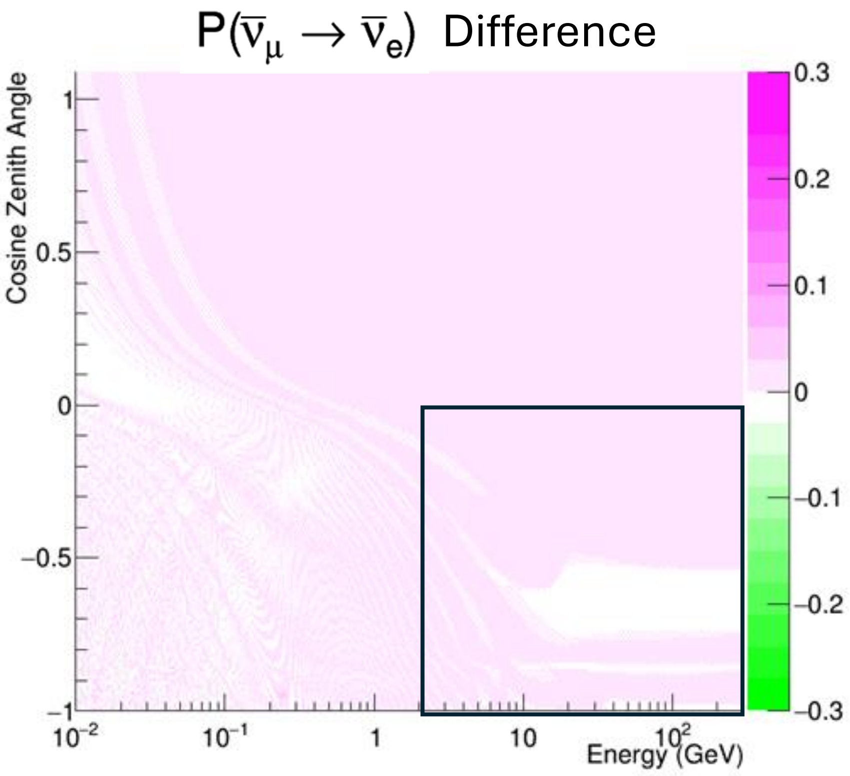
Unitary: $|U_{\mu 3}| = 0.663$

Non-Unitary $|U_{\mu 3}| = 0.653$

Higher Energy Neutrinos (>2 GeV)







Unitary: $|U_{\mu 3}| = 0.663$

Non-Unitary $|U_{\mu 3}| = 0.653$

Fitting Method

- Form grid of points in x number of parameters that want to fit and test each combination of parameters for (fake) data-MC agreement → grid becomes extremely large quickly for large x

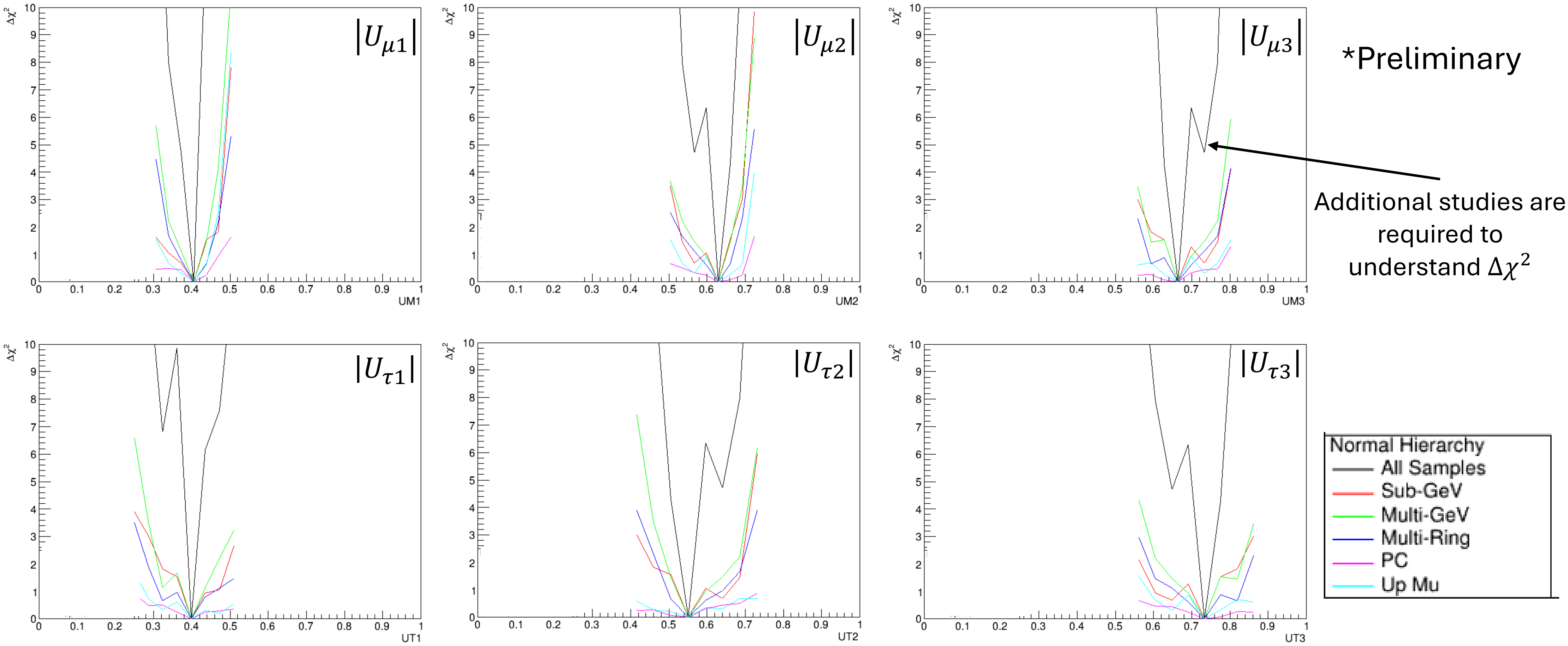
- Minimise for sum of n bins

$$\chi^2 = \sum_n \left(E_n - O_n + O_n \ln \frac{O_n}{E_n} \right) + \sum_i \epsilon_i^2$$

where:
O_n is observed events in nth bin
ϵ_i is ith systematic pull in units of σ
$E_n = E_n^0 \left(1 + \sum_i f_n^i \epsilon_i\right)$ is expected events in nth bin for nominal MC events E_n^0 scaled with fractional change f_n^i from systematics

$|U_{\mu i}|$ & $|U_{\tau i}|$ Magnitude-Only Sensitivity

Electron row fixed $|U_{ei}| = (0.823, 0.548, 0.148)$. CP phase δ=-1.75 included. Δm^2_{23}=0.0024 eV^2, Δm^2_{12}=7.53×10^{-5} eV^2.
Thus, only performing 6-dimensional sensitivity here where unitary case in normal hierarchy is nominal.



*Preliminary

Additional studies are required to understand $\Delta\chi^2$

Summary

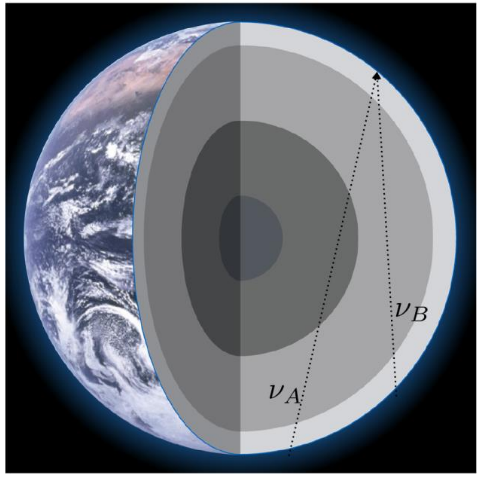
- Studies for lepton mixing matrix unitarity are increasing as neutrino experiments become more precise
- Can use atmospheric neutrinos to better measure muon and tau row elements
- Super-K has over 7000 live-days of atmospheric neutrino data available to provide sensitivity to the matrix elements

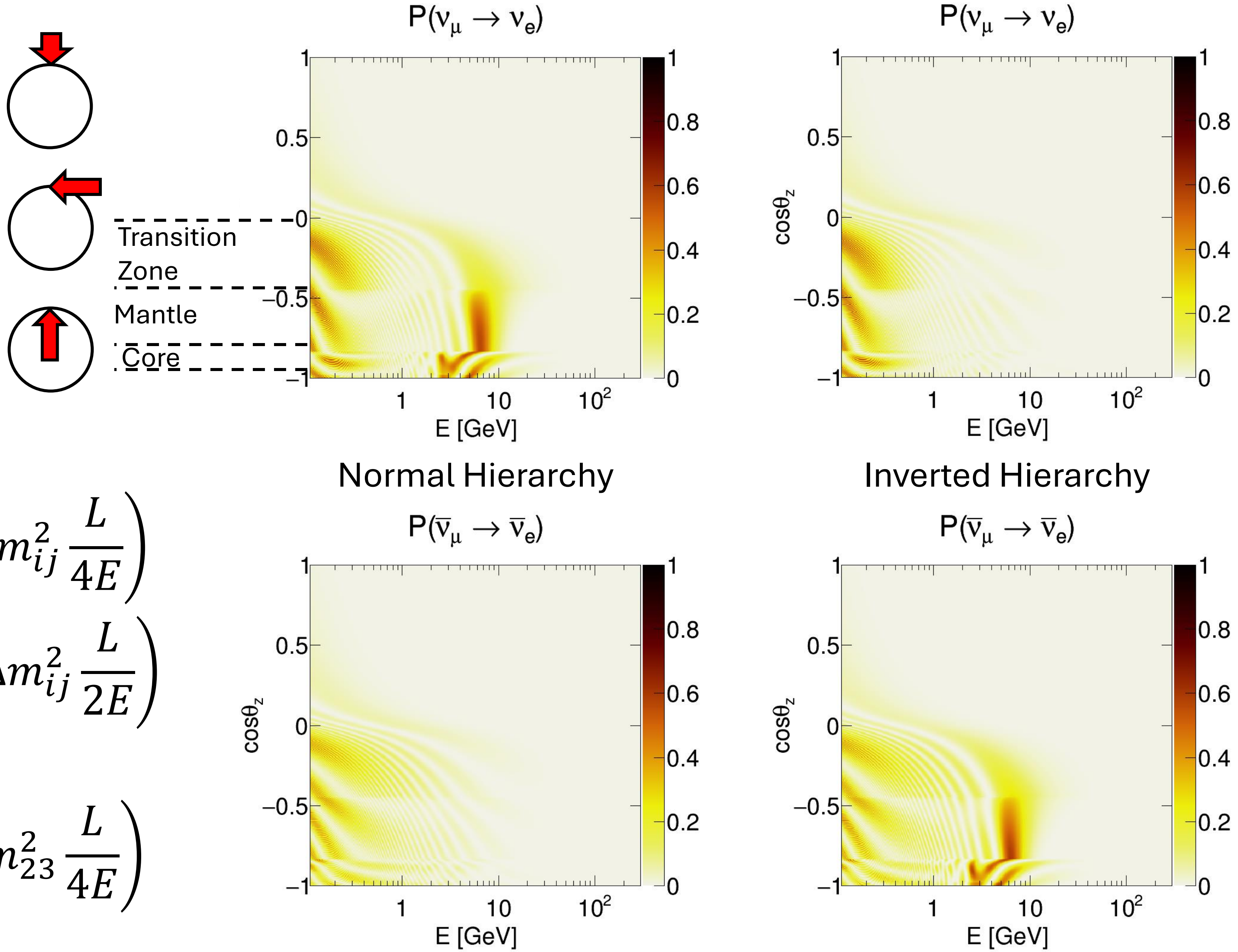
<u>Future Work</u>

- Measure more of the mixing matrix parameters simultaneously including complex phases
- Complete a fit using Super-K pure water data

Backup

Atmospheric Neutrino Oscillation Probability





$$P_{\alpha\to\beta} = \delta_{\alpha\beta}$$

$$-4\sum_{i>j} \mathrm{Re}\left(U^*_{\alpha i}U_{\beta i}U_{\alpha j}U^*_{\beta j}\right)\sin^2\left(\Delta m^2_{ij}\frac{L}{4E}\right)$$

$$\pm 2\sum_{i>j} \mathrm{Im}\left(U^*_{\alpha i}U_{\beta i}U_{\alpha j}U^*_{\beta j}\right)\sin^2\left(\Delta m^2_{ij}\frac{L}{2E}\right)$$

Eg

$$P_{\mu\leftrightarrow e} \approx \sin^2\theta_{23}\sin^2 2\theta_{13}\sin^2\left(\Delta m^2_{23}\frac{L}{4E}\right)$$

Modification to Atmospheric Oscillations

Since the $U_{\alpha i}$ no longer necessarily obey the unitarity conditions

→ get a 'zero-distance' effect: <u>L, E independent term</u>

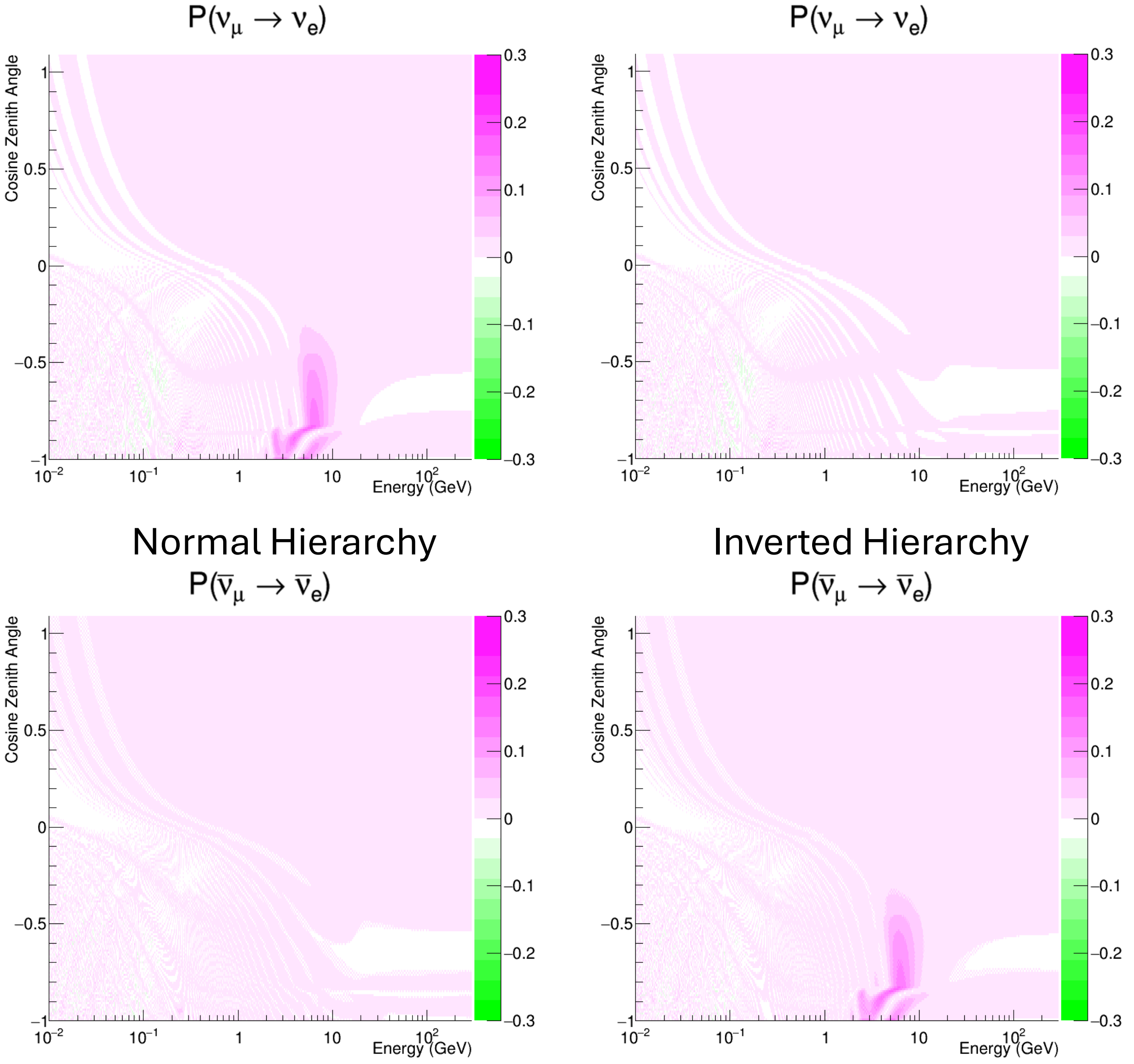
$$P_{\alpha\to\beta} = \left|\sum_i U^*_{\alpha i} U_{\beta i}\right|^2$$

$$-4\sum_{i>j} \mathrm{Re}\left(U^*_{\alpha i} U_{\beta i} U_{\alpha j} U^*_{\beta j}\right) \sin^2\left(\Delta m^2_{ij} \frac{L}{4E}\right)$$

$$\pm 2\sum_{i>j} \mathrm{Im}\left(U^*_{\alpha i} U_{\beta i} U_{\alpha j} U^*_{\beta j}\right) \sin^2\left(\Delta m^2_{ij} \frac{L}{2E}\right)$$

On the right, compare

$$P\left(|U_{\mu 3}| = 0.663\right) - P\left(|U_{\mu 3}| = 0.653\right)$$
$$P(\text{Unitary}) - P(\text{Non−Unitary})$$



In Context With Literature

NuFIT 5.3 (2024)

$$|U|_{3\sigma}^{\text{w/o SK-atm}} = \begin{pmatrix} 0.801 \to 0.842 & 0.518 \to 0.580 & 0.142 \to 0.155 \\ 0.236 \to 0.507 & 0.458 \to 0.691 & 0.630 \to 0.779 \\ 0.264 \to 0.527 & 0.471 \to 0.700 & 0.610 \to 0.762 \end{pmatrix}$$

$$|U|_{3\sigma}^{\text{with SK-atm}} = \begin{pmatrix} 0.801 \to 0.842 & 0.518 \to 0.580 & 0.143 \to 0.155 \\ 0.244 \to 0.500 & 0.498 \to 0.690 & 0.634 \to 0.770 \\ 0.276 \to 0.521 & 0.473 \to 0.672 & 0.621 \to 0.759 \end{pmatrix}$$

NuFit JHEP 09 (2020) 178 [arXiv:2007.14792]

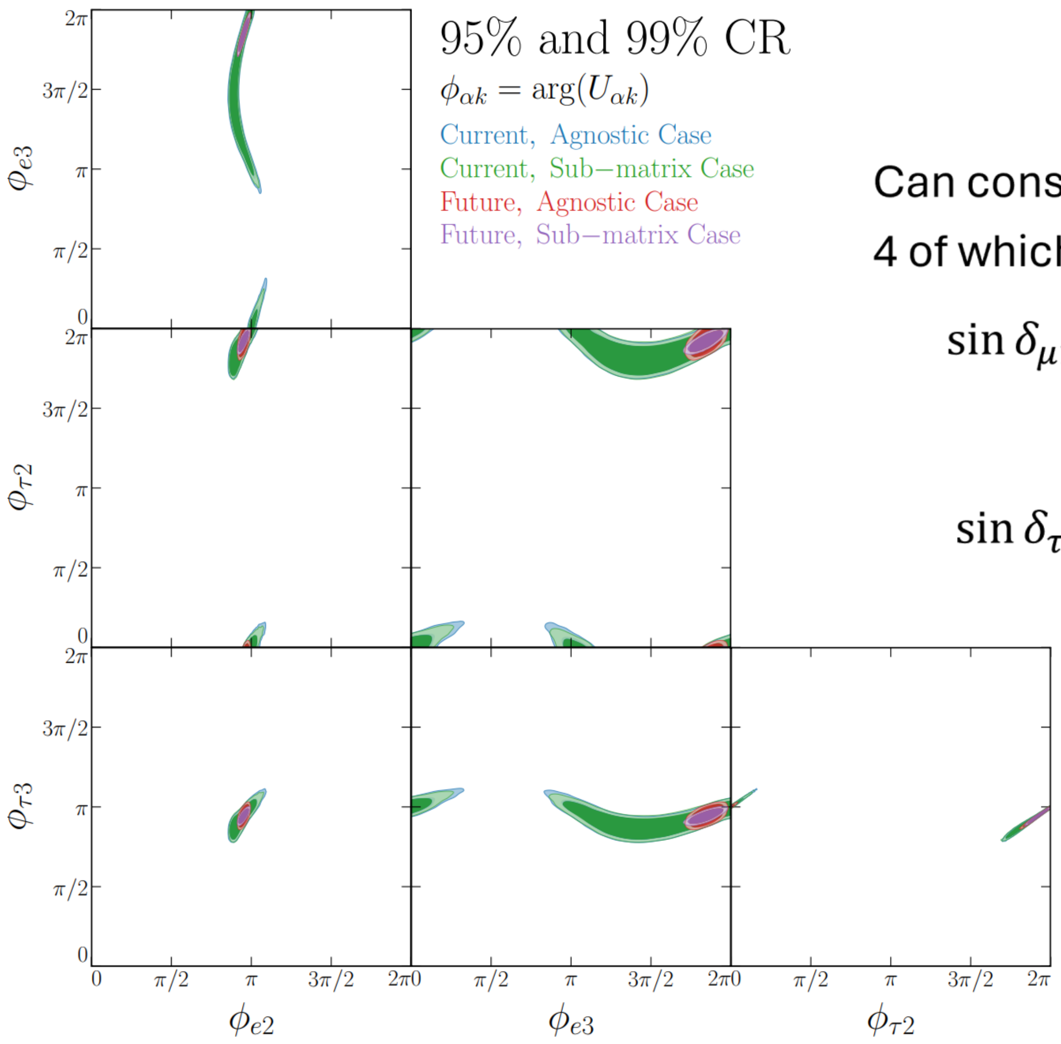
S. Parke, M. Ross-Lonergan arXiv:1508.05095v1 [hep-ph]

$$|U|_{3\sigma}^{\substack{\textbf{w/o Unitarity} \\ \text{(with Unitarity)}}} = \begin{pmatrix} \substack{0.76 \to 0.85 \\ (0.79 \to 0.85)} & \substack{0.50 \to 0.60 \\ (0.50 \to 0.59)} & \substack{0.13 \to 0.16 \\ (0.14 \to 0.16)} \\ \substack{0.21 \to 0.54 \\ (0.22 \to 0.52)} & \substack{0.42 \to 0.70 \\ (0.43 \to 0.70)} & \substack{0.61 \to 0.79 \\ (0.62 \to 0.79)} \\ \substack{0.18 \to 0.58 \\ (0.24 \to 0.54)} & \substack{0.38 \to 0.72 \\ (0.47 \to 0.72)} & \substack{0.40 \to 0.78 \\ (0.60 \to 0.77)} \end{pmatrix}.$$

SK sensitivity study so far...

$$|U|_{3\sigma} = \begin{pmatrix} 0.823 & 0.548 & 0.148 \\ 0.34 \to 0.42 & 0.53 \to 0.68 & 0.61 \to 0.78 \\ 0.30 \to 0.49 & 0.48 \to 0.69 & 0.60 \to 0.80 \end{pmatrix}$$

Complex Phases



S.A.R. Ellis, K.J. Kelly, S.W. Li. JHEP12 (2020) 068

Jarlskog Tensor

$$J \sum_{\gamma} \varepsilon_{\alpha\beta\gamma} \sum_{k} \varepsilon^{ijk} = \Im\left(U_{\alpha i} U_{\beta j} U^*_{\alpha j} U^*_{\beta i}\right)$$

Can construct 9 element tensor for $\alpha, \beta, \gamma \in e, \mu, \tau$ and $i, j, k \in 1,2,3$

4 of which are linear in $\sin \delta_{\mu,\tau,1,2}$

$$\sin \delta_{\mu 1} = \frac{J^{31}_{e\mu}}{|U_{e1}||U_{e3}||U_{\mu 1}||U_{\mu 3}|}, \sin \delta_{\mu 2} = \frac{-J^{23}_{e\mu}}{|U_{e2}||U_{e3}||U_{\mu 2}||U_{\mu 3}|}$$

$$\sin \delta_{\tau 1} = \frac{-J^{31}_{\tau e}}{|U_{e1}||U_{e3}||U_{\tau 1}||U_{\tau 3}|}, \sin \delta_{\tau 2} = \frac{J^{23}_{\tau e}}{|U_{e2}||U_{e3}||U_{\tau 2}||U_{\tau 3}|}$$

Under condition of unitarity one can prove all $J^{ij}_{\alpha\beta}$ are same size:

$$J_{PMNS} = c_{12} s_{12} c_{23} s_{23} c^2_{13} s_{13} \sin \delta_{CP}$$