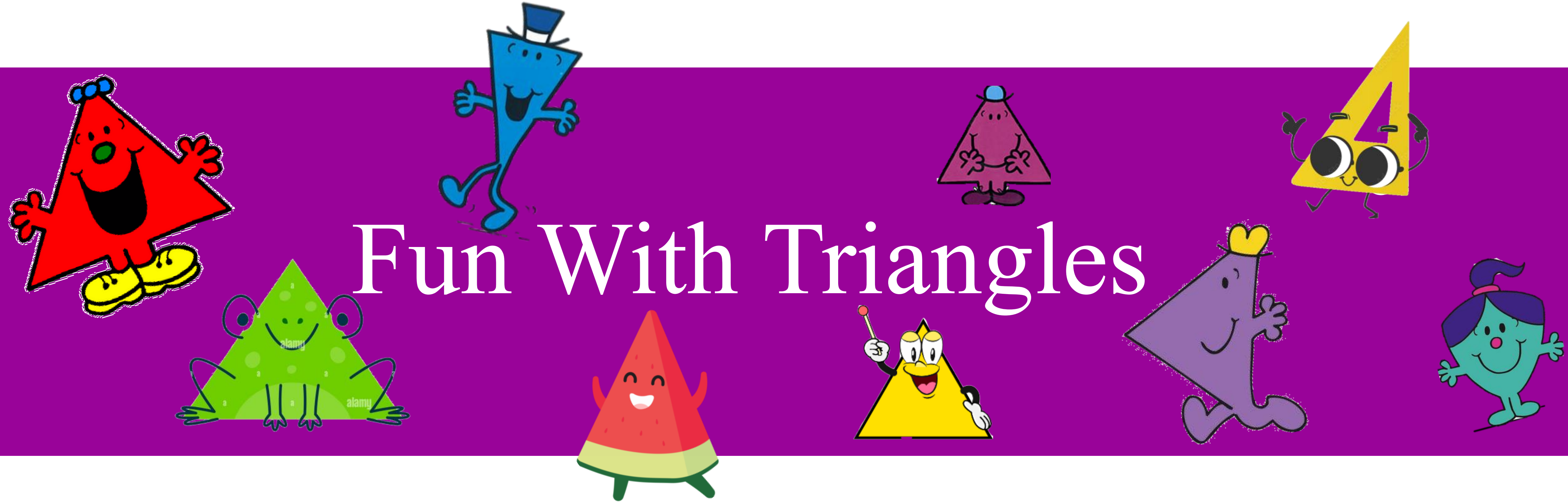

Fun With Triangles

An unbinned, Phase-Corrected measurement of the CKM angle γ

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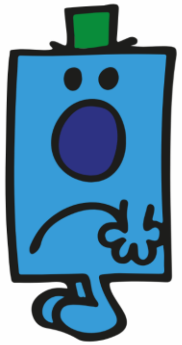
jozie.meldrum@bristol.ac.uk

LHCb · MONASH University · University of BRISTOL · UNI FREIBURG · BESIII

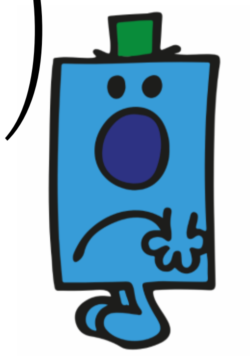
The triangle

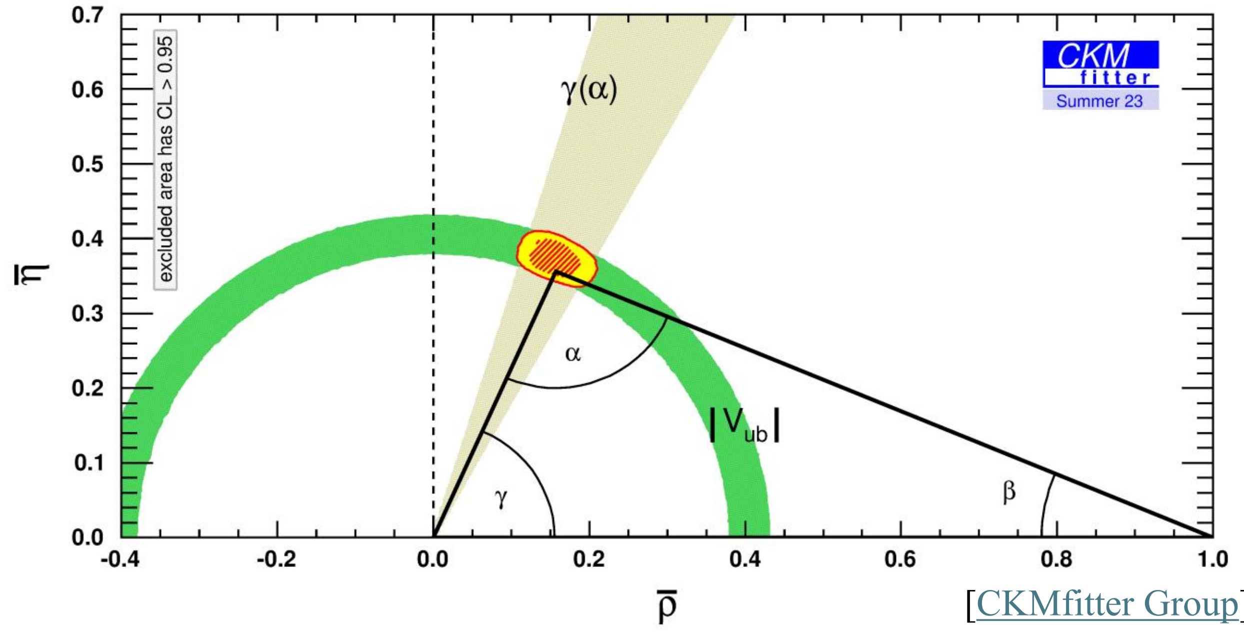
$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$



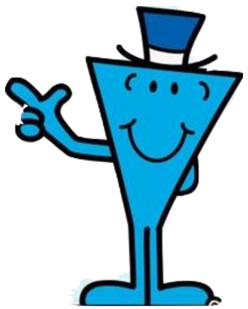
The triangle

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$



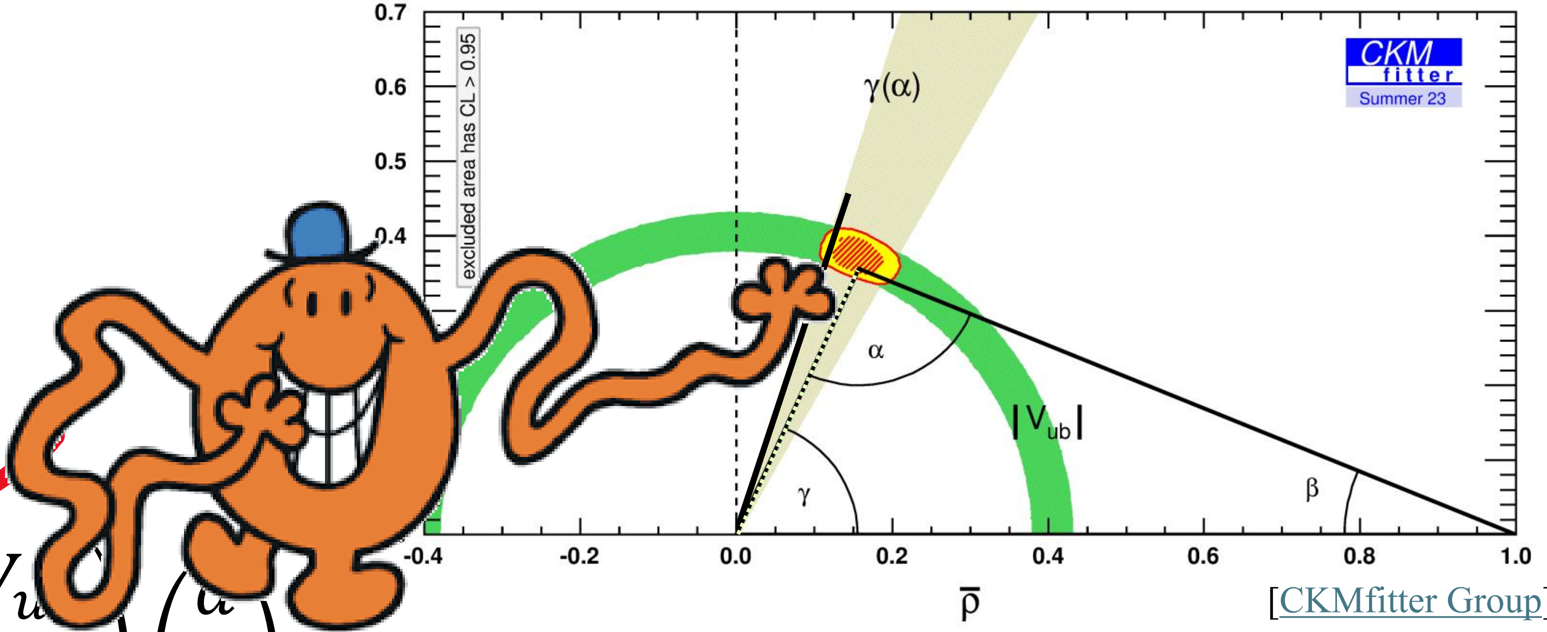


[CKMfitter Group]

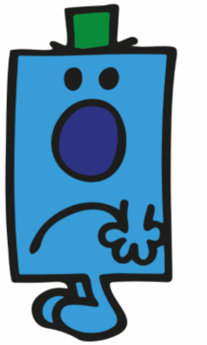


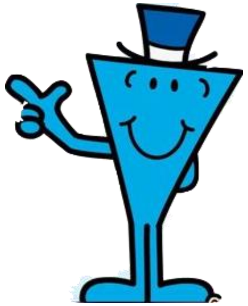
The triangle

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$



[CKMfitter Group]

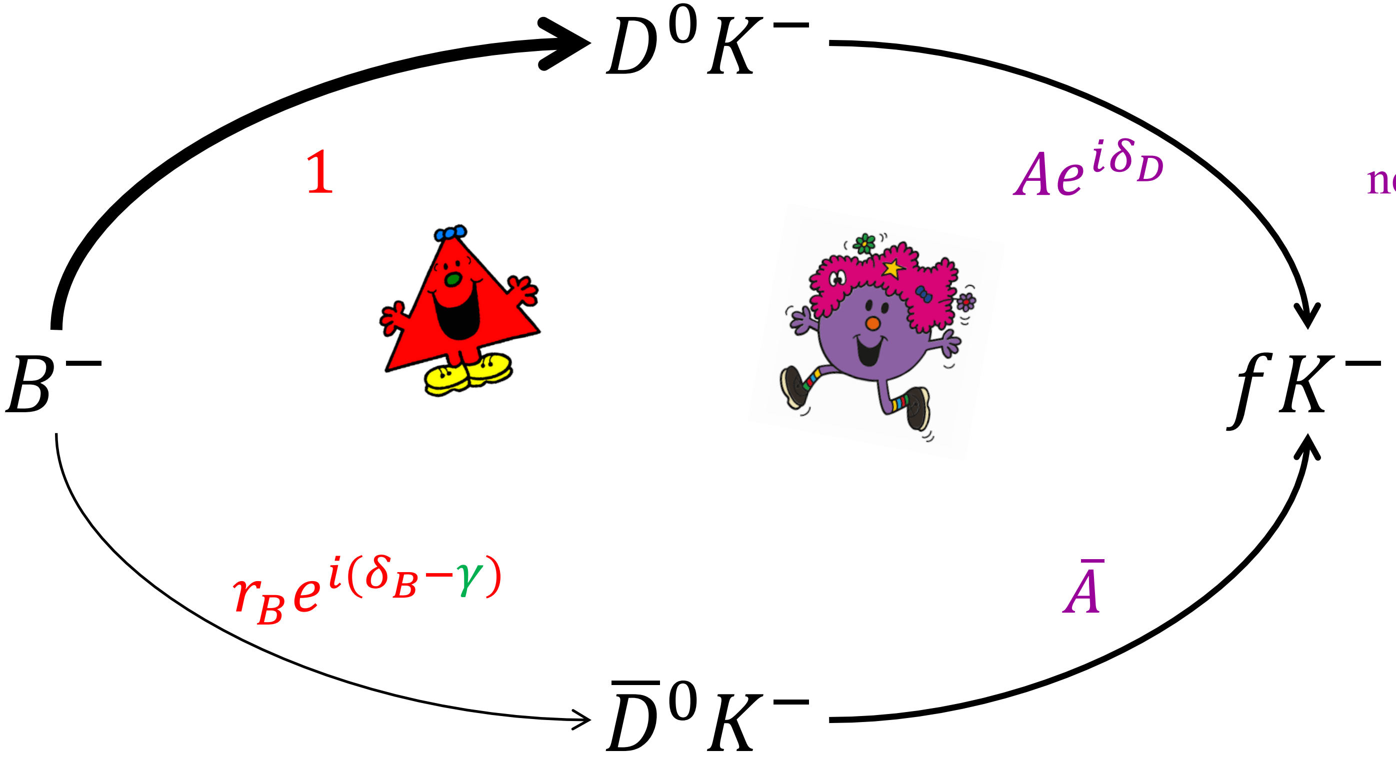




Direct measurements : BPGGSZ

***B*-decay amplitudes:**
contains γ, accessible with LHCb datasets, object of analysis

$\{r_B, \delta_B, \gamma\}$



***D*-decay amplitudes:**
need as input, accessible in both LHCb and BES-III datasets

$\{A, \bar{A}, \delta_D\}$

$$|\mathcal{A}^-_{tot}|^2 \propto A^2 + \bar{A}^2 r_B^2 + 2A\bar{A}r_B\cos(\delta_D - (\delta_B - \gamma))$$

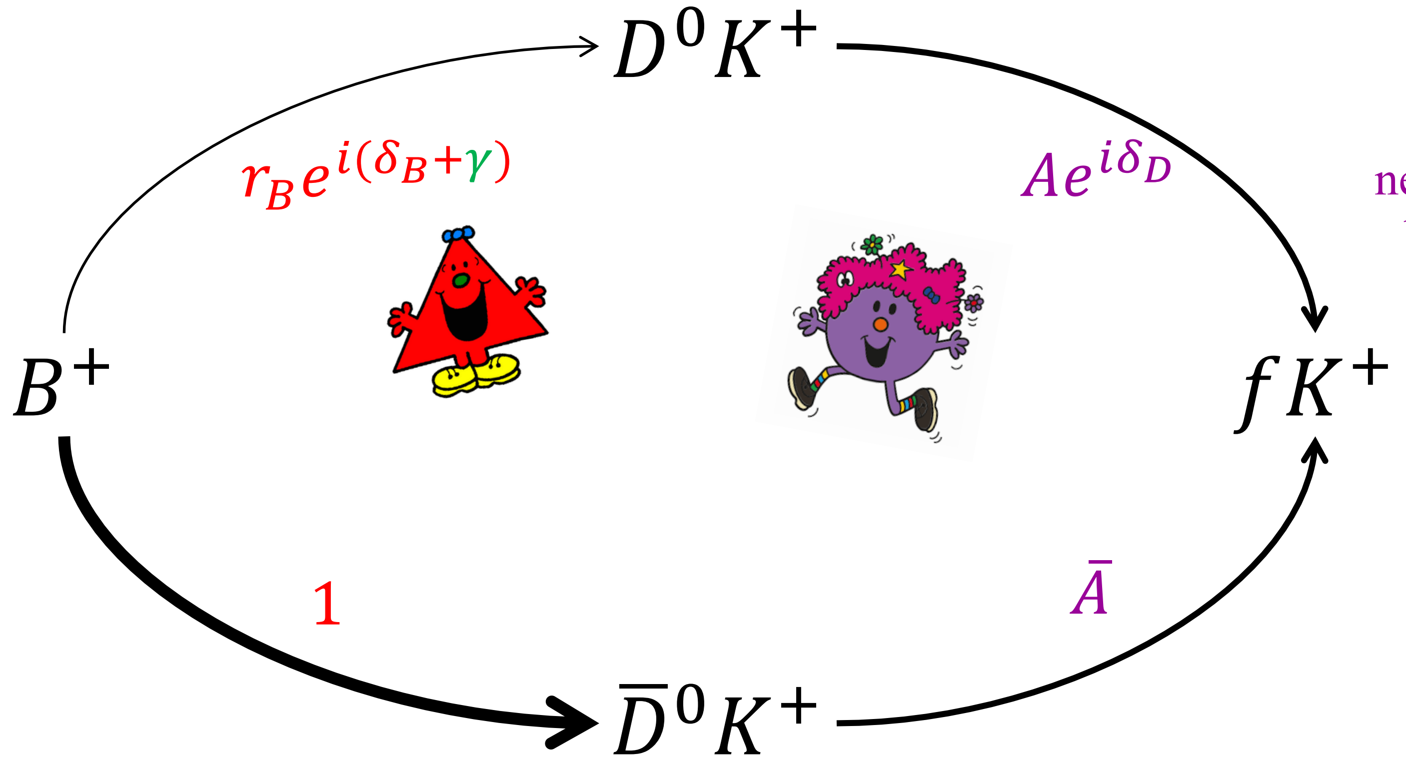
$$= A^2 + \bar{A}^2(x_-^2 + y_-^2) + 2A\bar{A}\,(x_-\cos(\delta_D) + y_-\sin(\delta_D))$$

$x_- = r_B\cos(\delta_B - \gamma)$,
$y_- = r_B\sin(\delta_B - \gamma)$

Direct measurements : BPGGSZ

***B*-decay amplitudes:**
contains γ, accessible with LHCb datasets, object of analysis

$\{r_B, \delta_B, \gamma\}$



***D*-decay amplitudes:**
need as input, accessible in both LHCb and BES-III datasets

$\{A, \bar{A}, \delta_D\}$

$$|\mathcal{A}^+_{tot}|^2 \propto A^2 r_B^2 + \bar{A}^2 + 2A\bar{A}r_B\cos(\delta_D + (\delta_B + \gamma))$$

$$= A^2(x_+^2 + y_+^2) + \bar{A}^2 + 2A\bar{A}\,(x_+\cos(\delta_D) - y_+\sin(\delta_D))$$

$x_+ = r_B\cos(\delta_B + \gamma)$,
$y_+ = r_B\sin(\delta_B + \gamma)$

Direct measurements : BPGGSZ

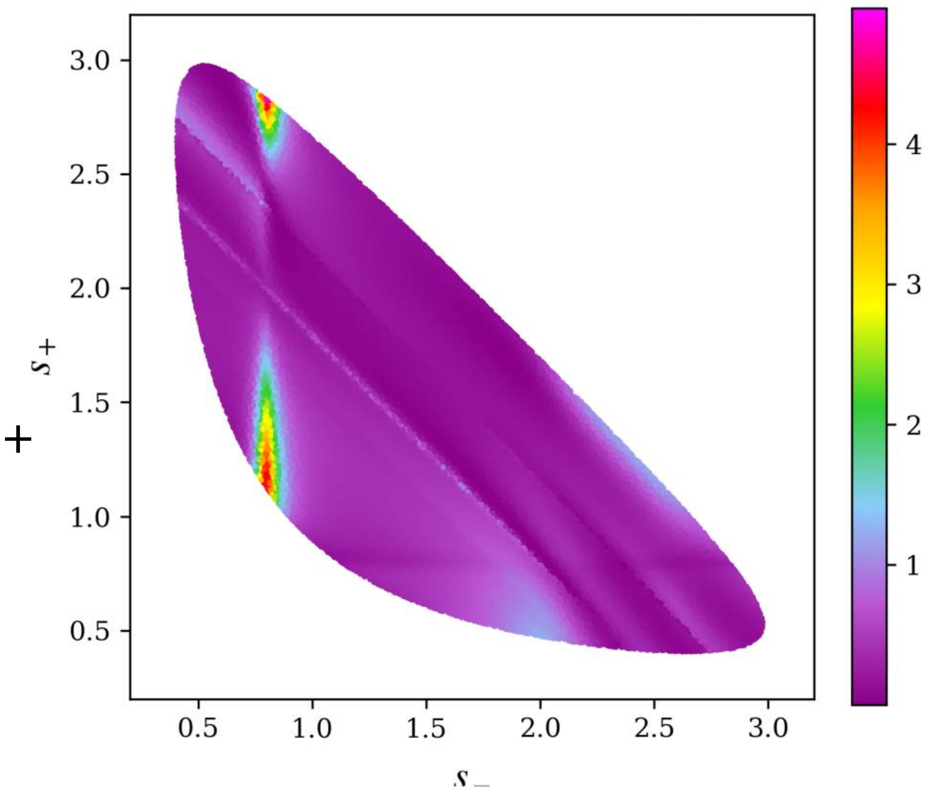
$$|\mathcal{A}_{tot}^{-}| \propto A^2 + \bar{A}^2(x_-^2 + y_-^2) + 2A\bar{A}\,(x_- \cos(\delta_D) + y_- \sin(\delta_D))$$

$$|\mathcal{A}_{tot}^{+}| \propto A^2(x_+^2 + y_+^2) + \bar{A}^2 + 2A\bar{A}\,(x_+ \cos(\delta_D) - y_+ \sin(\delta_D))$$

***B*-decay amplitude**:
4 free parameters,
Contains γ,
From fit to LHCb, $B \to [f]_D K$ data

***D*-decay amplitude**:
2 magnitudes, 1 phase
Depends on the final state f,
Needed as an input

D-decay input: $f = K_S\pi^+\pi^-$

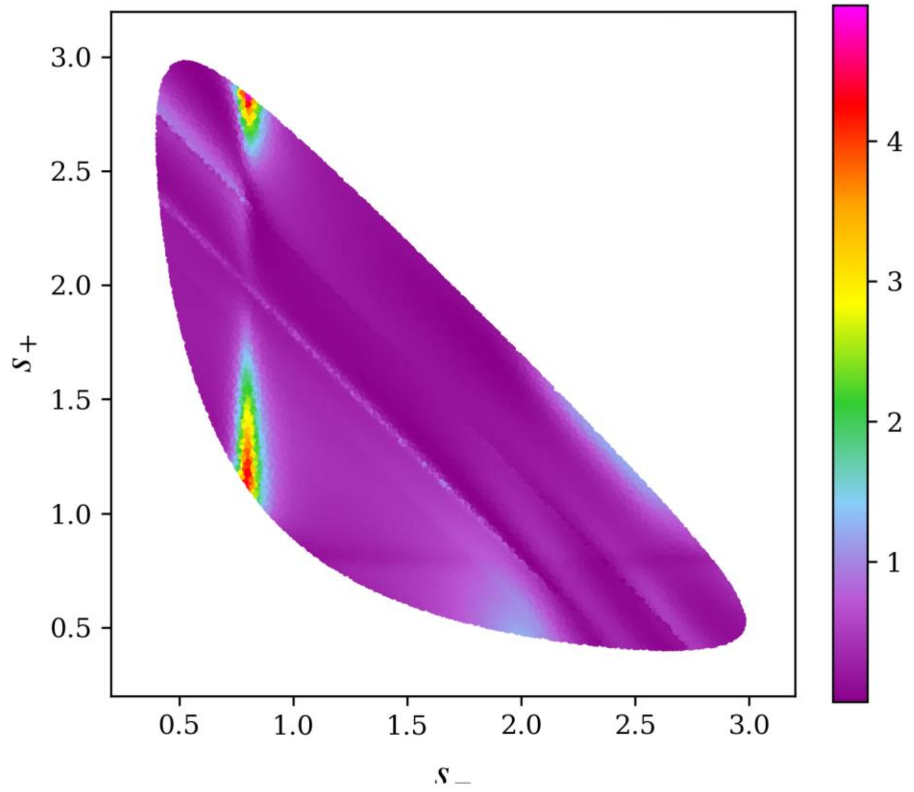


$K_S\pi^+$

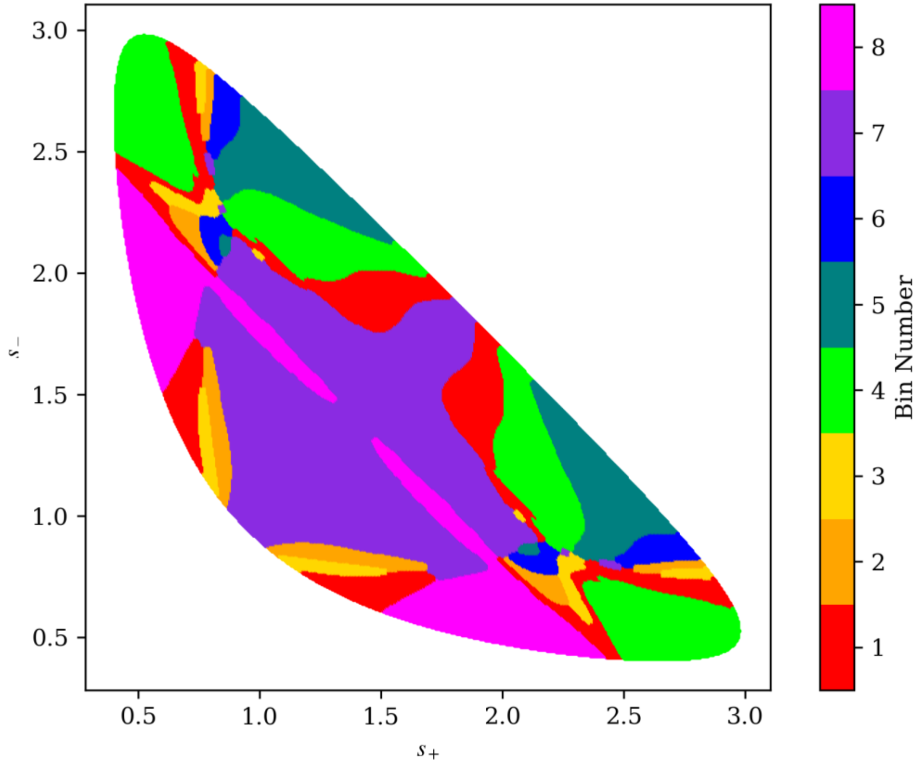
$K_S\pi^-$

D-decay input: $f = K_S\pi^+\pi^-$

e.g. $$s_i = \frac{\int_i A\bar{A}\sin(\delta_D)\, ds_+ds_-}{\sqrt{\int_i A^2 \; ds_+ds \int_i \bar{A}^2 \; ds_+ds_-}}$$



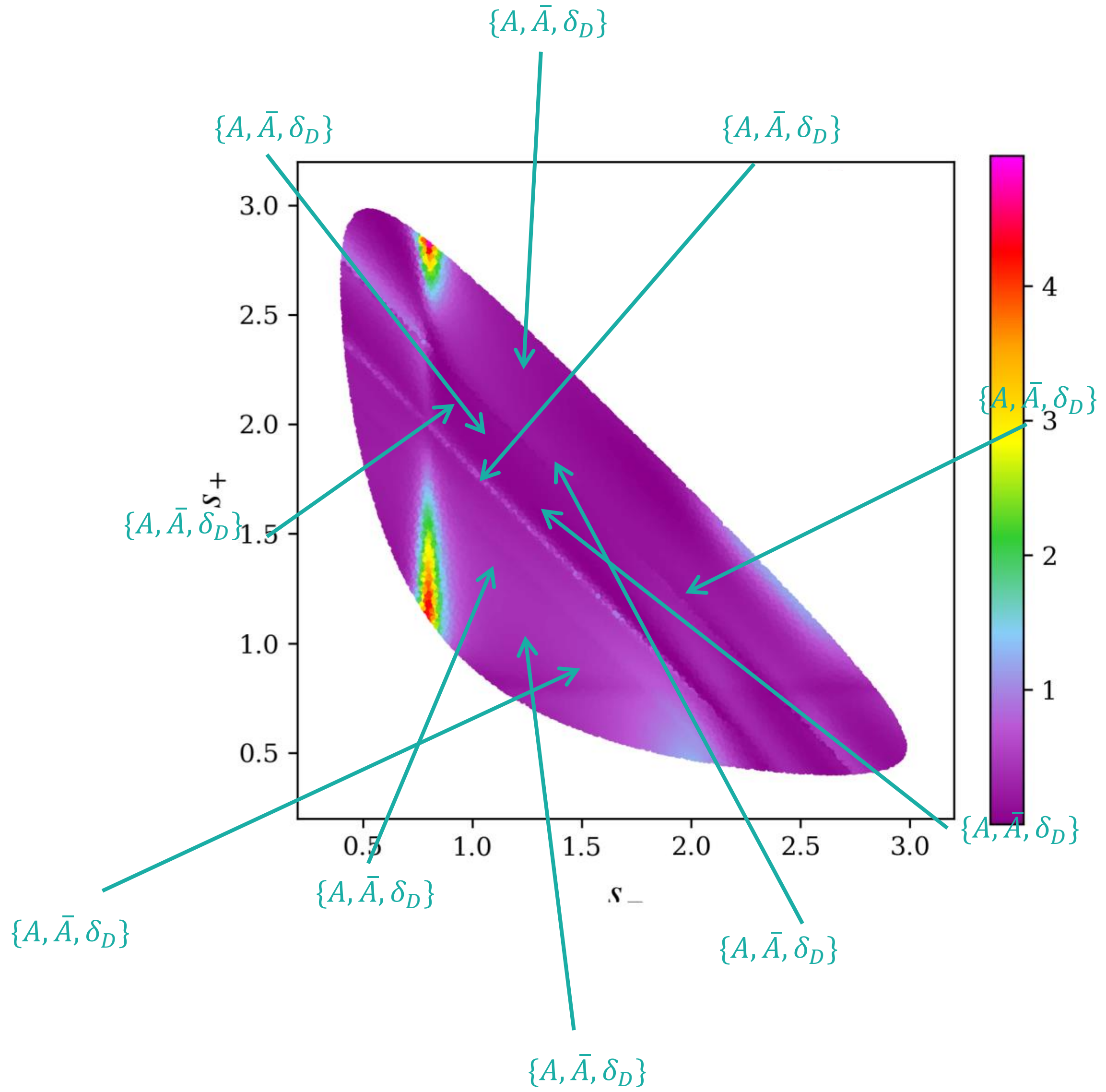
[PhysRevD.82.112006]



Most precise single measurement: $(68.7^{+5.2}_{-5.1})^\circ$ [LHCb-PAPER-2020-019]

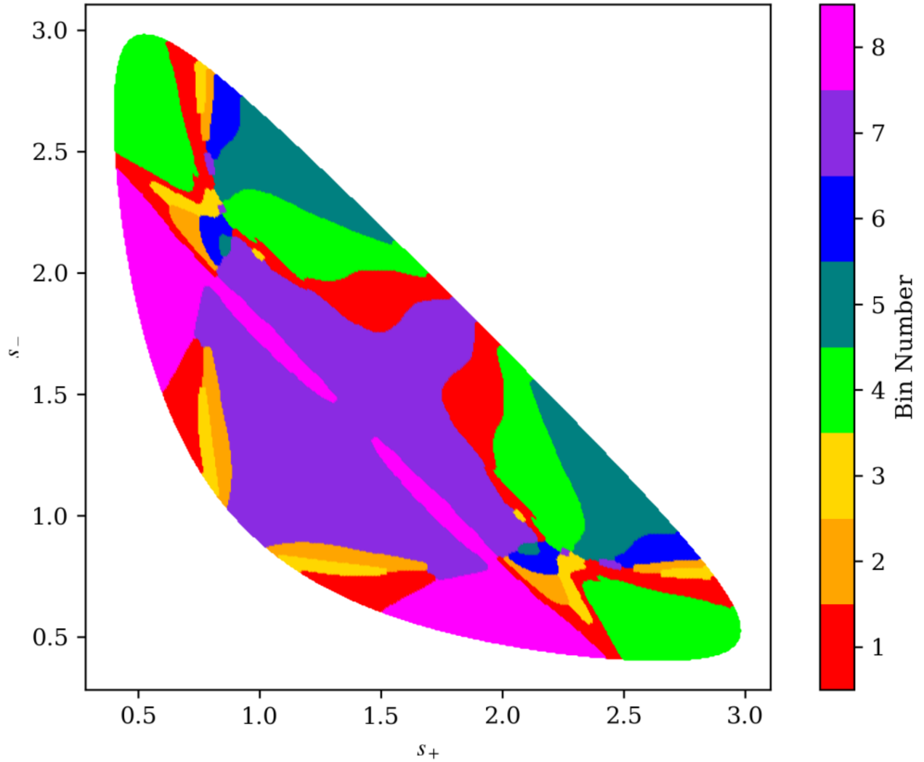
Current world average: $(66.4^{+2.8}_{-3.0})^\circ$ [HFLAV 2024]

D-decay input: $f = K_S\pi^+\pi^-$



e.g. $$s_i = \frac{\int_i A\bar{A}\sin(\delta_D)\, ds_+ ds_-}{\sqrt{\int_i A^2\ ds_+ ds \int_i \bar{A}^2\ ds_+ ds_-}}$$

[PhysRevD.82.112006]



Most precise single measurement: $(68.7^{+5.2}_{-5.1})°$ [LHCb-PAPER-2020-019]

Current world average: $(66.4^{+2.8}_{-3.0})°$ [HFLAV 2024]

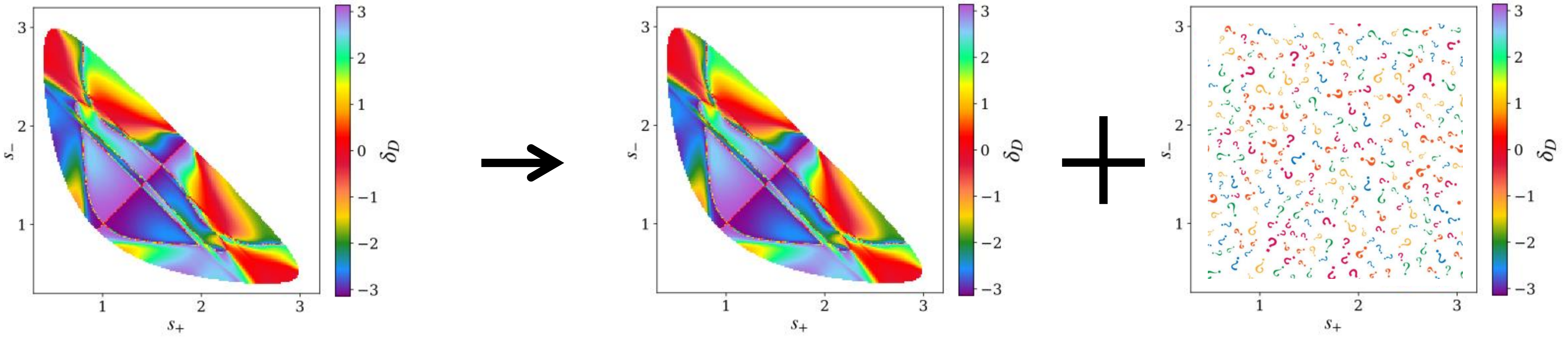
The Phase-Correction method

[JHEP 09 (2023) 007]

fixed value from a model

floating in fit, contains many parameters

$$\delta_D \rightarrow \delta_D + \delta_C$$



- No binning maximises statistical precision

- Phase-correction provides model-independency

The Phase-Correction method

[JHEP 09 (2023) 007]

fixed value from a model

floating in fit, contains many parameters

$$\delta_D \rightarrow \delta_D + \delta_C$$

$$|\mathcal{A}_{tot}^{-}| \propto A^2 + \bar{A}^2(x_-^2 + y_-^2) + 2A\bar{A}\,(x_- \cos(\delta_D + \delta_C) + y_- \sin(\delta_D + \delta_C))$$

δ_C shared between $B^{\pm}$ amplitudes

$$|\mathcal{A}_{tot}^{+}| \propto A^2(x_+^2 + y_+^2) + \bar{A}^2 + 2A\bar{A}\,(x_+ \cos(\delta_D + \delta_C) - y_+ \sin(\delta_D + \delta_C))$$

***B*-decay amplitude**: contains γ, from fit to LHCb data, unchanged

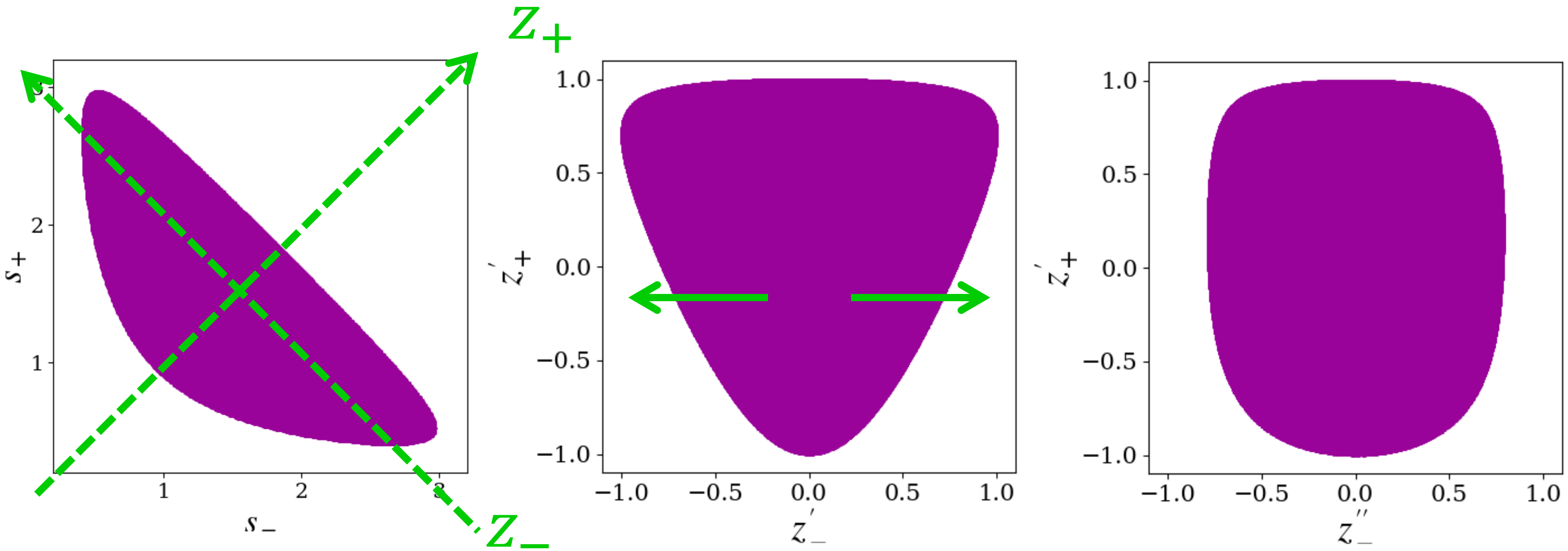
***D*-decay amplitude**: input from the amplitude model, magnitude unchanged

Still *D*-decay amplitude: from fit using both LHCb and BES-III data

- No binning maximises statistical precision

- Phase-correction provides model-independency

Finding the model-input difference



Preserving the symmetry
$\delta_D(s_-, s_+) = -\delta_D(s_+, s_-)$

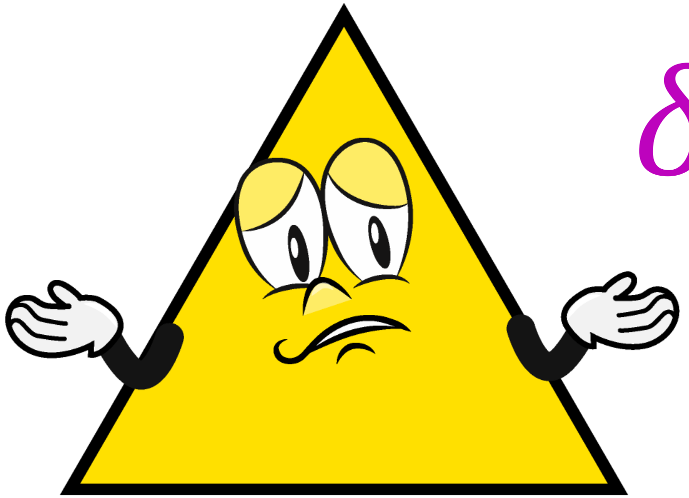
$$\delta_C = \sum_{i=0}^{N} \sum_{j=0}^{\frac{N-i}{2}} C_{i,2j+1} P_i(z'_+) P_{2j+1}(z''_-)$$

Coefficients, C: free in fit

Legendre polynomials, P: functions of Dalitz coordinate

δ_B γ

r_B δ_C

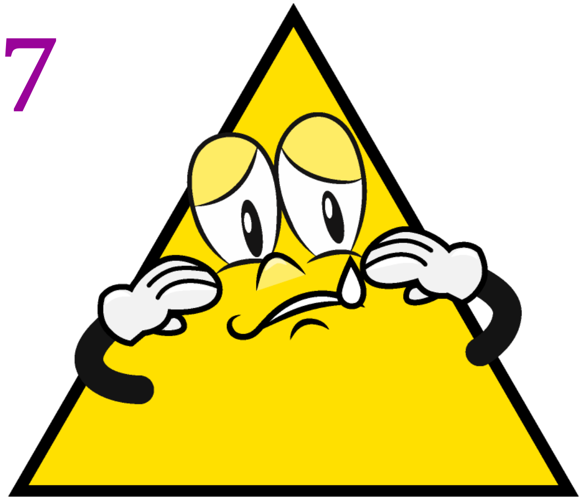


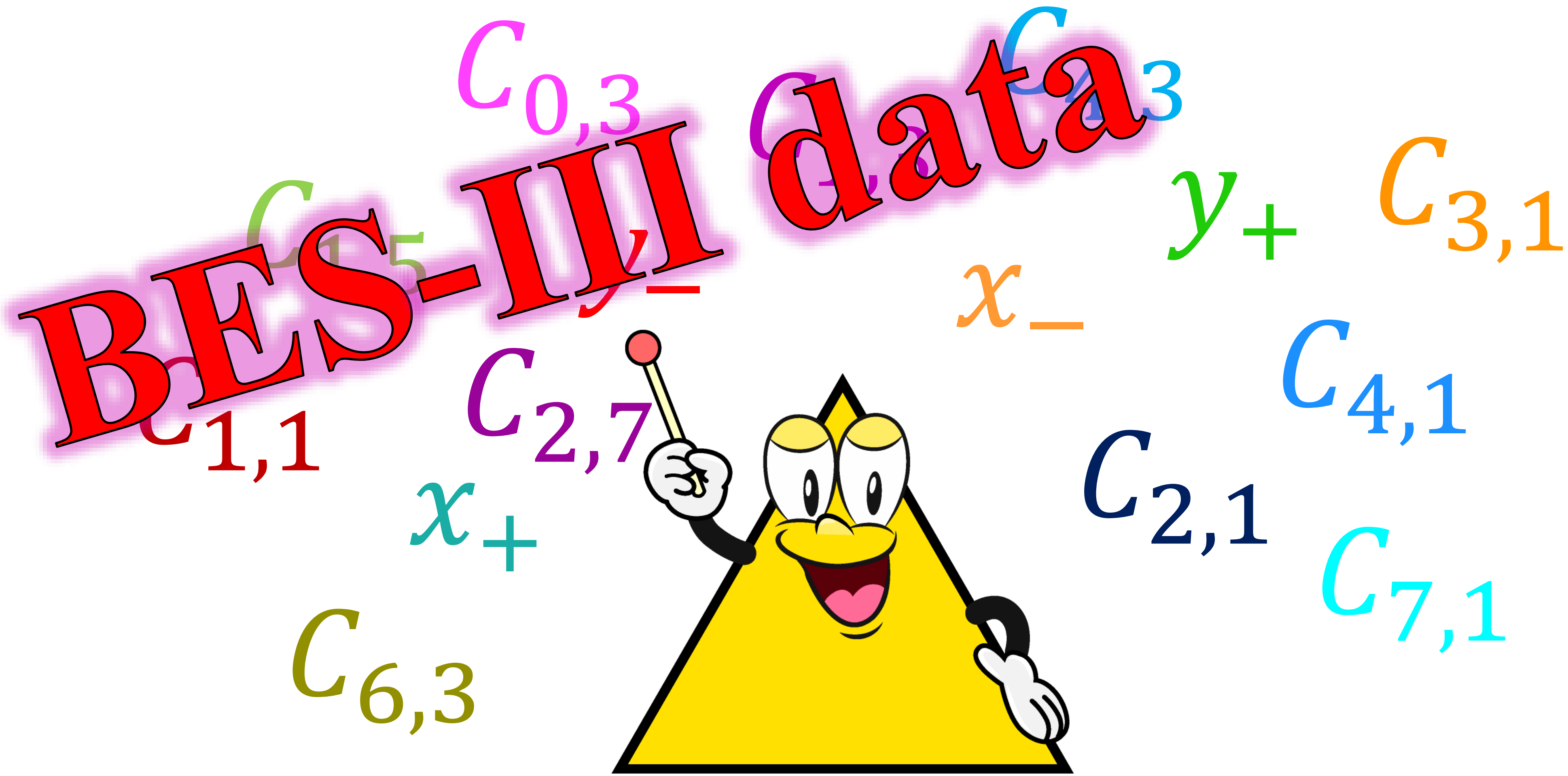
$C_{4,3}$ $C_{0,3}$ $C_{1,3}$

$C_{3,1}$ y_+ $C_{1,5}$ y_- x_-

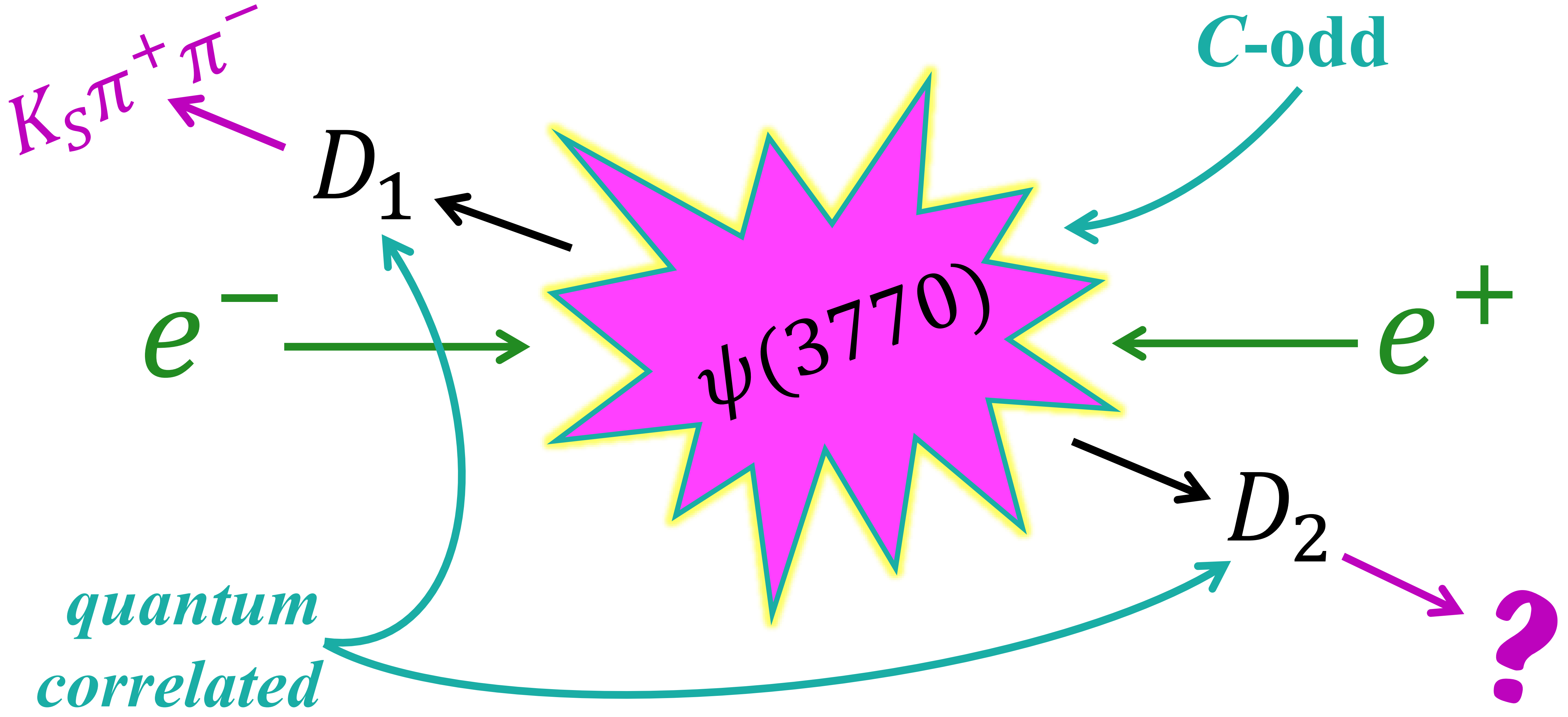
$C_{4,1}$ $C_{1,1}$ $C_{2,7}$ $C_{2,1}$

x_+ $C_{7,1}$ $C_{6,3}$

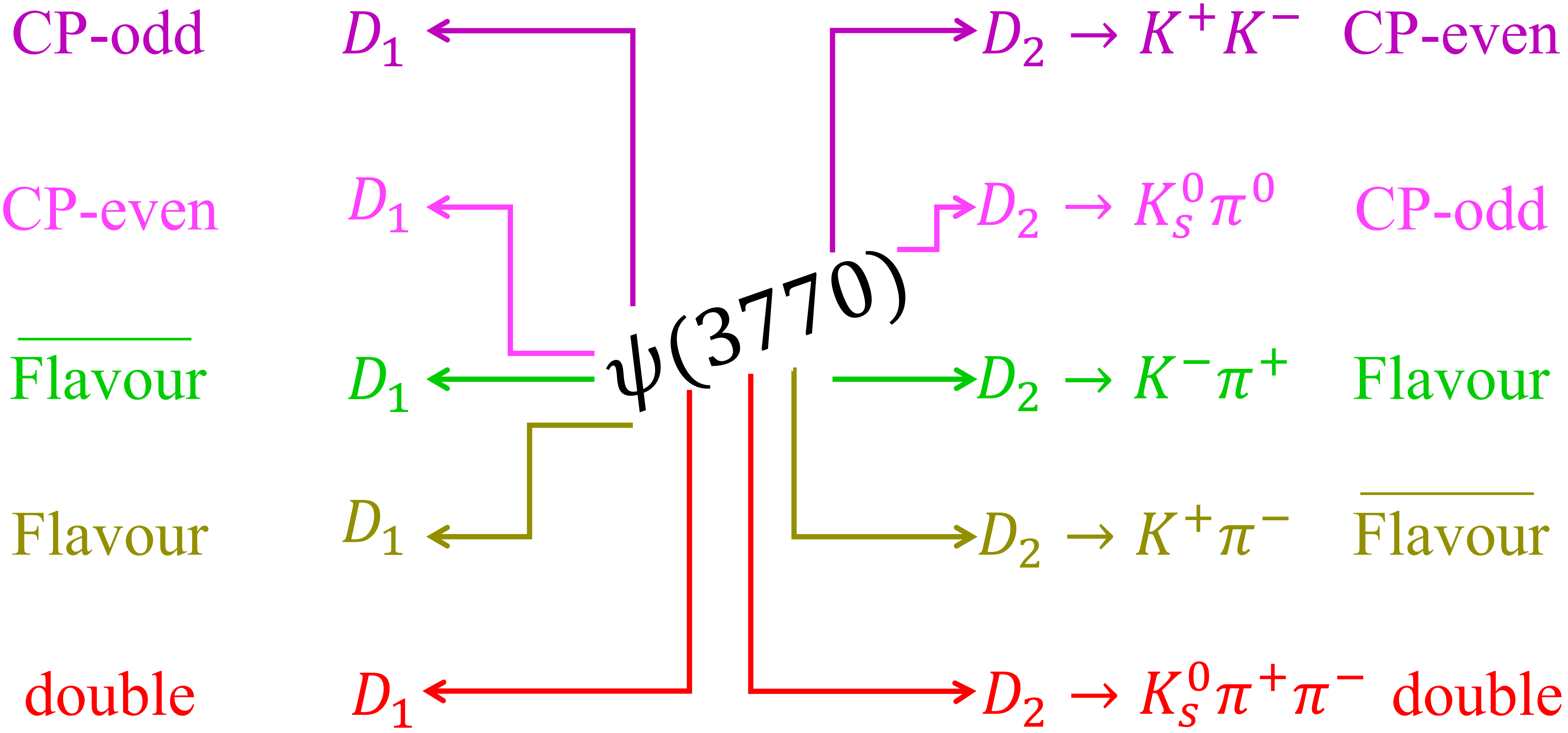



BES-III data
$c_{0,3}$
y_+
$c_{3,1}$
x_-
$c_{4,1}$
$c_{1,1}$
$c_{2,7}$
$c_{2,1}$
x_+
$c_{7,1}$
$c_{6,3}$

BES-III data



BES-III data



CP-odd
D_1
$D_2 \to K^+K^-$
CP-even
CP-even
D_1
$D_2 \to K_S^0\pi^0$
CP-odd
$\overline{\text{Flavour}}$
D_1
$D_2 \to K^-\pi^+$
Flavour
Flavour
D_1
$D_2 \to K^+\pi^-$
$\overline{\text{Flavour}}$
double
D_1
$D_2 \to K_S^0\pi^+\pi^-$
double
$\psi(3770)$

BES-III data

$$D_1^0 \overline{D}_2^0 - \overline{D}_1^0 D_2^0 \quad \Rightarrow \quad |\mathcal{A}_{tot}|^2 \propto \left|\mathcal{A}_{D_1} \bar{\mathcal{A}}_{D_2} - \bar{\mathcal{A}}_{D_1} \mathcal{A}_{D_2}\right|^2$$

CP-even: $A^2 - 2A^2\bar{A}^2\cos(\delta_D + \delta_C)$

CP-odd: $A^2 + 2A^2\bar{A}^2\cos(\delta_D + \delta_C)$

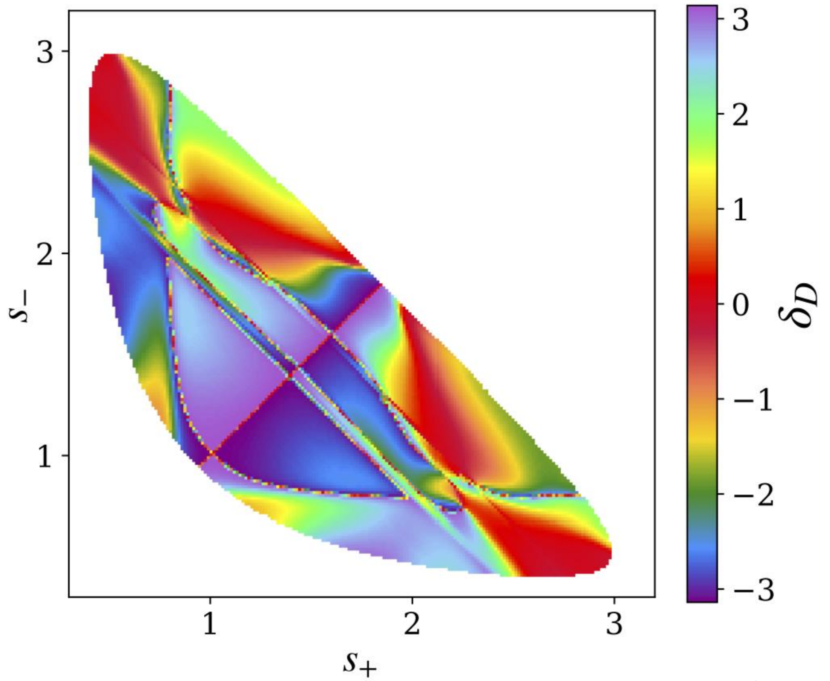
flavour: $\bar{A}^2$

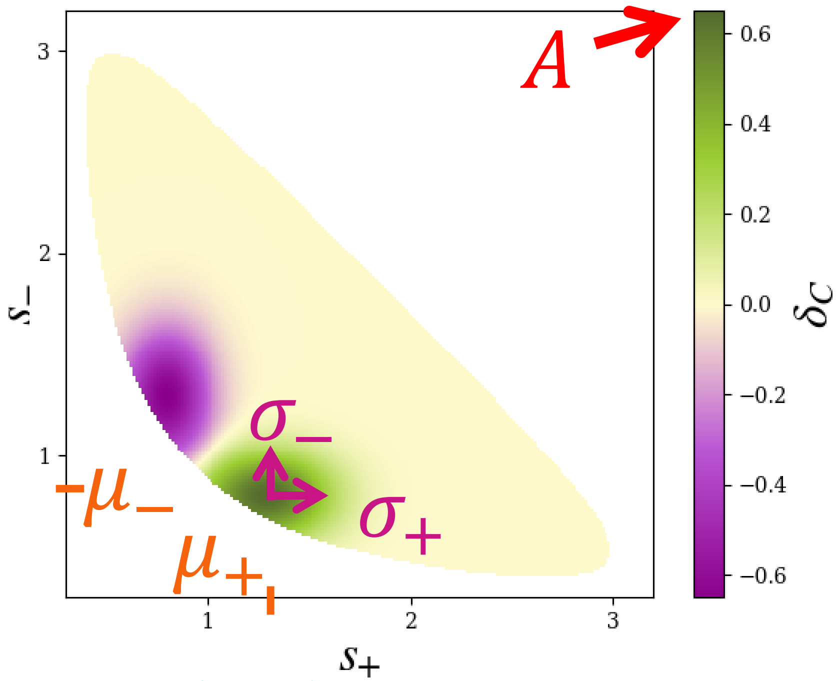
flavour-bar: A^2

double: $A_1^2\bar{A}_2^2 + \bar{A}_1^2 A_2^2 - 2A_1\bar{A}_1 A_2\bar{A}_2(\cos(\delta_{D_1} + \delta_{C_1})\cos(\delta_{D_2} + \delta_{C_2}) + \sin(\delta_{D_1} + \delta_{C_2})\sin(\delta_{D_2} + \delta_{C_2}))$

Input change: alter and test the model

$$\delta_D^{input} = \delta_D^{model} + \delta_C^{input}$$





peak on dalitz plane, using usual coordinates

magnitude *position of bias*

$$f\left(s_+, s_- \middle| \vec{A}, \vec{\varepsilon}, \overrightarrow{\mu_\pm}, \overrightarrow{\sigma_\pm}\right) = \sum_i A \cdot \operatorname{erf}\left(\frac{s_+ - s_-}{\varepsilon}\right) \cdot \exp\left(-\frac{(s_+ - \mu_+)^2}{\sigma_+^2} - \frac{(s_- - \mu_-)^2}{\sigma_-^2}\right)$$

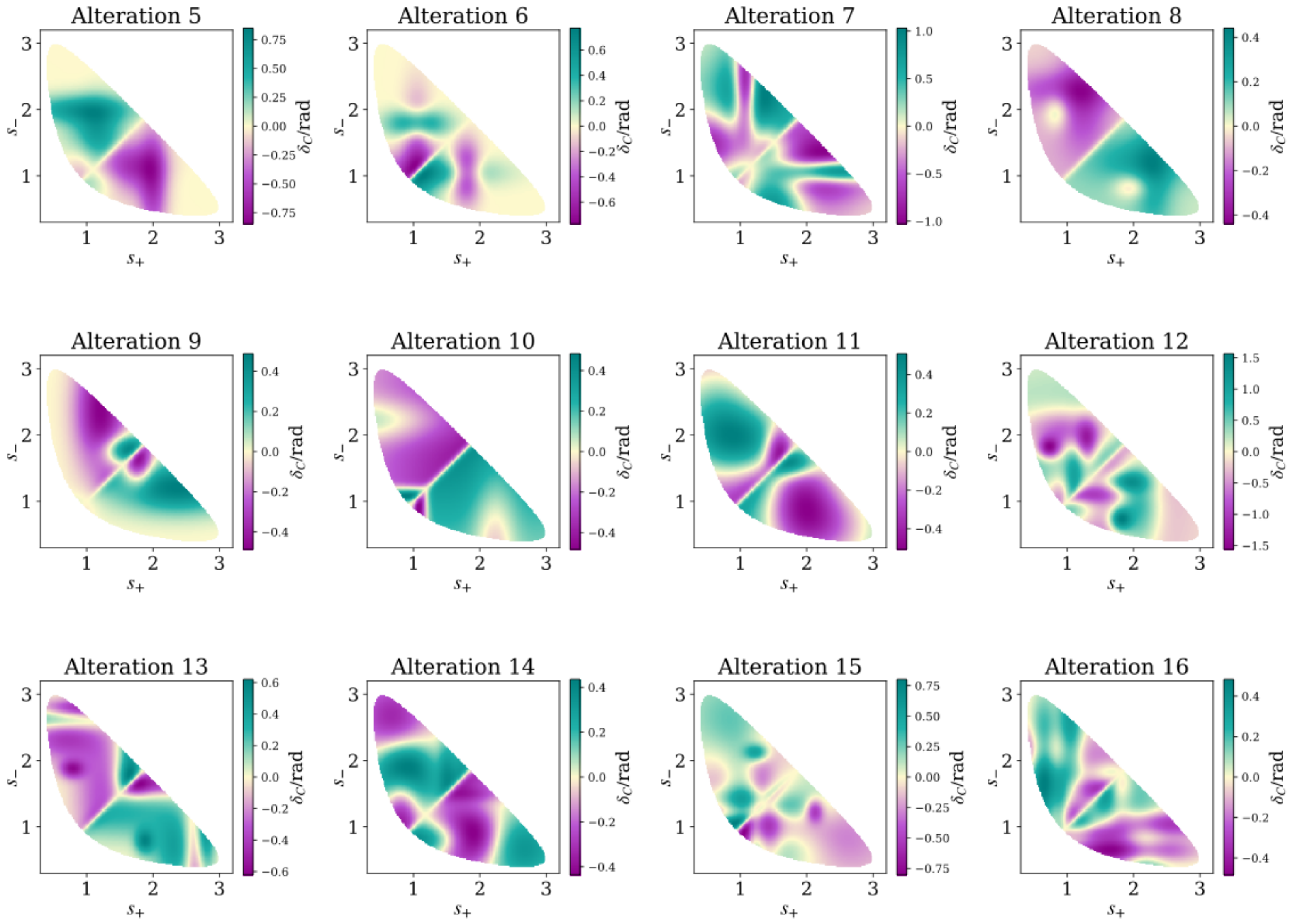
sum over many peaks *ensures δ_D symmetry + smooths the boundary* *width of bias*

Input change: alter and test the model

how to they effect a γ measurement?

can we keep the precision?

can we resolve complicated shapes?



are they realistic?

do all positions have an effect?

are different shapes harder?

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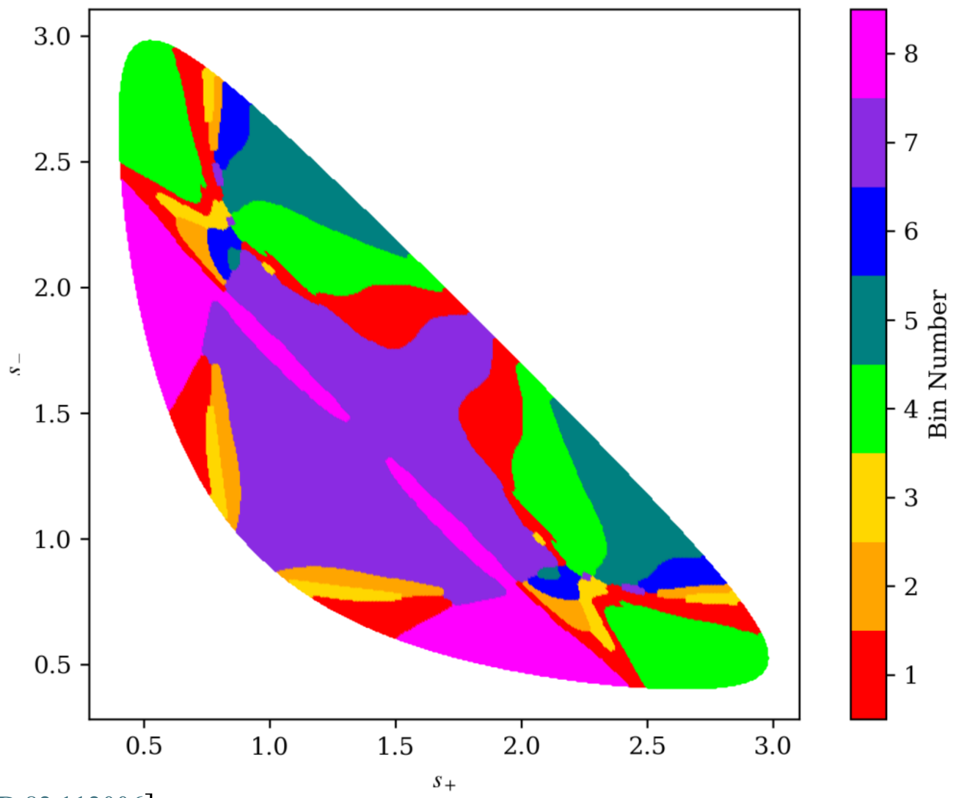
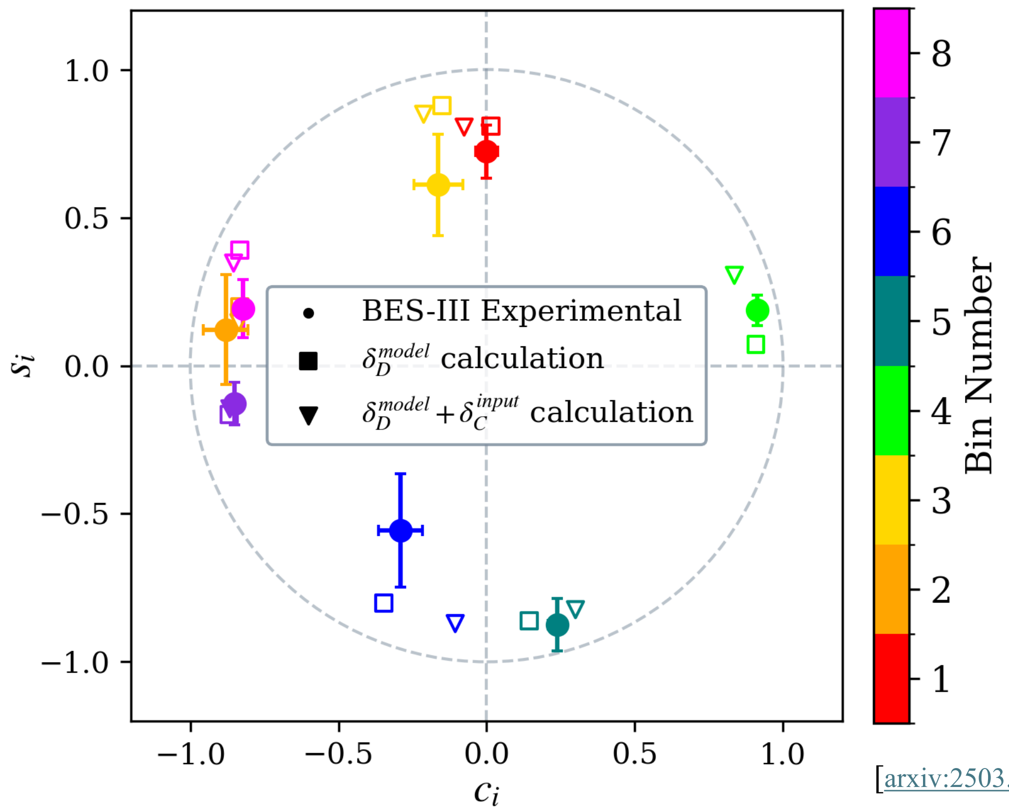
How realistic?



[PhysRevD.82.112006]

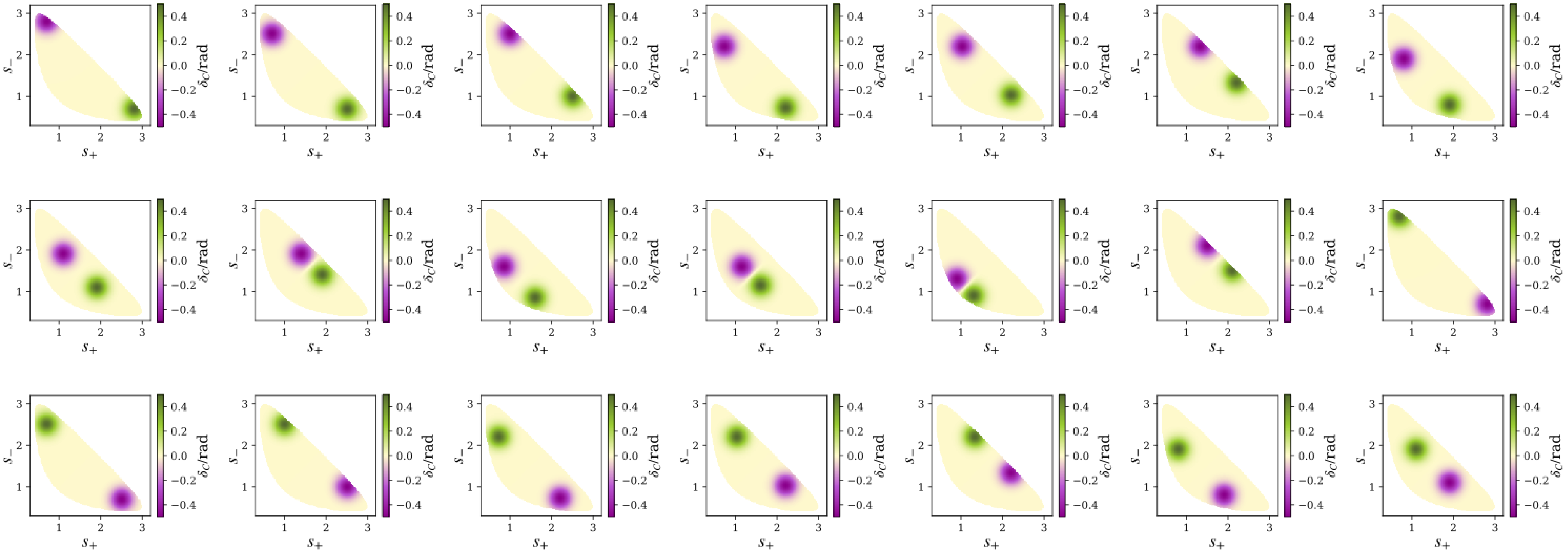


[arxiv:2503.22126]

[PhysRevD.98.112012]

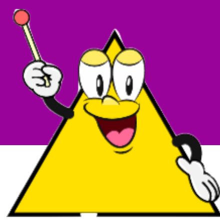
- Guide is from binned hadronic parameters measured experimentally
- Can quantify with a χ^2 between them and calculated ones

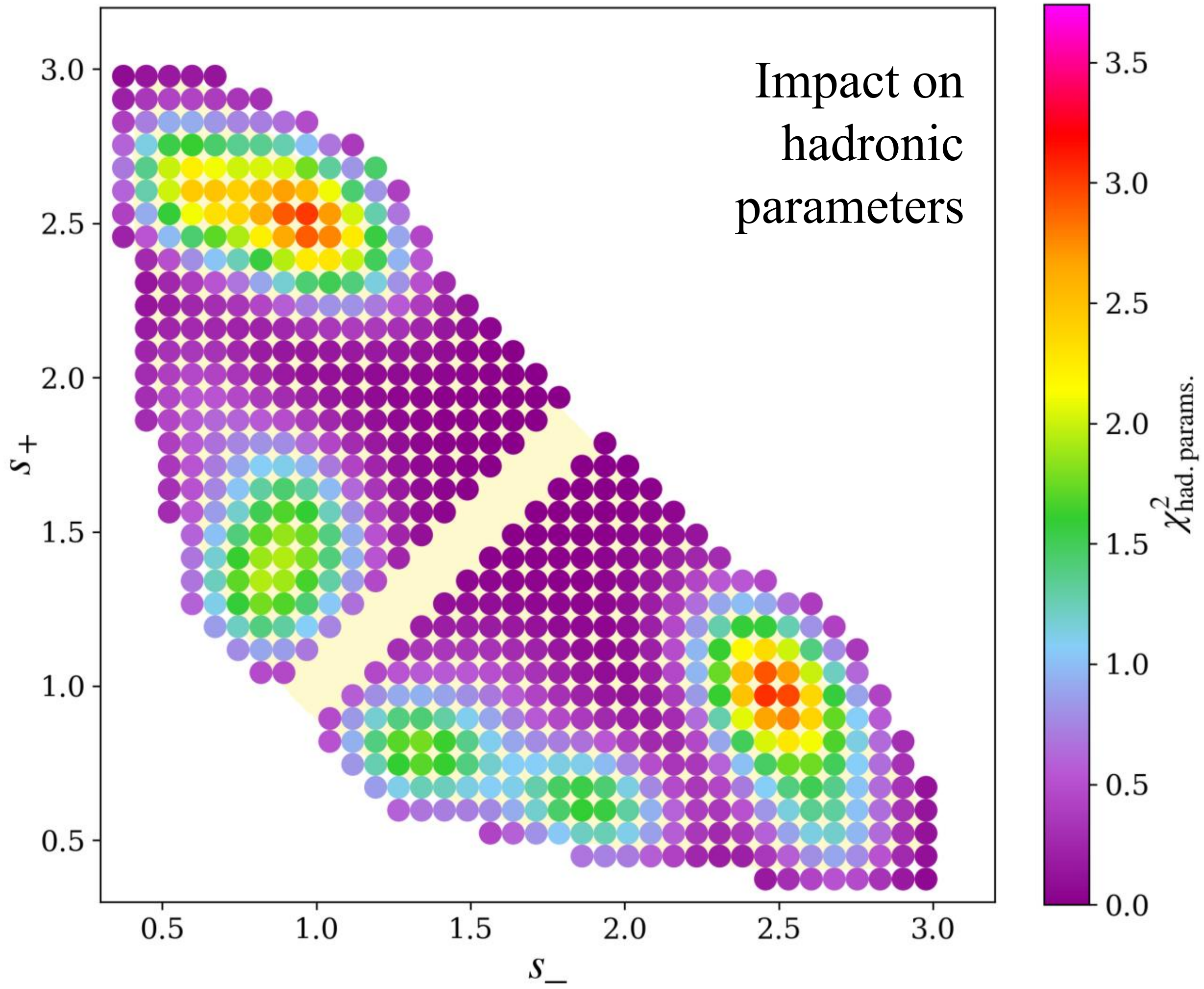
Is γ interested in the model-data difference (position)?

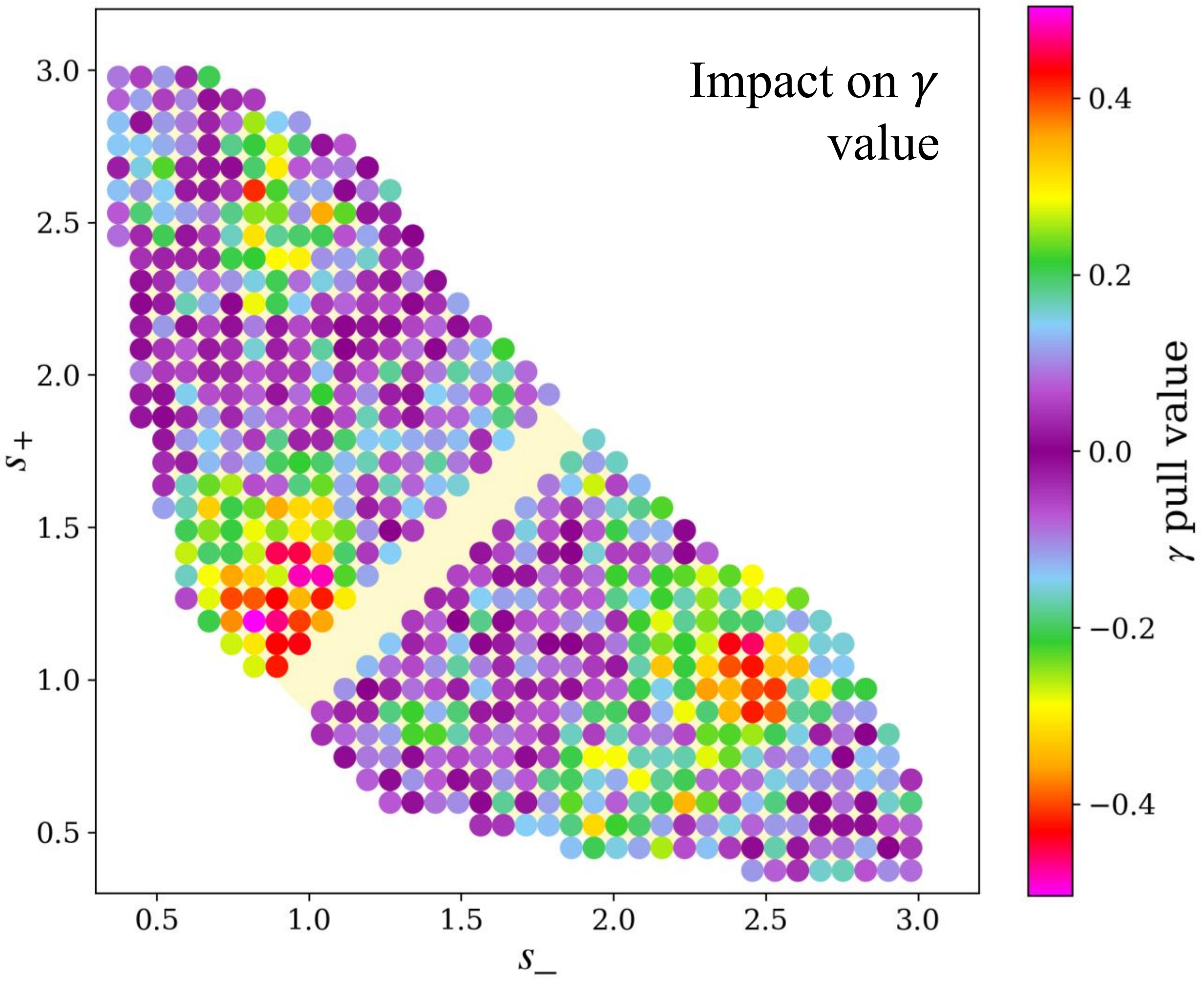


Change the model with δ_C^{input} at each point in phase space to see the effect on γ and whether this effect is visible in hadronic parameters

Is γ interested in the model-data difference (position)?

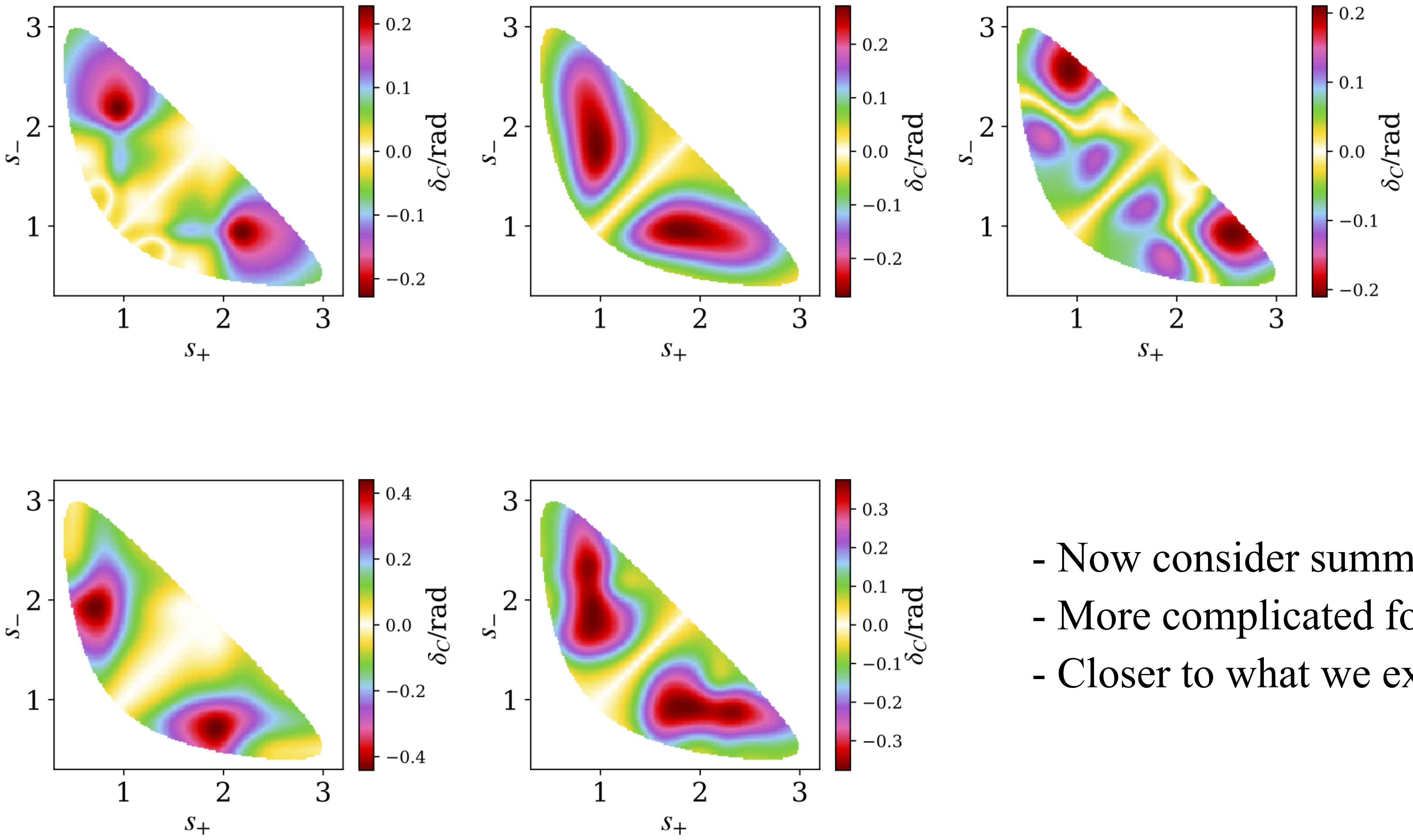






See a difference in the hadronic parameters $\Leftrightarrow$ γ will be biased

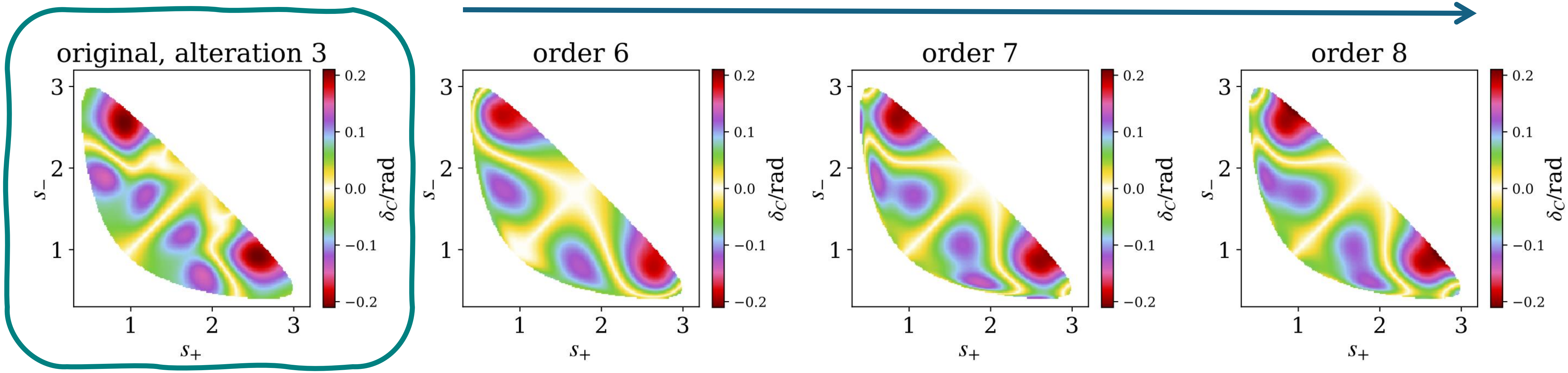
Can we resolve a complicated δ_C^{input}?



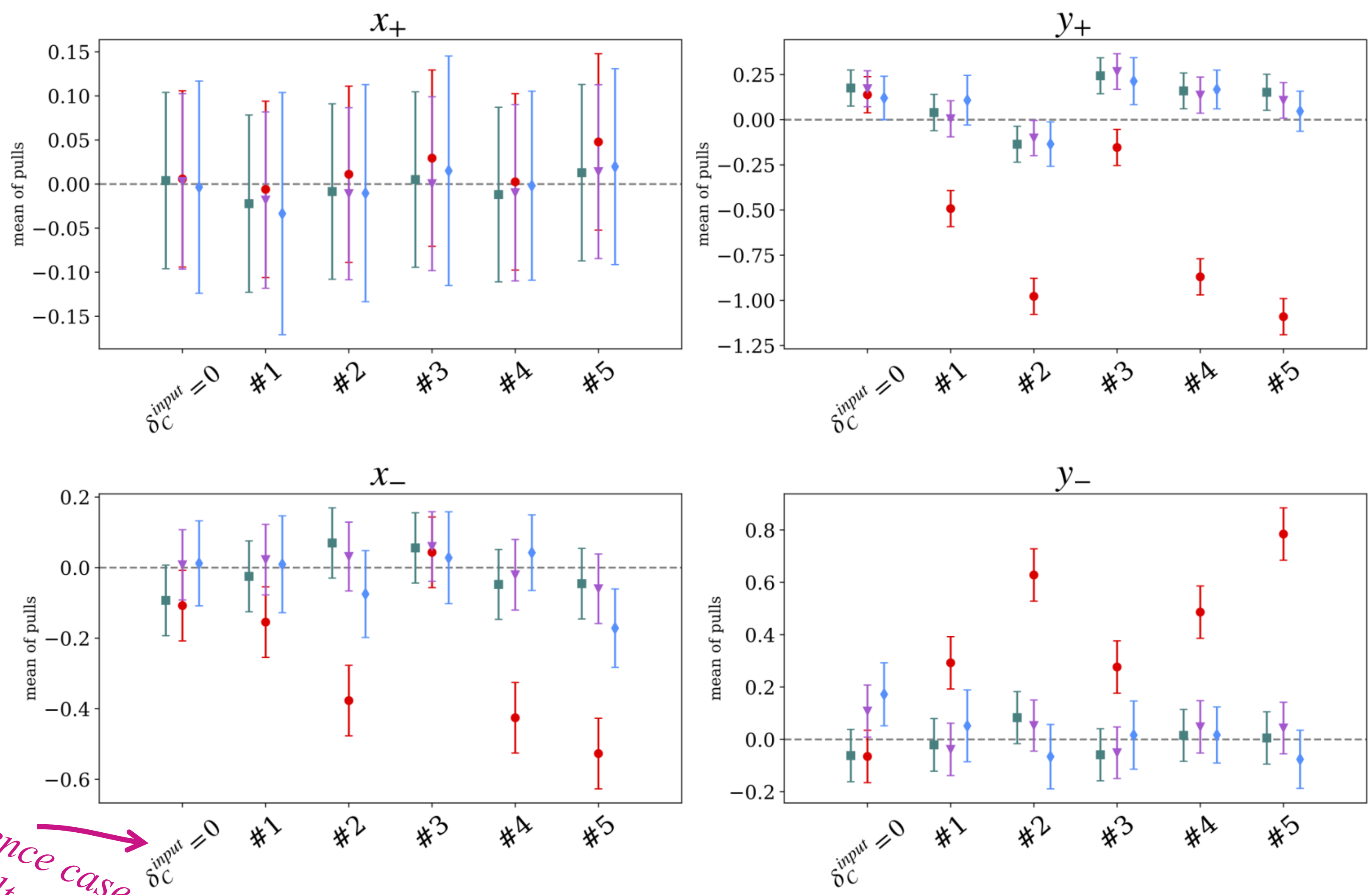
- Now consider summing many peaks
- More complicated for the polynomials
- Closer to what we expect from data

Can we resolve a complicated δ_C^{input}? Yes!

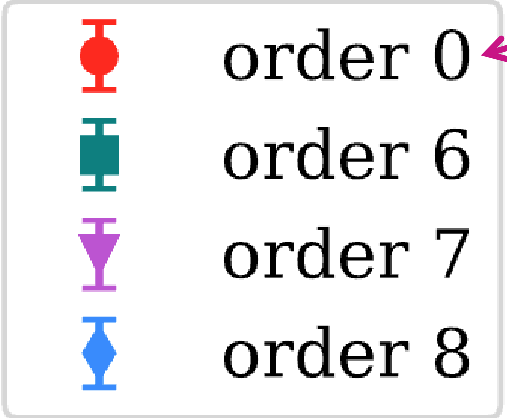
increase polynomial order



Can we resolve a complicated δ_C^{input}? Yes!



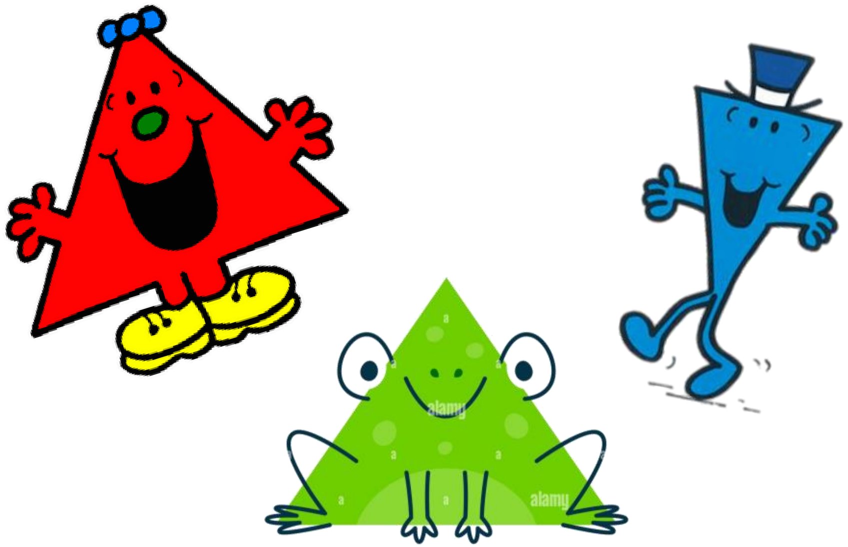
impact on γ



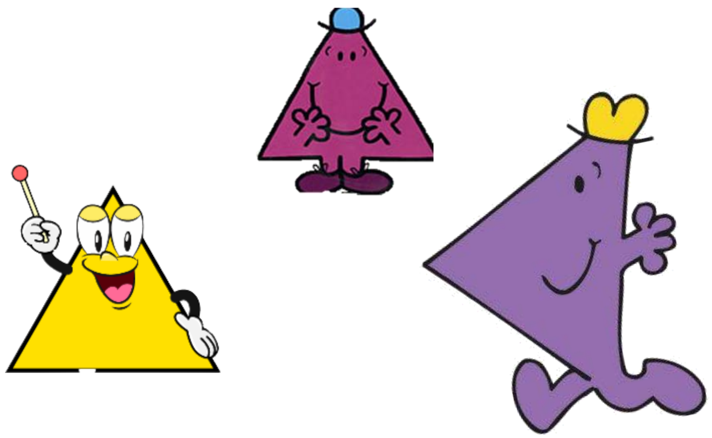
reference case of no alteration

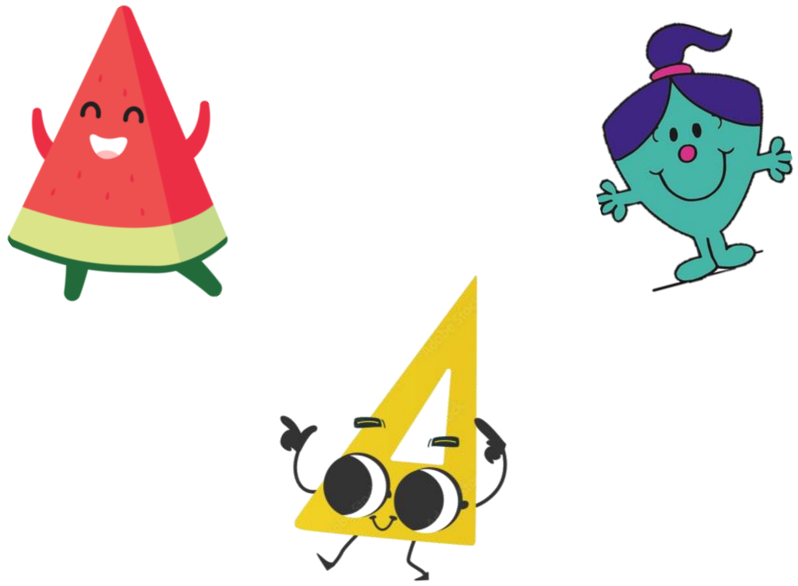
Conclusions and status

- Introduced the Phase-Correction Method
- Sensitivity studies show statistical-only uncertainty same as MD method
- Good progress to an unbinned MI gamma:
- Processed BES-III data ready for input
- Moving forwards with LHCb data processing and CP-fit



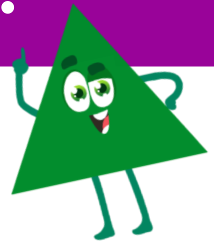
Thank you!





Backup

Can we keep the precision? – Yes!



Uncertainty/0.01:	$\sigma(x_+)$	$\sigma(y_+)$	$\sigma(x_-)$	$\sigma(y_-)$	$\sigma(\gamma)$
MD, unbinned	± 0.81	± 1.08	± 0.89	± 0.94	$\pm 4.3°$
MI, binned	± 0.96	± 1.22	± 1.08	± 1.45	$\pm 5.2°$
PC, unbinned	± 0.84	± 1.04	± 0.87	± 0.99	$\pm 4.4°$
Binned experimental	± 0.98	± 1.23	± 0.96	± 1.14	

All from same simulation system in AmpGen

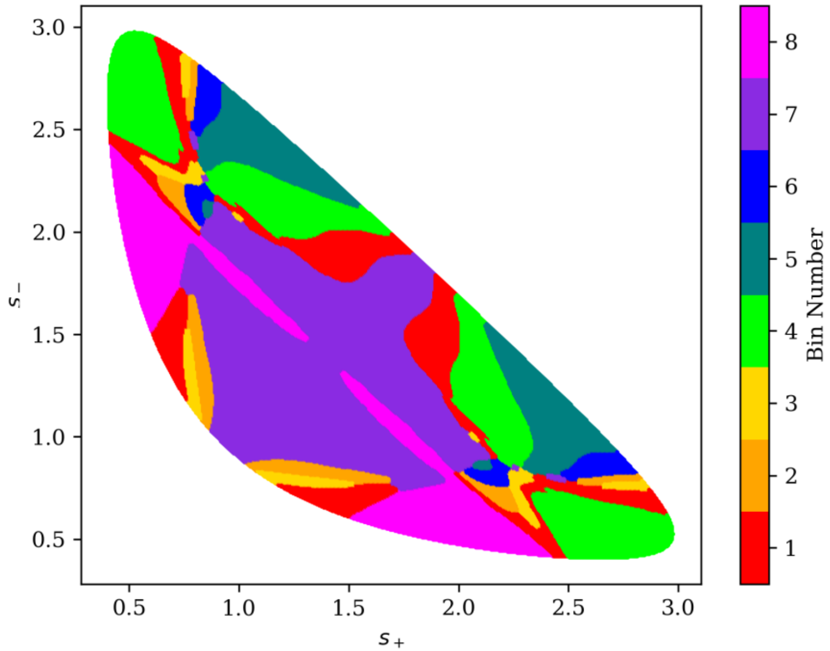
[LHCb-PAPER-2020-019]

In the phase-correction method, uncertainties consistent at different polynomial orders/biases

Uncertainties estimated from simulation study:
- Current LHCb/BES-III luminosities
- From 100 toys with a δ_C^{input}
- Statistical-only uncertainties
- For sensitivity only, pure signal toys

Binned hadronic parameters

$$\{A, \bar{A}, \delta_D\} \xrightarrow{\text{integration}} \{F_i, \bar{F}_i, c_i, s_i\}$$



$$F_i = \int_i A^2 \, ds_+ ds_+$$

$$\bar{F}_i = \int_i \bar{A}^2 \, ds_+ ds_-$$

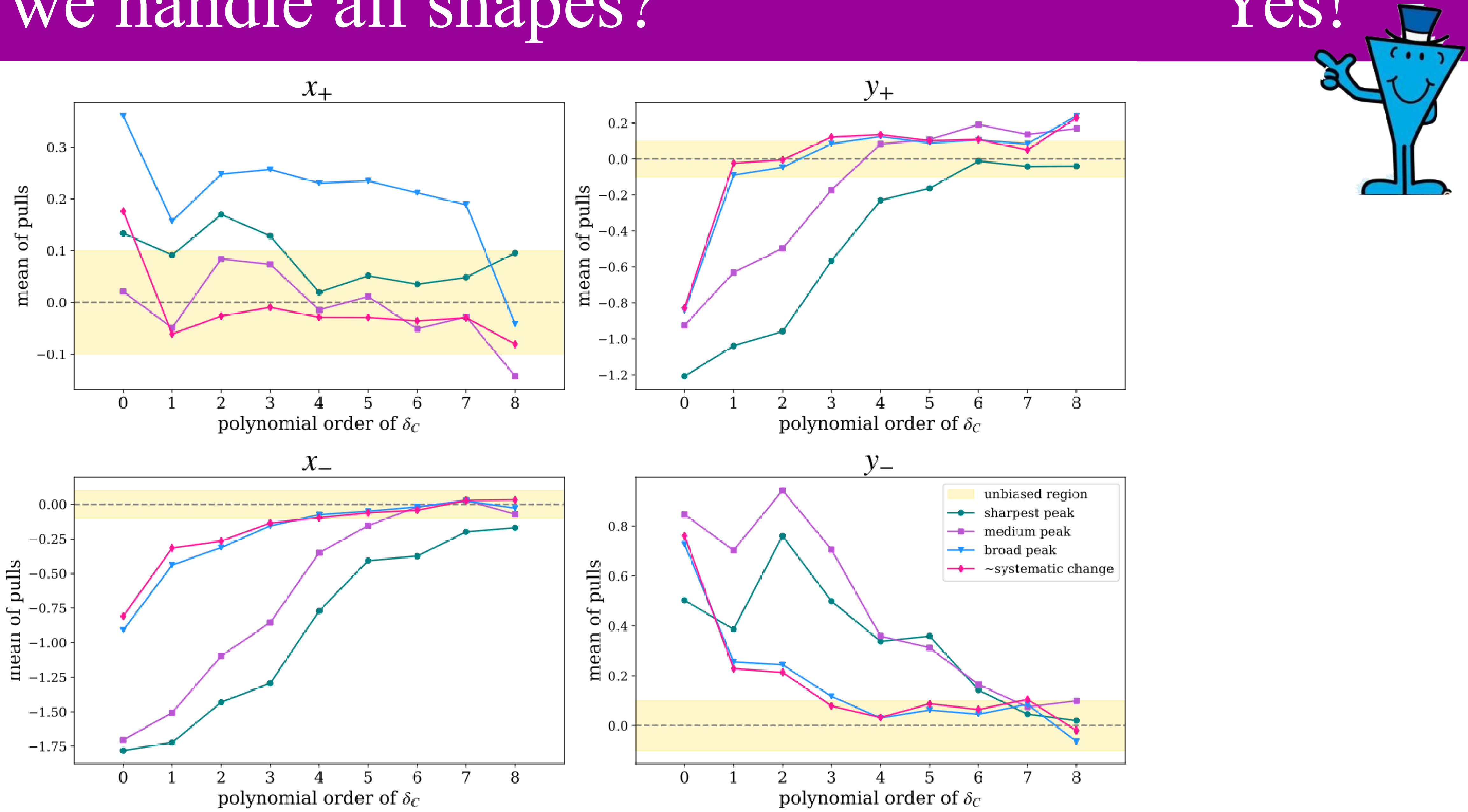
$$c_i = \frac{\int_i A\bar{A}\cos(\delta_D)\, ds_+ ds_-}{\sqrt{F_i \bar{F}_i}}$$

$$s_i = \frac{\int_i A\bar{A}\sin(\delta_D)\, ds_+ ds_-}{\sqrt{F_i \bar{F}_i}}$$

$$|\mathcal{A}_{tot}|^2 \propto A^2 r_B^2 + \bar{A}^2 + 2A\bar{A}\,(\mathrm{x}_- \cos(\delta_D) + y_- \sin(\delta_D))$$

$$\int_{\text{bin } i} |\mathcal{A}_{tot}|^2 \, ds_+ ds_- \propto \int_{\text{bin } i} \{A^2 r_B^2 + \bar{A}^2 + 2A\bar{A}\,(x_- \cos(\delta_D) + y_- \sin(\delta_D))\}\, ds_+ ds_- = F_i^2\, r_B^2 + \bar{F_i}^2 + 2F_i\bar{F_i}\,(x_- c_i + y_- s_i)$$

Can we handle all shapes? Yes!



Setup for final fit