IMPERIAL

Presenting Neutrino Oscillation Results in the Precision Era

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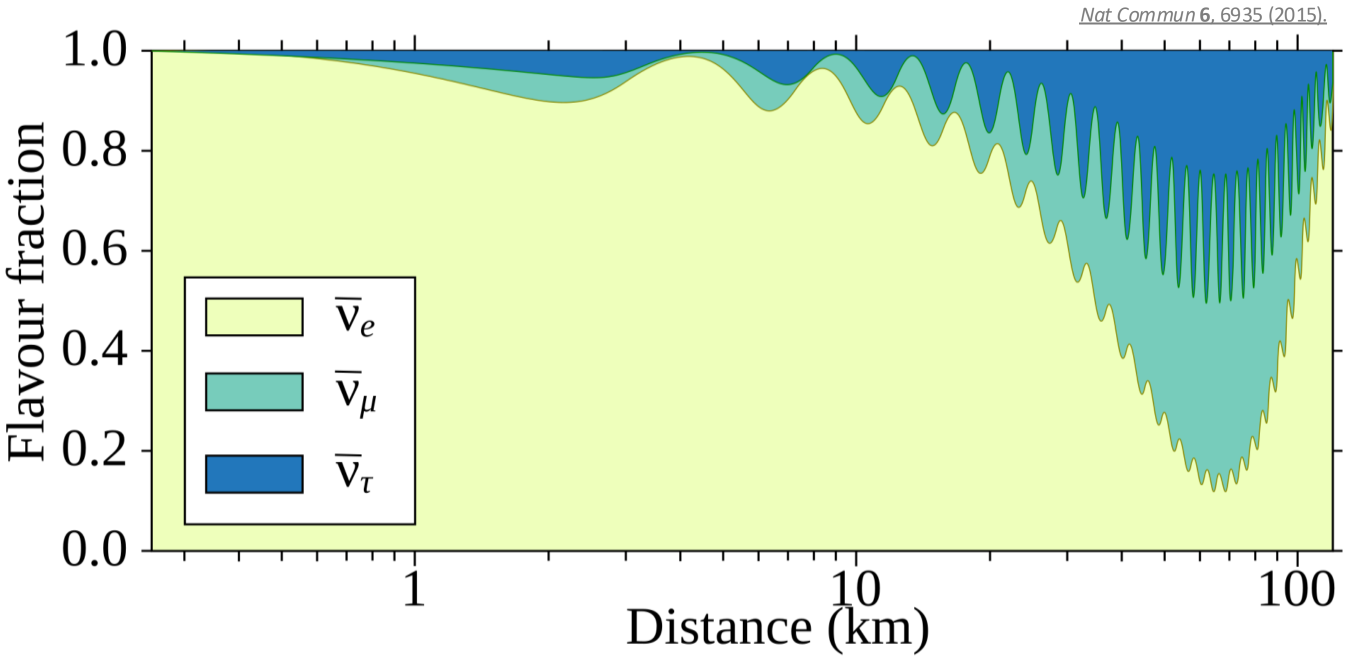
Neutrino Oscillations

- In the Standard Model:

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} U_{e1} & U_{\mu 1} & U_{\tau 1} \\ U_{e2} & U_{\mu 2} & U_{\tau 2} \\ U_{e3} & U_{\mu 3} & U_{\tau 3} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix} \quad + \quad \Delta m_{ij}^2 = m_i^2 - m_j^2 \neq 0$$



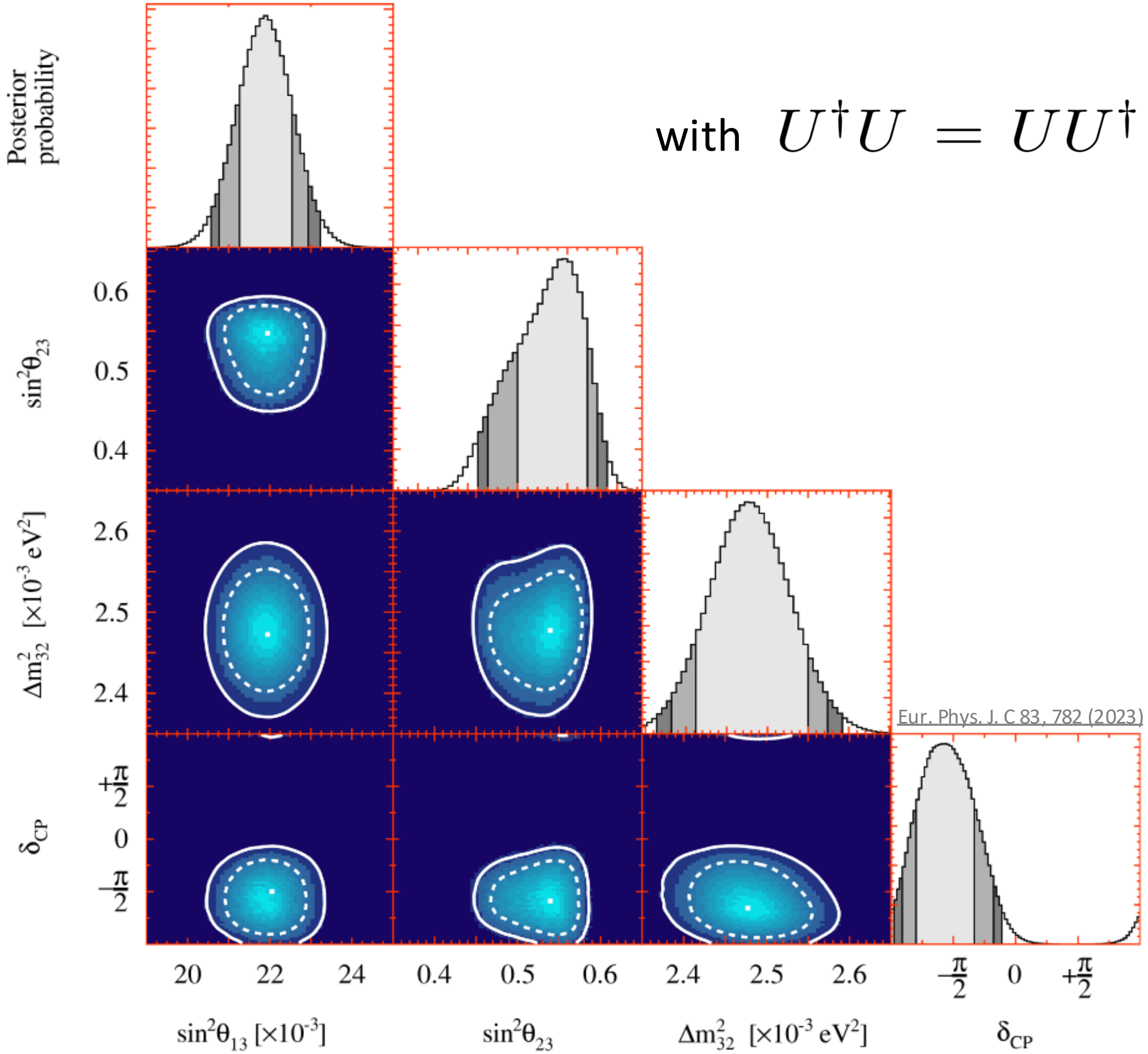
E = 4 MeV

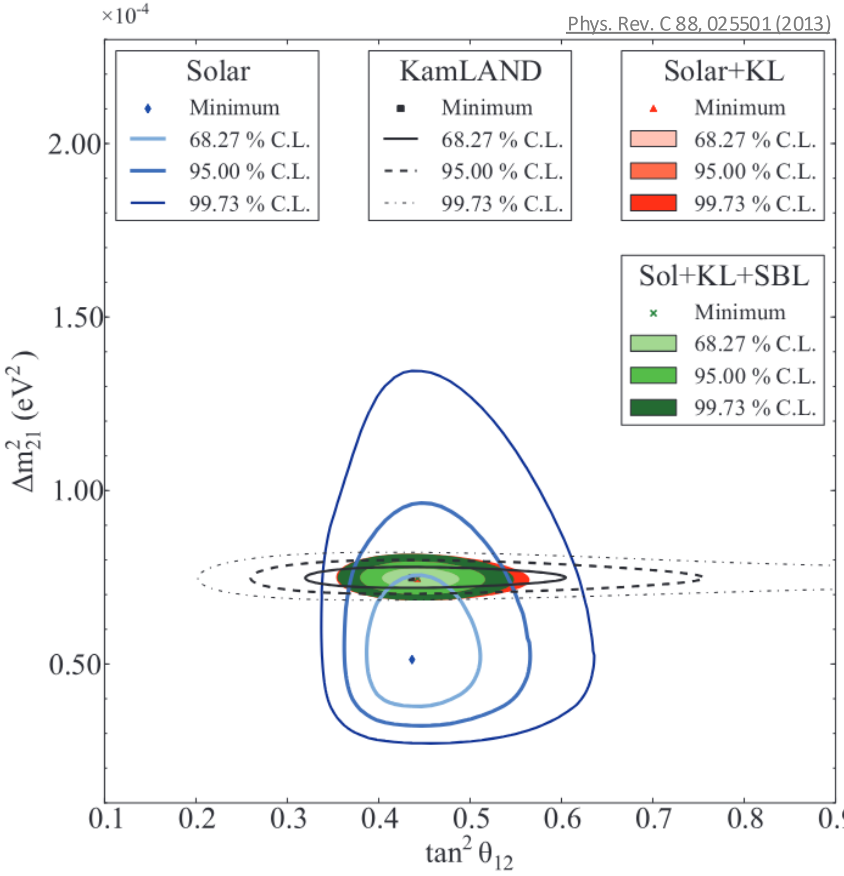
Leptonic Mixing Matrix

- Most common parameterisation: $U_{\rm PMNS} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$

$s_{ij} = \sin\theta_{ij},\ c_{ij} = \cos\theta_{ij}$ and $\delta = \delta_{\rm CP}$

with $U^\dagger U = UU^\dagger = \mathbb{I}$

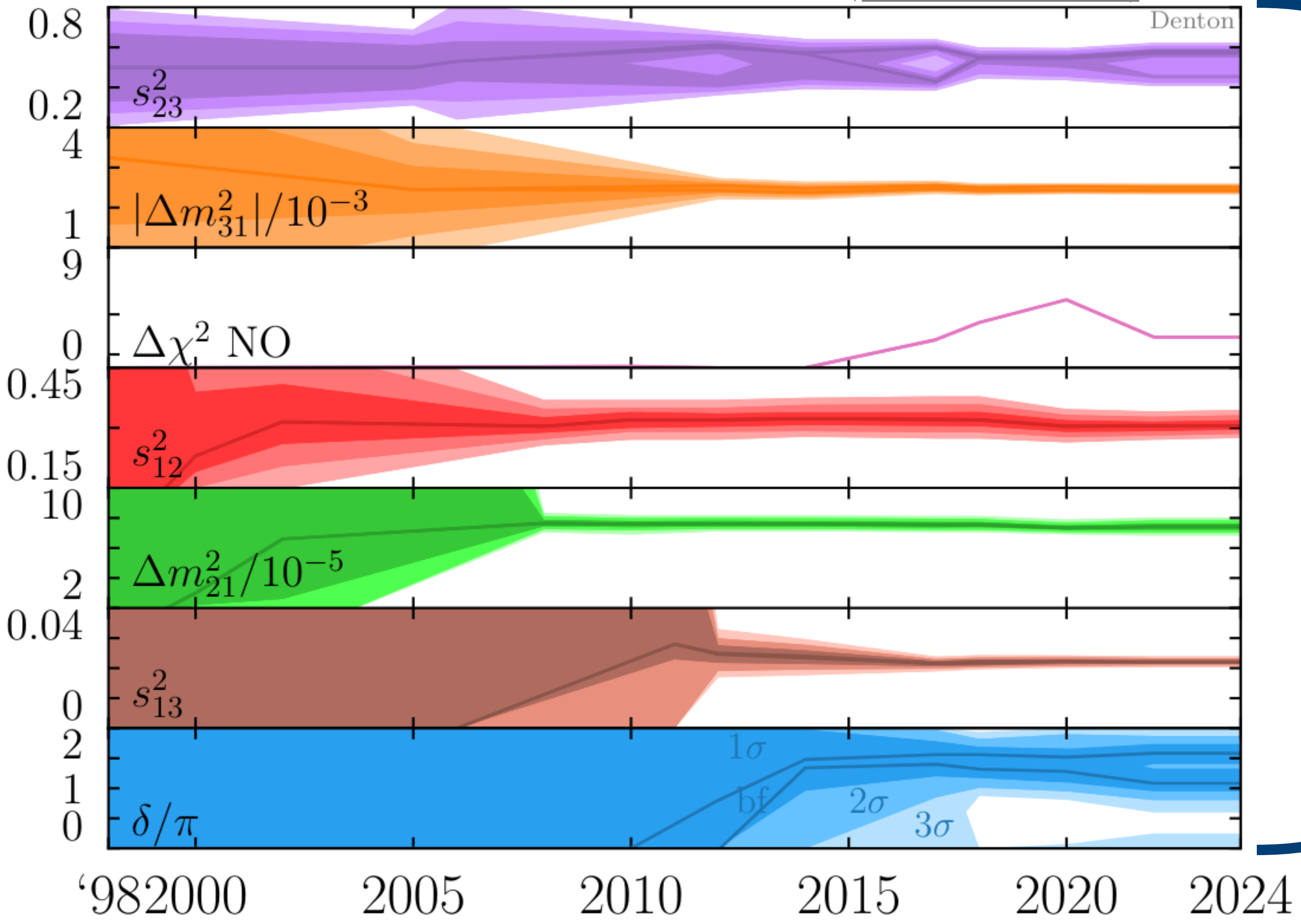




For newer results, see NuFIT 6.0

Entering the Precision Era of Neutrino Oscillations

P. Denton, Modern Neutrino Oscillation Theory, NuFact 2024



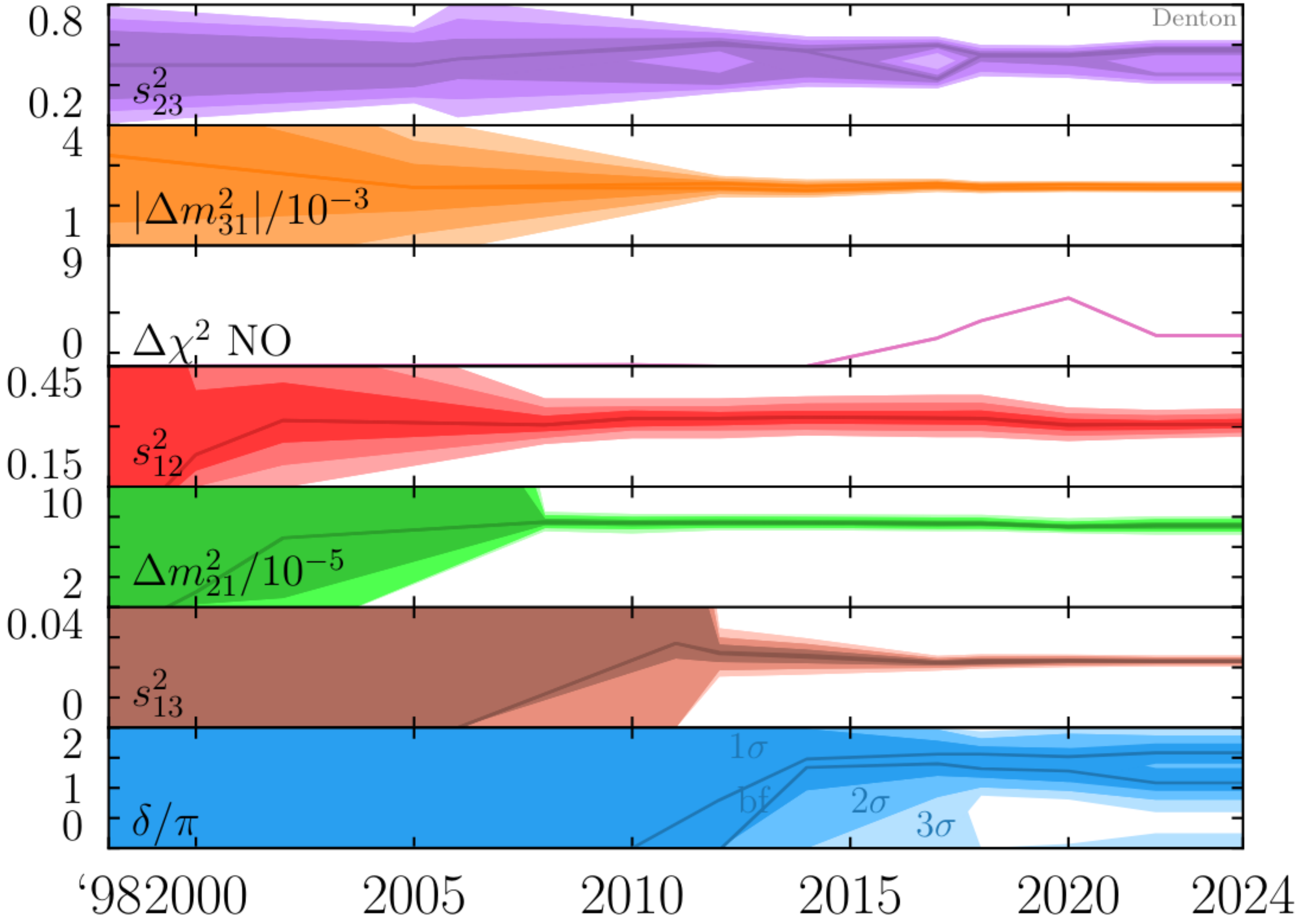
JUNO
Hyper-K
DUNE
+ others

(sub-)% precision

Are we done?

Entering the Precision Era of Neutrino Oscillations

P. Denton, Modern Neutrino Oscillation Theory, NuFact 2024



Open questions:

- *Octant*
- *Mass ordering*
- *CP violation*
- Absolute neutrino masses
- Neutrino mass generation
- Electromagnetic moments
- Non-standard interactions
- …

And:

$$U_{3+1} = \begin{bmatrix} U_{e1} & U_{e2} & U_{e3} & U_{e4} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} & U_{\mu 4} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} & U_{\tau 4} \\ U_{s1} & U_{s2} & U_{s3} & U_{s4} \end{bmatrix} ?$$

Leptonic Mixing Matrix – Revisited

- Most common parameterisation:

$$U_{\text{PMNS}} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

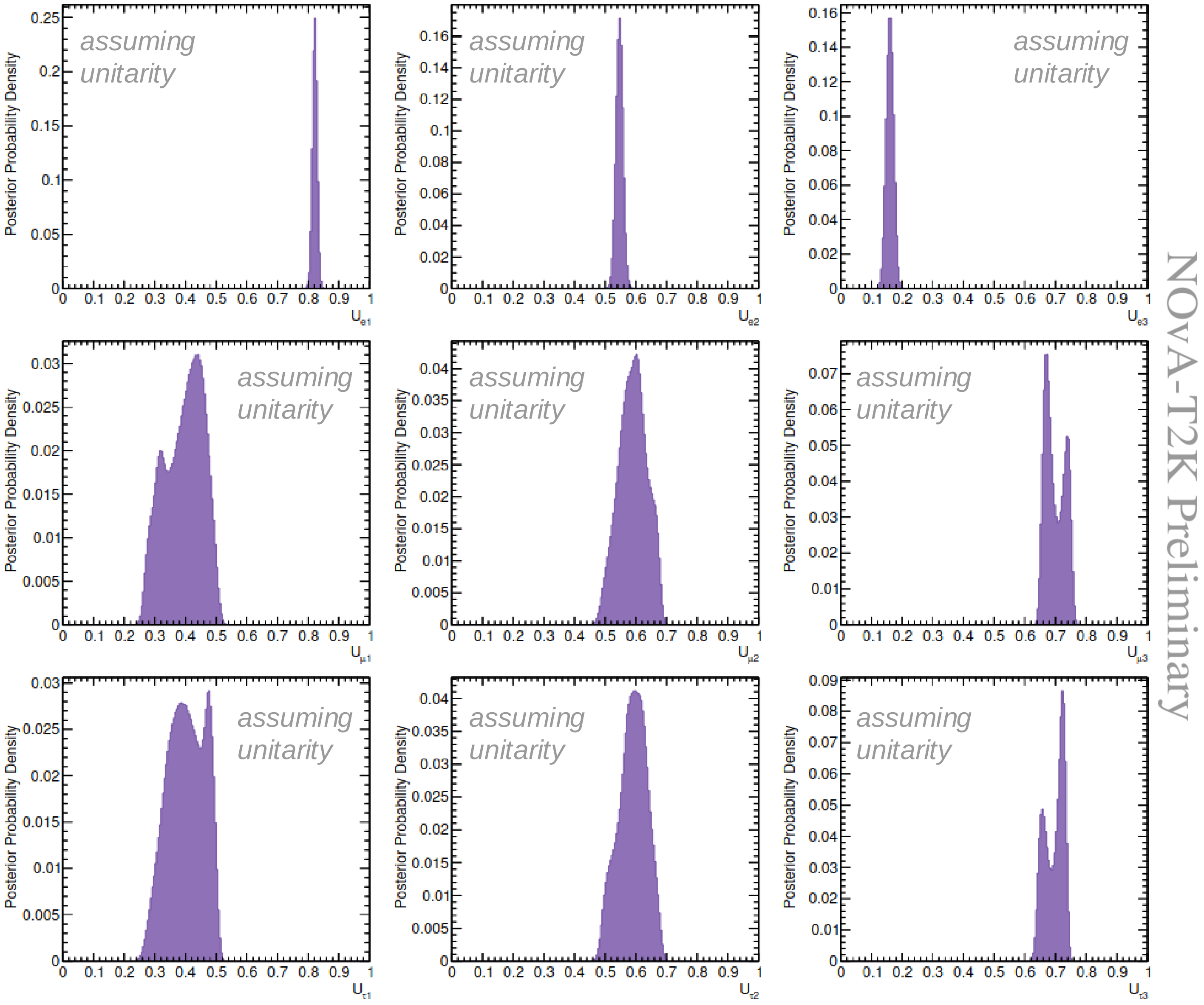
- One possible parameterisation without assuming unitarity:

Phys. Rev. D 102, 115027 (2020)

$$U_{LMM} = \begin{pmatrix} |U_{e1}| & |U_{e2}|e^{i\phi_{e2}} & |U_{e3}|e^{i\phi_{e3}} \\ |U_{\mu 1}| & |U_{\mu 2}| & |U_{\mu 3}| \\ |U_{\tau 1}| & |U_{\tau 2}|e^{i\phi_{\tau 2}} & |U_{\tau 3}|e^{i\phi_{\tau 3}} \end{pmatrix}$$

→ Need new presentations as well

Leptonic Mixing Matrix Element Moduli & Phases



Phys. Rev. D 102, 115027 (2020)

$$U_{LMM} = \begin{pmatrix} |U_{e1}| & |U_{e2}|e^{i\phi_{e2}} & |U_{e3}|e^{i\phi_{e3}} \\ |U_{\mu1}| & |U_{\mu2}| & |U_{\mu3}| \\ |U_{\tau1}| & |U_{\tau2}|e^{i\phi_{\tau2}} & |U_{\tau3}|e^{i\phi_{\tau3}} \end{pmatrix}$$

→ Moduli of the elements can already be used while assuming unitarity

Unitarity Triangles

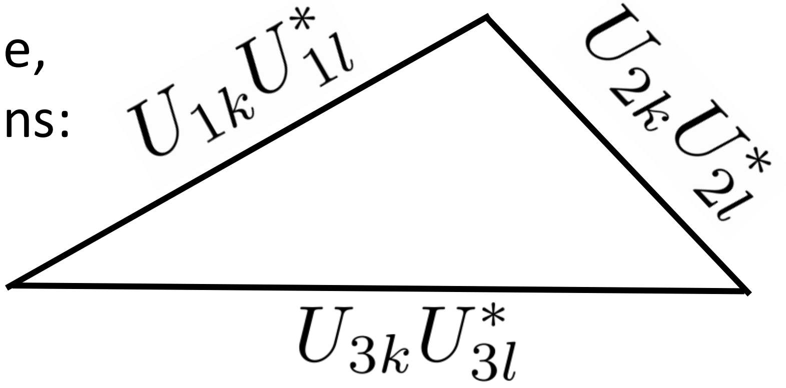
Unitarity condition and orthogonal vectors:

$$U^\dagger U = UU^\dagger = \mathbb{I}$$

→ 6 orthogonality conditions for 3x3 matrices:

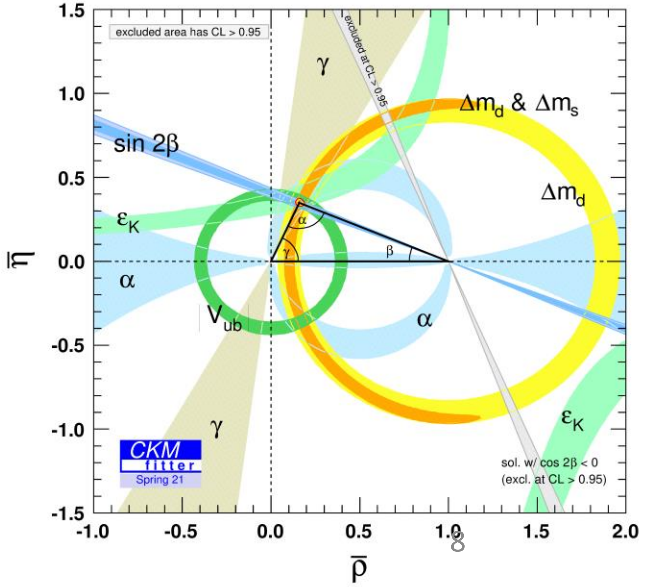
$$\sum_k U_{ik} U_{jk}^* = 0 \quad (i \neq j) \quad \text{(rows)} \qquad \sum_i U_{ik} U_{il}^* = 0 \quad (k \neq l) \quad \text{(columns)}$$

→ 6 unitarity triangles in complex plane,
e.g. for columns:



→ Powerful tool to test unitarity of mixing matrices in the SM

W. Qian, Recent CKMfitter updates on global fits of the CKM matrix, CKM 2021



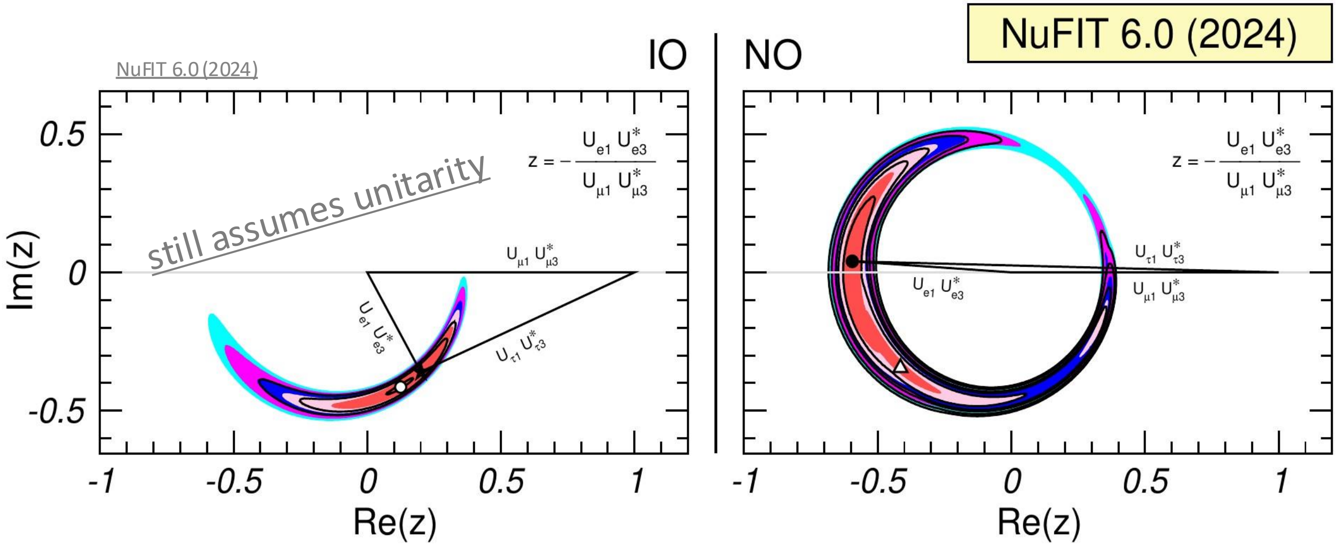
Unitarity Triangles for PMNS

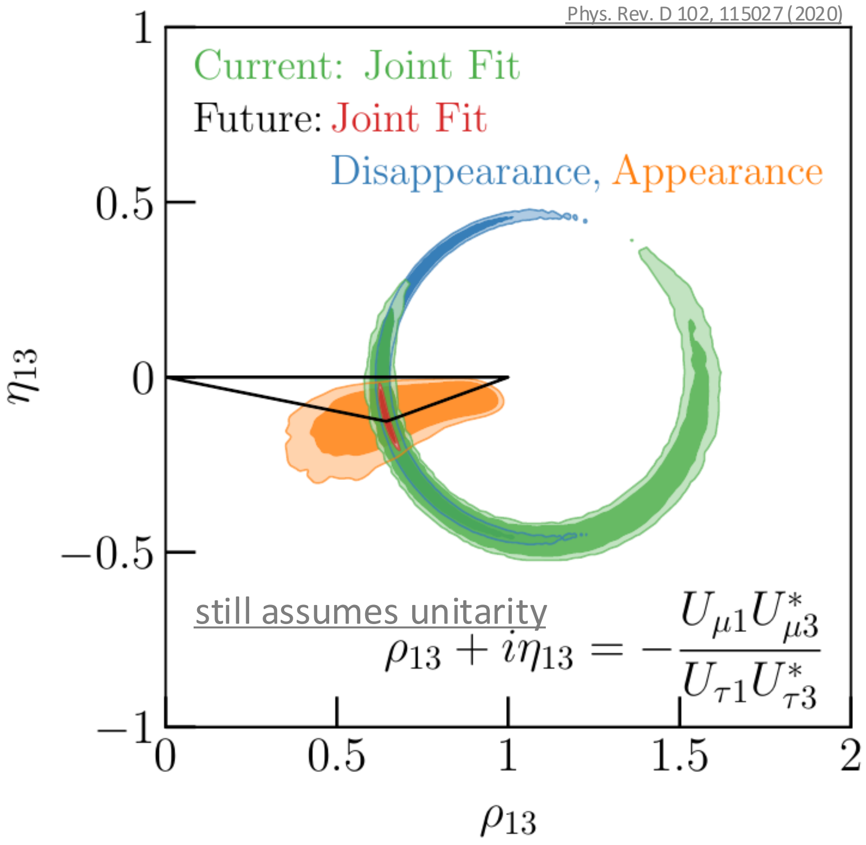
- Example triangle for PMNS:

$$U_{e1}U_{e3}^* + U_{\mu1}U_{\mu3}^* + U_{\tau1}U_{\tau3}^* = 0$$

- Arbitrary choice of normalisation

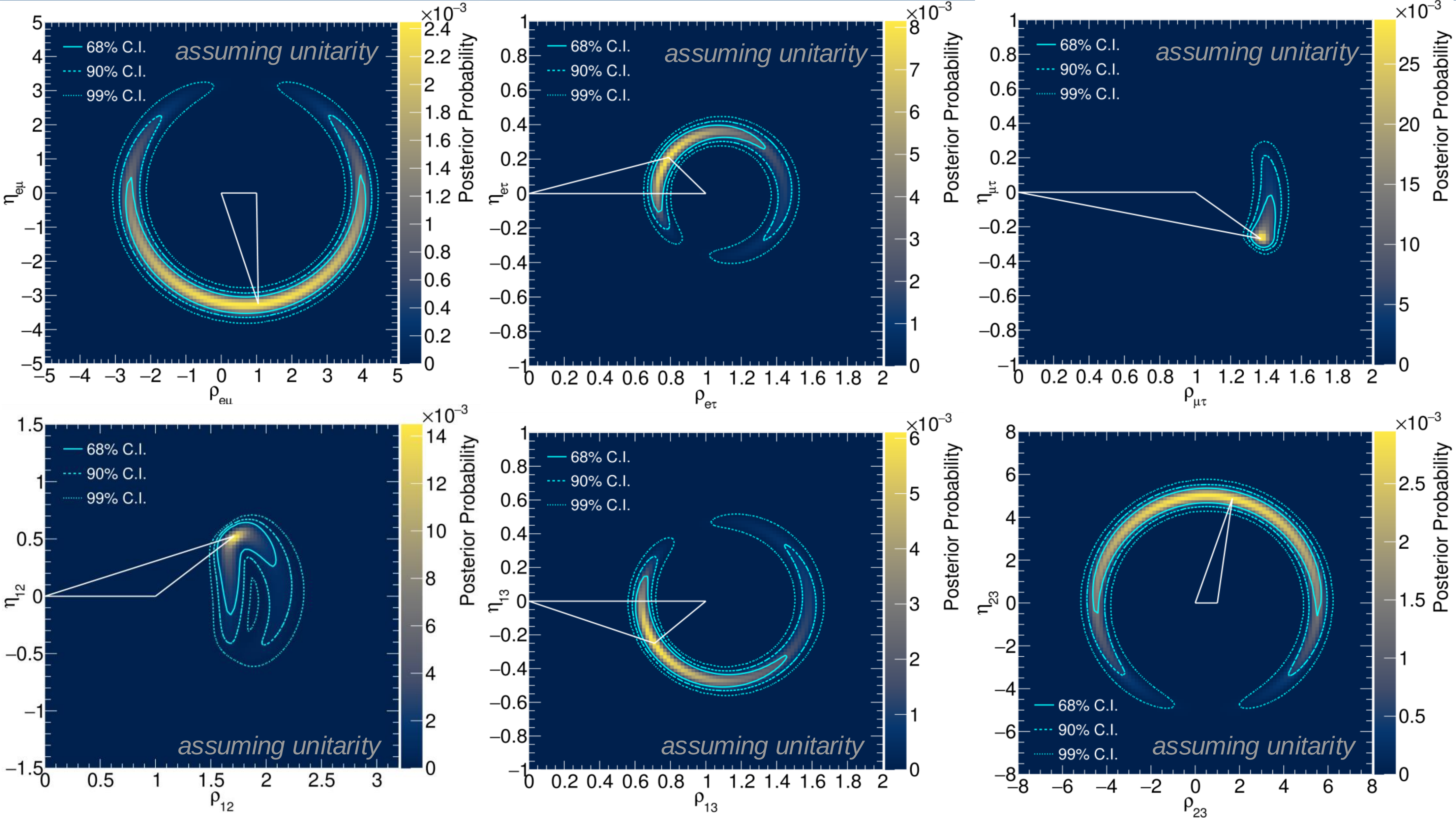
- Different choice, e.g. NuFIT:





Powerful tool even whilst assuming unitarity

Unitarity Triangles for PMNS



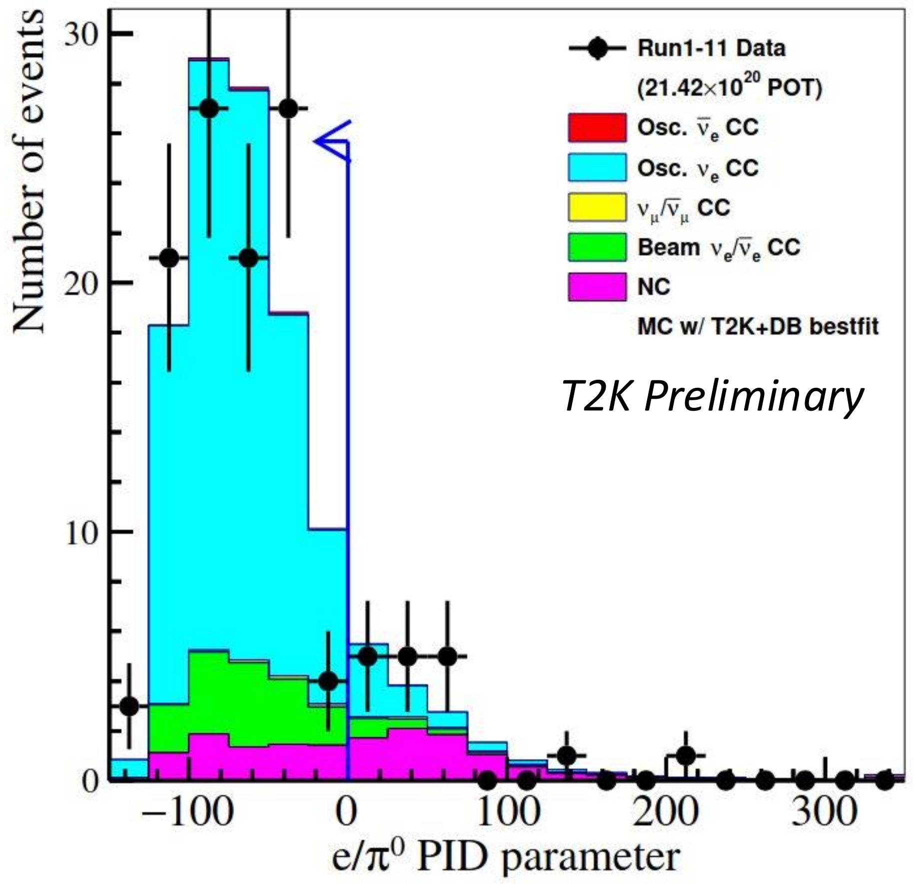
→ Triangle area proportional to CP violation

Note:
For visualisation purposes only; values for PMNS parameters generated loosely based on NuFIT 6.0

Constraints from Neutral Current Events

- Neutral current powerful tool to constrain overall flux
- Current efforts to include neutral current pi0 data in T2K oscillation analysis
- Additional charged current nue events contained in data sample give extra oscillation sensitivity

→ Can sample be understood well enough?

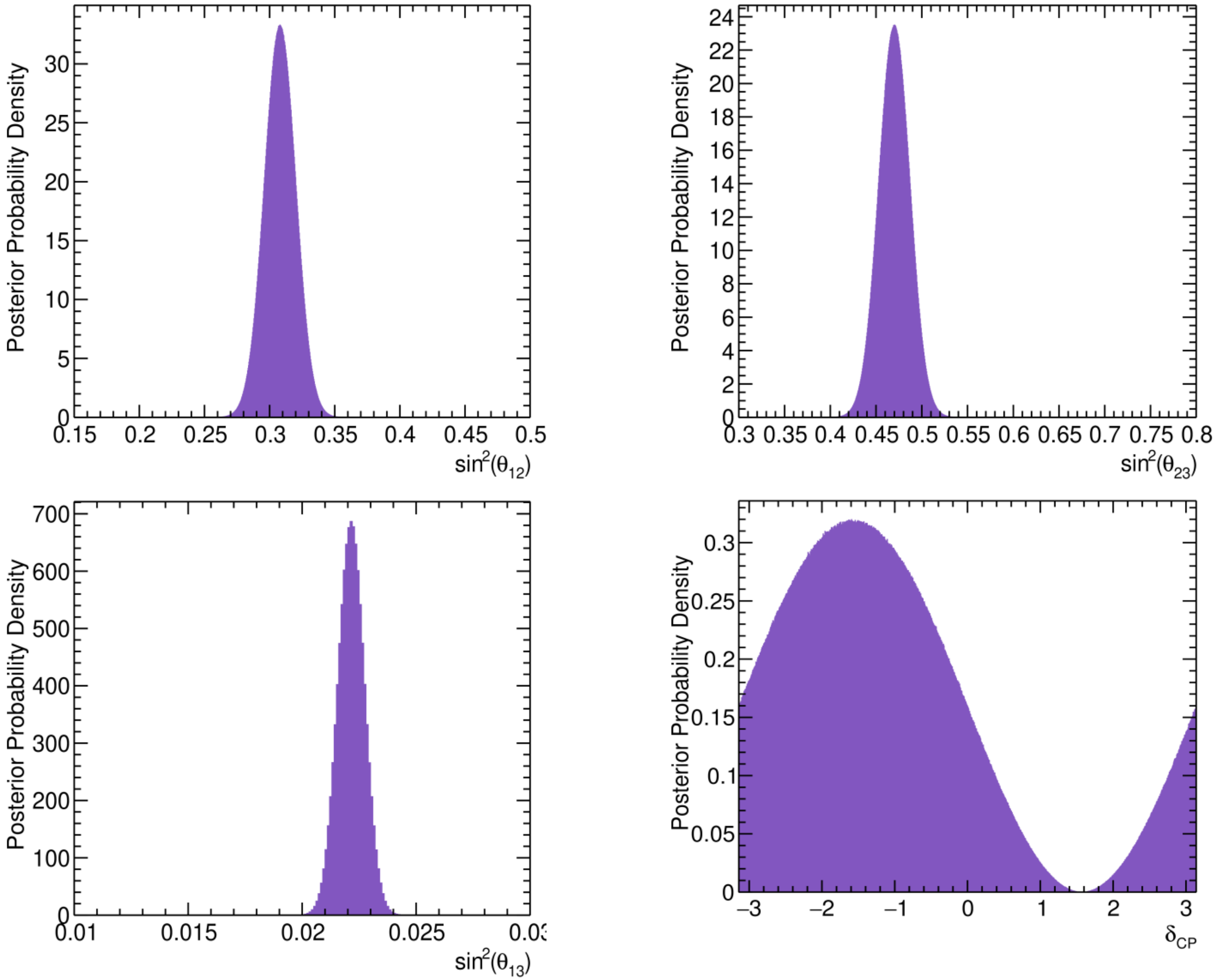


Summary & Conclusion

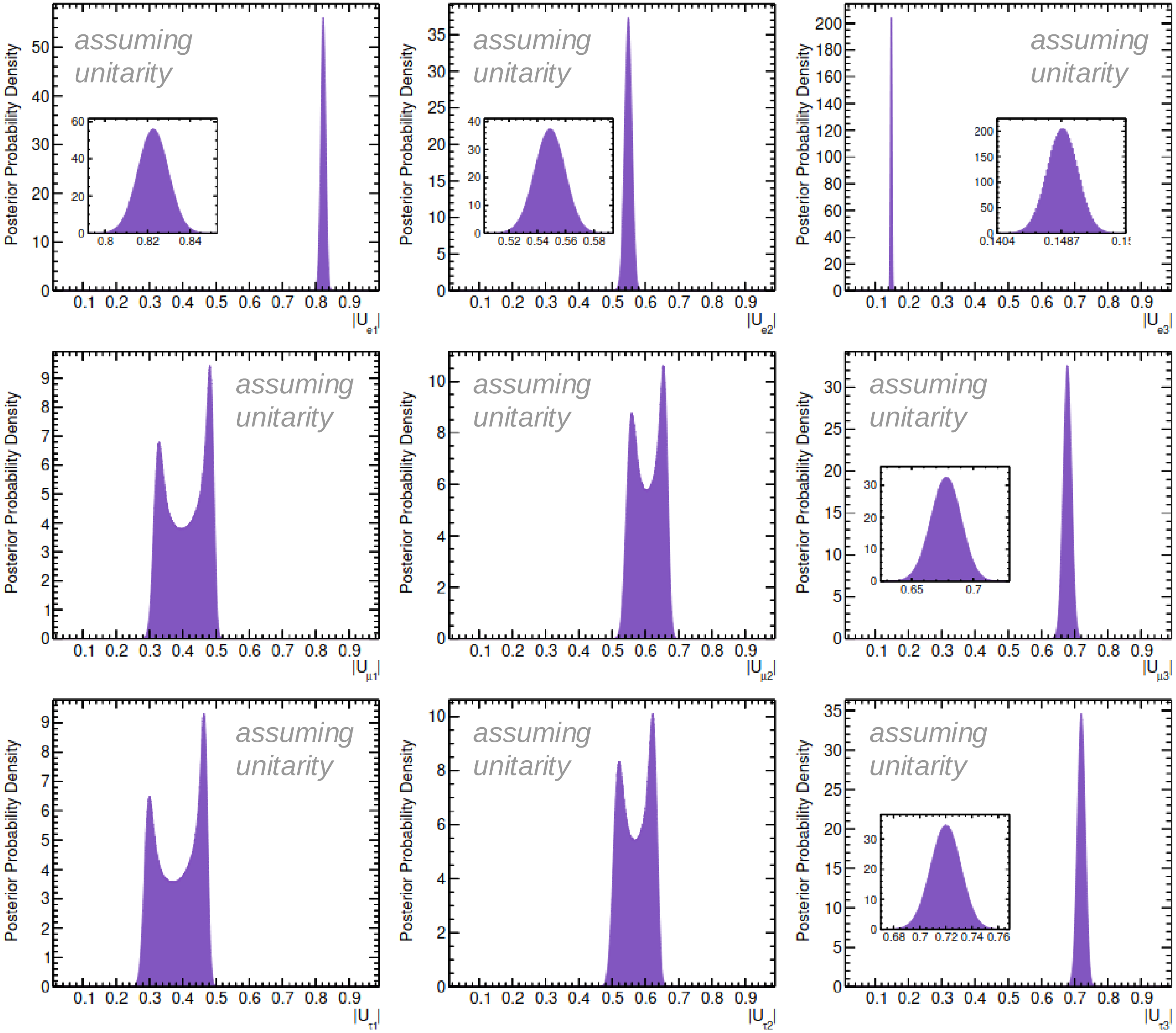
- Neutrino Physics is entering the precision era
- Soon it might be possible to test unitarity in long-baseline neutrino experiments
  → it will no longer be possible to solely rely on assumption of unitarity
- New presentations needed that don't assume unitarity or can test it
  → Should start using these now to accustom the community
- Using neutral current events as additional flux constraint might increase sensitivity
- First such sample is currently being implemented in the T2K oscillation analysis

Backup

PMNS Parameter Distributions



Leptonic Mixing Matrix Element Moduli & Phases



Note: *For visualisation purposes only; values for PMNS parameters generated loosely based on NuFIT 6.0*

PMNS Parameterisation

$$U_{\text{PMNS}} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{13}s_{23}e^{i\delta} & c_{12}c_{23} - s_{12}s_{13}s_{23}e^{i\delta} & c_{13}s_{23} \\ s_{12}s_{23} - c_{12}s_{13}c_{23}e^{i\delta} & -c_{12}s_{23} - s_{12}s_{13}c_{23}e^{i\delta} & c_{13}c_{23} \end{pmatrix}$$

Magnitudes & Phases (MP) Parameterisation

$$U_{LMM} = \begin{pmatrix} |U_{e1}| & |U_{e2}|e^{i\phi_{e2}} & |U_{e3}|e^{i\phi_{e3}} \\ |U_{\mu 1}| & |U_{\mu 2}| & |U_{\mu 3}| \\ |U_{\tau 1}| & |U_{\tau 2}|e^{i\phi_{\tau 2}} & |U_{\tau 3}|e^{i\phi_{\tau 3}} \end{pmatrix}$$

MP Parameterisation Vacuum Oscillation Probabilities

$$P(\nu_\alpha \to \nu_\alpha) = 1 - \frac{4\left|U_{\alpha 2}\right|^2\left(\left|U_{\alpha 1}\right|^2 + \left|U_{\alpha 3}\right|^2\right)}{N_\alpha^2}\sin^2\left(\frac{\Delta_{21}}{2}\right)$$
$$- \frac{4\left|U_{\alpha 3}\right|^2\left(\left|U_{\alpha 1}\right|^2 + \left|U_{\alpha 2}\right|^2\right)}{N_\alpha^2}\sin^2\left(\frac{\Delta_{31}}{2}\right)$$
$$+ \frac{8\left|U_{\alpha 2}\right|^2\left|U_{\alpha 3}\right|^2}{N_\alpha^2}\sin\left(\frac{\Delta_{21}}{2}\right)\sin\left(\frac{\Delta_{31}}{2}\right)\cos\left(\frac{\Delta_{32}}{2}\right)$$

MP Parameterisation Vacuum Oscillation Probabilities

$$P\left(\nu_\alpha \to \nu_\beta\right) = \frac{\left|t_{\alpha\beta}\right|^2}{N_\alpha N_\beta} + \frac{4\left|U_{\alpha 2}\right|^2\left|U_{\beta 2}\right|^2}{N_\alpha N_\beta}\sin^2\left(\frac{\Delta_{21}}{2}\right) + \frac{4\left|U_{\alpha 3}\right|^2\left|U_{\beta 3}\right|^2}{N_\alpha N_\beta}\sin^2\left(\frac{\Delta_{31}}{2}\right)$$

$$+ \frac{8\left|U_{\alpha 2}\right|\left|U_{\beta 2}\right|\left|U_{\alpha 3}\right|\left|U_{\beta 3}\right|}{N_\alpha N_\beta}\sin\left(\frac{\Delta_{21}}{2}\right)\sin\left(\frac{\Delta_{31}}{2}\right)\cos\left(\frac{\Delta_{32}}{2} + \varphi_2^{\alpha\beta} - \varphi_3^{\alpha\beta}\right)$$

$$+ \frac{4\left|t_{\alpha\beta}\right|}{N_\alpha N_\beta}\left[\left|U_{\alpha 2}\right|\left|U_{\beta 2}\right|\sin\left(\frac{\Delta_{21}}{2}\right)\sin\left(\frac{\Delta_{21}}{2} + \varphi^{\alpha\beta} - \varphi_2^{\alpha\beta}\right)\right.$$

$$\left. + \left|U_{\alpha 3}\right|\left|U_{\beta 3}\right|\sin\left(\frac{\Delta_{31}}{2}\right)\sin\left(\frac{\Delta_{31}}{2} + \varphi^{\alpha\beta} - \varphi_3^{\alpha\beta}\right)\right] ,$$

$$N_\alpha = \left|U_{\alpha 1}\right|^2 + \left|U_{\alpha 2}\right|^2 + \left|U_{\alpha 3}\right|^2 \text{ and } t_{\alpha\beta} = U_{\alpha 1}^* U_{\beta 1} + U_{\alpha 2}^* U_{\beta 2} + U_{\alpha 3}^* U_{\beta 3}$$

$$t_{\alpha\beta} = \left|t_{\alpha\beta}\right| e^{i\varphi^{\alpha\beta}} \qquad U_{\alpha k}^* U_{\beta k} = \left|U_{\alpha k}^* U_{\beta k}\right| e^{i\varphi_k^{\alpha\beta}}$$

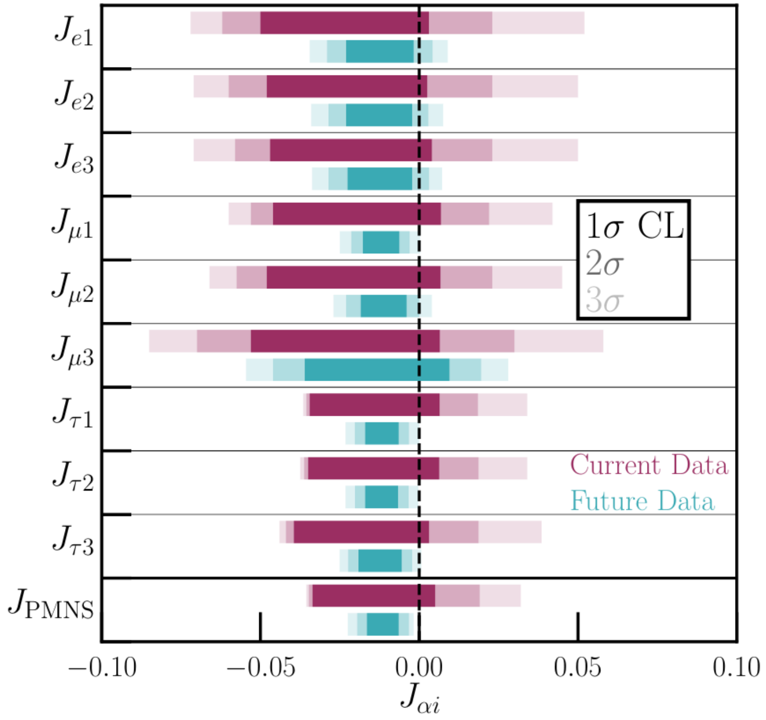
Jarlskog Factors

- 9 Jarlskog factors can be constructed as generalisations of the Jarlskog invariant according to:

$$\varepsilon_{\alpha\beta\gamma}\varepsilon_{ijk}J_{\alpha i} = \mathrm{Im}\left(U_{\beta j}U_{\gamma k}U^*_{\beta k}U^*_{\gamma j}\right)$$

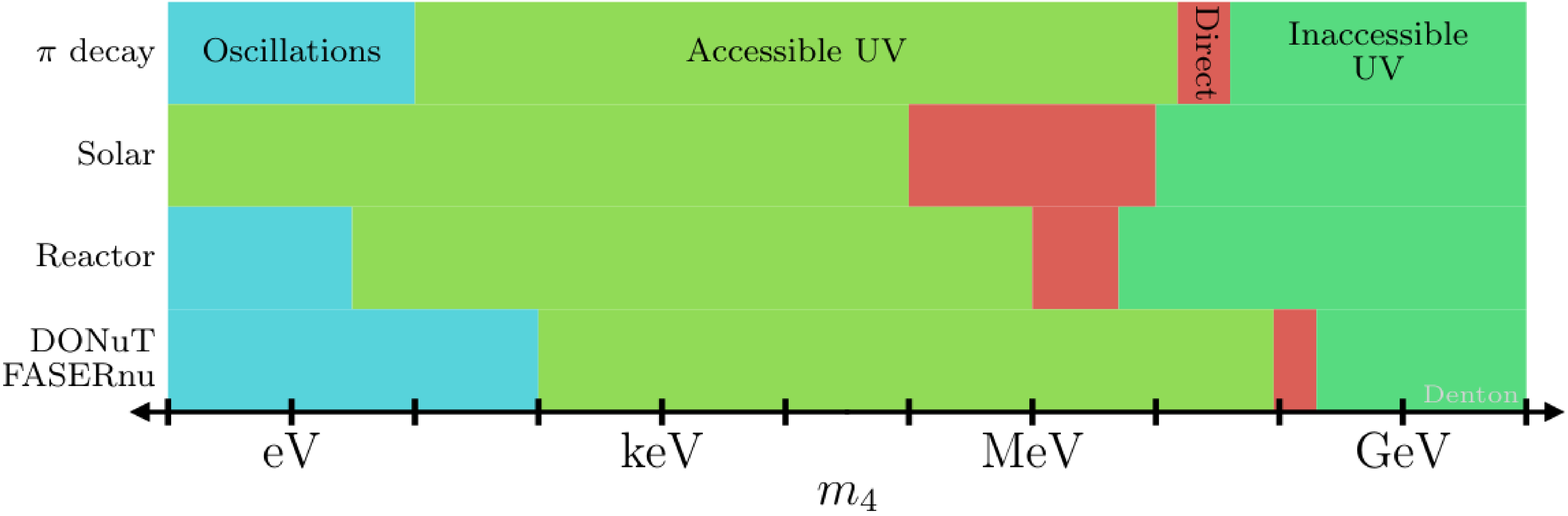
- They are directly related to the area of the unitarity triangles
- In case of unitarity, all Jarlskog factors are equal to the Jarlskog invariant
- In case of non-unitarity different Jarlskog factors relate to the different CP violating phases → full measure of CP violation in the case of non-unitarity

Phys. Rev. D 102, 115027 (2020)



Unitarity violation: a tale of four regimes

Taken from Peter Denton's talk
from the October DUNE Collab Call



*Details depends on the specific experiment/channel

Unitarity violation: mass ranges

Taken from Peter Denton's talk
from the October DUNE Collab Call

experiment	(4,4) (m_4)	(5,3) (m_4)
atmospheric ν_μ disappearance	$\in [10\text{ eV}, 15\text{ MeV}]$	$\gtrsim 40$ MeV
atmospheric ν_τ appearance	$\in [10\text{ eV}, 15\text{ MeV}]$	$\gtrsim 40$ MeV
astrophysical ν_τ appearance	$\lesssim 15$ MeV	$\gtrsim 40$ MeV
solar ^{8}B	$\lesssim 5$ MeV	$\gtrsim 20$ MeV
DONuT/FASERnu	$\in [100\text{ eV}, 90\text{ MeV}]$	$\gtrsim 200$ MeV
LBL ν_τ appearance (OPERA)	$\in [1\text{ eV}, 15\text{ MeV}]$	$\gtrsim 40$ MeV
LBL ν_τ appearance (DUNE)	$\in [0.1\text{ eV}, 15\text{ MeV}]$	$\gtrsim 40$ MeV
LBL ν_μ disappearance (DUNE)	$\in [0.1\text{ eV}, 15\text{ MeV}]$	$\gtrsim 40$ MeV
CEvNS	$\in [10\text{ eV}, 15\text{ MeV}]$	$\gtrsim 40$ MeV

(m, n): m total neutrinos, n accessible neutrinos

PBD, J. Gehrlein 2109.14575

Wolfenstein Parameterisation

See *Eur. Phys. J. C 21, 225–259 (2001)*

- CKM triangles shown as function of adapted Wolfenstein parameters:

$$(\bar\rho = \rho(1-\lambda^2/2),\ \bar\eta = \eta(1-\lambda^2/2))\,,$$ so that $$R_u = \left|\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*}\right| = \sqrt{\bar\rho^2+\bar\eta^2}$$

- Wolfenstein parameterisation is expansion possible due to diagonality of CKM matrix:

$$V \simeq \begin{pmatrix} 1-\frac{1}{2}\lambda^2-\frac{1}{8}\lambda^4 & \lambda & A\lambda^3(\rho-i\eta) \\ -\lambda\left[1+A^2\lambda^4\left(\rho+i\eta-\frac{1}{2}\right)\right] & 1-\frac{1}{2}\lambda^2-\frac{1}{8}\lambda^4(1+4A^2) & A\lambda^2 \\ A\lambda^3\left[1-(\rho+i\eta)\left(1-\frac{1}{2}\lambda^2\right)\right] & -A\lambda^2\left[1+\lambda^2\left(\rho+i\eta-\frac{1}{2}\right)\right] & 1-\frac{1}{2}A^2\lambda^4 \end{pmatrix}$$

with:

$$\begin{aligned} s_{12} &= \lambda\,, \\ s_{23} &= A\lambda^2\,, \\ s_{13}e^{-i\delta} &= A\lambda^3(\rho-i\eta)\,, \end{aligned}$$

which is valid to $O(|\lambda|^6) \simeq 0.01\%$.

Neutral CurrentNeutral Current

Phys. Rev. C 72, 055502 (2005)