A dynamical definition of the sphere of influence of the Earth*

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Abstract. Planetary close encounters can drastically modify the trajectories of small bodies, such as asteroids, comets, or spacecraft. The phenomenon is significant in different contexts, from the problem of impact monitoring to the design of interplanetary trajectories. One of the classical methods used to model close encounters is the patched-conic technique, consisting in approximating an n-body orbit as a sequence of two-body orbits, patched together at the boundary of the planetary spheres of influence. The concept of the planetary sphere of influence was firstly introduced by Laplace in 1805 in [1] to study the effects of close encounters of comets with Jupiter and the Earth. Since then, several definitions have been given, generating ambiguity. The most known are the definitions formulated by Laplace himself and Hill, both based on the features of the planet of interest, i.e. its mass and its heliocentric distance. The same holds also for the less known definition given by Chebotarev in [2]. Moreover, there exist more recent works (see [3, 4]), showing that the definition of the sphere of influence should also depend on the state vector of the small body involved, specifically on its velocity with respect to the planet. The purpose of this work is to determine the most suitable sphere of influence to be employed in the framework of the patched-conic method to reproduce the main features of close encounters and of the post-encounter trajectories with sufficient accuracy. The definition we propose takes into account both the position and the velocity of the small body with respect to the planet, in light of the literature. In particular, our study is focused on the sphere of influence of the Earth. It relies on an optimisation process, based on a comparison between the patched-conic orbits obtained by imposing different spheres of influence and the orbit computed in the framework of the circular restricted three-body problem; a suitable target function is minimised with respect to the radius of the sphere of influence. During the presentation, we describe

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the optimisation procedure implemented for the planar problem and we discuss the results obtained and the outcomes of some tests performed to validate our definition. Then, we show how to extend our selection technique to the three-dimensional case. Finally, we discuss a possible application for the study of resonant returns. We speak about resonant returns when a close encounter between an asteroid and a planet results in an orbital resonance between these two bodies so that a new close encounter will occur. There already exists an approximate analytical model based on an extended Öpik theory (see [5]). However, we show that it is possible to introduce a Hamiltonian description of the problem by employing the patched-conic method. For a suitable subspace of the phase space, a chain of canonical transformations can be built linking the encounter state before the first encounter to the state at the resonant return. We check that our model gives good approximations by comparison with a circular restricted three-body evolution. Finally, we describe the domain of our canonical transformation and its image at the second encounter.

Keywords: sphere of influence · patched-conic method · Earth.

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