# Search for the Charged Lepton Flavour Violating Decay $B^+ \to \pi^+ \mu^\pm e^\mp$

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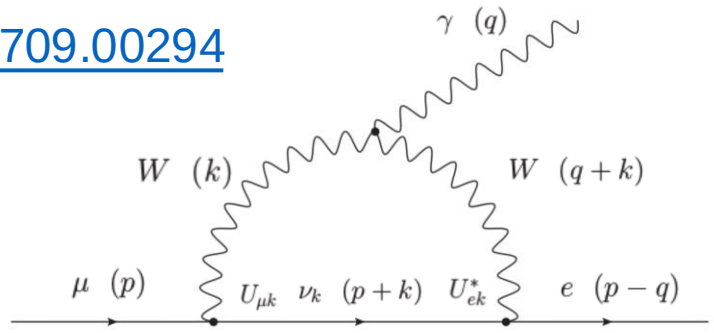
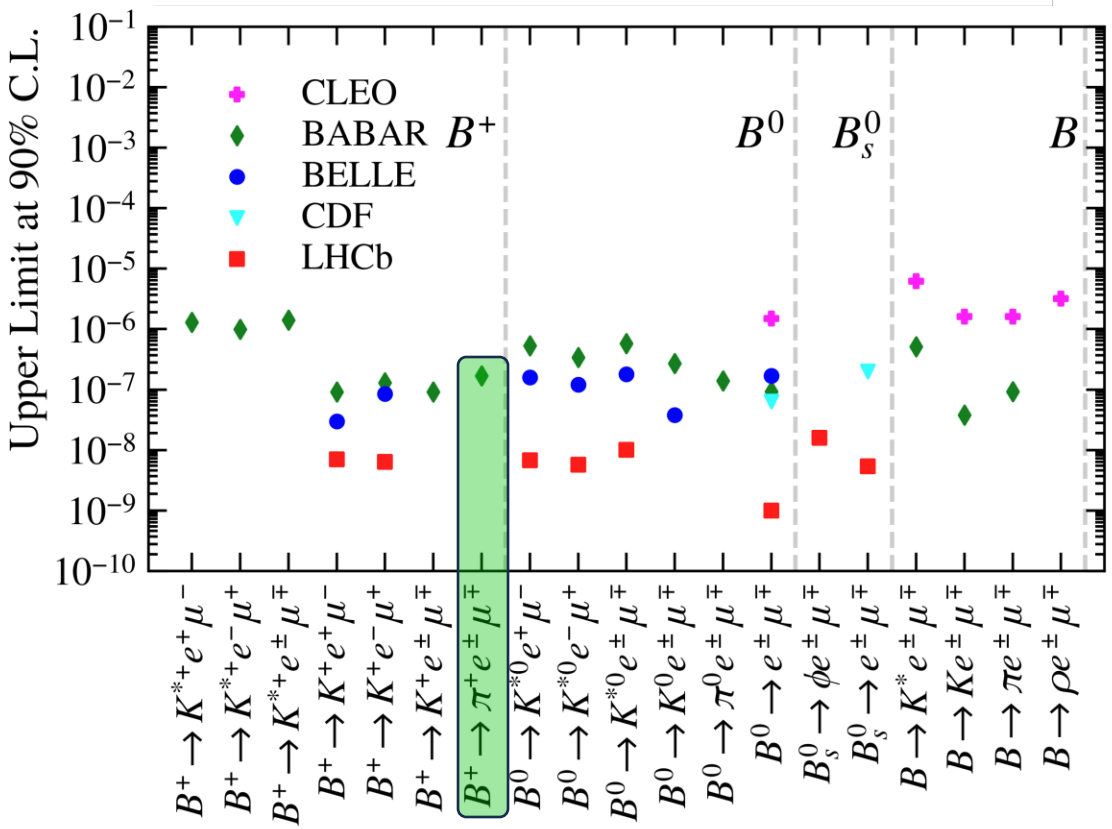
# Motivation

- Charged Lepton Flavour Violation (cLFV) only possible in SM via neutrino oscillation
  - BF ~ $10^{-54}$
- Observation of cLFV → Clear sign of New Physics (NP)!
- Some models that aim to explain B anomalies introduce cLFV (1411.0565)
  - BF enhanced to $\sim 10^{-10}$ within leptoquark model
- Search in $b \to d\ell\ell'$ channel can offer complimentary sensitivity to NP c.f $b \to s\ell\ell'$ processes due to double CKM suppression
- First search for cLFV in $b \to d$ transition @ LHC
- Previous limits set by BaBar at $1.7 \times 10^{-7}$ @ 90% CL
- LHCb can improve sensitivity by 2-3 orders of magnitude

arXiv:1709.00294

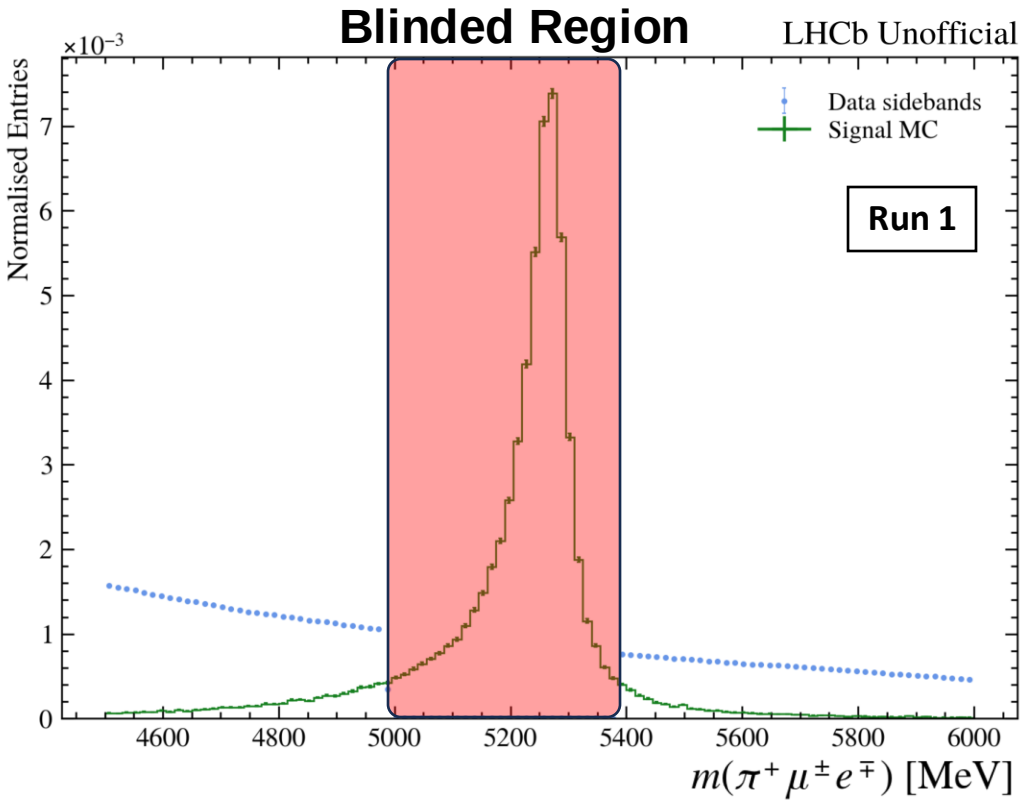



$$\mathrm{BR}(\mu \to e\gamma) \simeq \frac{\Gamma(\mu \to e\gamma)}{\Gamma(\mu \to e\nu\bar{\nu})} = \frac{3\alpha}{32\pi}\left|\sum_{k=1,3} \frac{U_{\mu k}U^*_{ek}m^2_{\nu_k}}{M^2_W}\right|^2$$



# Analysis Strategy

- Aim to observe or set upper limit on $B^+ \to \pi^+\mu^\mp e^\pm$ decay
- Use full 9fb$^{-1}$ dataset (**$10^{12}$ beauty quarks**) collected by LHCb detector
- Blind signal region $m_{B^\pm} \notin [4985, 5385]$ MeV
- Use $B^+ \to J/\psi\,(\to \mu^+\mu^-)K^+$ to normalise signal events to branching fraction
- Split into four analysis categories:
  - **R1 , R2**
  - **Bremsstrahlung photon(s)** associated to the electron

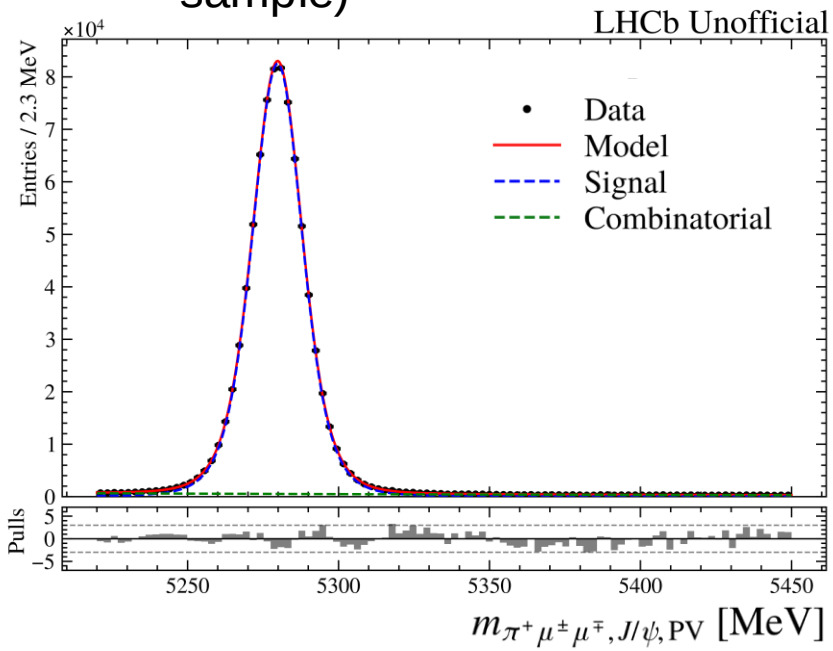



**Yields in data** **MC simulation** **PDG**

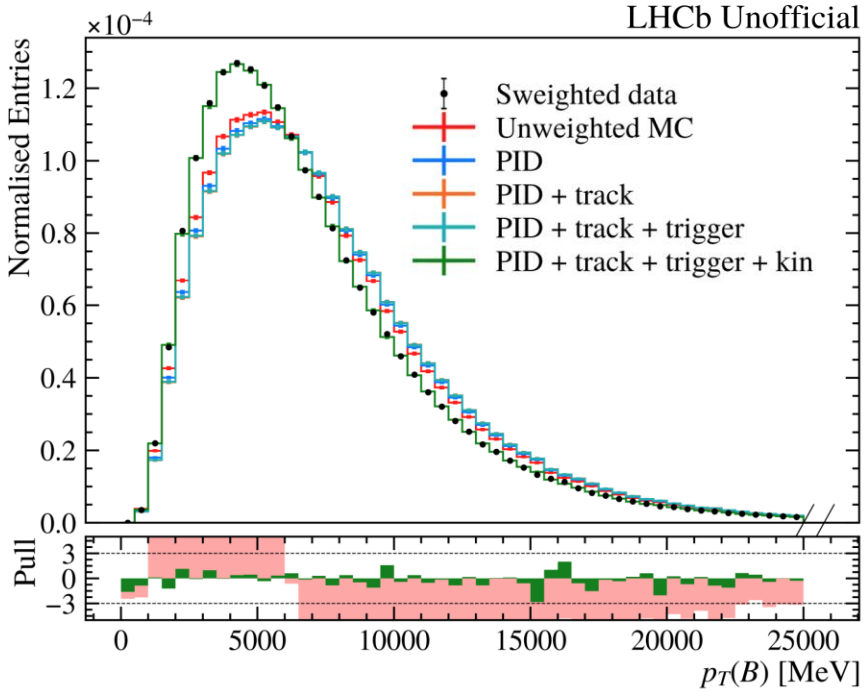
$$\mathcal{B}(B^+ \to \pi^+\mu^\pm e^\mp) = \frac{N(B^+ \to \pi^+\mu^\pm e^\mp)}{\varepsilon(B^+ \to \pi^+\mu^\pm e^\mp)} \cdot \frac{\varepsilon(B^+ \to J/\psi(\to \mu^+\mu^-)K^+)}{N(B^+ \to J/\psi(\to \mu^+\mu^-)K^+)} \cdot \mathcal{B}(B^+ \to J/\psi(\to \mu^+\mu^-)K^+)$$

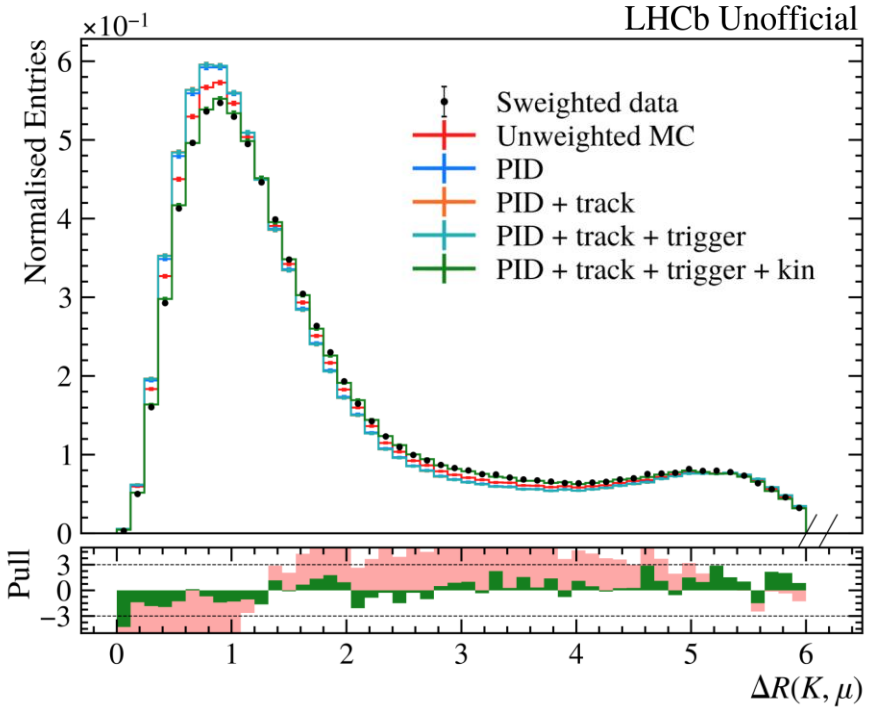
# Simulation Reweighting

- Monte Carlo (MC) simulation used to estimate efficiencies, background contribution and signal shapes
- Correct MC with data-driven methods
  - **Tracking efficiency** (tag-and-probe method)
  - **Trigger efficiency** (TISTOS method)
  - **$B^+$ production kinematics** (Gradient Boosted Reweighter trained on background-subtracted normalisation mode)
  - **PID efficiency** (background-subtracted large calibration sample)

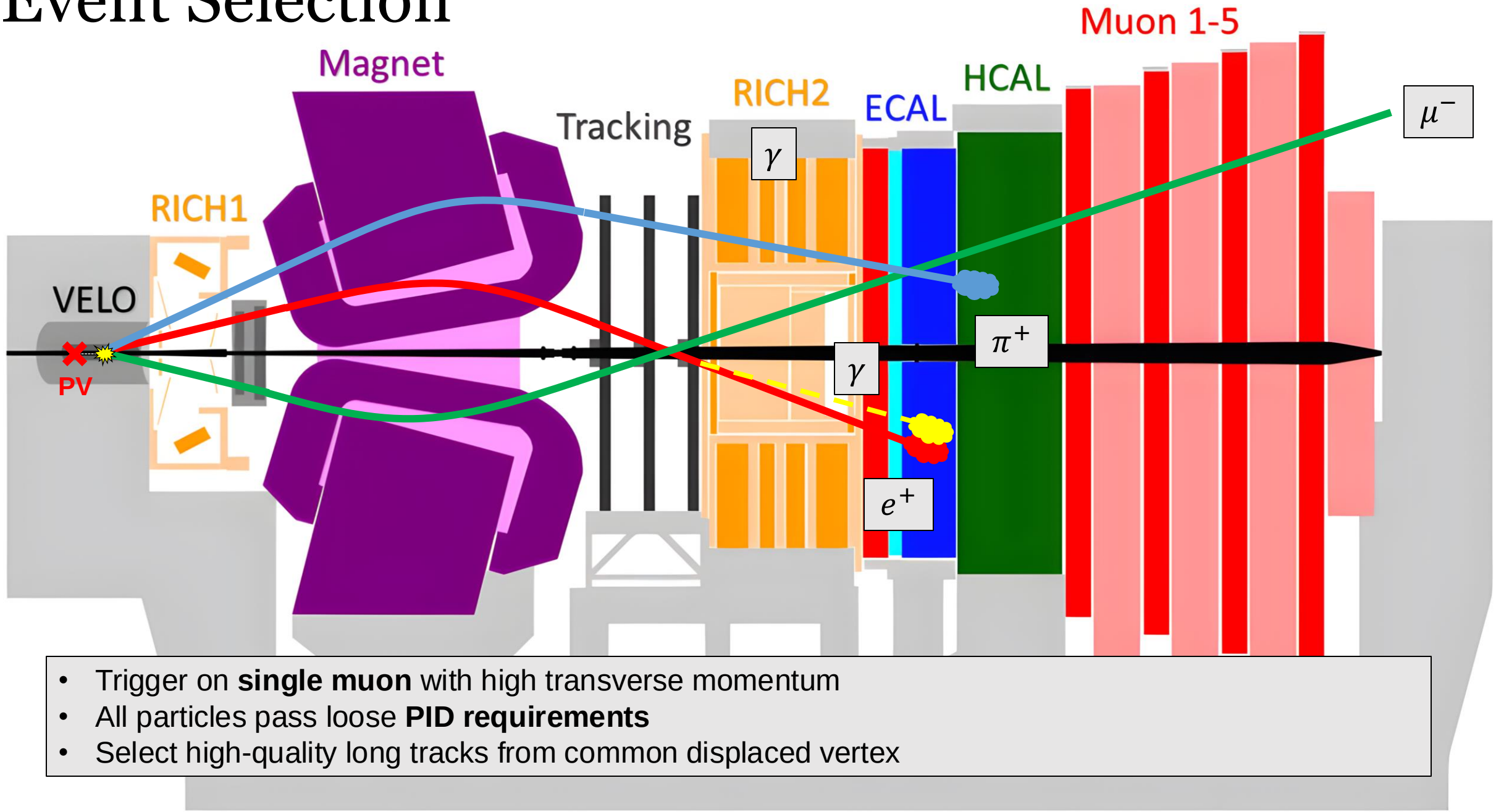



**Good agreement seen after reweighting!**



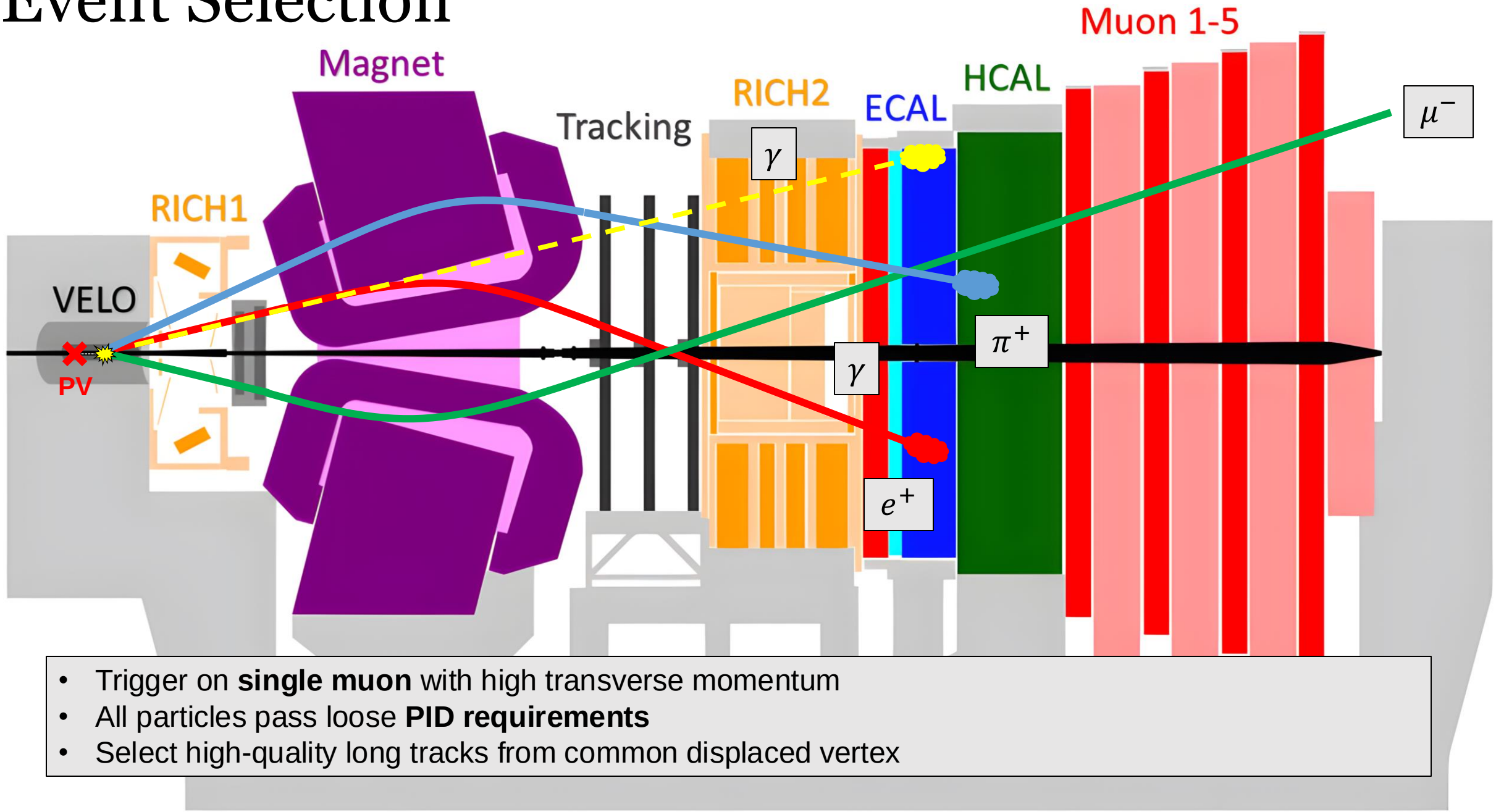



# Event Selection




- Trigger on **single muon** with high transverse momentum
- All particles pass loose **PID requirements**
- Select high-quality long tracks from common displaced vertex

# Event Selection

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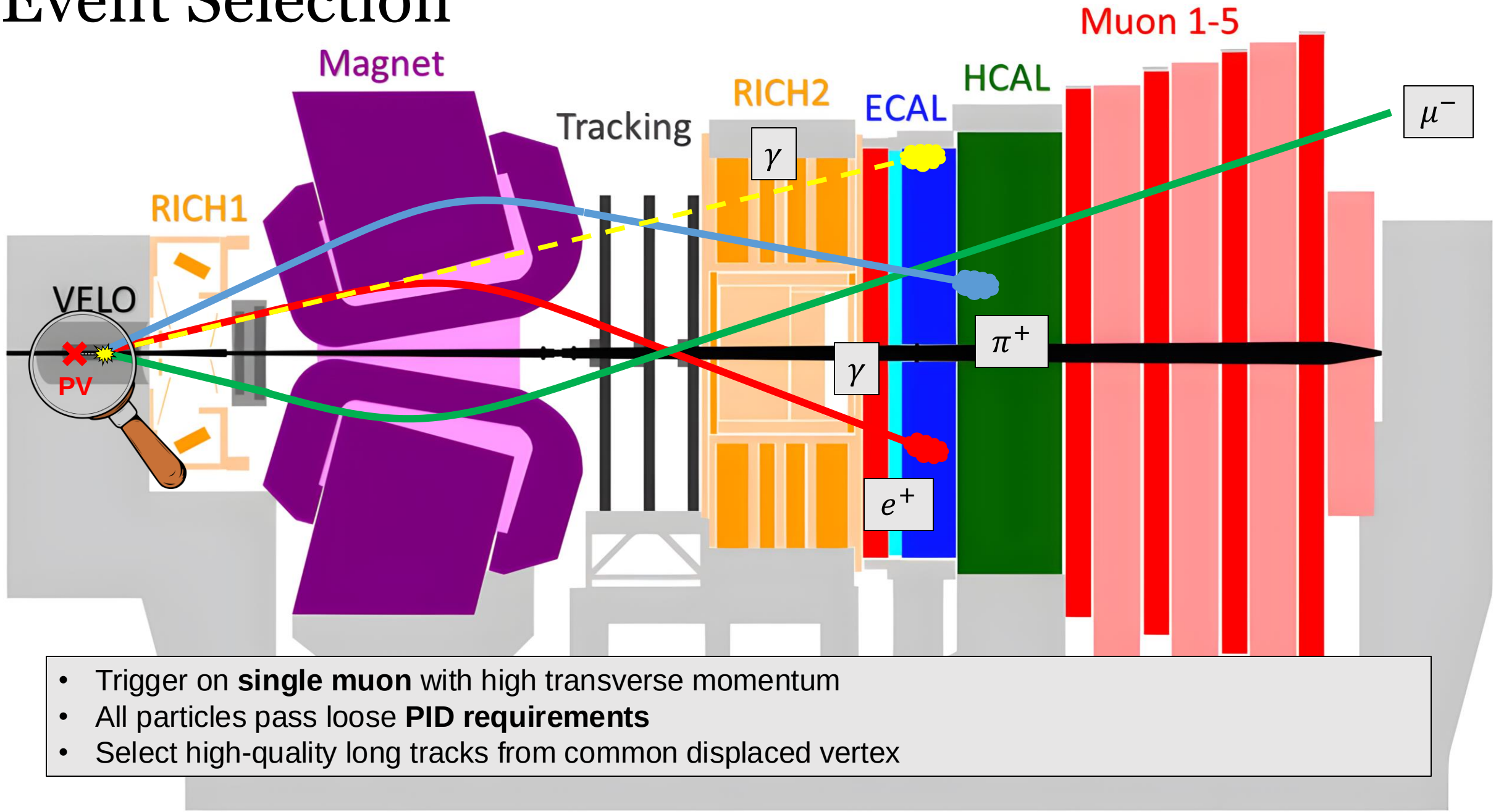
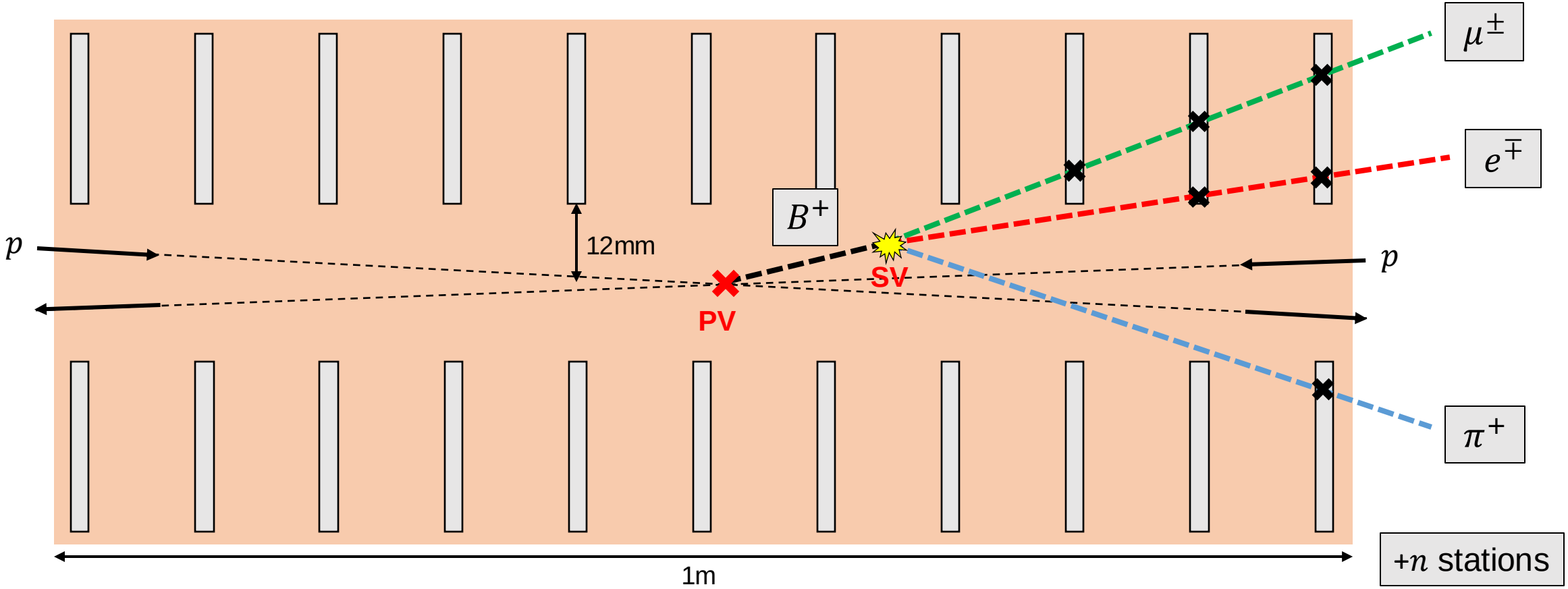



- Trigger on **single muon** with high transverse momentum
- All particles pass loose **PID requirements**
- Select high-quality long tracks from common displaced vertex

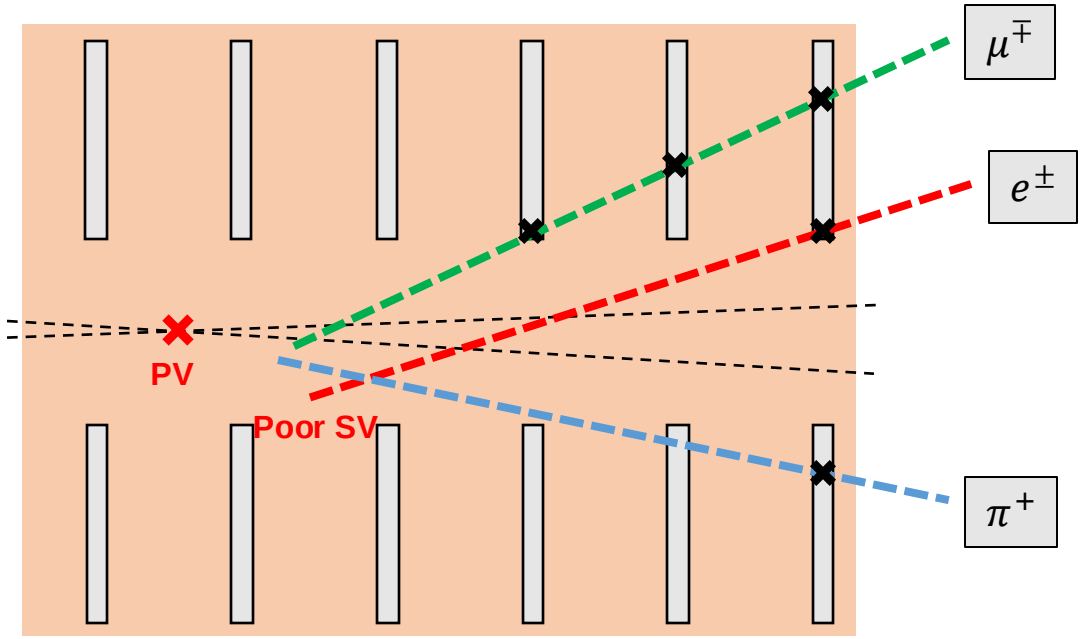
# Event Selection

- **Vertex Locator** (VELO) provides excellent vertexing
- Require all tracks originate from common displaced **Secondary Vertex (SV)**

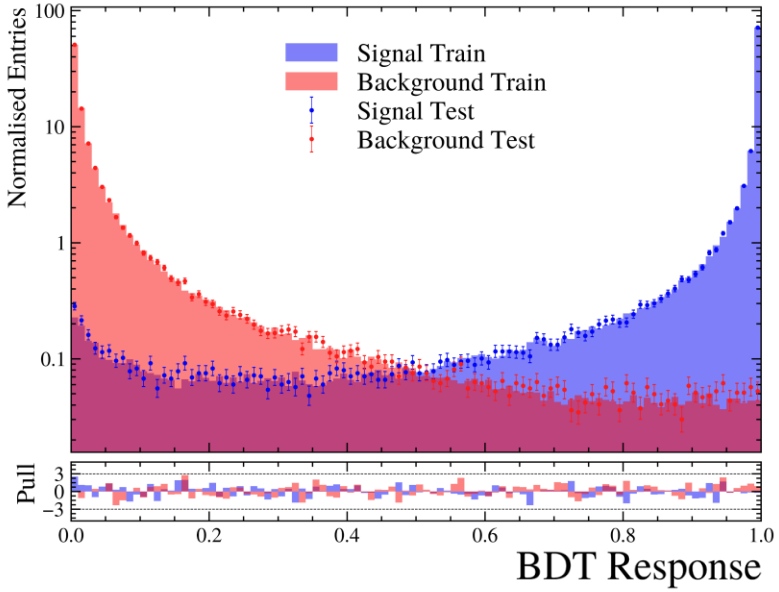


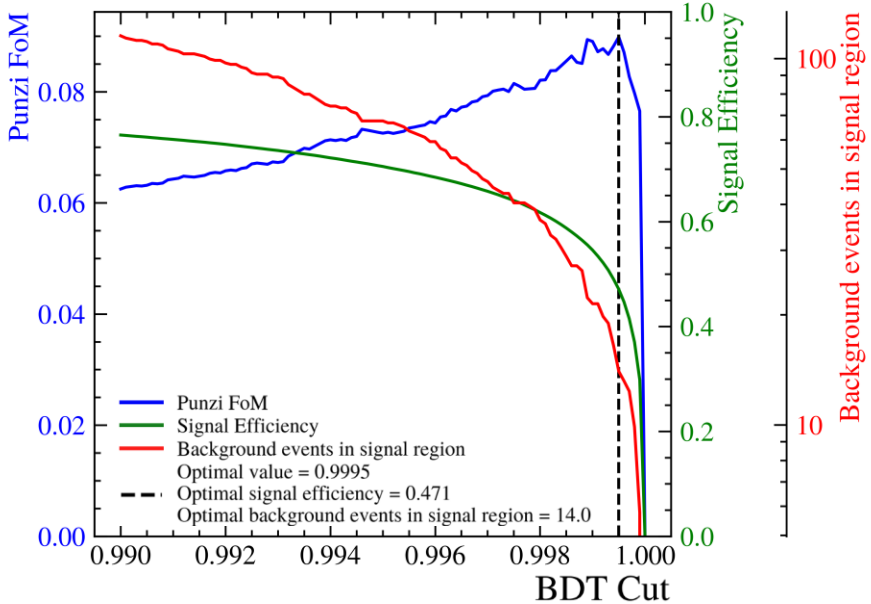
# Combinatorial Background

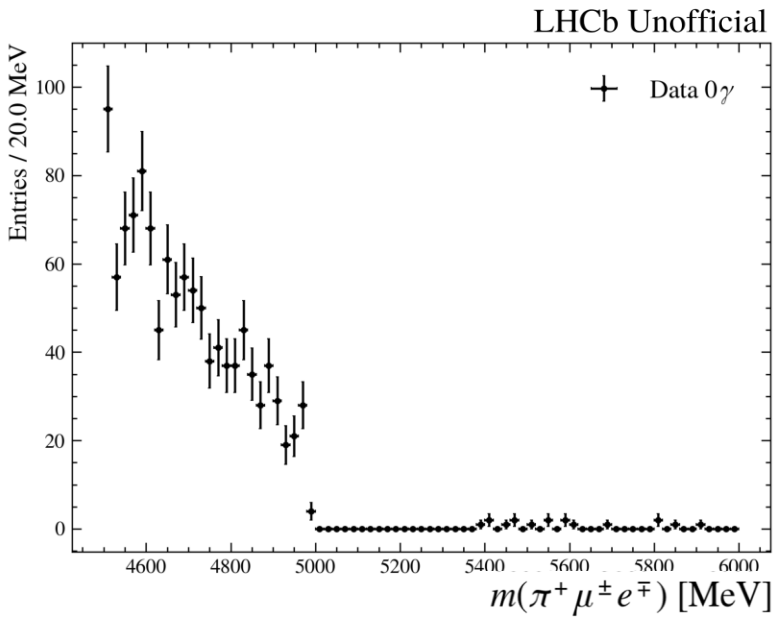
- Three real random tracks form poor quality vertex → combinatorial background
- Rejected with Gradient **Boosted Decision Tree** (BDT) from XGBoost package
- Trained on reweighted signal MC and data **upper sideband**
- Various discriminating topological and kinematic variables used
- Classifier hyperparameters optimised with Bayesian search of hyperparameter space




**Run 1 $0\gamma$**

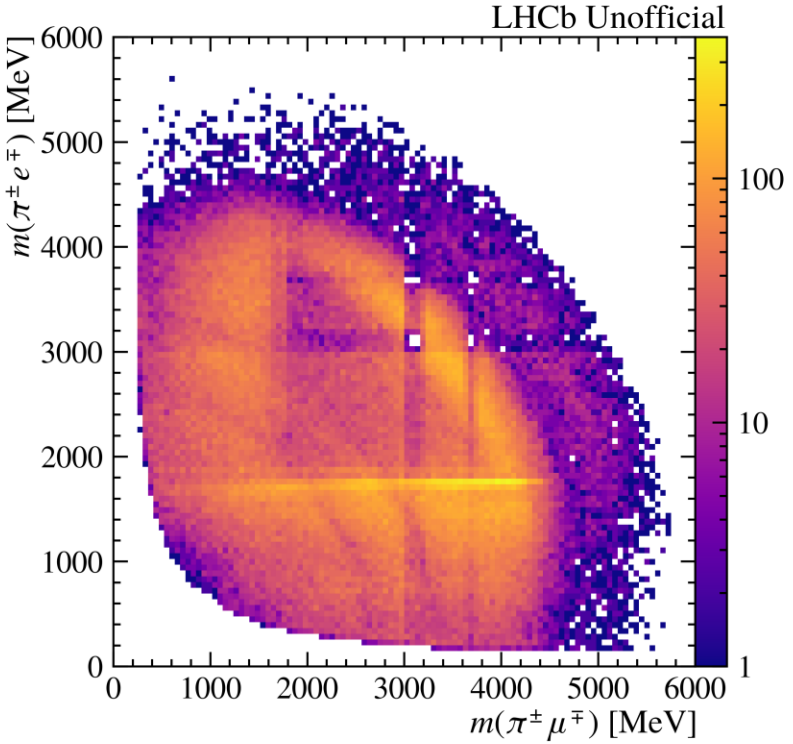


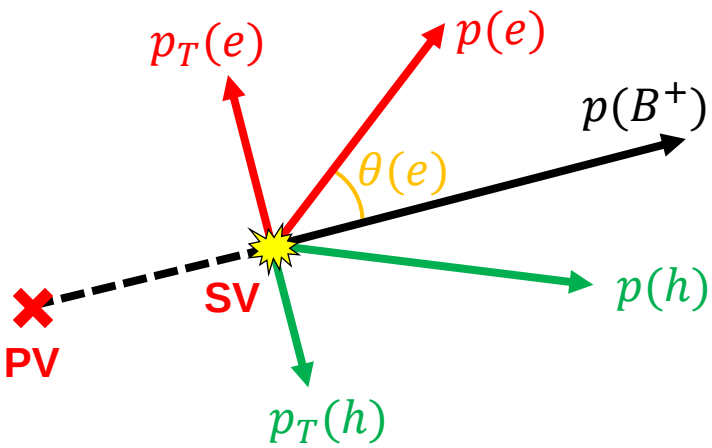



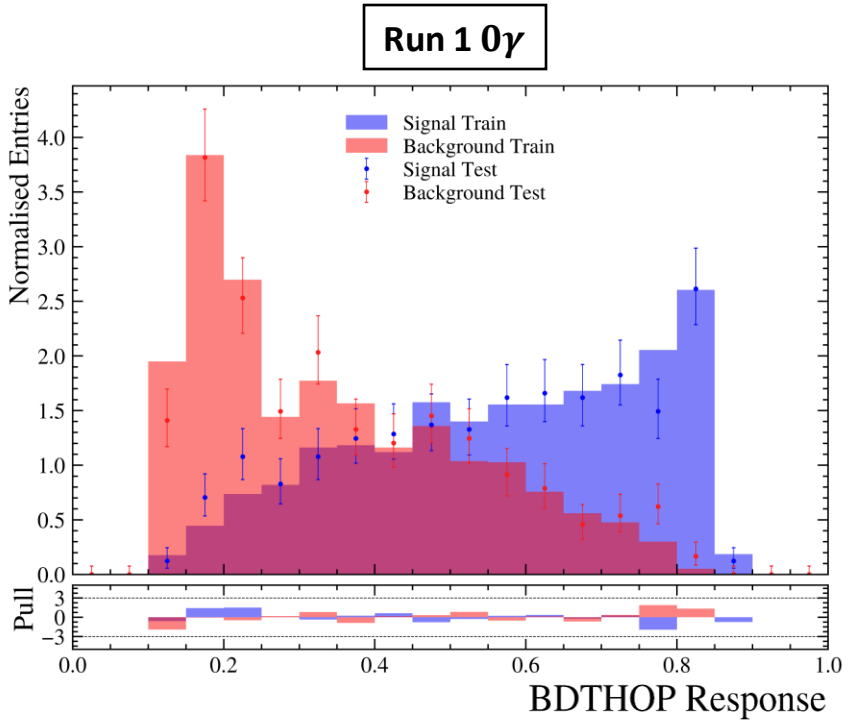



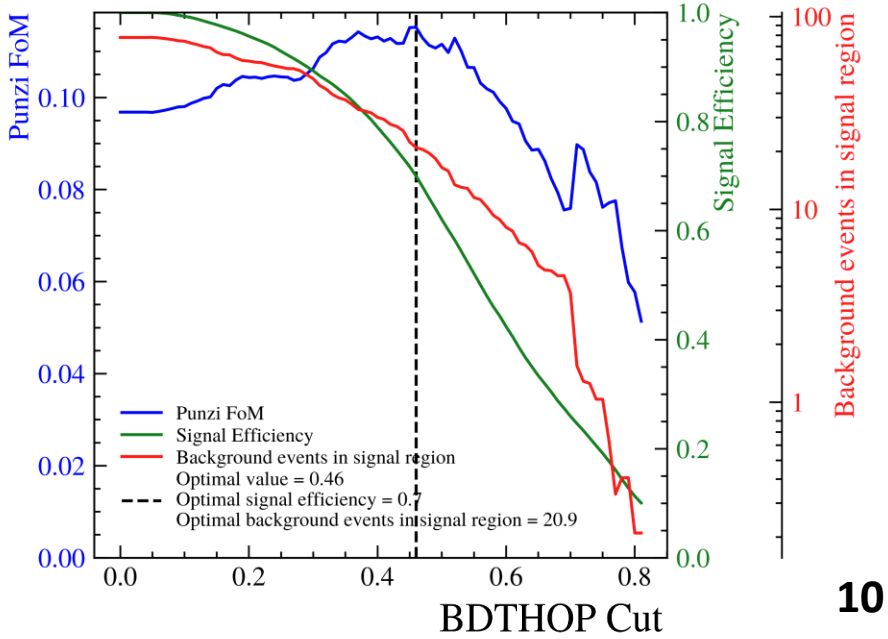
# Partially Reconstructed Backgrounds

- Partially reconstructed decays can easily produce signal tracks
- $B^+ \to \overline{D^0}(\to h^+\ell^-\nu_\ell)\ell'^+\nu_{\ell'}$ has BF ~$10^{-5}$ → Require $m_{h^\pm\ell^\mp} > 1885$ MeV
- Partially reconstructed backgrounds still pollute lower mass sideband
- Train second BDT on reweighted signal MC and lower mass sideband **after combinatorial BDT selection**
- Addition of kinematic variable suited for partially reconstructed decays



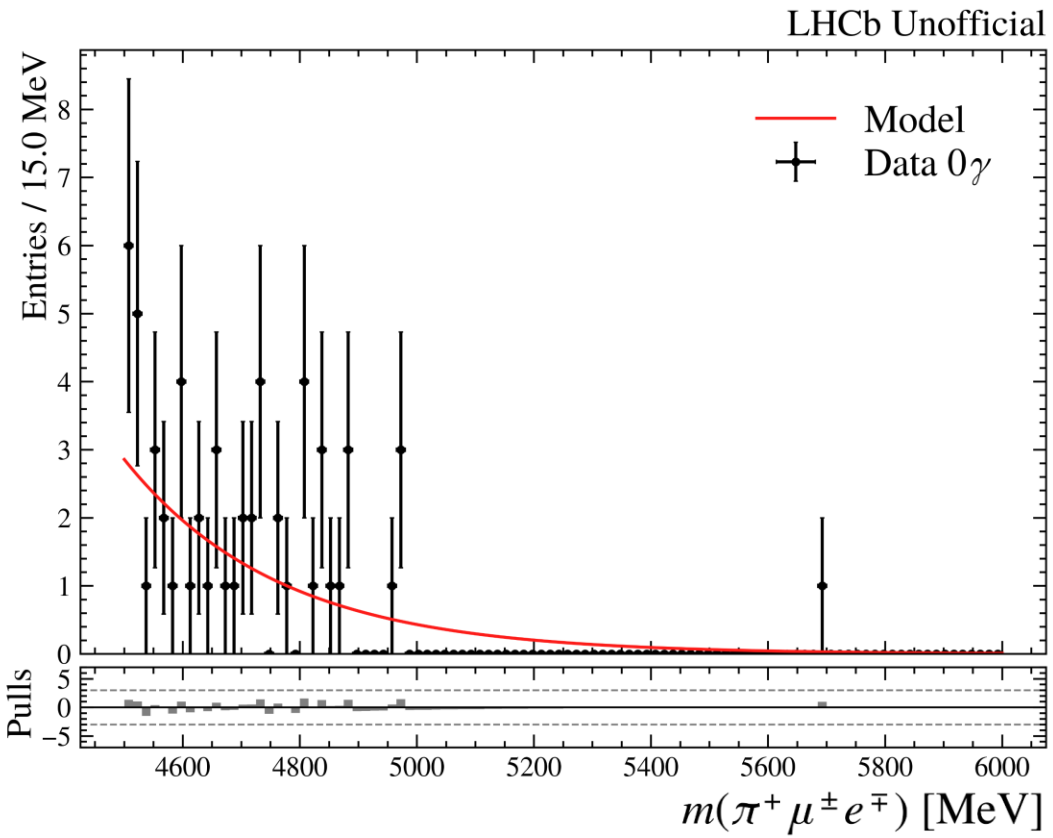



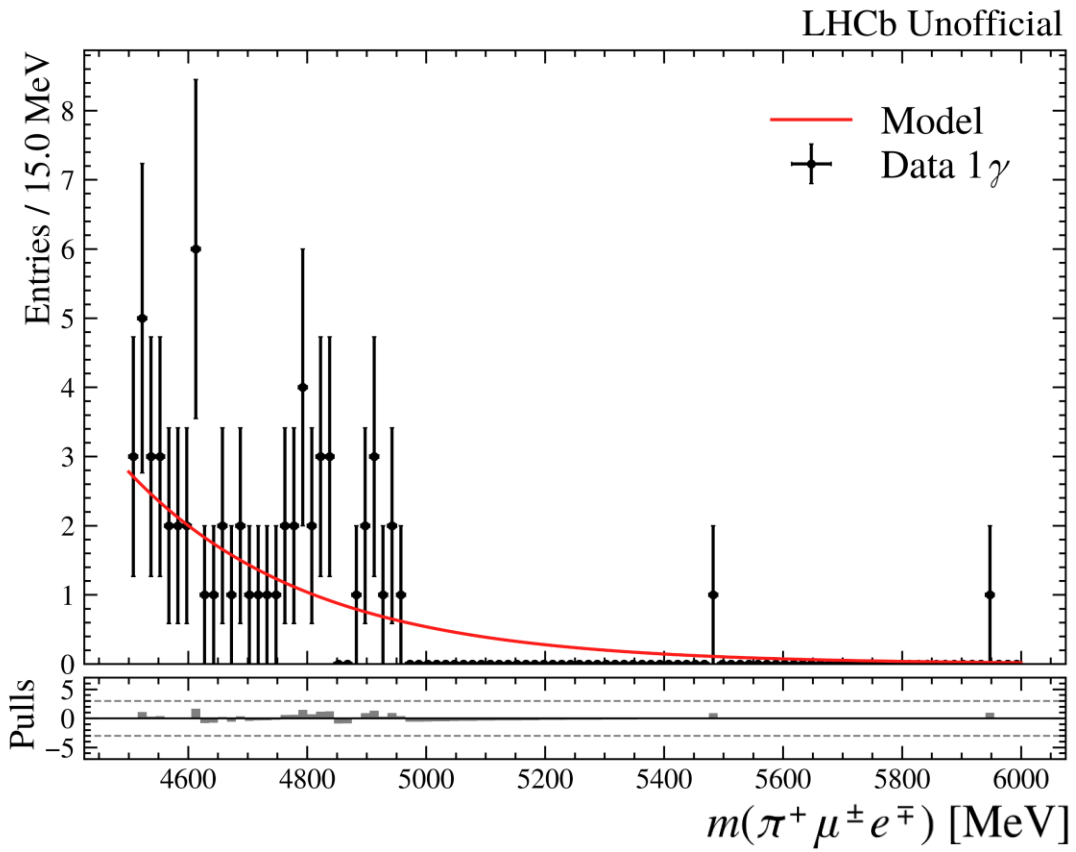



# Mis-identified Backgrounds

- Higher rate processes can mimic signal if one or more daughters are mis-identified
- $B^+ \to J/\psi\left(\to e^\pm e^\mp/\mu^\pm\mu^\mp\right)K/\pi^+$ have BF ~$10^{-5}$ → Require $\left|m_{\mu^\pm e^\mp_{\to\mu}} - m^{PDG}_{J/\psi}\right| > 100$ MeV + others
- $B^+ \to K/\pi^+\, e^\pm e^\mp/\mu^\pm\mu^\mp$ electroweak penguins and fully hadronic $B^+ \to h^+h^+h^-$ → Require **tight PID**



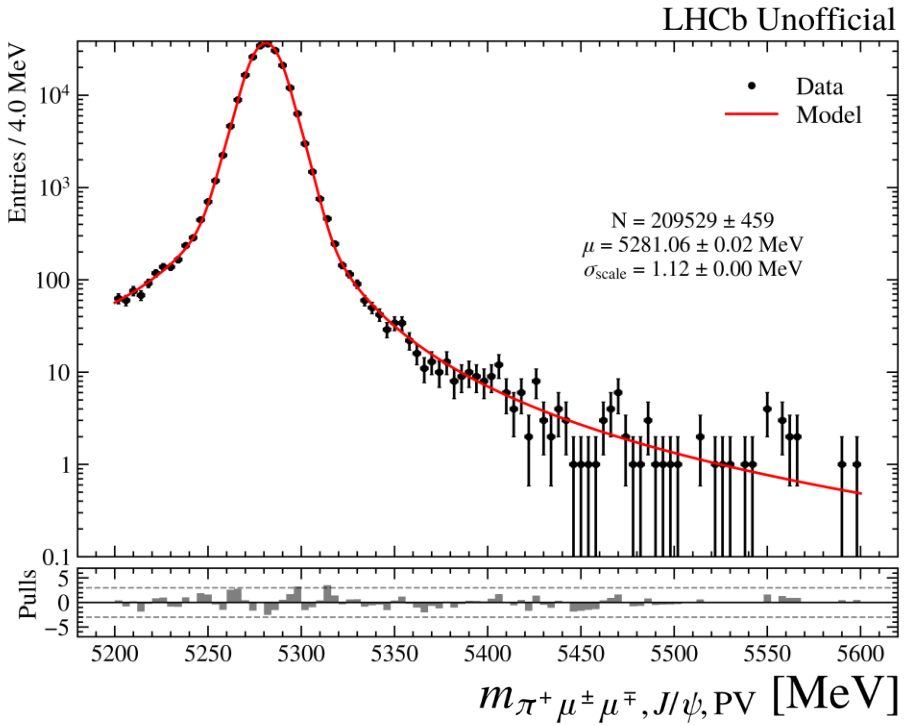

**Run 2**



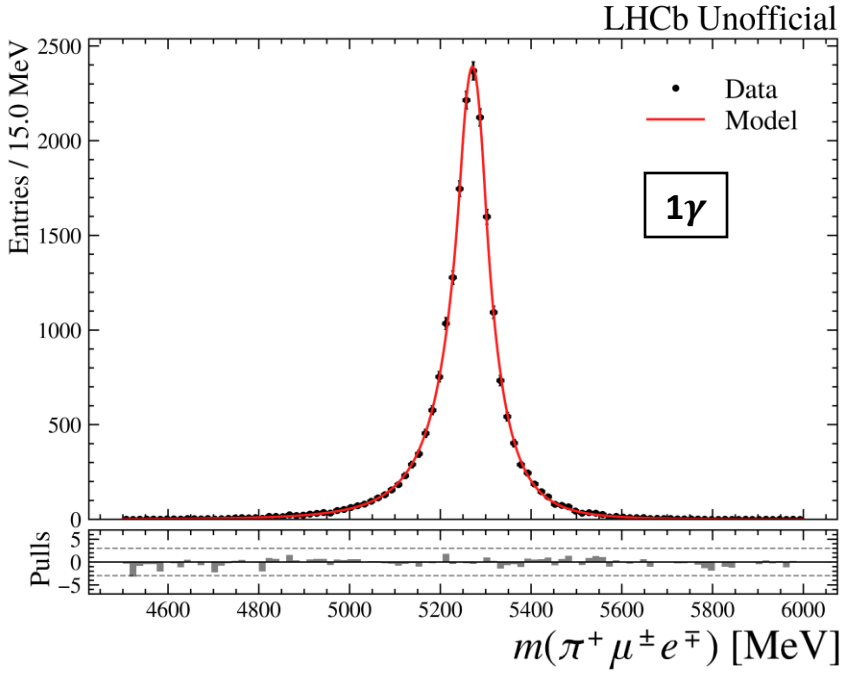
# Normalisation + Signal Shape

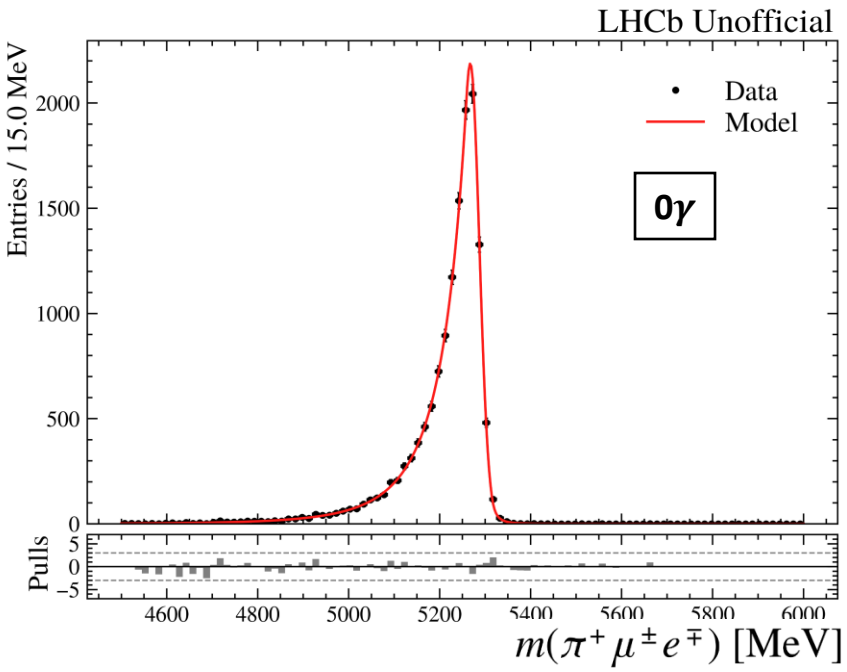
- Align signal and normalisation mode selection as best as possible
- Perform fit to extract number of candidates
- Efficiency of selections estimated with simulation
- Single Event Sensitivity (SES) assuming one signal event is observed

**Total SES:** $(3.20 \pm 0.06) \times 10^{-10}$



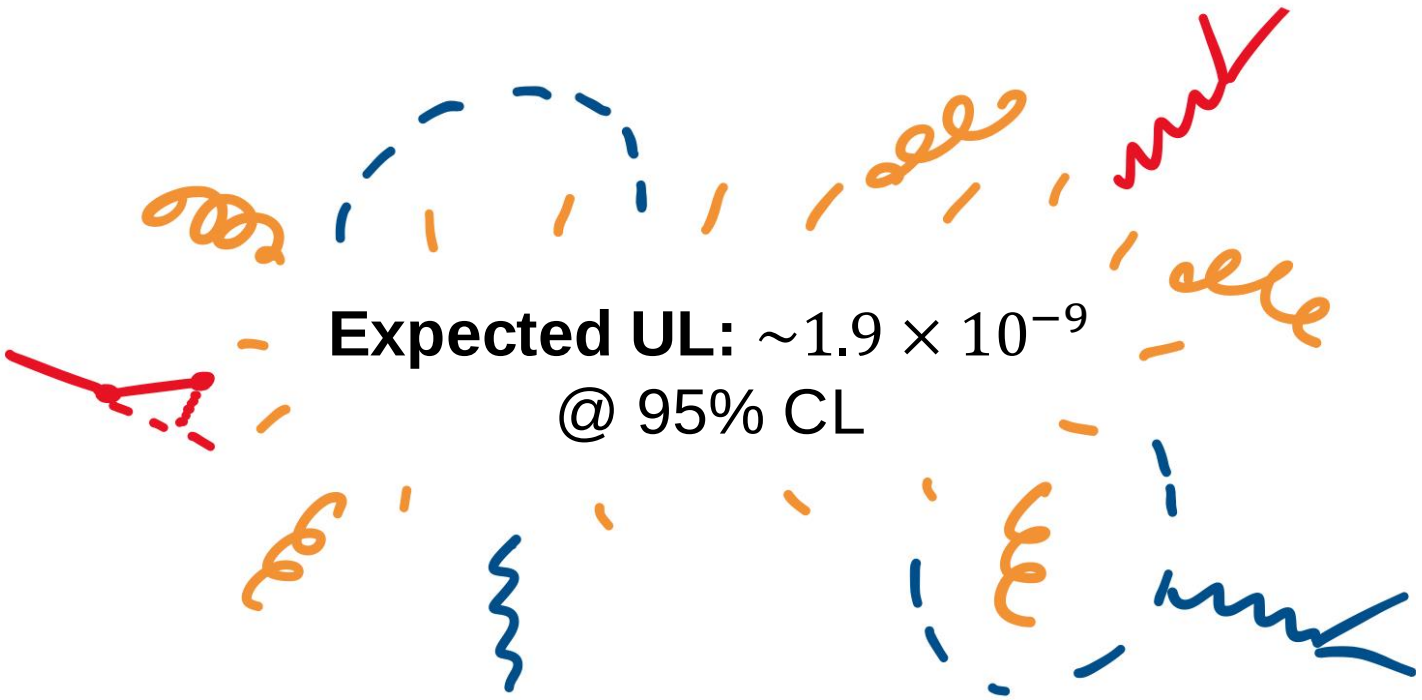

- For upper limit calculation, signal shape taken from reweighted MC



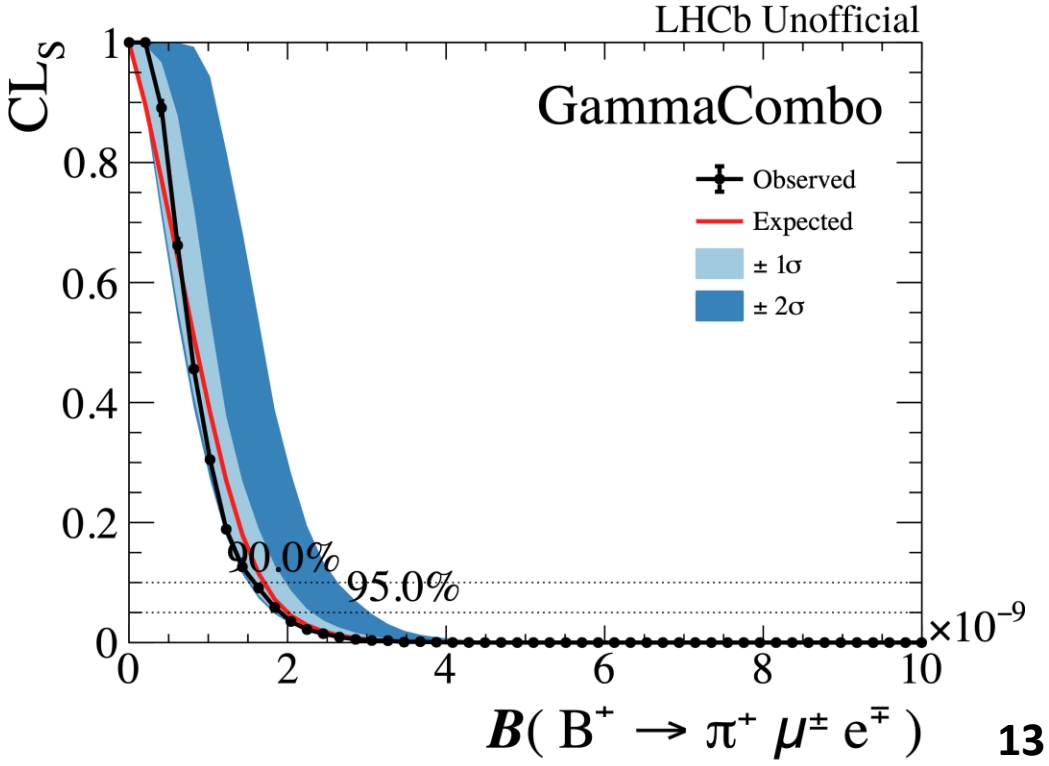



# Expected Upper Limit

- No signal? Set upper limit with **CLs method** with GammaCombo framework
- Generate and fit **background only** and **signal + background** toy distributions
- Build distributions of test statistic $\boldsymbol{Q = -2\ln\mathcal{L}}$
- CLs p-value is fraction of toy fit results which perform worse than the fit to measured data
- Signal region currently **remains blinded**, use background only hypothesis pseudo-experiments and repeat process 100 times

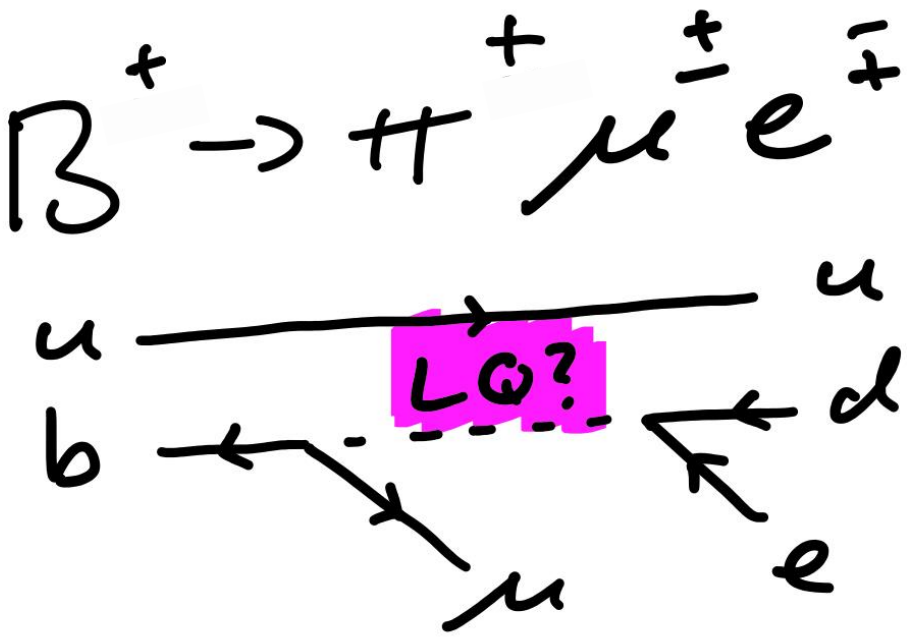


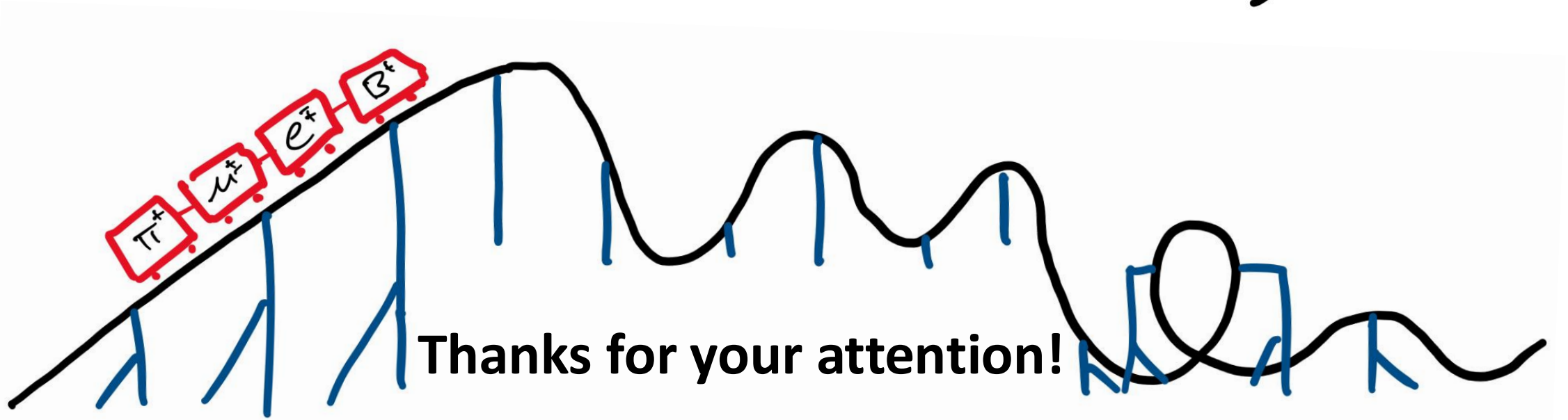
**Expected UL:** $\sim 1.9 \times 10^{-9}$
@ 95% CL



# Summary & Outlook

- First search for cLFV at the LHC in the $\boldsymbol{b} \rightarrow \boldsymbol{d}$ **transition**
- **Data-driven corrections** applied to MC simulation
- Event selection with two **BDTs** and **tight PID** requirements
- Expected upper limit $\sim \mathbf{1.9 \times 10^{-9}}$ → two orders of magnitude improvement c.f BaBar
- Dedicated background study remains WIP
- Evaluation of systematic effects remains TODO



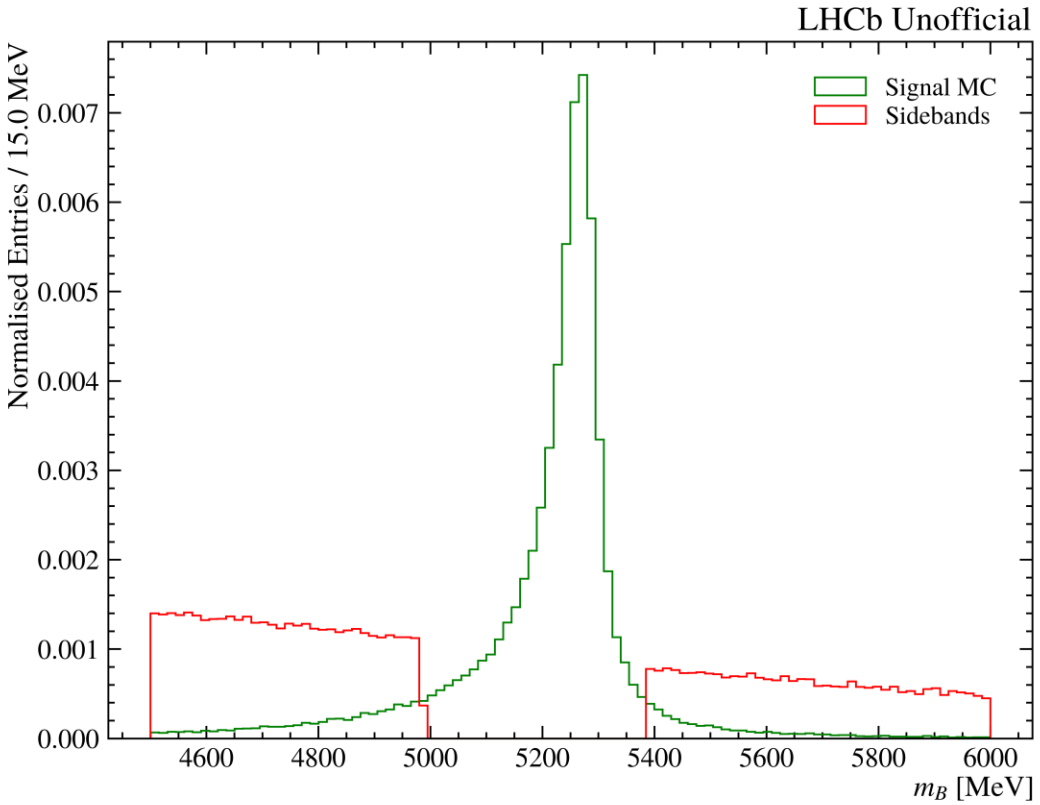



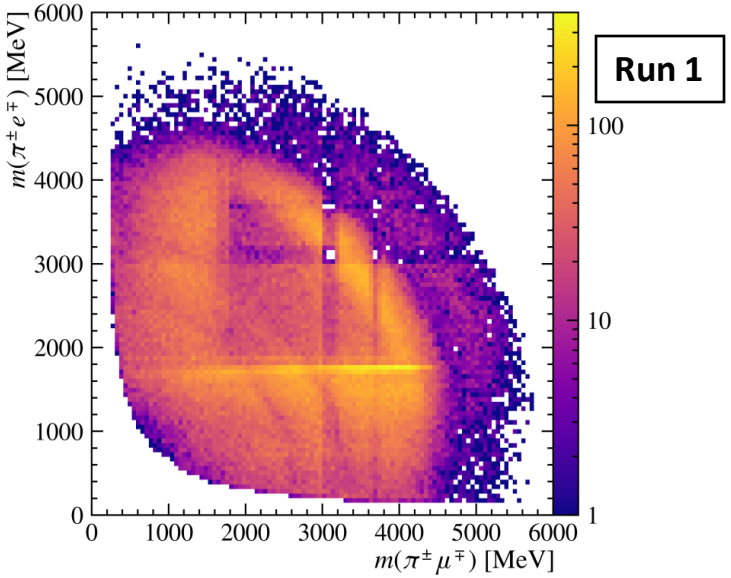

**Thanks for your attention!**

# Backups

# Event Selection

- Use Bu2KLL_ {mm,me(SS)}Line stripping lines
- L0 + HLT1 muon + HLT2 topo trigger lines aligned between signal and normalisation
- Loose fiducial selections applied to align with PIDCalib datasets
- Remove clone (ghost) tracks with track OA (ProbNNGhost) selection
- Apply truth matching (0|10|50|60) to MC samples
- Apply single candidate selection after full selection chain




- Apply dedicated vetoes to reject physics background
- $J/\psi\ (\psi(2S)) \to \ell^\pm\ell^\mp$ mis-ID: $q^2 \notin [9, 10.1]\ \&\ [13.0, 14.2]$ MeV
- SL $B^\pm \to D^0\ell^\pm\nu$ veto: $m_{\pi^\pm\ell^\mp} > 1885$ MeV
- $D^0 \to K^\pm\pi^\mp$ veto: $\left|m_{\pi^\pm_{\to K}\ell^\mp_{\to\pi}} - m_{D^0}\right| > 25$ MeV

# Event Selection

## Preselection requirements

| Description | Particles or Event | Requirement |
|---|---|---|
| $B^{\pm}$ mass window | $B$ | $4500 < m_{\pi^+\mu^+} < 6000\,\text{MeV}$ |
| Kinematic | Event | nTracks $< 300$ |
| | $B$ | $2 < \eta < 5$ |
| | $\pi$ | $p_T > 250\,\text{MeV}$<br>$2000 < p < 200000\,\text{MeV}$<br>$1.7 < \eta < 5.3$ |
| | $\mu$ | $800 < p_T < 30000\,\text{MeV}$<br>$3000 < p < 200000\,\text{MeV}$ |
| | $e$ | $500 < p_T < 30000\,\text{MeV}$<br>$3000 < p < 200000\,\text{MeV}$ |
| Quality | $\pi$ | hasRich $== 1$<br>$\chi^2_{\text{track}} < 3$<br>`Track_GhostProb` $< 0.4$ |
| | $\mu$ | InAccMuon $== 1$<br>hasRich $== 1$<br>hasMuon $== 1$<br>$\chi^2_{\text{track}} < 3$<br>`Track_GhostProb` $< 0.4$ |
| | $e$ | hasRich $== 1$<br>hasCalo $== 1$<br>$\chi^2_{\text{track}} < 3$<br>`Track_GhostProb` $< 0.4$ |
| | $\pi\mu$ pair | Opening Angle $> 0.0005$ |
| | $\pi e$ pair | Opening Angle $> 0.0005$ |
| | $\mu e$ pair | Opening Angle $> 0.0005$ |

## Stripping line requirements

| Particle | Requirement |
|---|---|
| $B^{\pm}$ | $\lvert m - m_{\text{PDG}}\rvert < 1500\,\text{MeV}$<br>$\chi^2_{\text{vtx}}/\text{dof} < 9$<br>$\chi^2_{\text{FD}} > 100$<br>$\chi^2_{\text{IP}} < 25$<br>DIRA $> 0.9995$ |
| $K^{\pm}/\pi^{\pm}$ | $p_T > 400\,\text{MeV}$<br>$\chi^2_{\text{IP}} > 9$ |
| $e^{\pm}$ | $p_T > 300\,\text{MeV}$<br>$\chi^2_{\text{IP}} > 9$<br>PID$e > 0$ |
| $\mu^{\pm}$ | $p_T > 300\,\text{MeV}$<br>$\chi^2_{\text{IP}} > 9$<br>hasMuon $== 1$<br>isMuon $== 1$ |
| $\mu e(\mu\mu, ee)$ pair | $p_T > 0\,\text{MeV}$<br>$m_{\mu e(\mu\mu,ee)} < 5500\,\text{MeV}$<br>$p_T > 0\,\text{MeV}$<br>$\chi^2_{\text{vtx}}/\text{dof} < 9$<br>$\chi^2_{\text{FD}} > 16$<br>$\chi^2_{\text{IP}} > 0$ |

## Charmonium vetoes

| Mass Veto [ MeV ] |
|---|
| $\lvert m_{\mu^\pm e^\mp \to \mu} - m_{J/\psi}\rvert > 100$ |
| $\lvert m_{\mu^\pm e^\mp \to \mu} - m_{\psi(2S)}\rvert > 55$ |
| $\lvert m_{\mu^\pm \to e} e^\mp - m_{J/\psi}\rvert > 100$ |
| $\lvert m_{\mu^\pm \to e} e^\mp - m_{\psi(2S)}\rvert > 55$ |
| $\lvert m_{\pi^\pm \to \mu} \mu^\mp - m_{J/\psi}\rvert > 100$ |
| $\lvert m_{\pi^\pm \to \mu} \mu^\mp - m_{\psi(2S)}\rvert > 55$ |
| $\lvert m_{\pi^\pm \to e} e^\mp - m_{J/\psi}\rvert > 100$ |
| $\lvert m_{\pi^\pm \to e} e^\mp - m_{\psi(2S)}\rvert > 55$ |

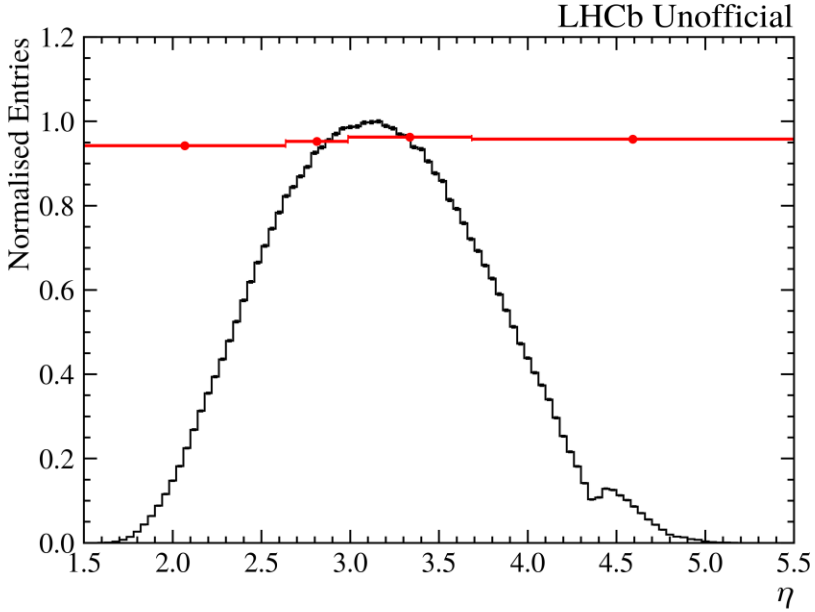
# Event Selection

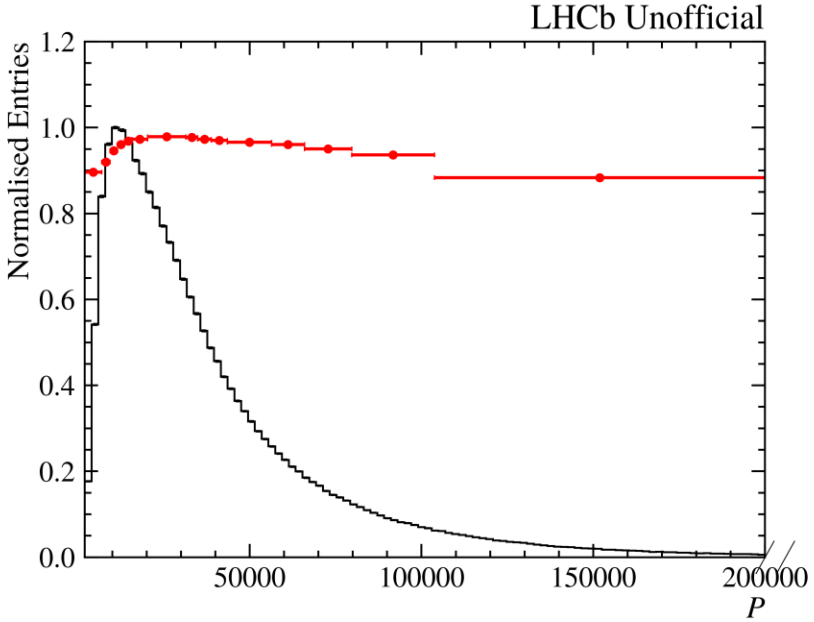
- Only TOS triggers considered
- Inclusion of L0Electron trigger improves signal sensitivity by ~10%
  - Prefer not to do this to mis-align search and normalisation channel

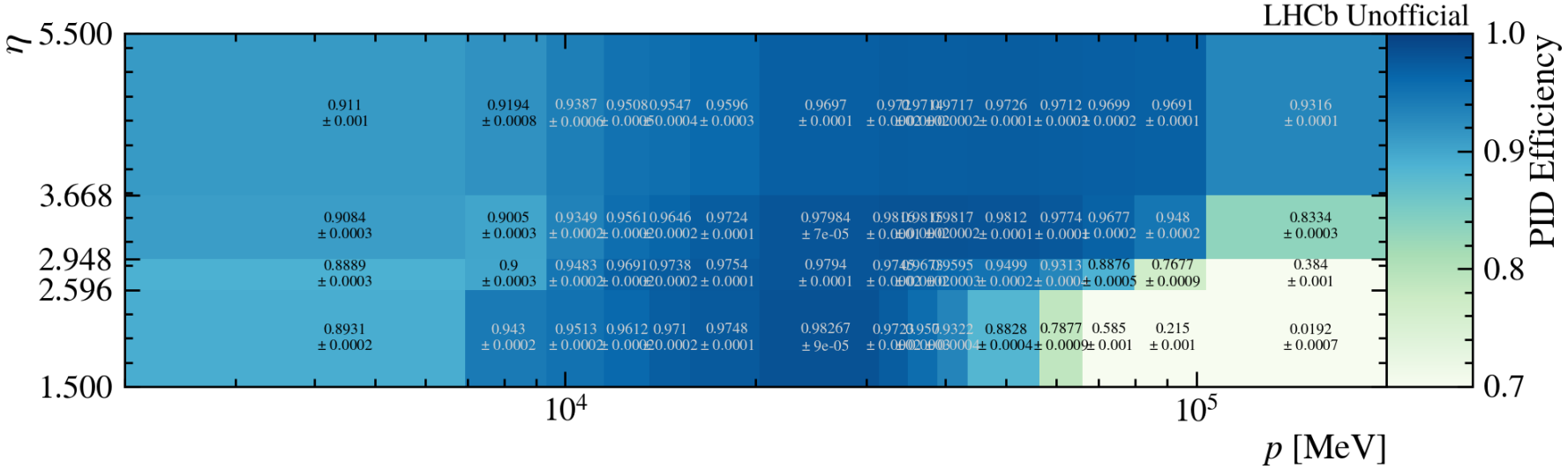
| **Trigger Level** | **Trigger Lines** | |
|---|---|---|
| | **Run 1** | **Run 2** |
| **L0** ($\mu^{\pm}$) | L0MuonDecision_TOS | |
| **HLT1** ($B^{\pm}$) | TrackAllL0Decision_TOS<br>TrackMuonDecision_TOS | TrackMVADecision_TOS<br>TwoTrackMVADecision_TOS |
| **HLT2** ($B^{\pm}$) | Topo2BodyBBDTDecision_TOS<br>Topo3BodyBBDTDecision_TOS<br>TopoMu2BodyBBDTDecision_TOS<br>TopoMu3BodyBBDTDecision_TOS | Topo2BodyDecision_TOS<br>Topo3BodyDecision_TOS<br>TopoMu2BodyDecision_TOS<br>TopoMu3BodyDecision_TOS |

# MC Reweighting - PID

- Re-strip MC samples to remove stripping PID requirements
- PID efficiencies for various PID selections calculated using pidcalib bins of $p$ and $\eta$
- Bin optimisation with $R_x$ method
  - Merge iso-populated bins if adjacent bin values are < 5σ apart




**2018 MU**

$\pi$_ProbNNpi > 0.2

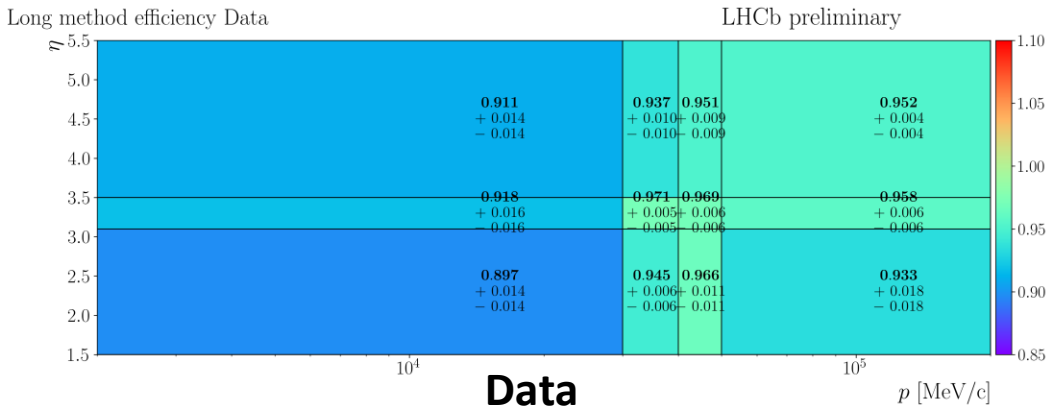
$\pi$_ProbNNk < 0.8

$\pi$_ProbNNp < 0.8

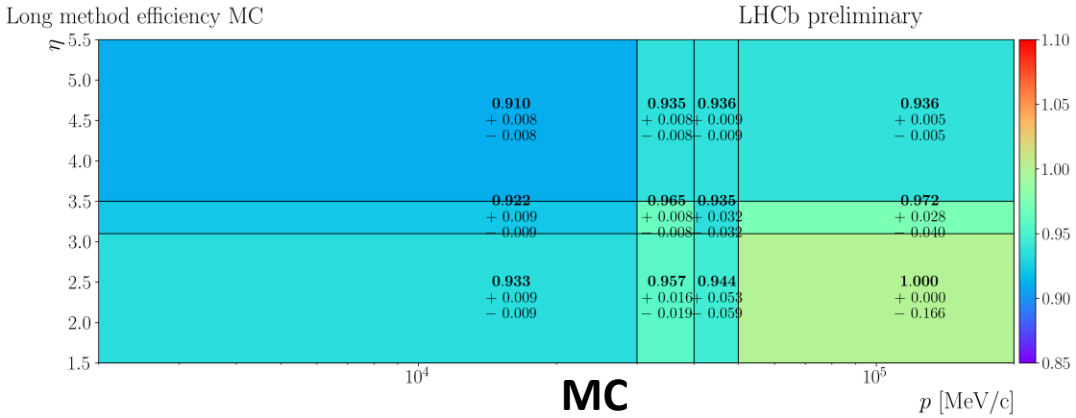
$\pi$_isMuon == 0

# MC Reweighting - Tracking

- Tracking efficiencies calculated using trackcalib2 in bins of $p$ and $\eta$
- Calibration sample selection aligned to analysis selection

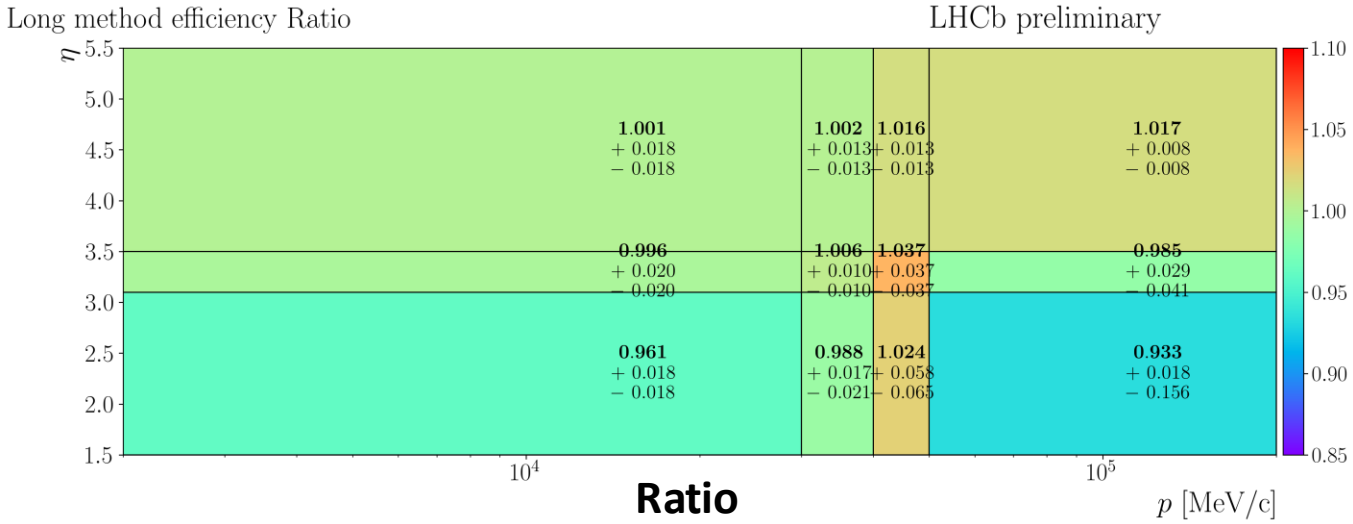



**Data**




**MC**

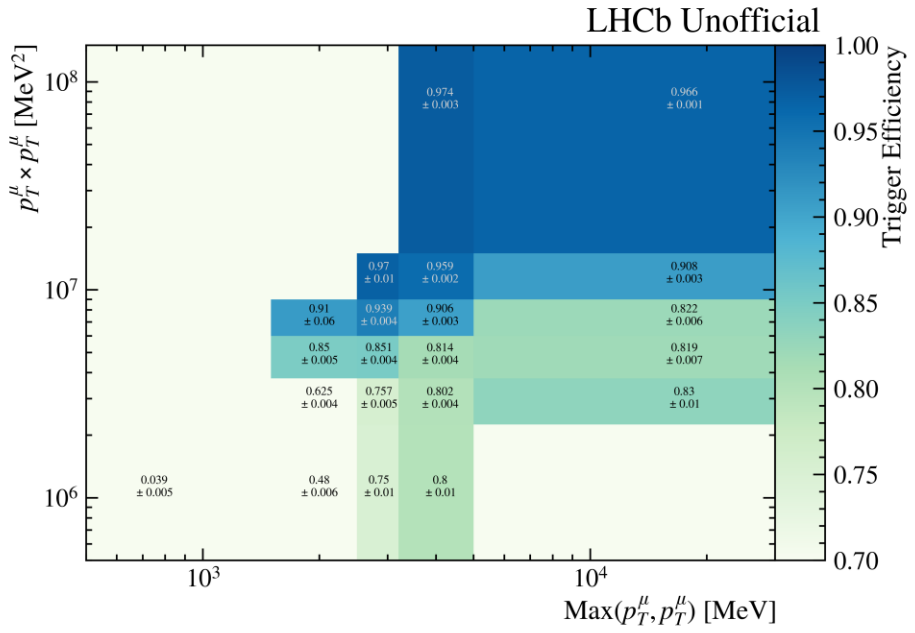
**2018**




**Ratio**

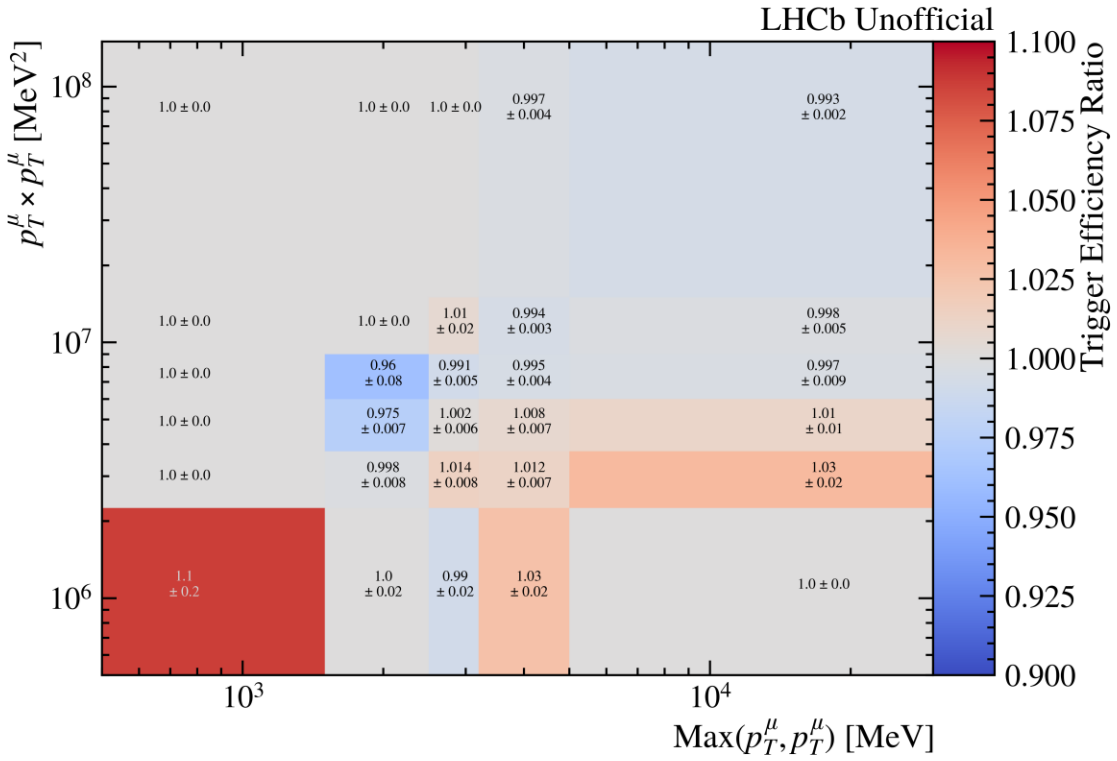
# MC Reweighting - Trigger

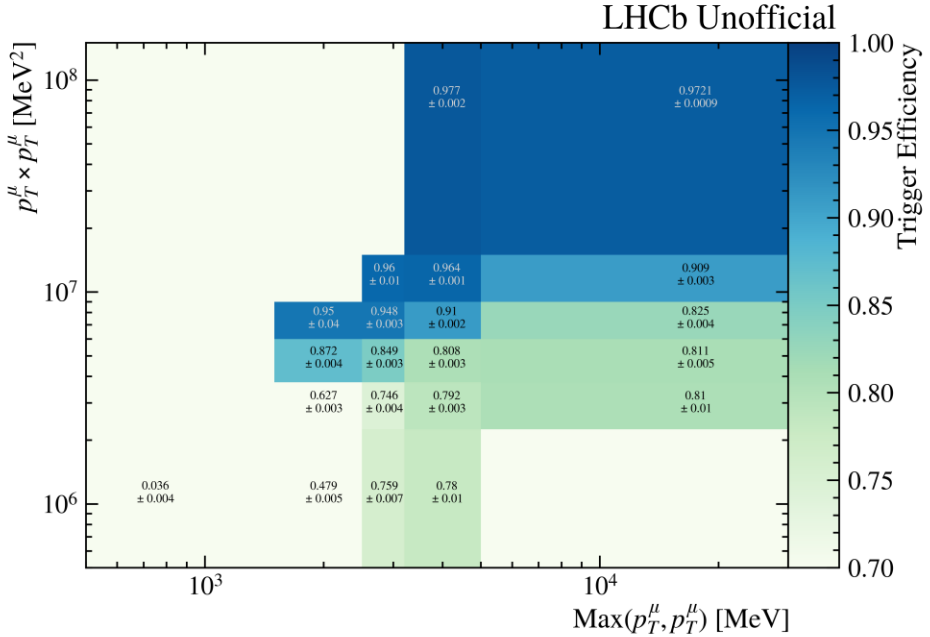
- TISTOS method to correct trigger efficiency using $B^{\pm} \to J/\psi\, K^{\pm}$ calculated in bins of $\max({p_T}^{l_1}, {p_T}^{l_2})$ and ${p_T}^{l_1} \times {p_T}^{l_2}$




**Data**

**2018**

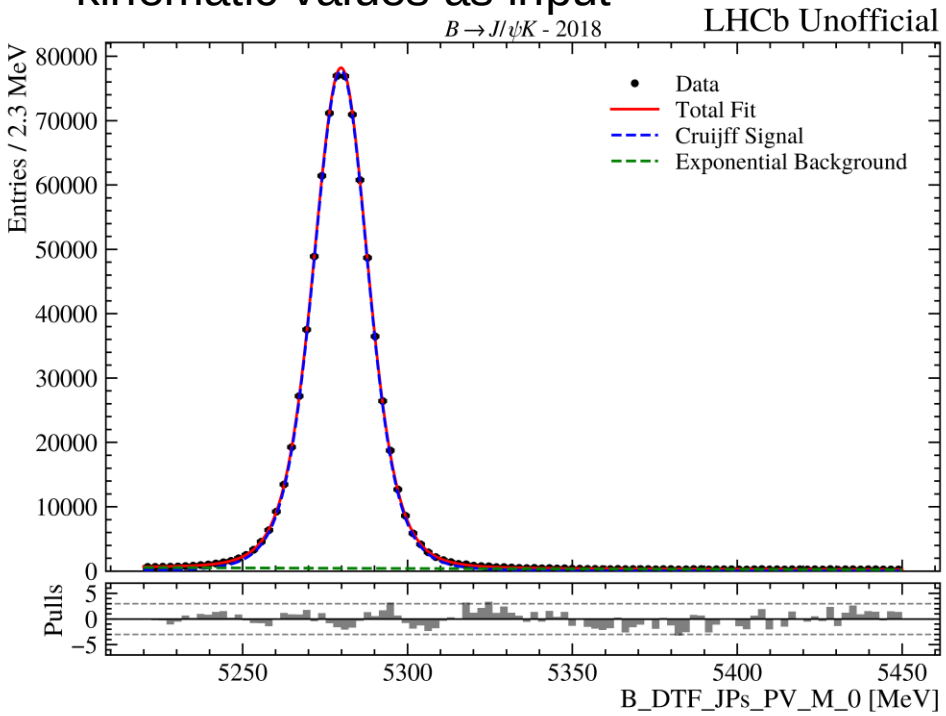


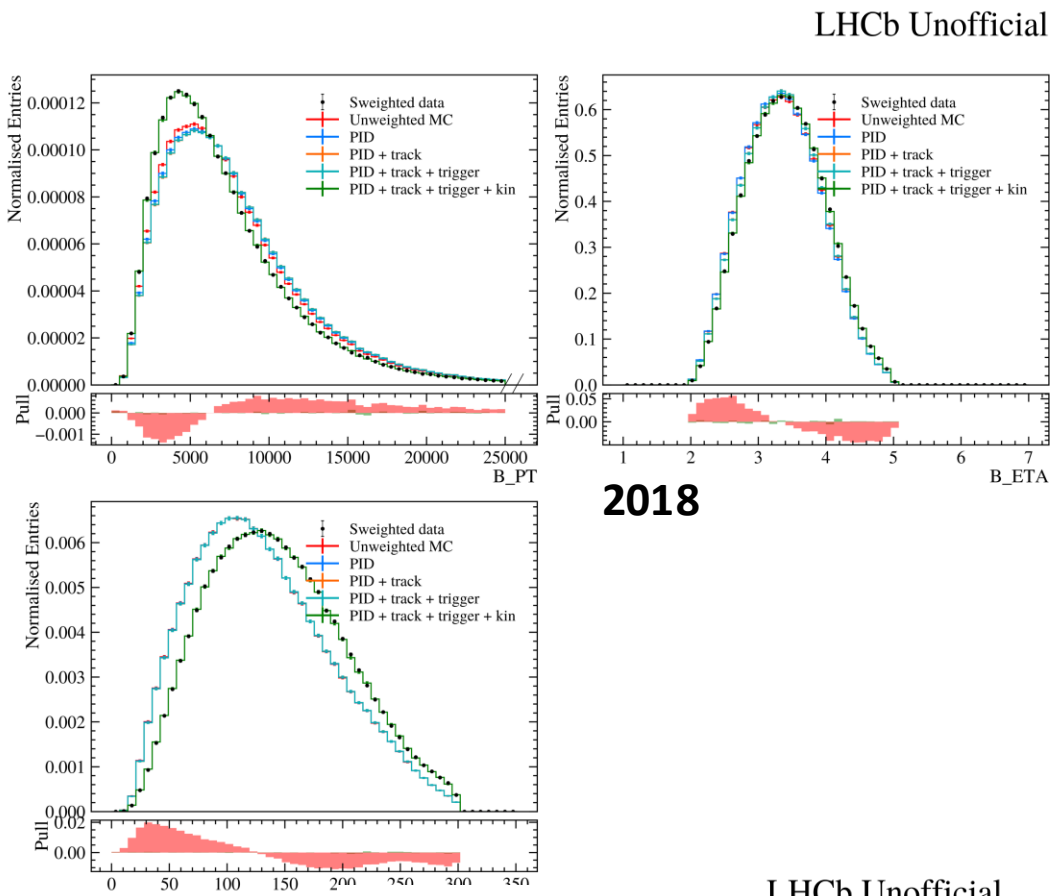



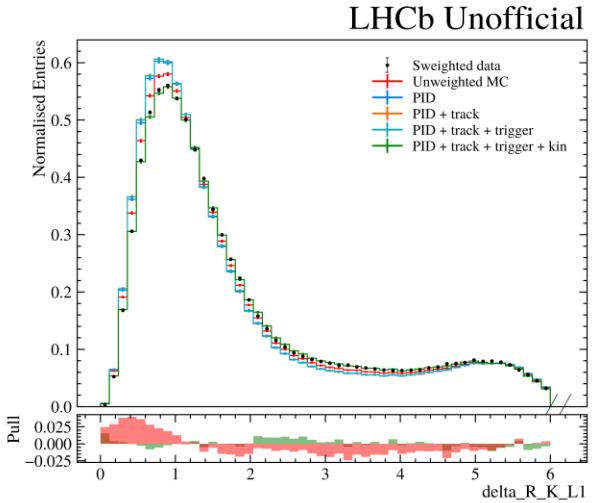

**MC**

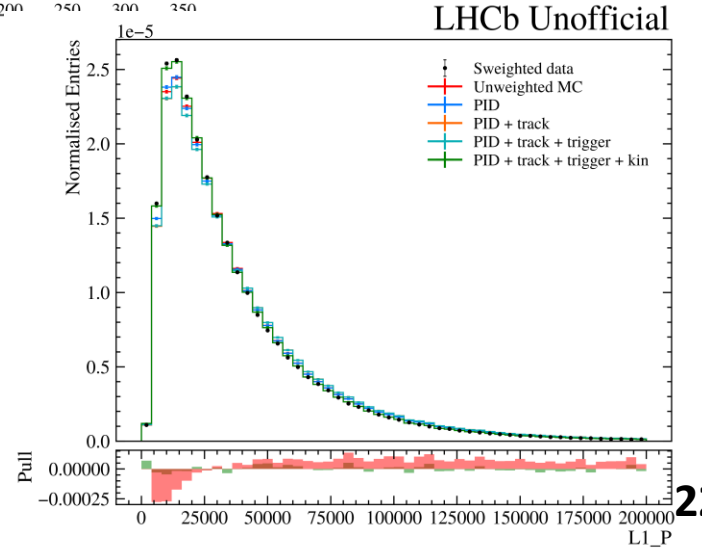
# MC Reweighting - Kinematic

- Apply a loose PID selection to $B^{\pm} \to J/\psi\left(\to \mu^{\mp}\mu^{\pm}\right)K^{\pm}$
- Fit to Cruijff + exponential to extract sWeights
- Train GBReweighter to correct $B^{+}$ production kinematics with reweighted (track, trigger, PID) MC and sWeighted data
- Train per year with input variables $P_T(B),\ \eta(B),\ \mathrm{nTracks}$
- Apply same trained model to gen level samples with true kinematic values as input



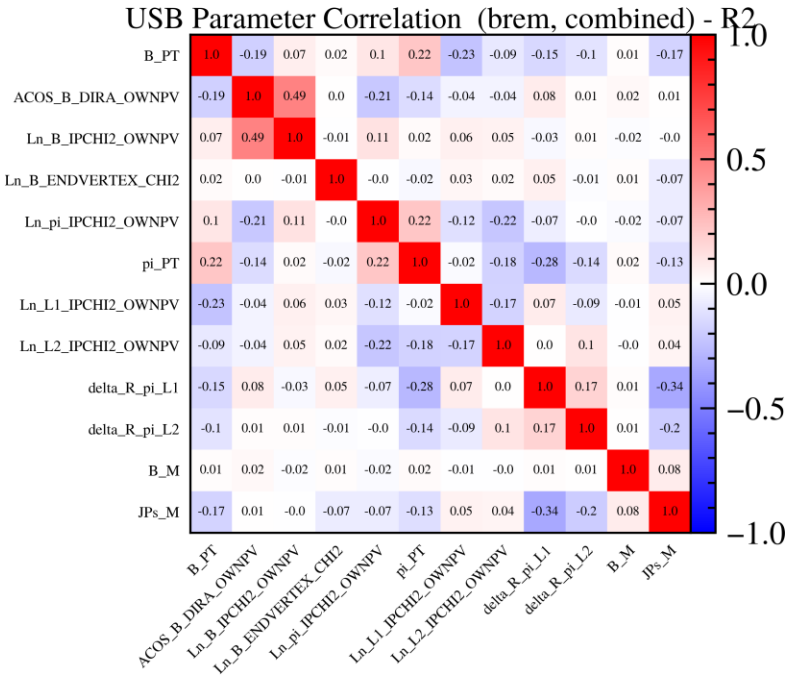



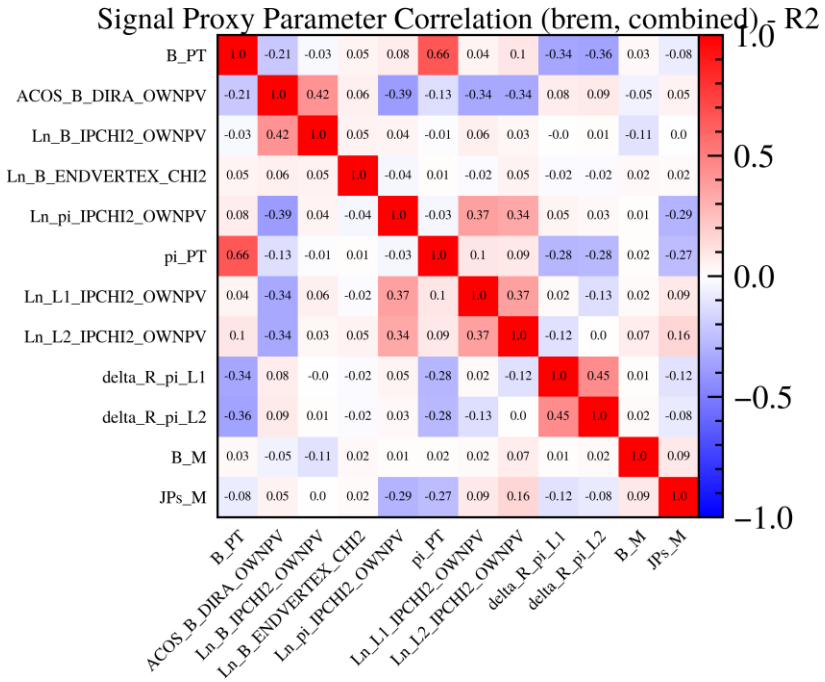



# BDT Training

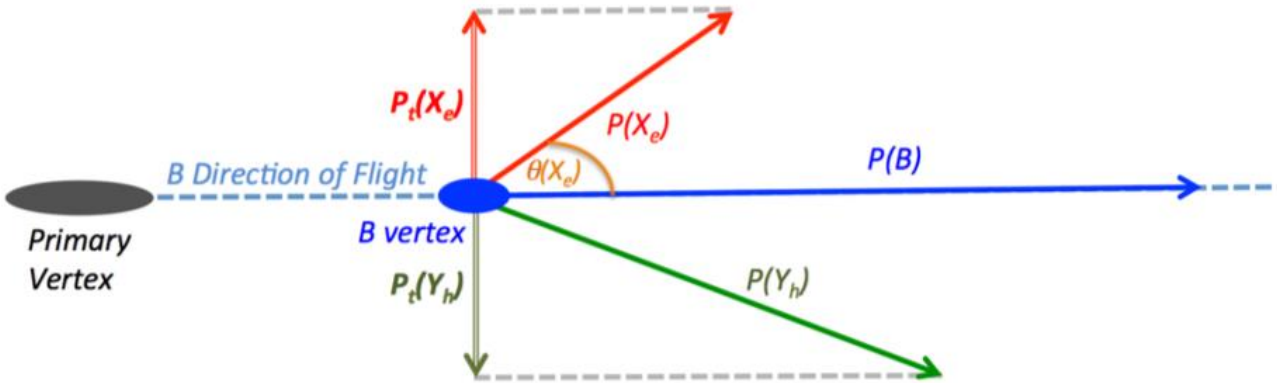
- Apply loose PID selection to data
- Train XGBoost BDT on truth matched reweighted signal MC against data upper sideband to reject combinatorial background
  - Train in each analysis category
  - 5 k-folds
  - Various kinematic and topological variables of mother and daughter particles
- Model parameters optimised with Bayesian search of hyperparameter space using Optuna package
  - Maximise $\mathrm{FoM} = \overline{\mathrm{AUC}_{\mathrm{Test}}}\left(1 - \frac{|\overline{\mathrm{AUC}_{\mathrm{Train}}} - \overline{\mathrm{AUC}_{\mathrm{Test}}}|}{\overline{\mathrm{AUC}_{\mathrm{Train}}}}\right)$
  - $B^{\pm}\{p_T, \mathrm{DIRA}, \chi^2_{\mathrm{IP}}, \chi^2_{\mathrm{vtx}}\}$
  - $\pi^{\pm}\{p_T, \chi^2_{\mathrm{IP}}\}$
  - $\mu^{\pm}(\chi^2_{\mathrm{IP}})$
  - $e^{\pm}(\chi^2_{\mathrm{IP}})$
  - $\Delta R(\pi, \mu) = \sqrt{(\phi(\pi) - \phi(\mu))^2 + (\eta(\pi) - \eta(\mu))^2}$
  - $\Delta R(\pi, e) = \sqrt{(\phi(\pi) - \phi(e))^2 + (\eta(\pi) - \eta(e))^2}$

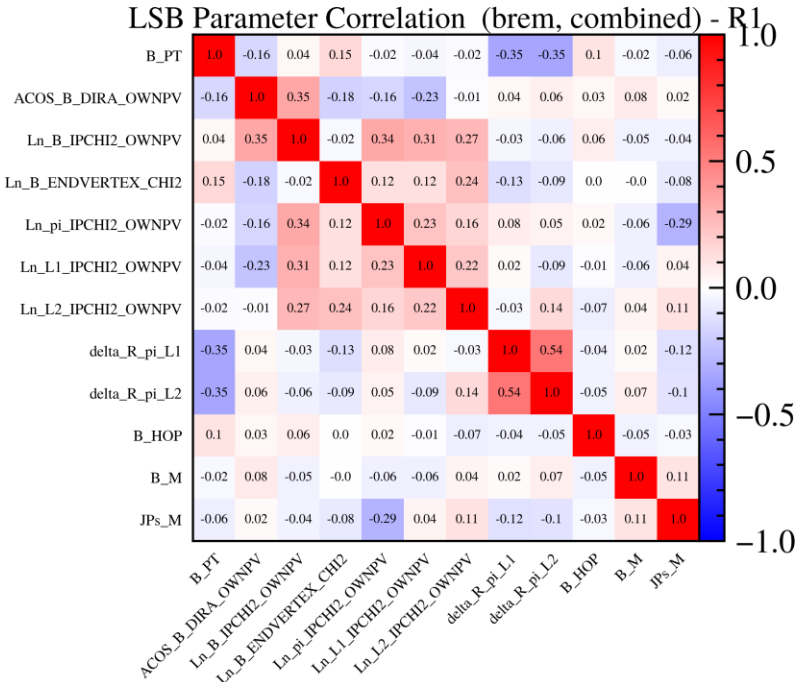


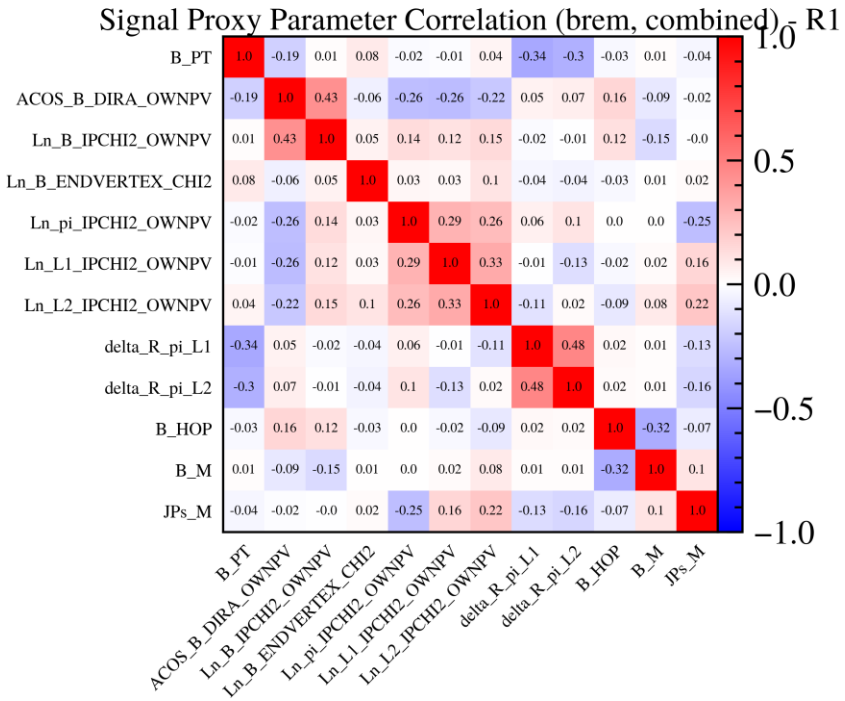



# BDTHOP Training

- After BDT selection, lower sideband is polluted with partially reconstructed background
- Train BDTHOP against this background **after BDT selection**
  - Same variables as BDT, but with addition of $B_{\mathrm{HOP}}$
  - 5 k-folds
- Reduced statistics and increased similarity between datasets
  - Model complexity is reduced
  - Hyperparameters optimised with same method as BDT



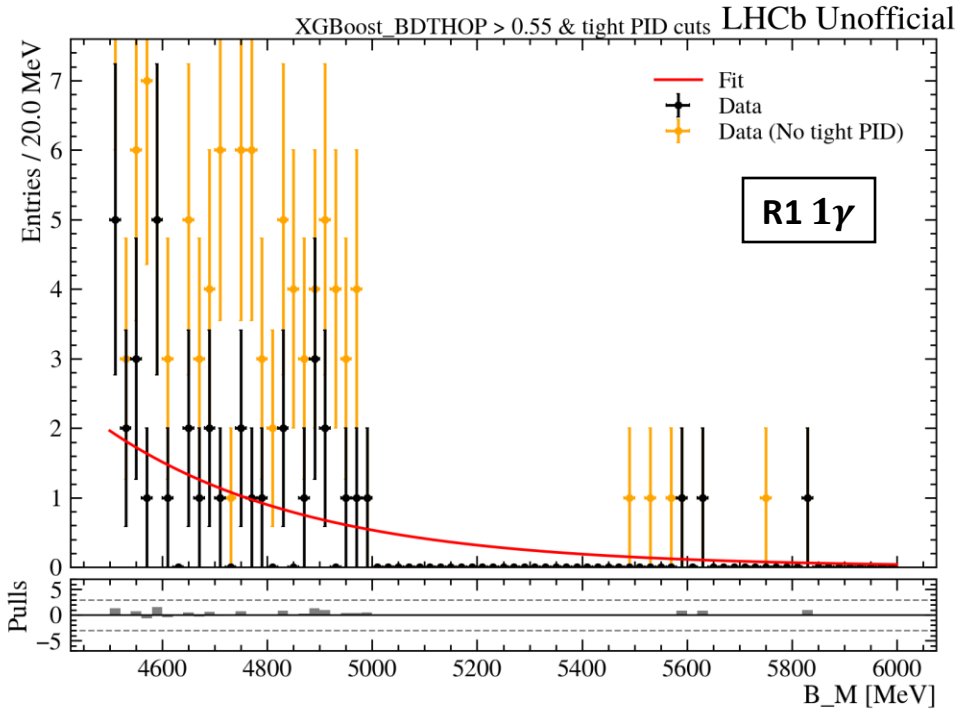



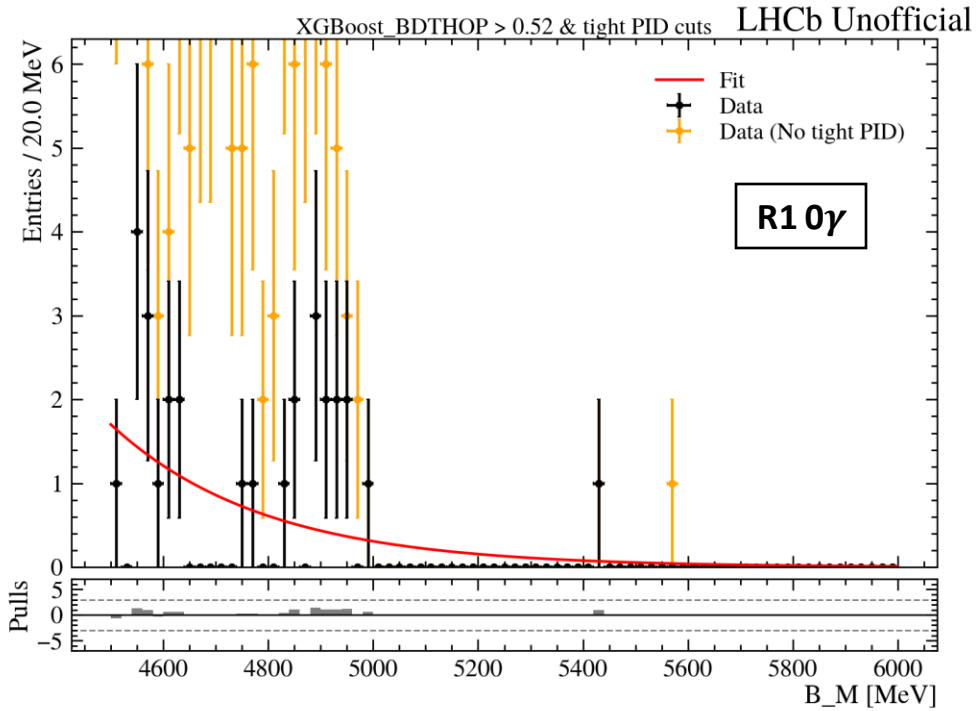



# PID Selection

- Tight PID cuts applied to reduce physics background
- Essential due to much higher rate $b \rightarrow s$ processes
- No optimisation at present time
- Will likely be tweaked depending on results of dedicated background study
- Effective at removing remaining mis-ID in LSB

| Particle | Tight PID Selection |
|---|---|
| $\pi^{\pm}$ | ProbNNpi > 0.8<br>ProbNNk < 0.2<br>ProbNNp < 0.2 |
| $\mu^{\pm}$ | ProbNNmu > 0.8 |
| $e^{\pm}$ | ProbNNe > 0.8<br>PIDe > 4 |



# Efficiencies

## Normalisation

| **Efficiency** (%) Categories | Analysis Categories Run 1 | Run 2 |
|---|---|---|
| Generator | $16.13 \pm 0.02$ | $17.37 \pm 0.02$ |
| Strip. + Reco. + Truth | $36.56 \pm 0.02$ | $32.69 \pm 0.01$ |
| B Mass Window | $92.96 \pm 0.01$ | $96.109 \pm 0.008$ |
| Kinematic | $79.45 \pm 0.02$ | $81.79 \pm 0.02$ |
| Quality | $86.13 \pm 0.02$ | $86.85 \pm 0.02$ |
| $q^2$ Selection | $91.22 \pm 0.02$ | $91.34 \pm 0.01$ |
| cc Veto | $100.0 \pm 3e-05$ | $100.0 \pm 2e-05$ |
| DTF Status | $99.99994 \pm 7e-05$ | $99.99985 \pm 7e-05$ |
| L0 | $86.43 \pm 0.03$ | $80.19 \pm 0.02$ |
| HLT1 | $93.03 \pm 0.02$ | $97.85 \pm 0.01$ |
| HLT2 | $89.46 \pm 0.03$ | $92.71 \pm 0.02$ |
| BDT | $56.0 \pm 0.04$ | $63.28 \pm 0.03$ |
| Tight PID | $44.35 \pm 0.06$ | $40.37 \pm 0.04$ |
| Single Candidate | $99.936 \pm 0.005$ | $99.928 \pm 0.004$ |
| Total | $0.592 \pm 0.001$ | $0.5133 \pm 0.0009$ |

## Signal

| **Efficiency** (%) Categories | Analysis Categories Run 1 $1\gamma$ | Run 1 $0\gamma$ | Run 2 $1\gamma$ | Run 2 $0\gamma$ |
|---|---|---|---|---|
| Generator | $15.98 \pm 0.01$ | $15.98 \pm 0.01$ | $17.24 \pm 0.01$ | $17.24 \pm 0.01$ |
| Strip. + Reco. + Truth | $14.88 \pm 0.02$ | $20.61 \pm 0.02$ | $13.0 \pm 0.01$ | $16.36 \pm 0.01$ |
| B Mass Window | $89.55 \pm 0.05$ | $77.57 \pm 0.05$ | $92.42 \pm 0.03$ | $85.37 \pm 0.03$ |
| Kinematic | $76.7 \pm 0.07$ | $66.84 \pm 0.07$ | $80.85 \pm 0.04$ | $74.54 \pm 0.04$ |
| Quality | $89.84 \pm 0.05$ | $82.19 \pm 0.07$ | $90.08 \pm 0.04$ | $85.76 \pm 0.04$ |
| $q^2$ Selection | $99.55 \pm 0.01$ | $99.47 \pm 0.01$ | $99.551 \pm 0.008$ | $99.457 \pm 0.009$ |
| cc Veto | $80.33 \pm 0.07$ | $81.12 \pm 0.08$ | $80.3 \pm 0.05$ | $80.98 \pm 0.05$ |
| DTF Status | $100.0 \pm 0.0002$ | $100.0 \pm 0.0002$ | $100.0 \pm 0.0001$ | $100.0 \pm 0.0001$ |
| D0 Veto | $75.47 \pm 0.09$ | $74.01 \pm 0.09$ | $75.43 \pm 0.06$ | $74.13 \pm 0.06$ |
| L0 | $56.8 \pm 0.1$ | $51.0 \pm 0.1$ | $52.63 \pm 0.08$ | $47.89 \pm 0.08$ |
| HLT1 | $89.1 \pm 0.1$ | $88.8 \pm 0.1$ | $96.66 \pm 0.04$ | $97.45 \pm 0.04$ |
| HLT2 | $82.8 \pm 0.1$ | $87.4 \pm 0.1$ | $90.55 \pm 0.07$ | $91.29 \pm 0.07$ |
| BDT | $52.2 \pm 0.2$ | $45.0 \pm 0.2$ | $46.9 \pm 0.1$ | $46.4 \pm 0.1$ |
| BDT HOP | $65.9 \pm 0.2$ | $64.1 \pm 0.3$ | $66.2 \pm 0.2$ | $80.3 \pm 0.1$ |
| Tight PID | $59.9 \pm 0.3$ | $54.6 \pm 0.4$ | $52.4 \pm 0.2$ | $44.2 \pm 0.2$ |
| Single Candidate | $99.95 \pm 0.02$ | $99.33 \pm 0.08$ | $99.93 \pm 0.02$ | $99.33 \pm 0.05$ |
| Total | $0.0735 \pm 0.0006$ | $0.0524 \pm 0.0005$ | $0.0605 \pm 0.0004$ | $0.0595 \pm 0.0004$ |

$$\alpha_{\mathrm{SES}} = \frac{1}{\varepsilon_{\mathrm{sig}}} \times \frac{\varepsilon_{\mathrm{norm}}}{N_{\mathrm{norm}}} \times BF_{\mathrm{norm}}$$

| **Run** | **$0\gamma$ SES** | **$1\gamma$ SES** |
|---|---|---|
| **R1** | $(2.80 \pm 0.06) \times 10^{-9}$ | $(2.00 \pm 0.04) \times 10^{-9}$ |
| **R2** | $(8.91 \pm 0.18) \times 10^{-10}$ | $(8.77 \pm 0.18) \times 10^{-10}$ |

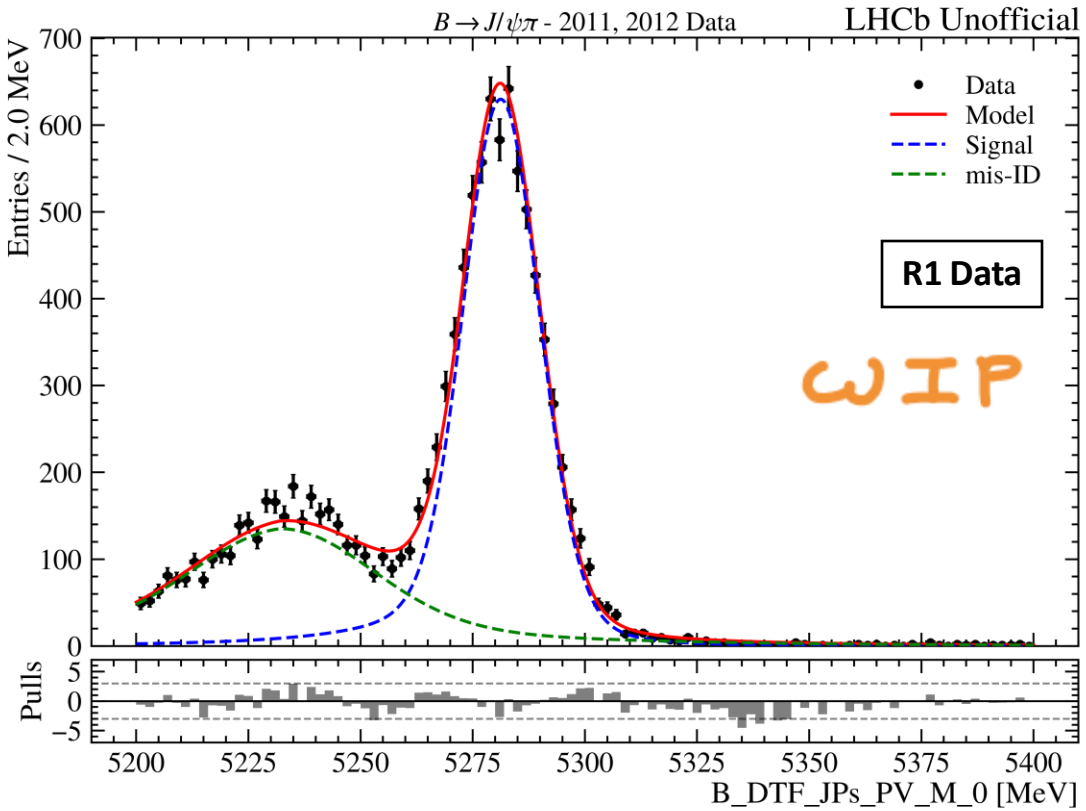
**Total SES:** $(3.20 \pm 0.06) \times 10^{-10}$

# Efficiencies Crosscheck

- Crosscheck our efficiencies with $B^{\pm} \to J/\psi\,(\to \mu^{\mp}\mu^{\pm})\pi^{\pm}$
- Calculate $R = \frac{\mathrm{BF}(B^{\pm}\to J/\psi\pi^{\pm})}{\mathrm{BF}(B^{\pm}\to J/\psi K^{\pm})} = \frac{N(K)/\varepsilon(K)}{N(\pi)/\varepsilon(\pi)}$
- $R_{\mathrm{PDG}} = 0.0384 \pm 0.0011$
- $R_{\mathrm{calculated}} = 0.0380 \pm 0.0005$
- Results differ by 0.38σ

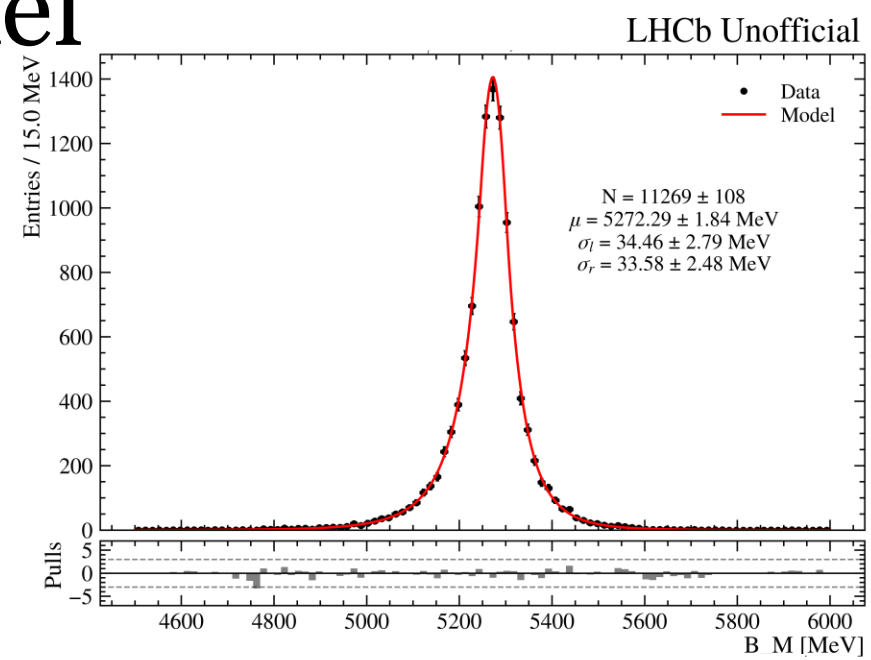
| **Efficiency (%)** Categories | Analysis Categories Run 1 0γ | Run 2 0γ |
|---|---|---|
| Generator | $15.6 \pm 0.01$ | $16.7 \pm 0.5$ |
| Strip. + Reco. + Truth | $37.45 \pm 0.02$ | $33.79 \pm 0.01$ |
| B Mass Window | $93.54 \pm 0.02$ | $96.443 \pm 0.008$ |
| Kinematic | $79.36 \pm 0.03$ | $81.78 \pm 0.02$ |
| Quality | $79.03 \pm 0.04$ | $79.72 \pm 0.02$ |
| $q^2$ Selection | $91.32 \pm 0.03$ | $91.4 \pm 0.01$ |
| cc Veto | $100.0 \pm 6e-05$ | $100.0 \pm 2e-05$ |
| DTF Status | $100.0 \pm 6e-05$ | $99.99991 \pm 6e-05$ |
| L0 | $87.03 \pm 0.04$ | $79.62 \pm 0.02$ |
| HLT1 | $93.33 \pm 0.03$ | $97.97 \pm 0.009$ |
| HLT2 | $89.84 \pm 0.04$ | $92.38 \pm 0.02$ |
| BDT | $32.64 \pm 0.06$ | $26.73 \pm 0.03$ |
| Tight PID | $63.4 \pm 0.1$ | $63.55 \pm 0.06$ |
| Single Candidate | $99.935 \pm 0.007$ | $99.922 \pm 0.004$ |
| Total | $0.452 \pm 0.001$ | $0.36 \pm 0.01$ |

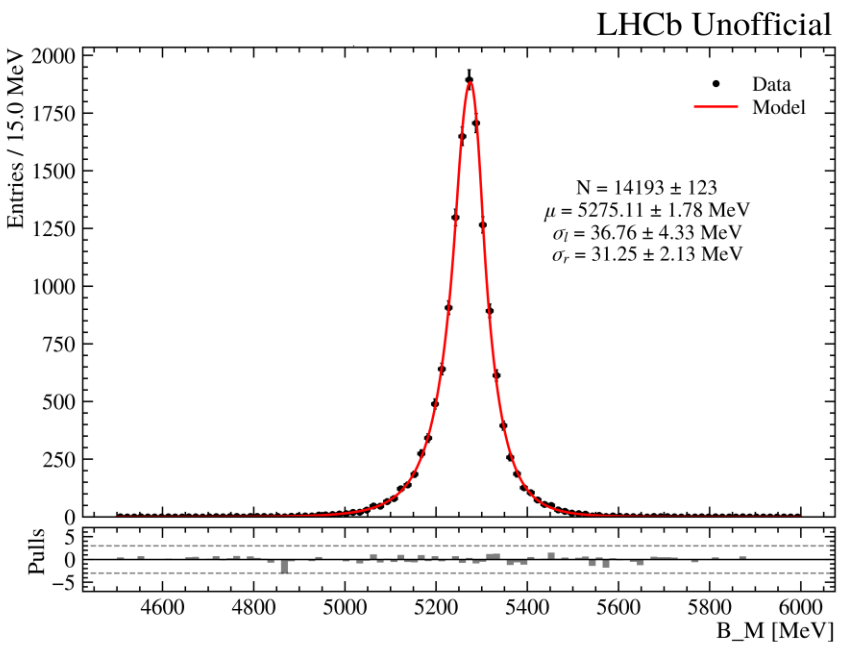
- Signal shape DSCB + Gaussian core
- Include JpsK mis-ID component
- Shape taken from MC with PID weights applied from pidcalib2
- Shape fixed apart from sigmas



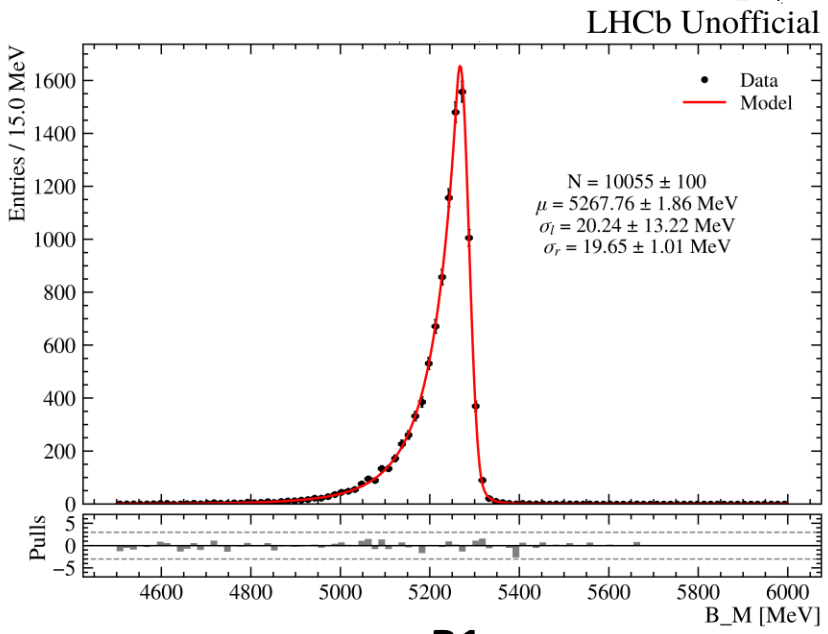
# Signal Fit Model

- Signal MC fits in each analysis category after full selection
- Signal shape described well by DSCB

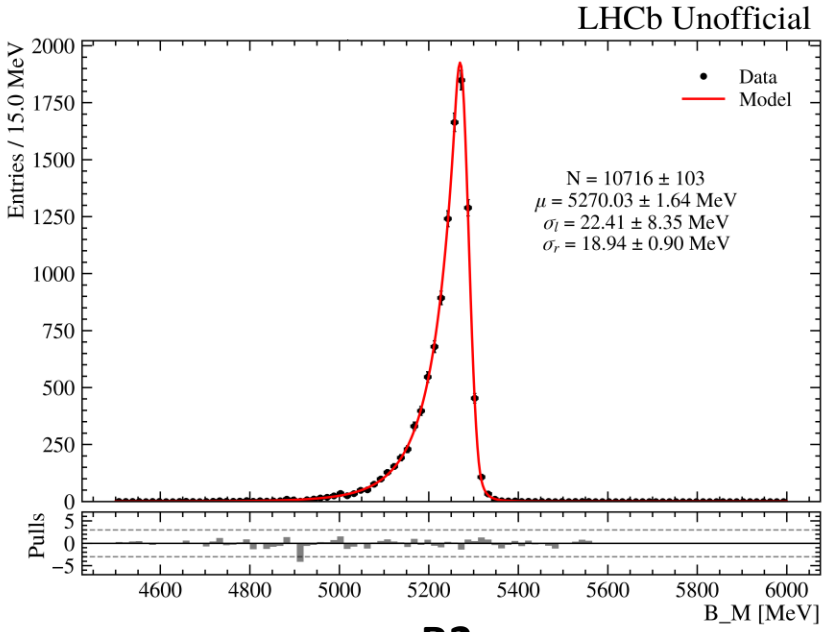



**1$\gamma$**




**R1**

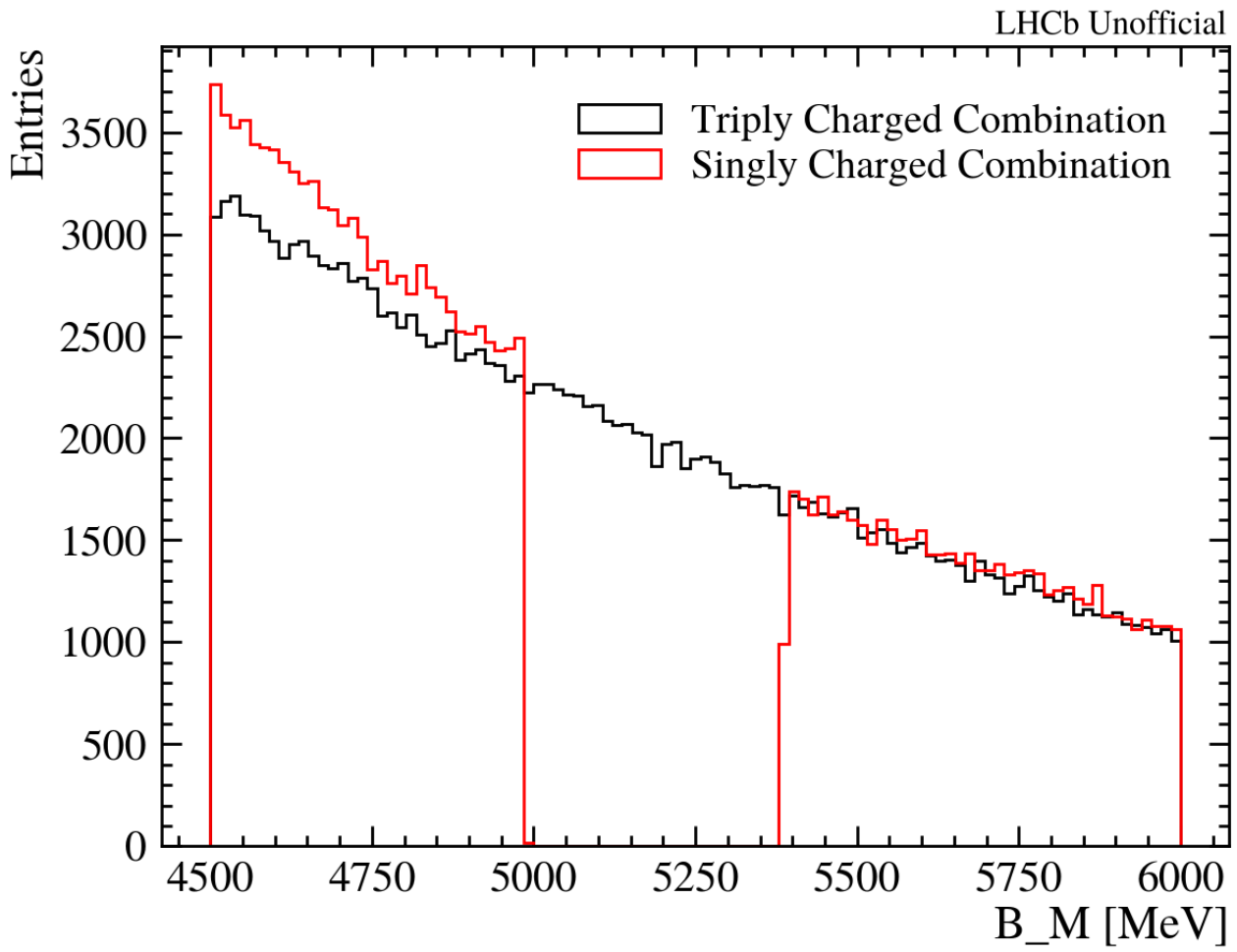



**R2**

**0$\gamma$**

# Combinatorial Proxy

- Use Triply Charged (TC) SS lepton sample with B mass < 6000MeV
- Do this to remove any possibility of unblinding HNL analysis




**HNL Analysis**

Opposite Sign (OS) leptons

→ **Dirac and Majorana neutrino (LFV)**

$B^+_{(c)} \to \mu^+ N(\to e^- \pi^+)$, $B^+_{(c)} \to e^+ N(\to \mu^- \pi^+)$

Same Sign (SS) leptons

→ **only Majorana neutrino (LFV and LNV)**

$B^+_{(c)} \to \mu^+ N(\to e^+ \pi^-)$, $B^+_{(c)} \to e^+ N(\to \mu^+ \pi^-)$