



Measurements of charge and CP asymmetries in b -hadron decays using top-quark events collected by the ATLAS detector in pp collisions at $\sqrt{s} = 13\text{TeV}$

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IOP HEPP parallel talk 7/4/2025

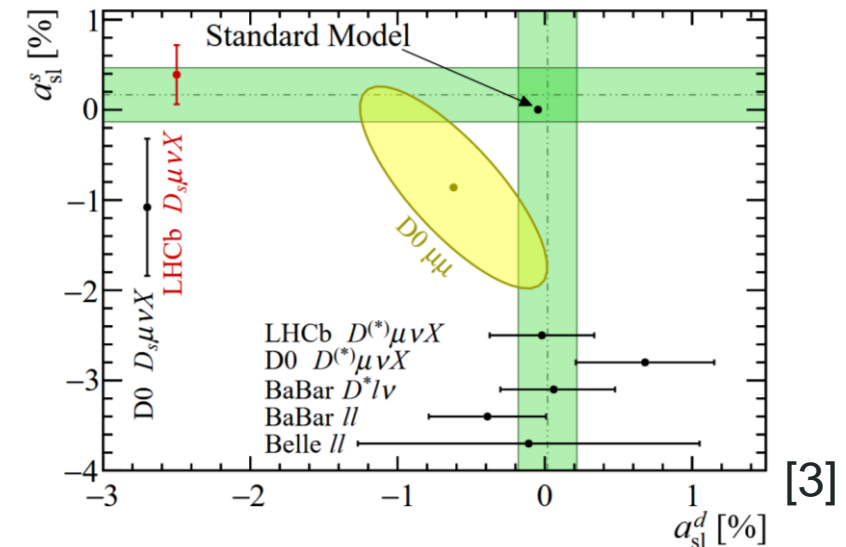


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Introduction: Theory and background



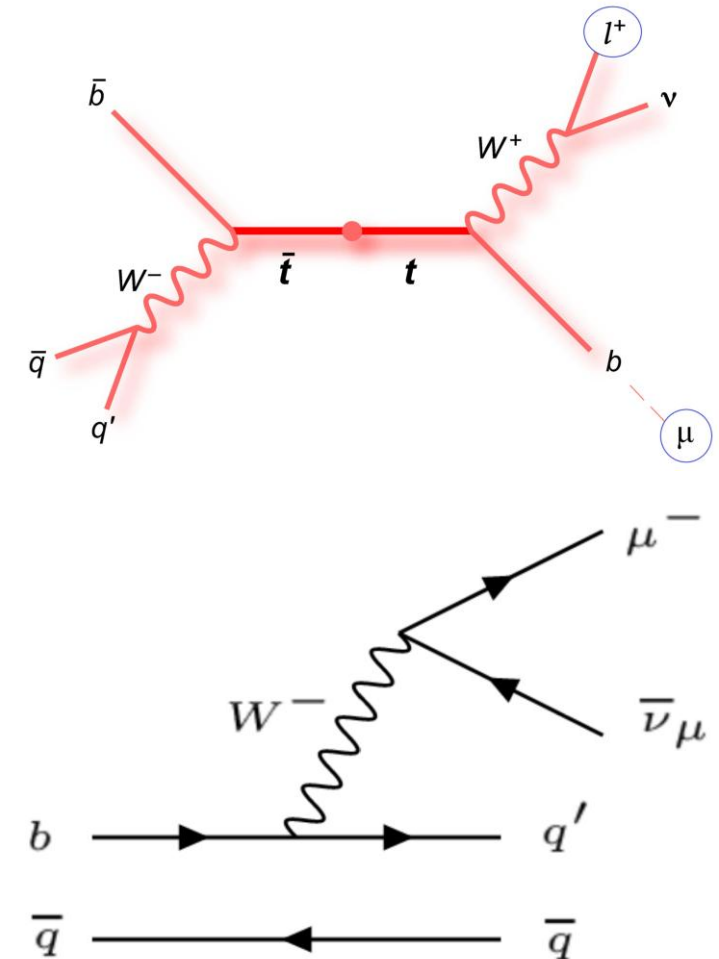
- CPV (Charge-Parity Violation) generates **matter-antimatter asymmetry**
- CPV in the **Standard Model (SM) is insufficient** to explain matter dominated universe
- In 2010, DØ reported an **anomalous asymmetry** from semi leptonic b-hadron decays, **3.6σ from SM** [1]
- Can use **top quark decays as a source of b-hadrons** to measure this at ATLAS
- Following from **statistically-limited** analysis using 20.3 fb⁻¹ Run 1 dataset [2]
- 140 fb⁻¹ Run 2 dataset will **improve statistical uncertainties** and updates to analysis techniques will **improve systematics**
- LHCb has **excluded the DØ measurement as result of indirect (from mixing) CPV** [3] but we are **able to probe the direct case**



Charge tagging and soft muons



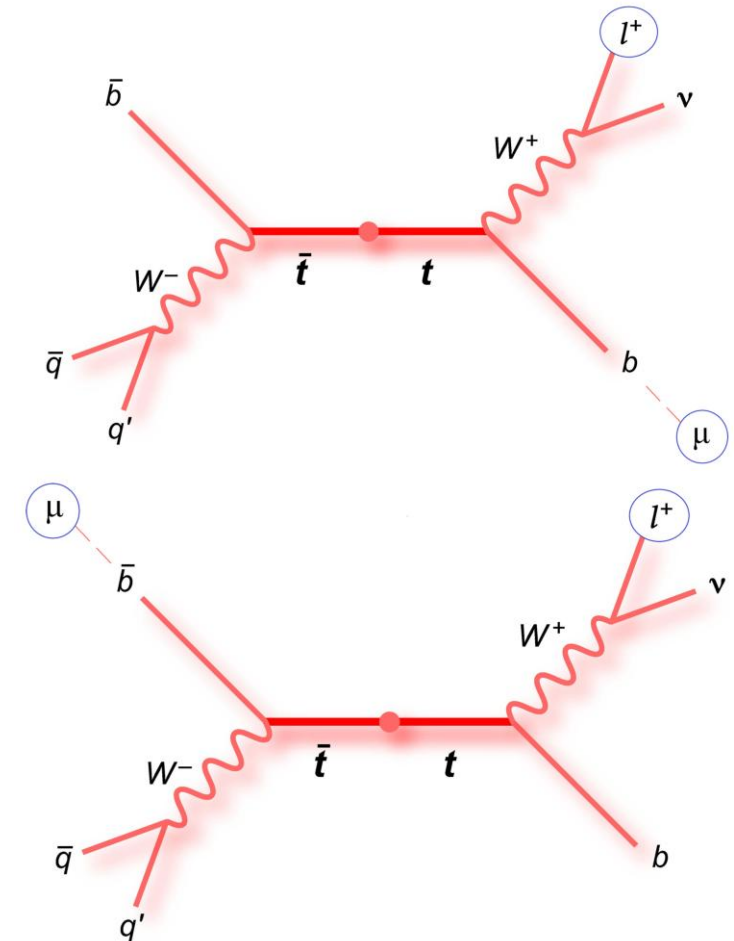
- Use **$t\bar{t}$ lepton + jets channel** with exactly **one prompt lepton**. Also require events to have a **soft muon**.
- Prompt lepton from W -boson decay **tags the b-quark charge at production**
- Soft muons are produced by **semi-leptonic decay of hadrons within b-jets** (Lower p_T hence “soft”). **Tags the b-quark charge at decay**
- Measure these values and **unfold** to build **charge and CPV asymmetries**
- For events where the prompt lepton and soft muon come from opposite sides of the $t\bar{t}$ system, identify and flip the prompt lepton charge. This is known as **same-top/different-top assignment**



Event selection



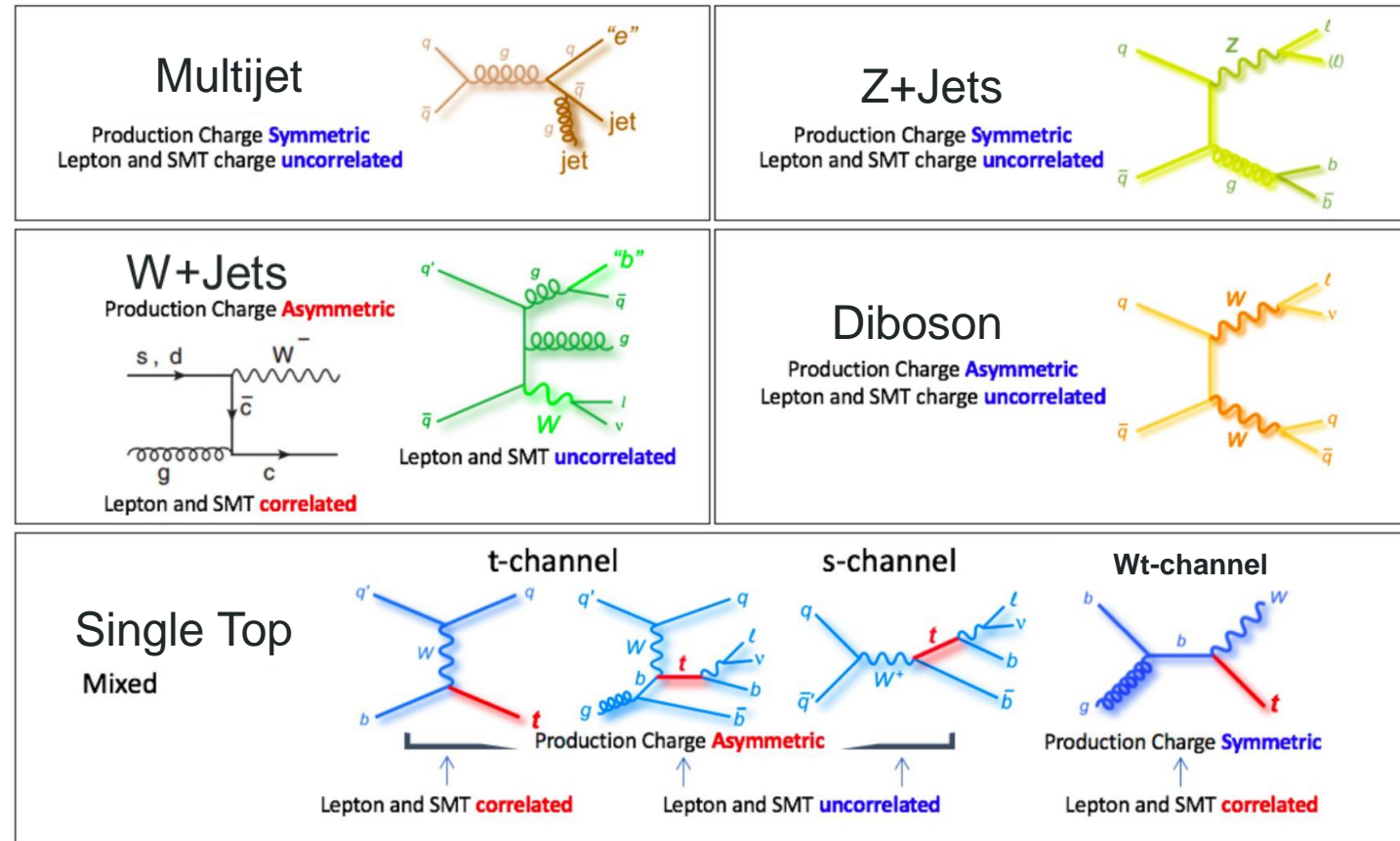
- Use $t\bar{t}$ lepton + jets channel with **exactly one prompt lepton**
- Also require events to have a **soft muon ($p_T > 4$ GeV)** coming from the semileptonic decay of a b -hadron or c -hadron originating from the b -quark or \bar{b} -quark
- The jet producing the soft muon is considered **Soft Muon Tagged (SMT)**
- We require **≥ 4 jets with $p_T > 25$ GeV**, and **≥ 1 b-jet** (at 77% efficiency working point)
- Most often the **SMT jet and b-tagged jet are the same**



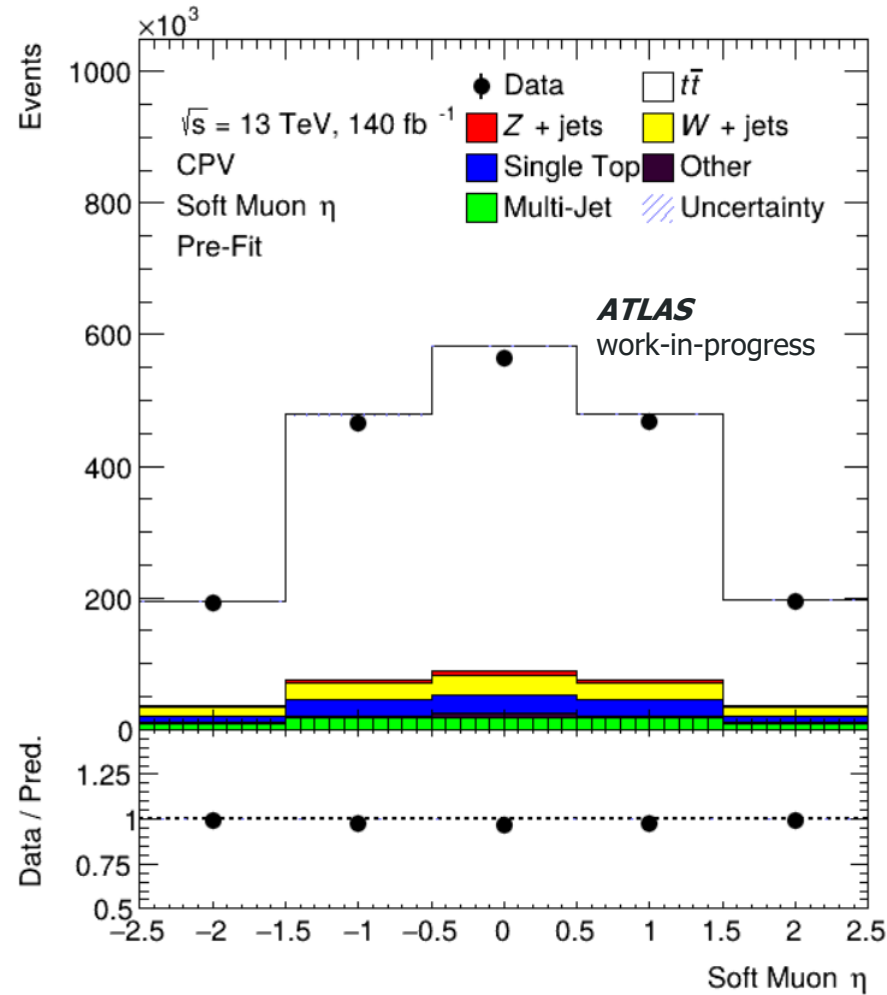
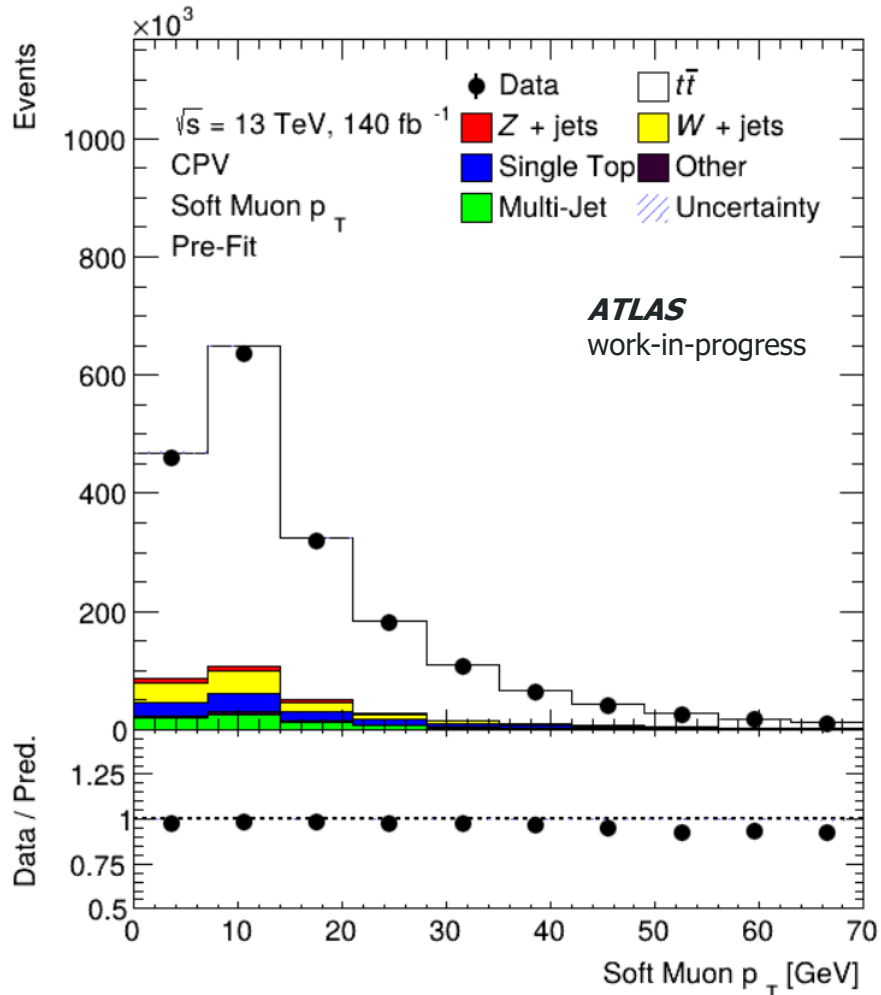
Backgrounds



- Lepton + jets $t\bar{t}$ events with soft muons can be imitated by:
 - Single top
 - W+jets
 - Z+jets
 - Multijet
 - Diboson
 - ttV/ttH
- Multijet yield estimated using **Asymptotic Matrix method**
- Also have **background from within $t\bar{t}$ due to misidentification between prompt and soft muons** – e.g. prompt muons produced close to jets and passing soft muon selection

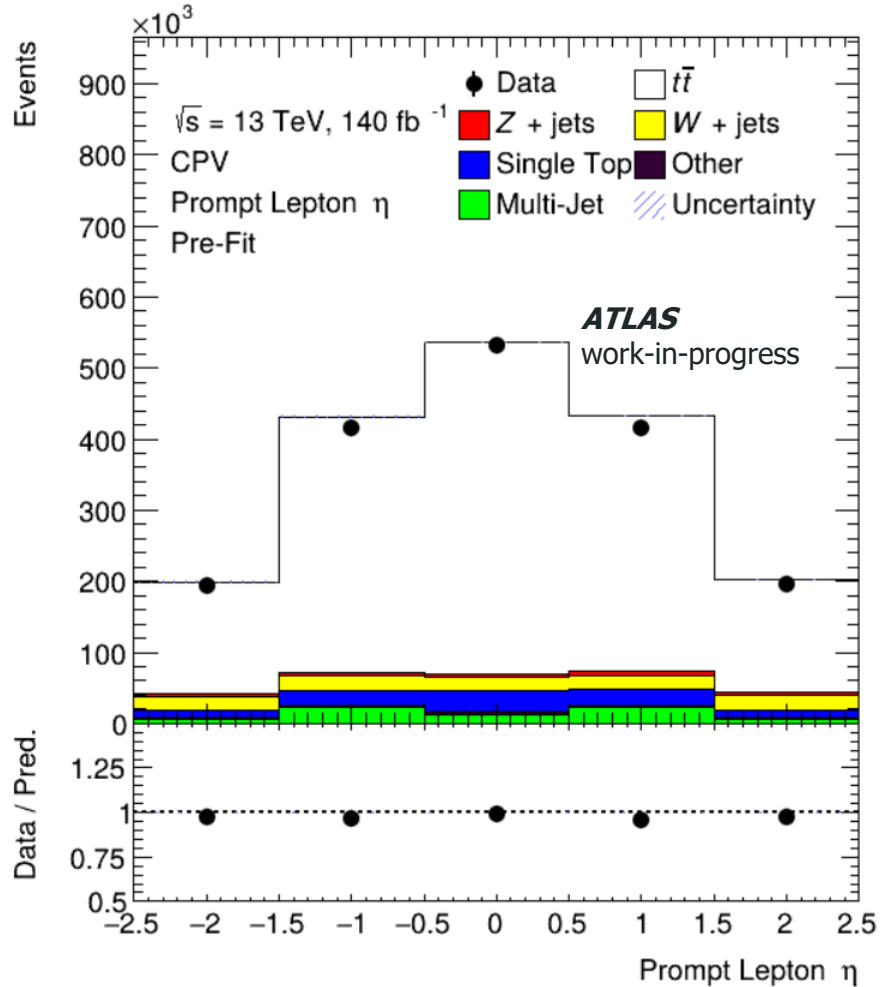
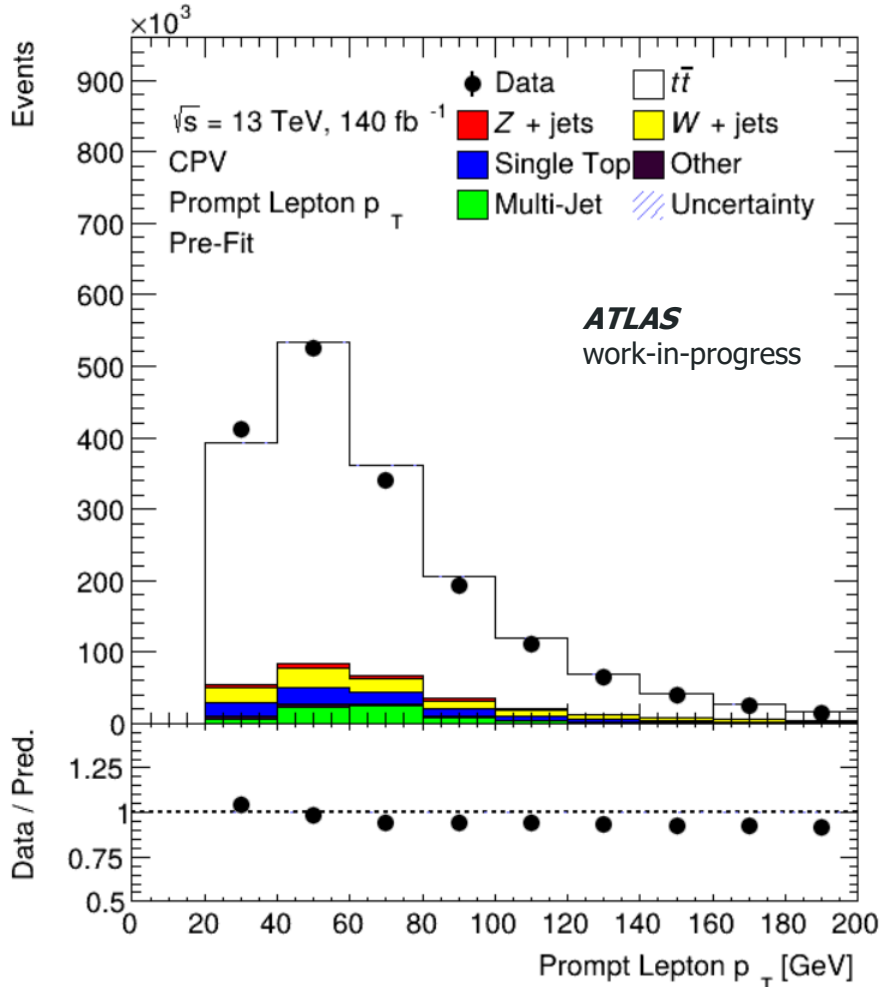


Control Plots: Soft muons

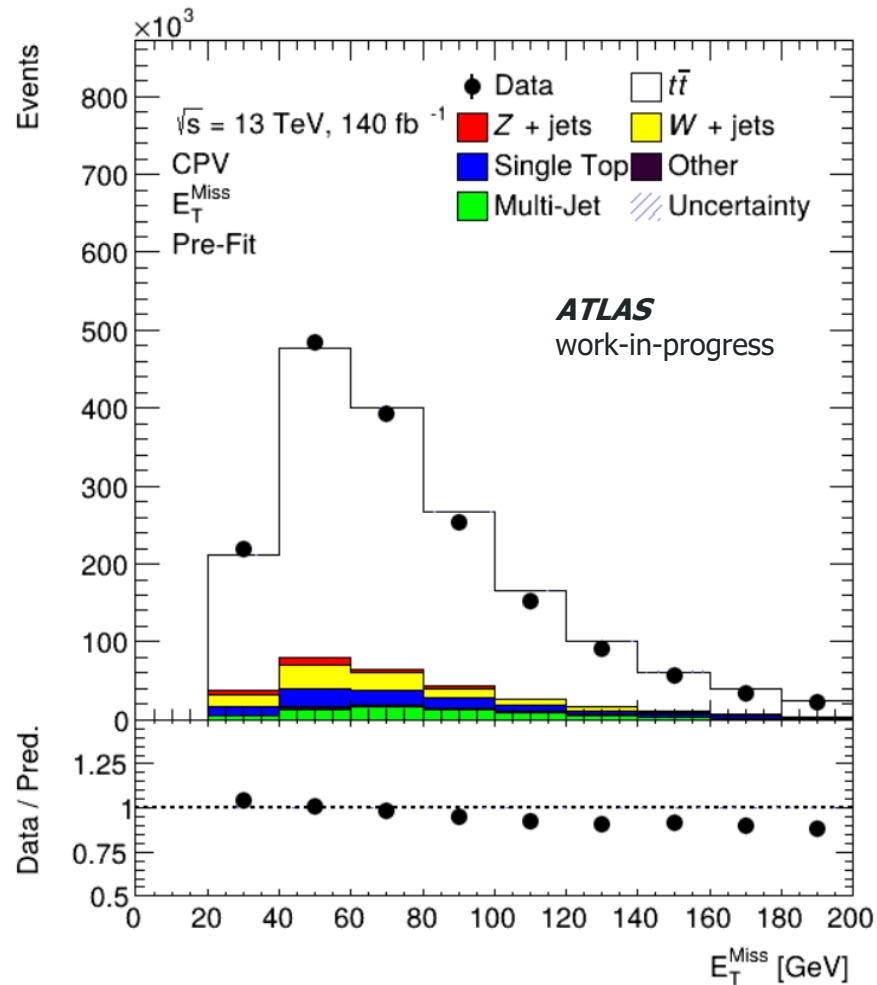


- Good Data-MC agreement
- Distributions follow expected trends

Control Plots: Prompt Leptons



Control Plots: E_T^{Miss} and yields



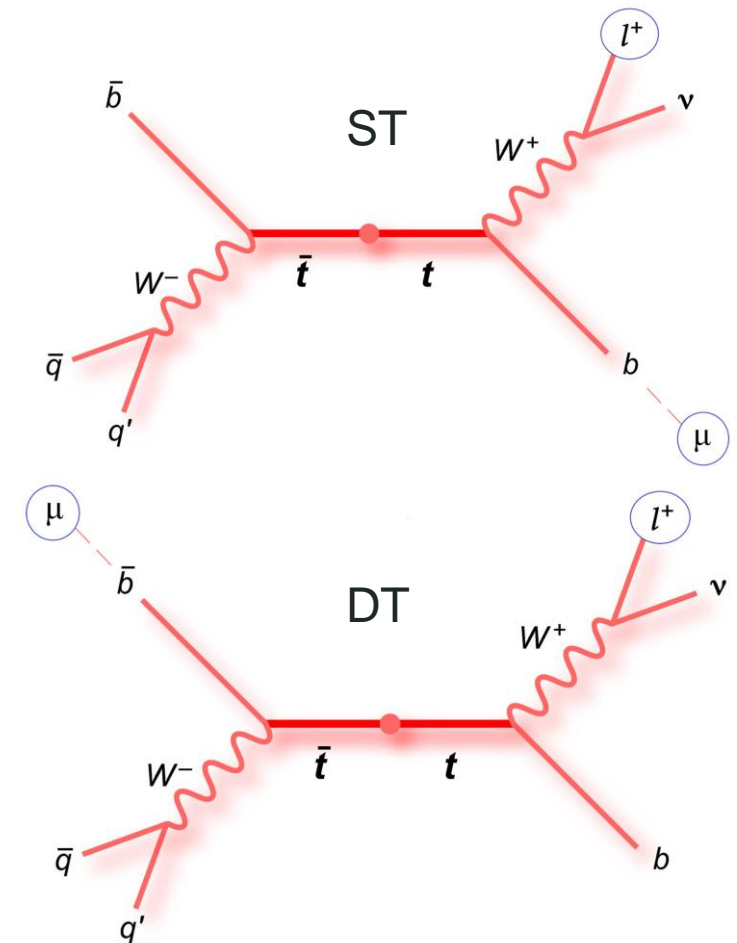
- Event selection **dominated by signal**
- W+Jets and combined single top channels are largest background contributions

Sample	Contribution	Percentage Contribution
$t\bar{t}$	$(1.6168 \pm 0.0005) \times 10^6$	83.7%
W+jets	$(1.0581 \pm 0.0003) \times 10^5$	5.5%
Multijet	$(7.0584 \pm 0.0023) \times 10^4$	3.7%
Single Top (tW -channel)	$(6.6541 \pm 0.0021) \times 10^4$	3.4%
Single Top (t -channel)	$(2.7150 \pm 0.0008) \times 10^4$	1.4%
Z+jets	$(2.6143 \pm 0.0008) \times 10^4$	1.4%
$t\bar{t}V$	$(7.4444 \pm 0.0024) \times 10^3$	0.4%
Diboson	$(5.0524 \pm 0.0016) \times 10^3$	0.3%
$t\bar{t}H$	$(3.8227 \pm 0.0012) \times 10^3$	0.2%
Single Top (s -channel)	$(1.9449 \pm 0.0006) \times 10^3$	0.1%
Total MC	$(1.9313 \pm 0.0006) \times 10^6$	100%
Experimental Data	$(1.8846 \pm 0.0013) \times 10^6$	–

Same Top/Different Top (ST/DT) assignment



- Essential to determine if prompt lepton and soft muon come from the **same-top (ST)** or **different-top (DT)**
- At 8 TeV, assignment was **sensitive to changes in initial and final state radiation** resulting in **large systematic uncertainty** so **3 new methods are being tested**:
 - **Cut on ΔR** between the prompt lepton and soft muon
 - **SKM (Simple Kinematic Method)**, a simple “top mass” reconstruction
 - **BDT (Boosted Decision Tree)** trained on 4 different feature sets
- Performance evaluated considering efficiency, ε , and purity, ρ . Ultimately, **how systematics are affected will be the most important**



ST/DT: Angular separation (ΔR)

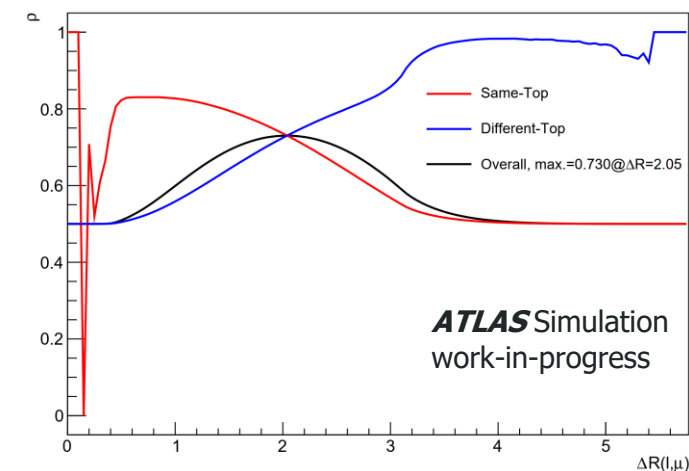
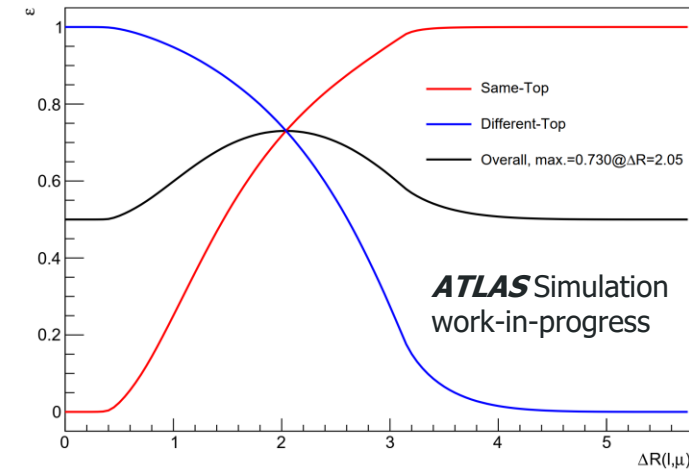
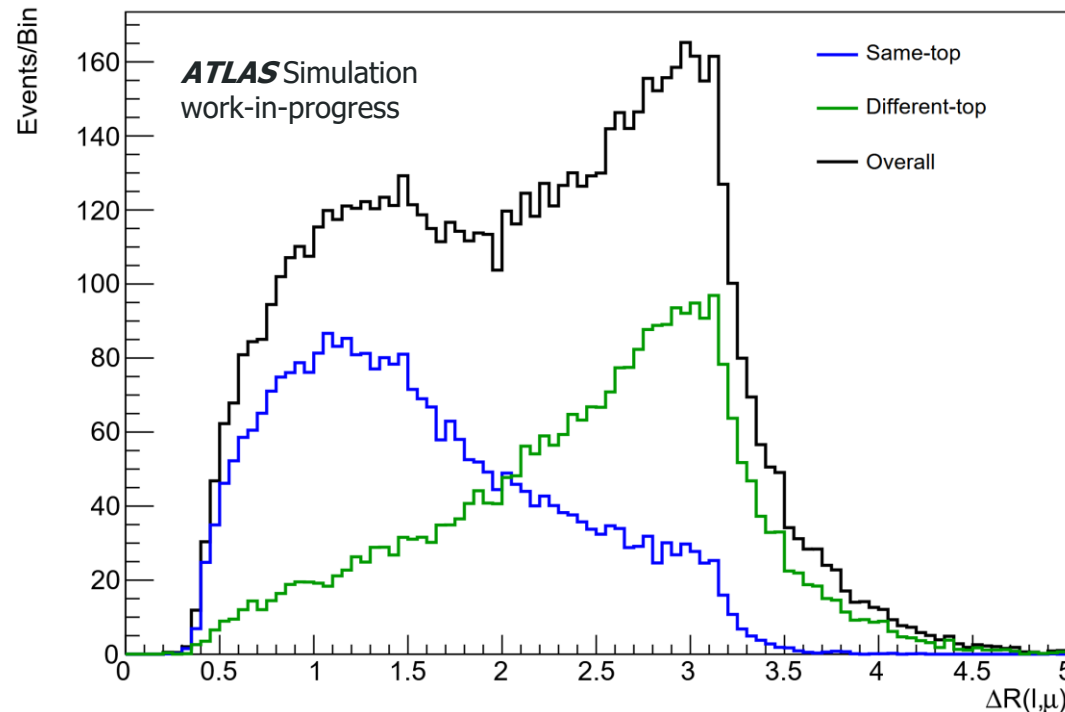


- Place a cut on the ΔR between the prompt lepton and soft muon

- $$\Delta R(l, \mu) = \sqrt{\Delta\phi(l, \mu)^2 + \Delta\eta(l, \mu)^2}$$

- Optimised cut at $\Delta R = 2.05$

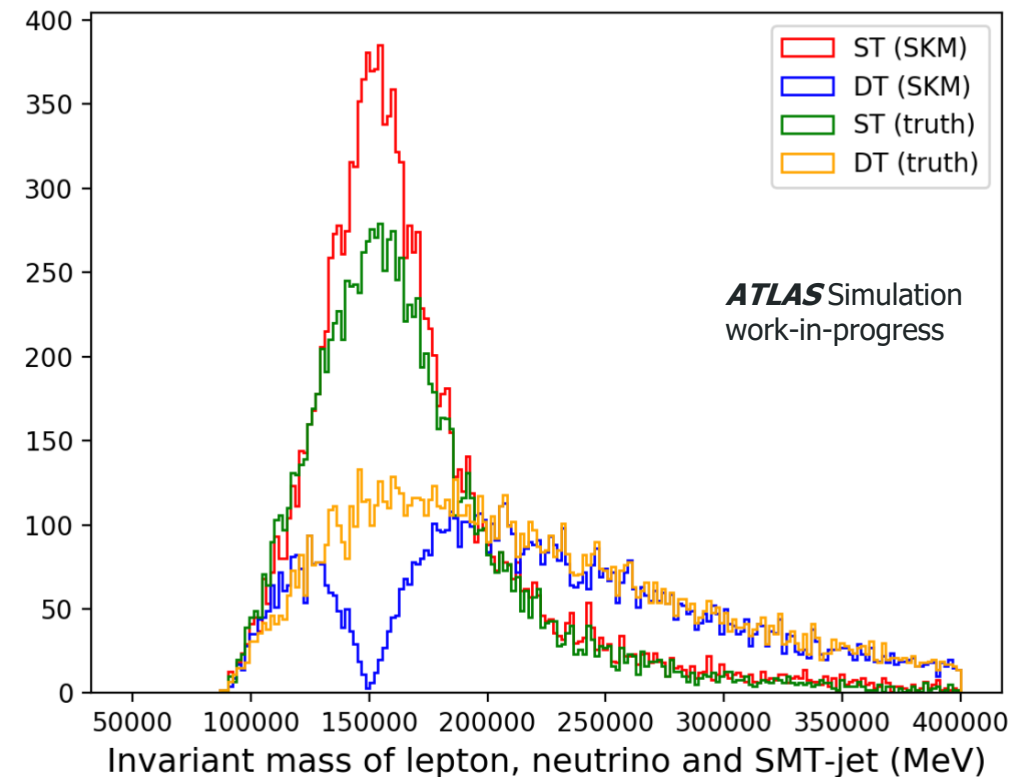
- Yields: $\epsilon = \rho = 0.730$



ST/DT: Simple Kinematic Method (SKM)



- Method:
 1. Take **SMT jet** and **jet with the highest b -tagging score** as the b -jets
 2. Using the constraint $m_W = m_{l\nu}$, the **z -component** of the **neutrino's four momentum is calculated**
 3. Use the b -jets to **reconstruct the leptonic top**
 4. Choose b -jet that more closely constructs the "top mass". **If the chosen b -jet is an SMT-jet, then this event is assigned ST (if not, then DT).**
- Top mass used is **150 GeV** due to m_W constraint. Tests show **improved efficiency and purity** if this value is used
- Yields: **$\rho = 0.678$**



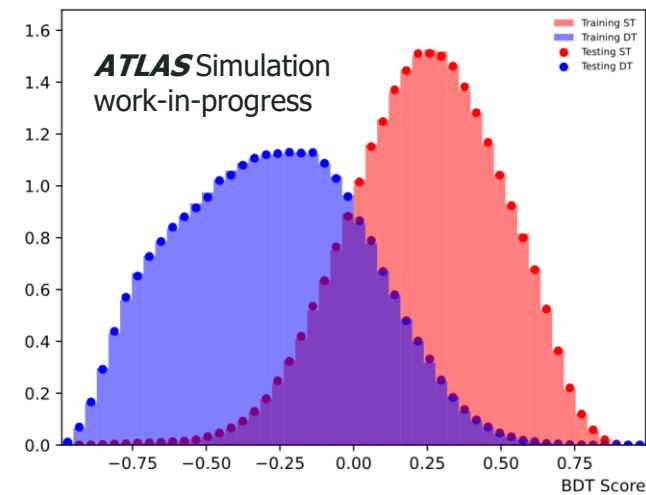
ST/DT assignment: BDT



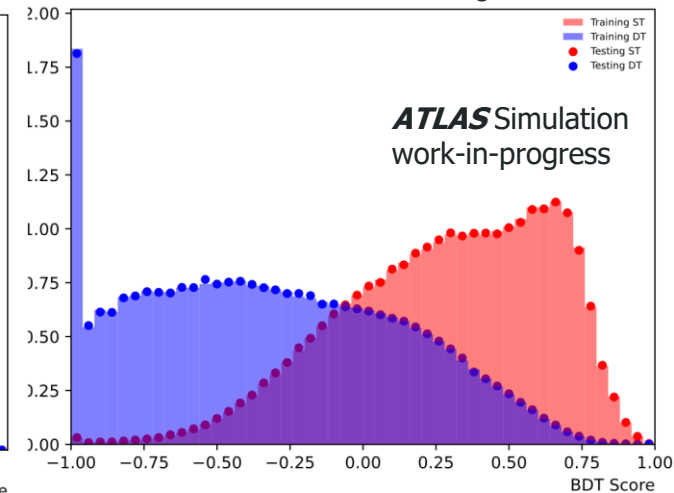
- BDT is trained on **4 feature sets**:
With/without jet variables and
with/without prompt lepton and soft muon charges
- Aim is to **reduce initial and final state radiation systematic** and investigate **off diagonal elements in migration matrix**

STDT method	Overall purity
BDT (Leptonic Features, charge included)	0.764
BDT (Leptonic Features, no charge)	0.741
BDT (Jet Features, charge included)	0.823
BDT (Jet Features, no charge)	0.767

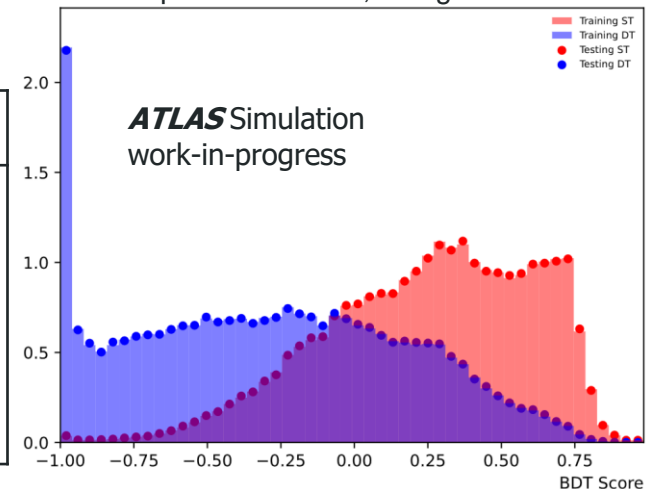
Jet Features, charge included



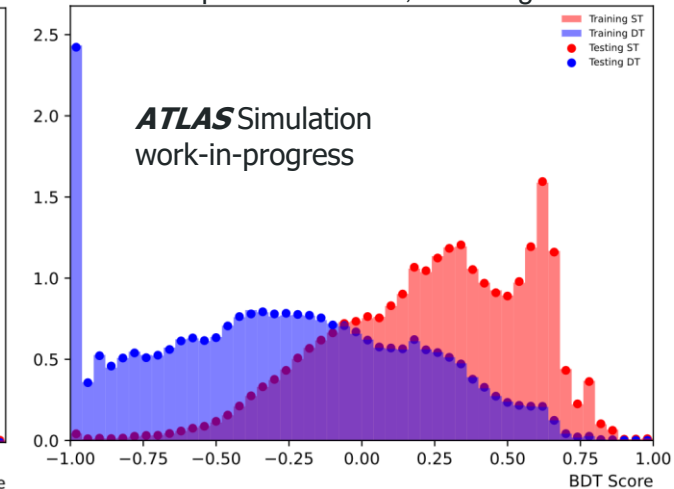
Jet Features, no charge



Leptonic Features, charge included



Leptonic Features, no charge



Unfolding

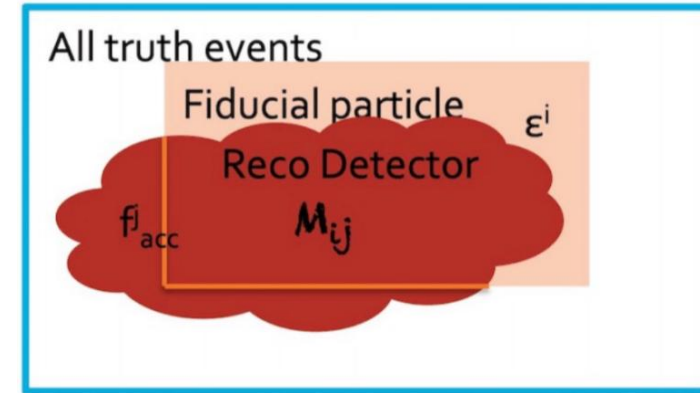


- We **measure the prompt lepton and soft muon charge combination**, and count $N^{l\mu}$ for $l\mu \in \{++,- -,+ -, - +\}$ to build charge and CP asymmetries

$$A^{SS} \equiv \frac{P(b \rightarrow l^+) - P(\bar{b} \rightarrow l^-)}{P(b \rightarrow l^+) + P(\bar{b} \rightarrow l^-)} = \frac{\frac{N^{++}}{N^+} - \frac{N^{--}}{N^-}}{\frac{N^{++}}{N^+} + \frac{N^{--}}{N^-}},$$

$$A^{OS} \equiv \frac{P(b \rightarrow l^-) - P(\bar{b} \rightarrow l^+)}{P(b \rightarrow l^-) + P(\bar{b} \rightarrow l^+)} = \frac{\frac{N^{+-}}{N^+} - \frac{N^{-+}}{N^-}}{\frac{N^{+-}}{N^+} + \frac{N^{-+}}{N^-}}$$

- Use unfolding technique to **recover the true distribution** from a reconstructed one



$$N_{\text{unfolded}}^j = \frac{1}{\epsilon^j} \cdot \sum_i \mathcal{M}_{ij}^{-1} \cdot f_{\text{acc}}^i \cdot (N_{\text{data}}^i - N_{\text{bkg}}^i);$$

$$\epsilon^j = \left(\frac{n(\text{reco-level} \cap \text{particle-level})}{n(\text{particle-level})} \right)^j,$$

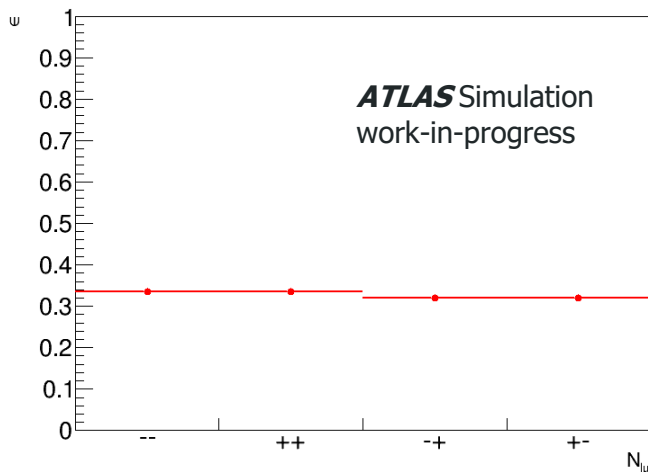
$$f_{\text{acc}}^i = \left(\frac{n(\text{reco-level} \cap \text{particle-level})}{n(\text{reco-level})} \right)^i.$$

Unfolding (2)

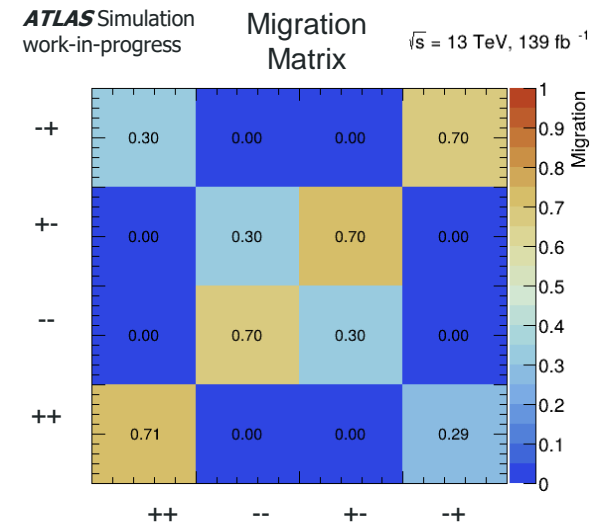
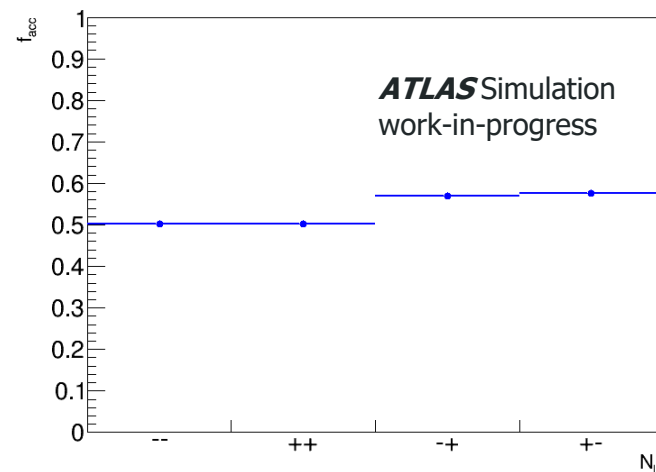


- Previously used **unregularized matrix inversion** [4]
- Testing **Profile Likelihood** method to better handle systematics [5]

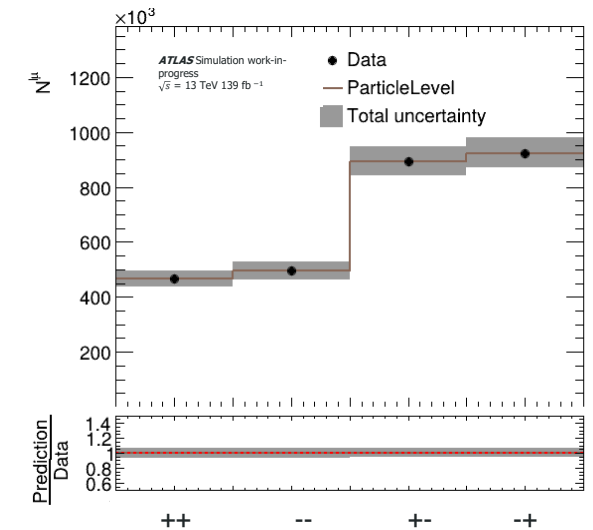
Efficiency



Acceptance



Unfolded distribution



[4] <https://gitlab.cern.ch/RooUnfold/RooUnfold> [5] <https://gitlab.cern.ch/TRExStats/TRExFitter>

Unfolding: Unregularized matrix inversion



- Comparison between all ST/DT methods. Despite **large performance differences**, **uncertainties are within a few percent**
- Still **missing largest systematics (signal modelling and extra radiation)**

A^{SS}	ΔR	SKM	BDT (jet)	BDT (jet, no charge)	BDT (lep)	BDT (lep, no charge)
Purity	73.0%	67.8%	82.3%	76.7%	76.4%	74.1%
Stat. Uncertainty	16%	17%	14%	15%	15%	15%
Syst. Uncertainty	7.4%	7.8%	5.6%	6.2%	6.7%	7.2%
Total Uncertainty	18%	19%	15%	16%	17%	17%

A^{OS}	ΔR	SKM	BDT (jet)	BDT (jet, no charge)	BDT (lep)	BDT (lep, no charge)
Purity	73.0%	67.8%	82.3%	76.7%	76.4%	74.1%
Stat. Uncertainty	16%	17%	14%	15%	15%	15%
Syst. Uncertainty	6.8%	7.5%	5.1%	5.8%	6.3%	6.8%
Total Uncertainty	17%	18%	15%	16%	16%	17%

Unfolding: Profile Likelihood

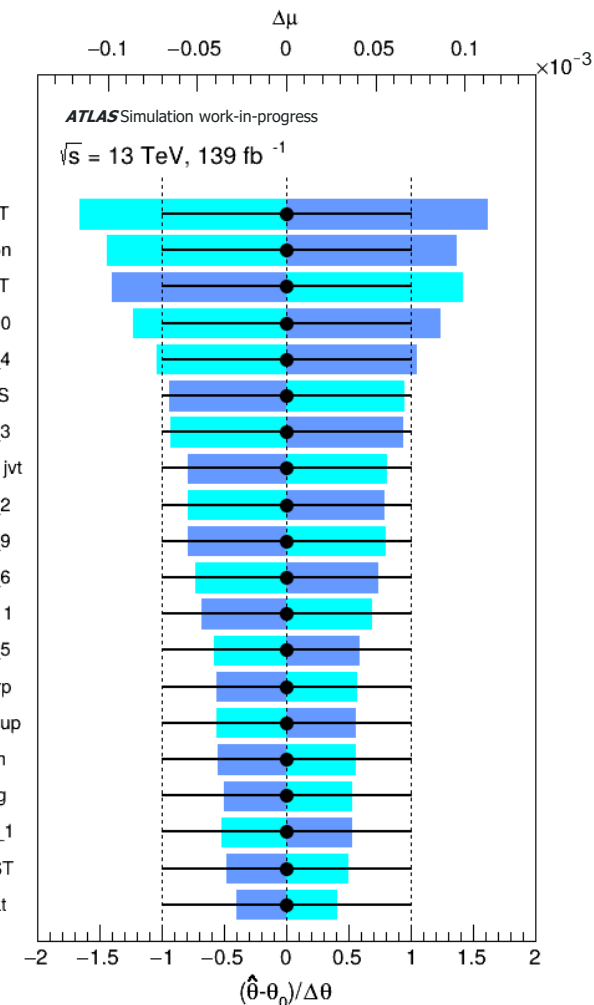


- Mostly **compatible with unregularized matrix inversion**

A^{SS}	ΔR	SKM	BDT (jet)	BDT (jet, no charge)	BDT (lep)	BDT (lep, no charge)
Purity	73.0%	67.8%	82.3%	76.7%	76.4%	74.1%
Total Uncertainty (Unregularized matrix inversion)	18%	19%	15%	16%	17%	17%
Total Uncertainty (Profile Likelihood)	18%	22%	15%	16%	17%	17%

Pre-fit impact on μ :
 $\square \theta = \hat{\theta} + \Delta\theta$ $\square \theta = \hat{\theta} - \Delta\theta$
 Post-fit impact on μ :
 $\blacksquare \theta = \hat{\theta} + \hat{\Delta}\theta$ $\blacksquare \theta = \hat{\theta} - \hat{\Delta}\theta$
 — Nuis. Param. Pull

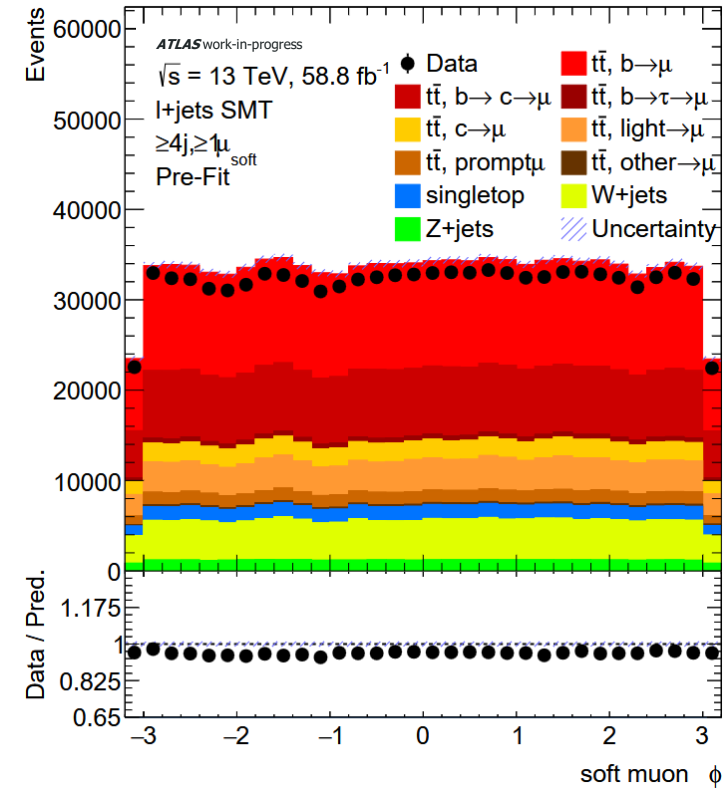
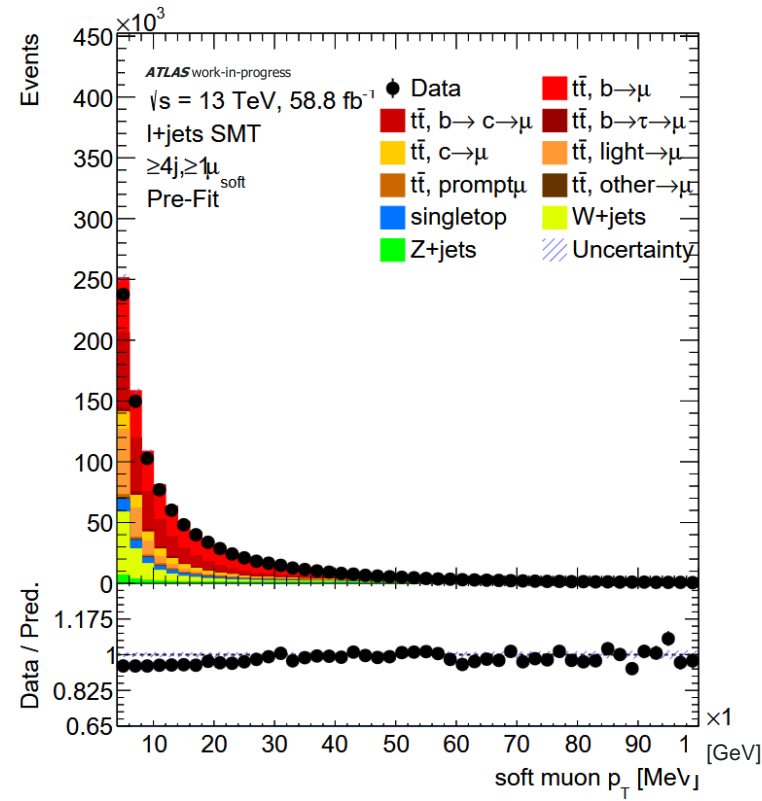
FT_MU_ID_SYST_LOWPT
 JET_Flavor_Composition
 FT_MU_ID_STAT_LOWPT
 JET_JER_EffectiveNP_10
 JET_JER_EffectiveNP_4
 MUON_SAGITTA_RESBIAS
 JET_JER_EffectiveNP_3
 jvt
 JET_JER_EffectiveNP_2
 JET_JER_EffectiveNP_9
 JET_JER_EffectiveNP_6
 JET_JER_EffectiveNP_11
 JET_JER_EffectiveNP_5
 MET_SoftTrk_ResoPerp
 pileup
 JET_JER_EffectiveNP_12restTerm
 JET_EtaIntercalibration_Modelling
 JET_JER_EffectiveNP_1
 FT_MU_Isol_SYST
 JET_EtaIntercalibration_TotalStat



Current work and future



- Working on **adding remaining systematics**
- Working on **migration to new software release**
 - Optimising event selection
 - Improved soft muon selections





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Thank you for listening



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Backup

Full Event selection



Prompt electron:

- Tight likelihood
- Gradient isolation
- $p_T > 15 \text{ GeV}$
- $d_0^{\text{sig}} < 5$
- $|z_0 \sin \theta| < 0.5 \text{ mm}$
- $|\eta| < 2.47$
- $1.37 < |\eta| < 1.52$ excluded

Prompt muon:

- $p_T > 25 \text{ GeV}$
- $|\eta| < 2.5$
- $|d_0^{\text{sig}}| < 3$
- $|z_0 \sin \theta| < 0.5 \text{ mm}$
- $\Delta R > 0.4$ from nearest jet
- Gradient isolation
- Medium quality

Soft muon:

- $p_T > 4 \text{ GeV}$
- $|\eta| < 2.5$
- $|d_0| < 3 \text{ mm}$
- $|z_0 \sin \theta| < 3 \text{ mm}$
- $\Delta R < 0.4$ from nearest jet
- Only keep highest p_T muon for each jet
- Not prompt
- Tight quality

Jets:

- Particle flow algorithm
- $p_T > 25 \text{ GeV}$
- $|\eta| < 2.5$
- $JVT > 0.59$ if $p_T < 60 \text{ GeV}$, $|\eta| < 2.4$
- ≥ 4 jets with $p_T > 25 \text{ GeV}$ (excl. SMT-tagged jet)
- ≥ 1 b-jet (DL1r at 77% efficiency working point)

MET:

- $\text{MET} > 30 \text{ GeV}$
- $\text{MET} + \text{MT}(W) > 60 \text{ GeV}$

Multijet background estimation



- Hard to model in MC so a **data-driven technique** is used: The **asymptotic matrix method**

$$\begin{pmatrix} N^t \\ N^l \end{pmatrix} = \begin{pmatrix} \varepsilon_r & \varepsilon_f \\ 1 - \varepsilon_r & 1 - \varepsilon_f \end{pmatrix} \begin{pmatrix} N_r^l \\ N_f^l \end{pmatrix}$$

t = tight lepton selection, l = loose lepton selection

- Real efficiency, ε_r , calculated from $Z \rightarrow ee$ **tag and probe** (centrally provided). Fake efficiency, ε_f , **depends on event selection**
- Invert to get the fake background:

$$N_f^t = \varepsilon_f N_f^l = \frac{\varepsilon_f}{\varepsilon_r - \varepsilon_f} [(\varepsilon_r - 1)N^t + \varepsilon_r N^l]$$

- Apply 'fake' weight to each event:

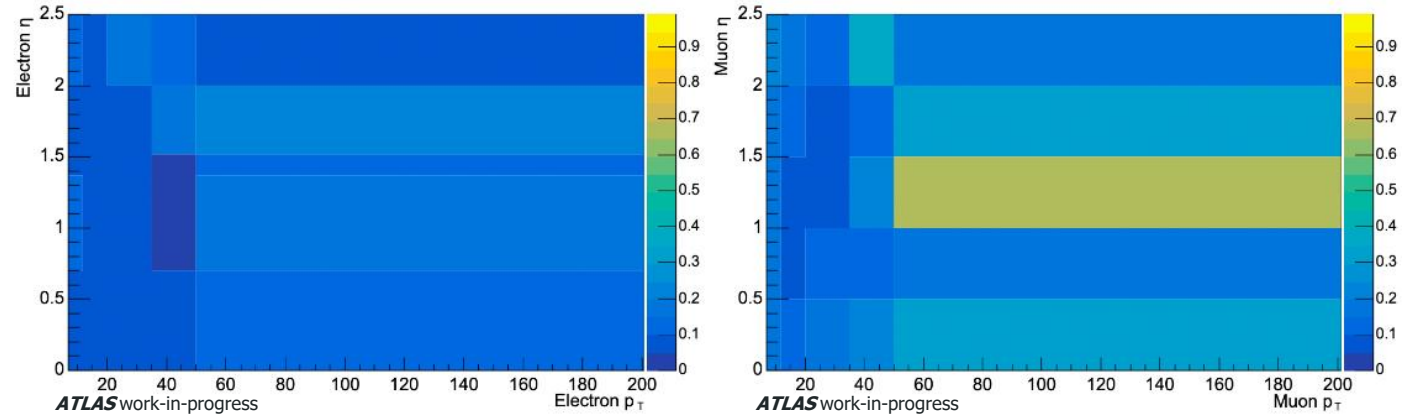
$$w_i = \begin{cases} \frac{\varepsilon_{f,i}}{\varepsilon_{r,i} - \varepsilon_{f,i}} (\varepsilon_{r,i} - 1) & \text{if } N^t = 1, N^l = 0 \\ \frac{\varepsilon_{f,i}}{\varepsilon_{r,i} - \varepsilon_{f,i}} \varepsilon_{r,i} & \text{if } N^t = 0, N^l = 1 \end{cases}, \quad N_f^t = \sum_i w_i$$

Multijet background estimation (cont.)

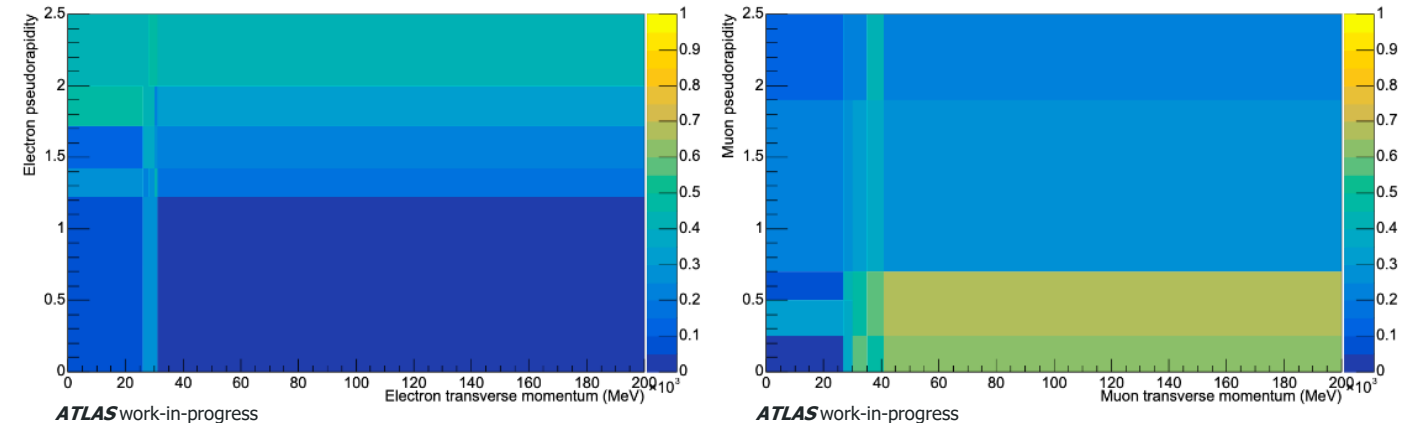


- Asymptotic since statistical uncertainty of N_f^t is **only valid for a large number of events**
- To obtain Fake efficiency:
 - Apply the event selection to data** but with an **inverted E_t^{miss} cut** and the **$(E_t^{miss} + m_T(W)) > 60 GeV$ cut removed**
 - Take the **ratio of events passing the tight selection vs loose selection** after subtracting the MC signal and other backgrounds
- Minimal difference, **will use central values for R25**

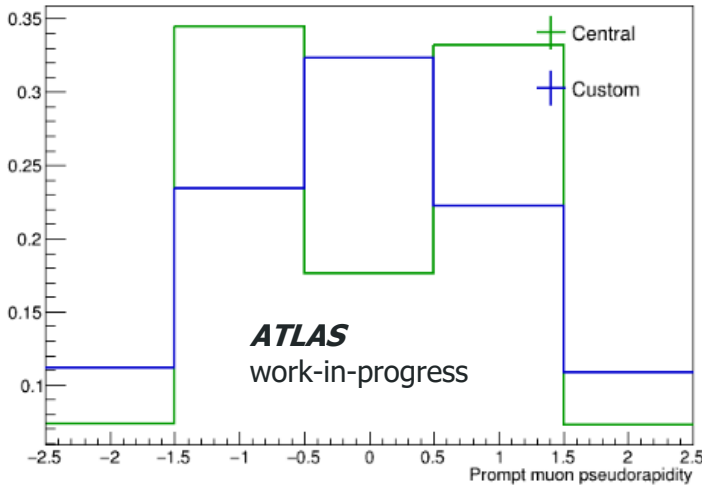
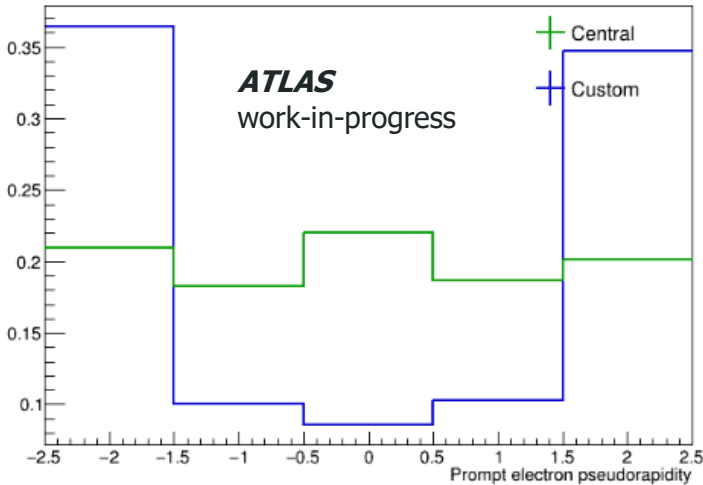
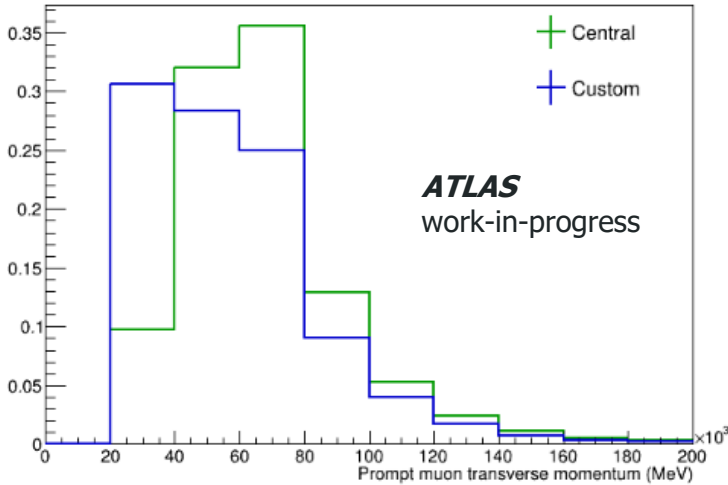
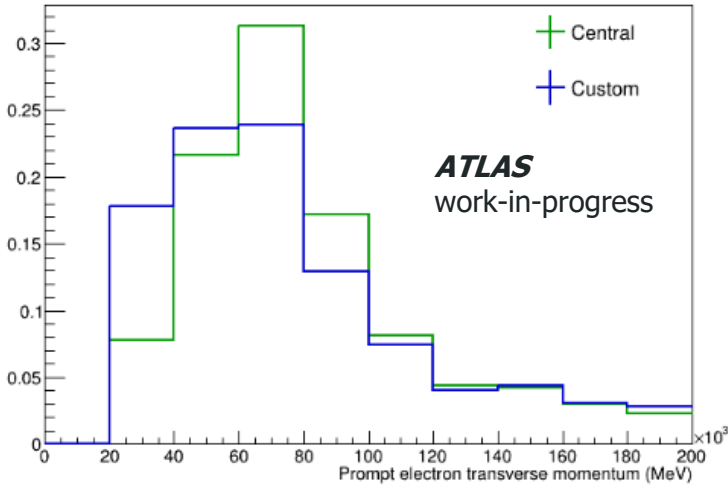
Obtained from central efficiencies



Obtained from custom efficiencies



Multijet background: Central vs Custom



SKM Parameterisation



- Neutrino P_z^ν component is calculated as follows:

$$P_z^\nu = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

where $a = E_l^2 - (P_z^l)^2$, $b = -2P_z^l(m_{diff} + P_T^{\nu l})$, $c = (E_T^{miss})^2 E_l - m_{diff}^2 - (P_T^{\nu l})^2 - 2m_{diff}P_T^{\nu l}$

$$\text{and } m_{diff} = \frac{1}{2} \left((m_W^{PDG})^2 - (m_l^{PDG})^2 \right), P_T^{\nu l} = P_x^\nu P_x^l + P_y^\nu P_y^l$$

- Top quark mass is reconstructed by:

$$M_t^2 = E_t^2 - P_t^2 = (E_l + E_\nu + E_{jet})^2 - (P_l + P_\nu + P_{jet})^2$$

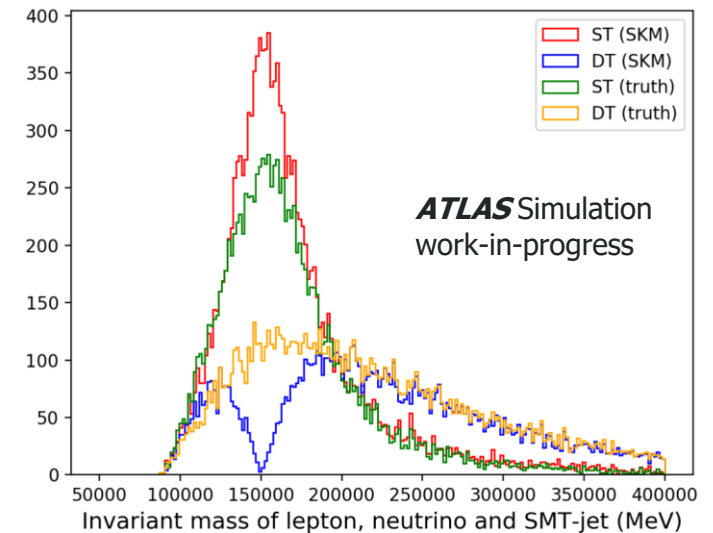
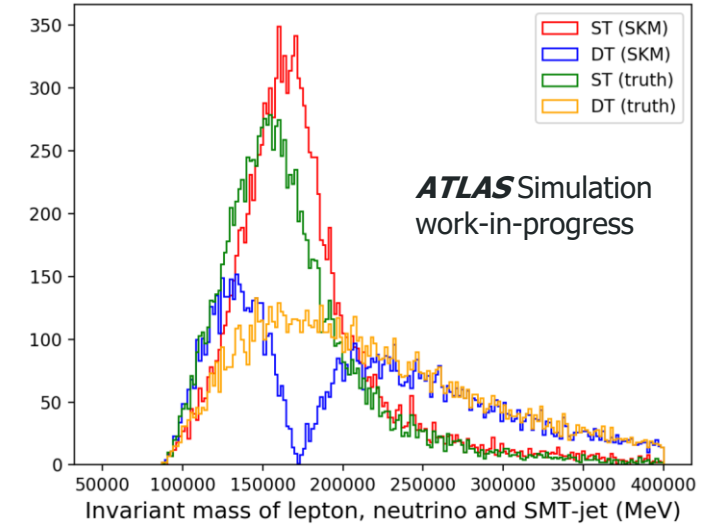
SKM tests



- Compared purity with 150 GeV and 172.57 GeV top masses with 4 methods of choosing additional jets
- Plots show 172.57 GeV (top) and 150 GeV (bottom) comparison

SKM method ($m_t = 172.57$ GeV)	ST purity	DT purity	Overall purity (accuracy)	ST accuracy	DT accuracy
SMT & highest-pt b-jet	0.631 ± 0.004	0.694 ± 0.005	0.657 ± 0.005	0.755 ± 0.004	0.558 ± 0.004
SMT & 1 DL1r jet	0.652 ± 0.004	0.684 ± 0.004	0.666 ± 0.005	0.716 ± 0.004	0.616 ± 0.004
SMT & 2 DL1r jets	0.673 ± 0.005	0.616 ± 0.004	0.639 ± 0.005	0.544 ± 0.004	0.735 ± 0.004
SMT & 3 DL1r jets	0.683 ± 0.005	0.587 ± 0.004	0.618 ± 0.005	0.445 ± 0.004	0.792 ± 0.004

SKM method ($m_t = 150$ GeV)	ST purity	DT purity	Overall purity (accuracy)	ST accuracy	DT accuracy
SMT & highest-pt b-jet	0.630 ± 0.004	0.729 ± 0.005	0.666 ± 0.005	0.805 ± 0.004	0.526 ± 0.004
SMT & 1 DL1r jet	0.653 ± 0.004	0.713 ± 0.004	0.678 ± 0.005	0.762 ± 0.004	0.594 ± 0.004
SMT & 2 DL1r jets	0.688 ± 0.004	0.652 ± 0.004	0.668 ± 0.005	0.617 ± 0.004	0.719 ± 0.004
SMT & 3 DL1r jets	0.712 ± 0.005	0.624 ± 0.004	0.656 ± 0.005	0.527 ± 0.005	0.786 ± 0.004

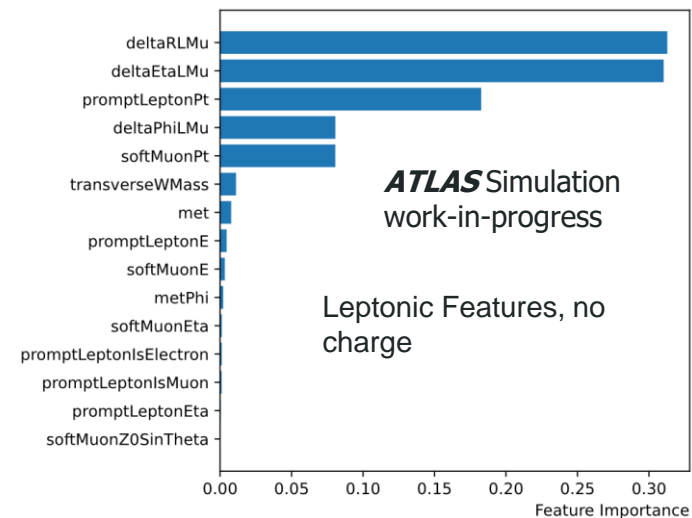
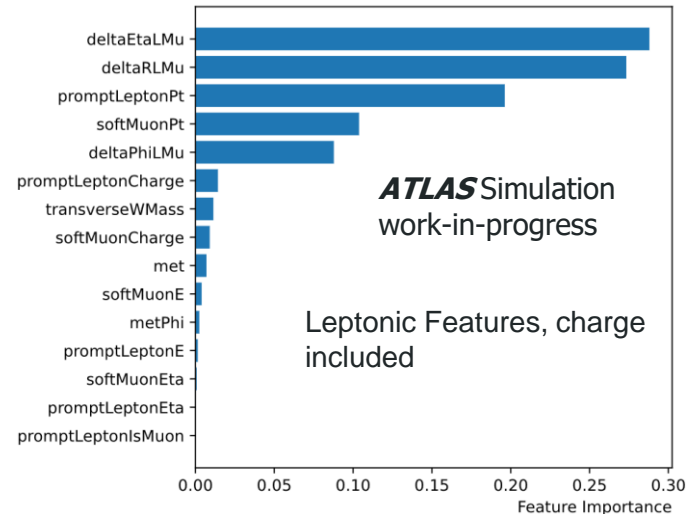
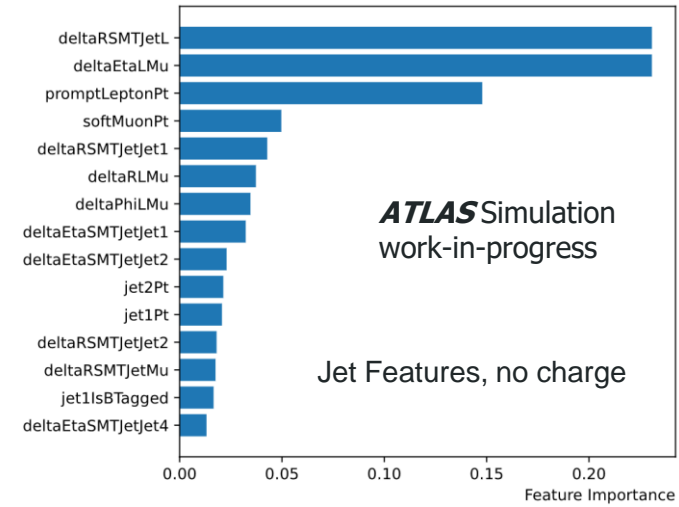
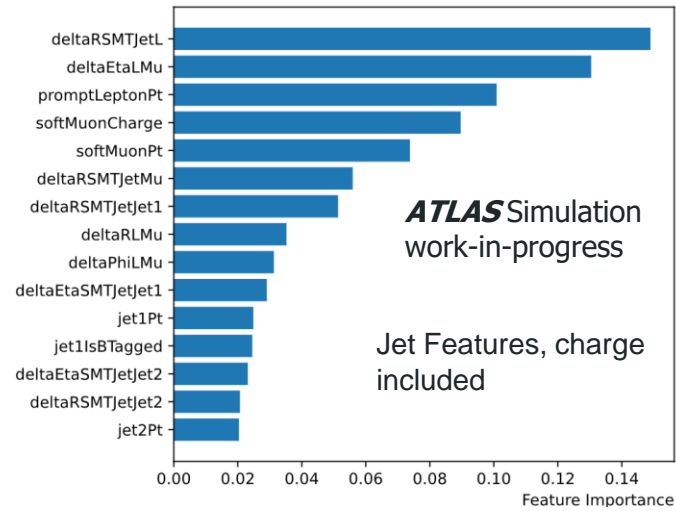


BDT hyperparameter optimisation



BDT type	Number of estimators	criterion	Maximum depth	Learning rate
Leptonic features, charge included	50	entropy	5	0.3
Leptonic features, no charge	100	gini	5	0.2
Jet features, charge included	100	gini	5	0.3
Jet features, no charge included	50	entropy	5	0.2

BDT feature importance



ST/DT: comparing purities



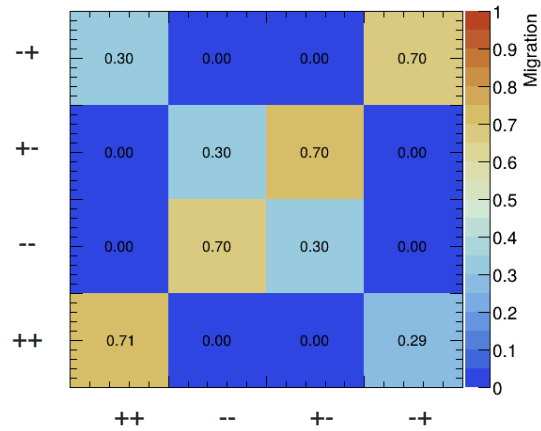
ST/DT method	Overall purity (\pm MC stat)
ΔR Cut	0.730 ± 0.005
SKM	0.678 ± 0.005
BDT (Leptonic Features, charge included)	0.764 ± 0.001
BDT (Leptonic Features, no charge)	0.741 ± 0.001
BDT (Jet Features, charge included)	0.823 ± 0.001
BDT (Jet Features, no charge)	0.767 ± 0.001

Unfolding: Migration matrices



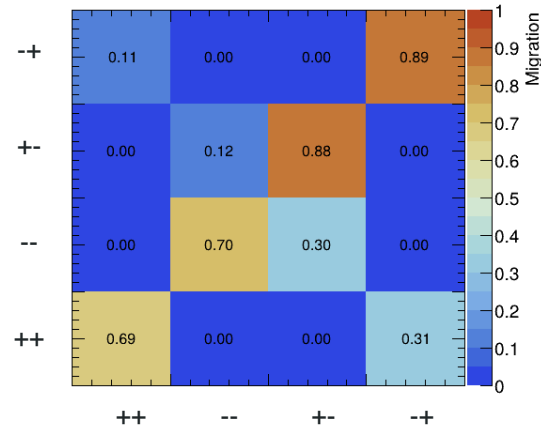
ATLAS Simulation
work-in-progress

$\sqrt{s} = 13 \text{ TeV}, 139 \text{ fb}^{-1}$



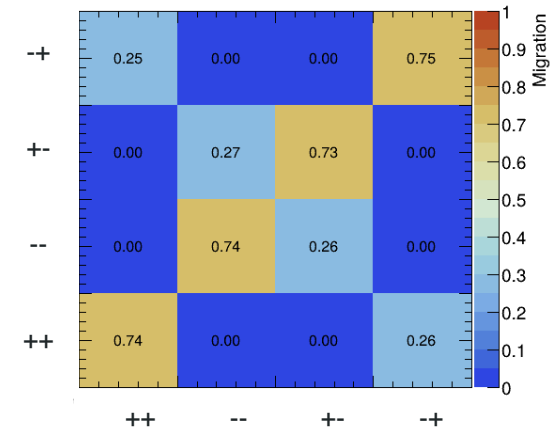
ATLAS Simulation
work-in-progress

$\sqrt{s} = 13 \text{ TeV}, 139 \text{ fb}^{-1}$



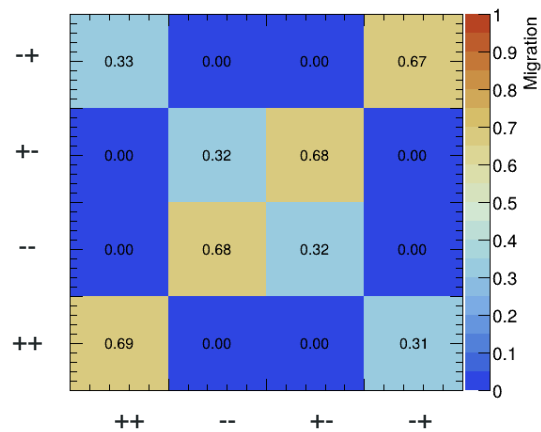
ATLAS Simulation
work-in-progress

$\sqrt{s} = 13 \text{ TeV}, 139 \text{ fb}^{-1}$



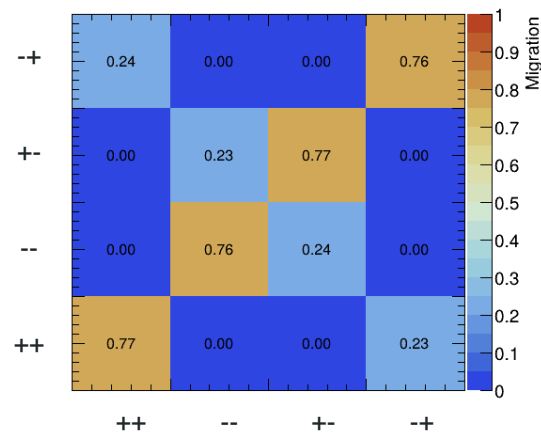
ATLAS Simulation
work-in-progress

$\sqrt{s} = 13 \text{ TeV}, 139 \text{ fb}^{-1}$



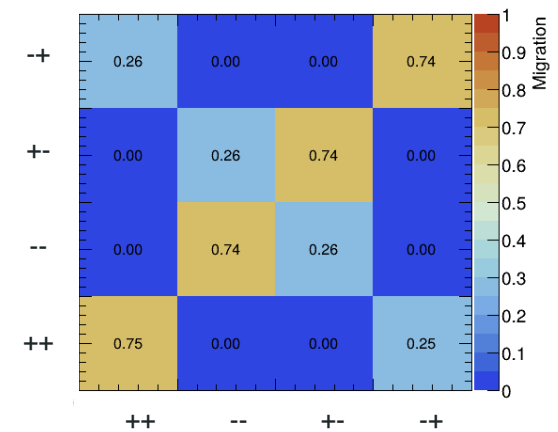
ATLAS Simulation
work-in-progress

$\sqrt{s} = 13 \text{ TeV}, 139 \text{ fb}^{-1}$

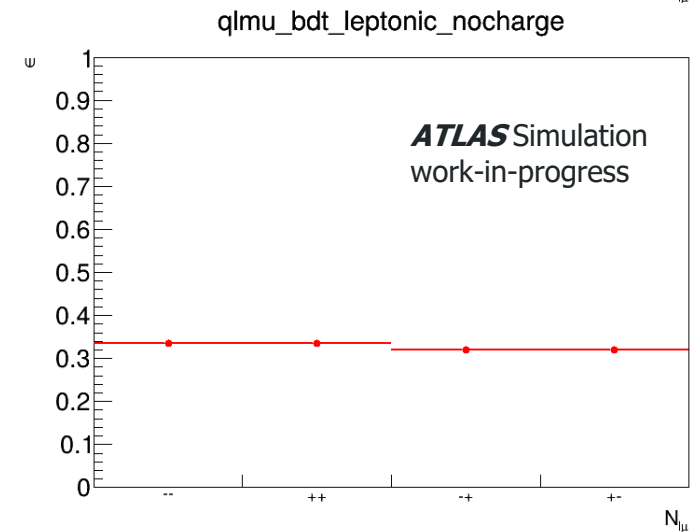
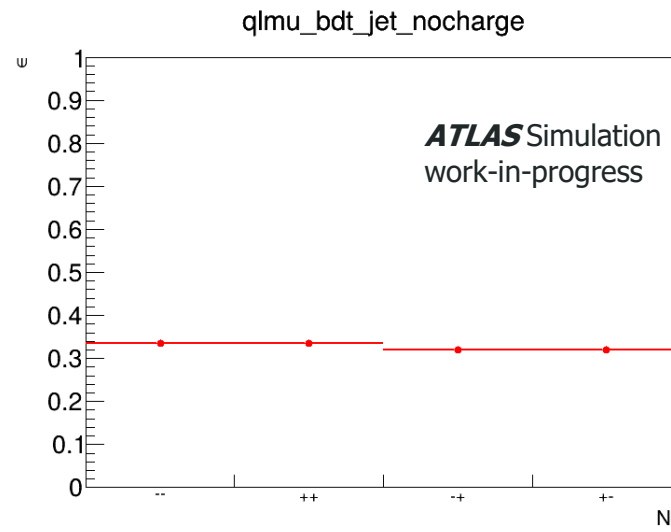
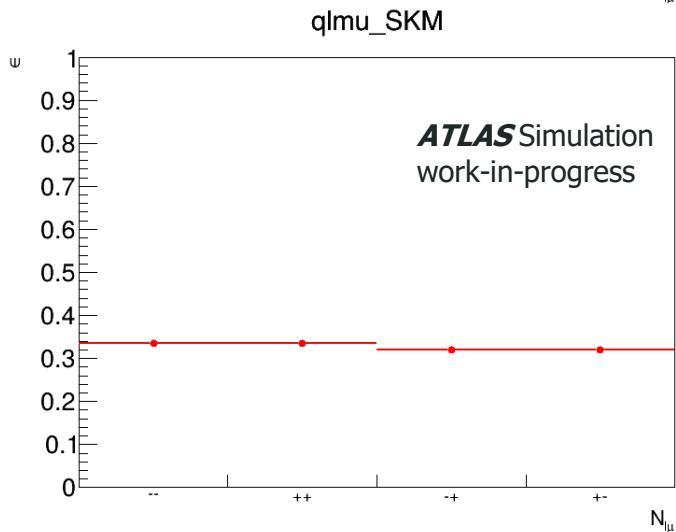
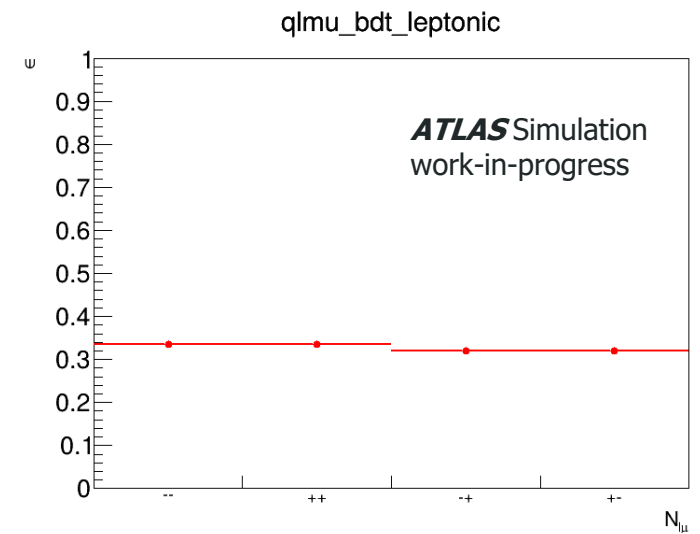
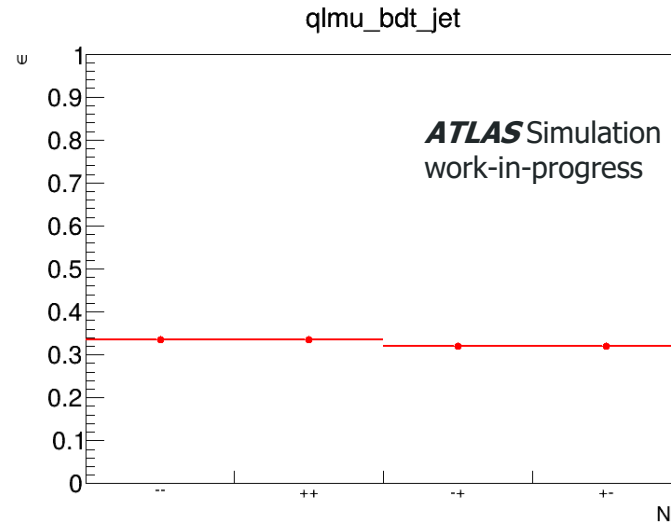
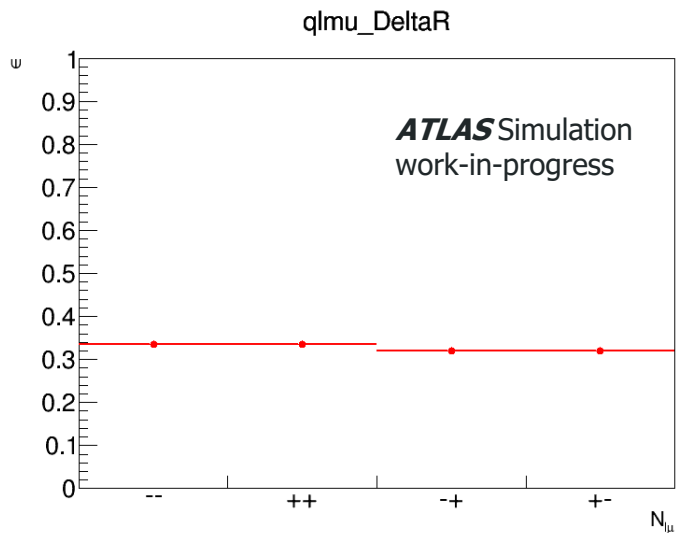


ATLAS Simulation
work-in-progress

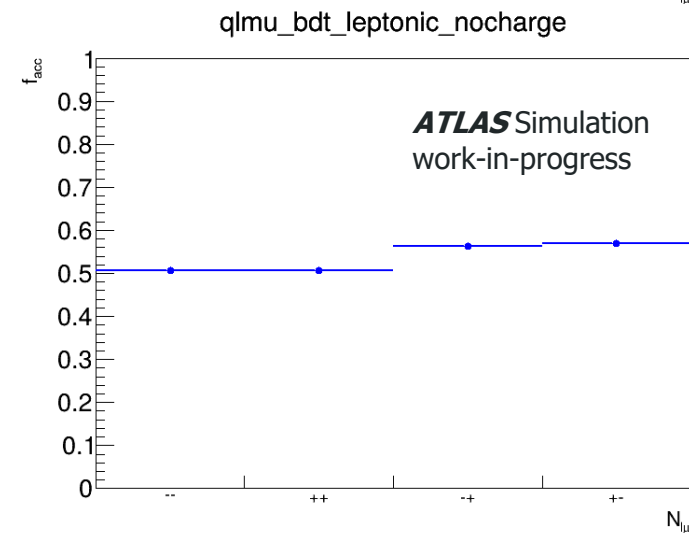
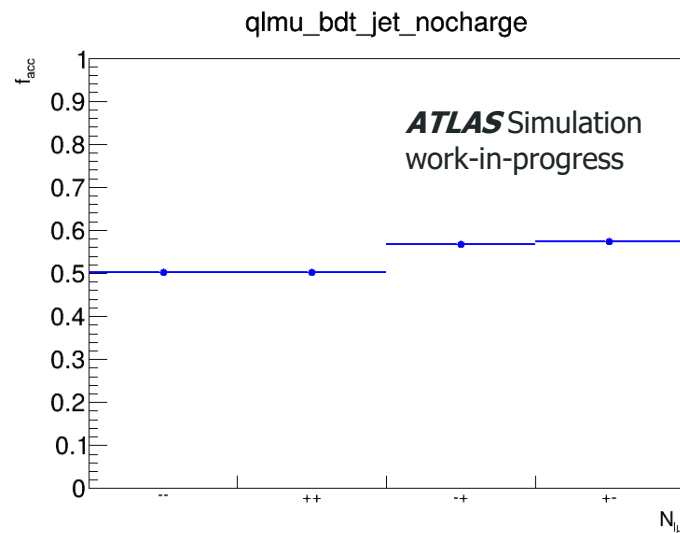
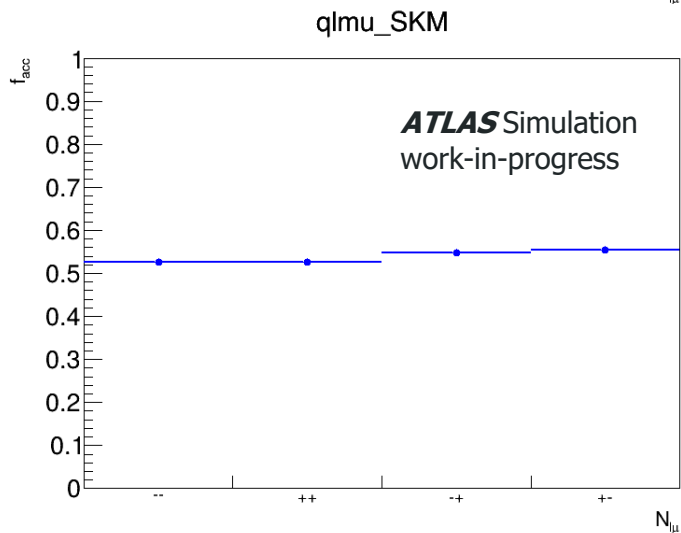
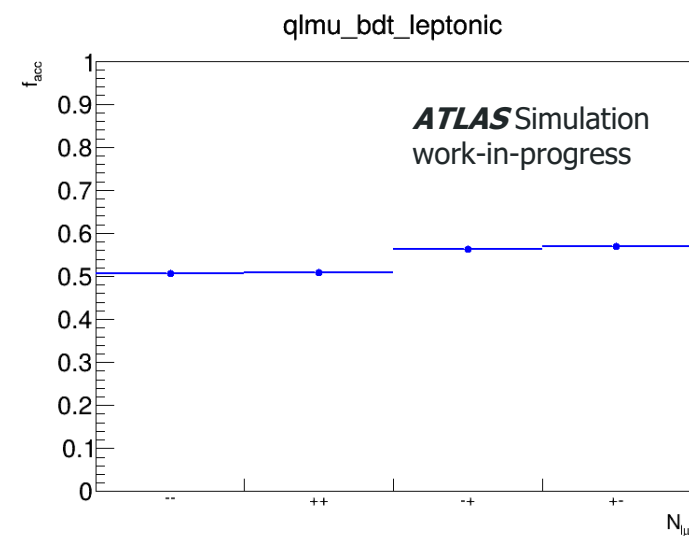
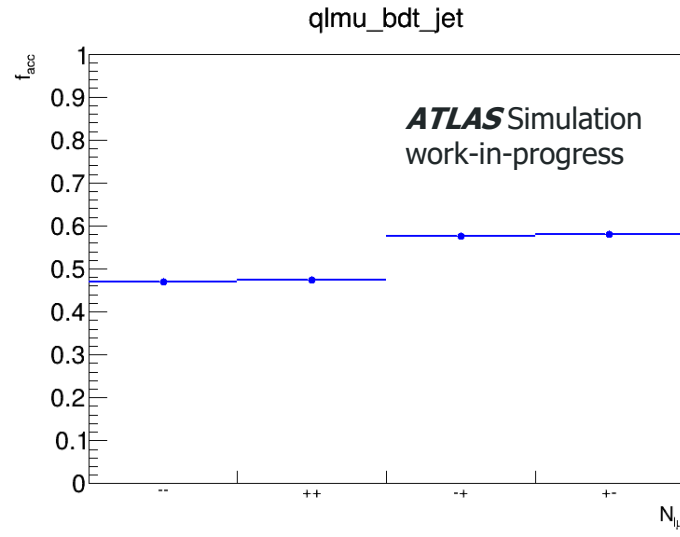
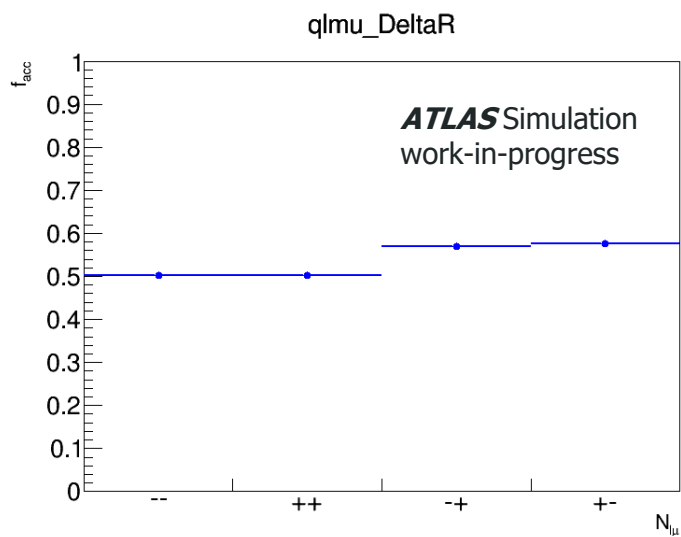
$\sqrt{s} = 13 \text{ TeV}, 139 \text{ fb}^{-1}$



Unfolding: Efficiencies



Unfolding: Acceptances



Charge and CP asymmetries



- The number of events in each of the decay chains are $N_{r_b}, N_{r_c}, N_{r_{c\bar{c}}}, N_{\tilde{r}_b}, N_{\tilde{r}_c}, N_{\tilde{r}_{c\bar{c}}}$ respectively, which are measured in MC simulations.
- These are used to derive the decay chain fractions:

$$r_b = \frac{N_{r_b}}{N_{r_b} + N_{r_c} + N_{r_{c\bar{c}}}}, \dots$$

- From a small MC sample (dependent on the fiducial particle level volume):

	r_b	r_c	$r_{c\bar{c}}$	\tilde{r}_b	\tilde{r}_c	$\tilde{r}_{c\bar{c}}$
Results	0.230	0.743	0.026	0.830	0.071	0.099

- These values are similar to r values obtained in 8 TeV analysis. They will eventually have full systematics applied.

Decay chains contributing to A^{SS} :

$$N_{r_b}: \quad t \rightarrow \ell^+ \nu (b \rightarrow \bar{b}) \rightarrow \ell^+ \mu^+ X,$$

$$N_{r_c}: \quad t \rightarrow \ell^+ \nu (b \rightarrow c) \rightarrow \ell^+ \mu^+ X,$$

$$N_{r_{c\bar{c}}}: \quad t \rightarrow \ell^+ \nu (b \rightarrow \bar{b} \rightarrow c\bar{c}) \rightarrow \ell^+ \mu^+ X,$$

Decay chains contributing to A^{OS} :

$$N_{\tilde{r}_b}: \quad t \rightarrow \ell^+ \nu b \rightarrow \ell^+ \mu^- X,$$

$$N_{\tilde{r}_c}: \quad t \rightarrow \ell^+ \nu (b \rightarrow \bar{b} \rightarrow \bar{c}) \rightarrow \ell^+ \mu^- X,$$

$$N_{\tilde{r}_{c\bar{c}}}: \quad t \rightarrow \ell^+ \nu (b \rightarrow c\bar{c}) \rightarrow \ell^+ \mu^- X,$$

Charge and CP asymmetries (cont)



- A relationship between observable charge asymmetries, A^{SS} , A^{OS} , and underlying CPV asymmetries is defined using the decay chain fractions, r_b , etc. :

$$A^{SS} = r_b A_{mix}^{bl} + r_c (A_{dir}^{bc} - A_{dir}^{cl}) + r_{c\bar{c}} (A_{mix}^{bc} - A_{dir}^{cl})$$

$$A^{OS} = \tilde{r}_b A_{dir}^{bl} + \tilde{r}_c (A_{mix}^{bc} + A_{dir}^{cl}) + \tilde{r}_{c\bar{c}} (A_{dir}^{cl})$$

$$A_{mix}^{bl} = \frac{\Gamma(b \rightarrow \bar{b} \rightarrow \ell^+ X) - \Gamma(\bar{b} \rightarrow b \rightarrow \ell^- X)}{\Gamma(b \rightarrow \bar{b} \rightarrow \ell^+ X) + \Gamma(\bar{b} \rightarrow b \rightarrow \ell^- X)},$$

$$A_{mix}^{bc} = \frac{\Gamma(b \rightarrow \bar{b} \rightarrow \bar{c} X) - \Gamma(\bar{b} \rightarrow b \rightarrow c X)}{\Gamma(b \rightarrow \bar{b} \rightarrow \bar{c} X) + \Gamma(\bar{b} \rightarrow b \rightarrow c X)},$$

$$A_{dir}^{bl} = \frac{\Gamma(b \rightarrow \ell^- X) - \Gamma(\bar{b} \rightarrow \ell^+ X)}{\Gamma(b \rightarrow \ell^- X) + \Gamma(\bar{b} \rightarrow \ell^+ X)},$$

$$A_{dir}^{cl} = \frac{\Gamma(\bar{c} \rightarrow \ell^- X_L) - \Gamma(c \rightarrow \ell^+ X_L)}{\Gamma(\bar{c} \rightarrow \ell^- X_L) + \Gamma(c \rightarrow \ell^+ X_L)},$$

$$A_{dir}^{bc} = \frac{\Gamma(b \rightarrow c X_L) - \Gamma(\bar{b} \rightarrow \bar{c} X_L)}{\Gamma(b \rightarrow c X_L) + \Gamma(\bar{b} \rightarrow \bar{c} X_L)},$$