# Dawn's probability of capture into 1:1 ground-track resonance around Vesta *

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## 1 Introduction

The DAWN mission demonstrates the possibility of using low-thrust propulsion for most of the mission duration and it approaches the asteroid Vesta from the high-altitude mission orbit (HAMO) to the low-altitude mission orbit (LAMO). As the spacecraft slowly advances toward the asteroid, there is a possibility that it is captured by the 1:1 ground-track resonance (GTR). The spacecraft at each revolution encounters the same gravitational configuration, which causes the orbit eccentricity and inclination to change noticeably. Small variations in the initial state of the spacecraft make a difference in whether the spacecraft escapes the resonance from if the spacecraft remains trapped in it despite the continuous thrusting. Since the application of low-thrust propulsion is the future tendency, the study of the probability of capture (PoC) into resonance of a spacecraft around an asteroid needs to be investigated.

The Adiabatic Invariant Theory (AIT) is a useful semi-analytical approach to estimate the PoC into resonance of a dynamical system if the system's Hamiltonian is dependent on slowly changing parameters over time, for example, a pendulum whose length slowly changes with time. The theory is based on the fact that the trajectory of the non-autonomous system is close to the trajectory of the autonomous system, but slowly drifting from it.

## 2 Methodologies

The model considered for the numerical simulations is the perturbed two-body problem with perturbations from Vesta's irregular gravitational field, truncated to the 4th order and degree, and the low-thrust, constant in magnitude and always in the opposite direction of Dawn's velocity. Using Monte Carlo simulations, the PoC is estimated and compared with the PoC obtained using the

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AIT. By providing realistic assumptions on how the parameters change over time, the AIT can provide precise information regarding the dynamical system's long-term evolution. As the system evolves, the separatrices move, expanding the libration region in the phase space. This area variation is directly correlated with the probability of capture as

$$Pr = -\frac{dA_{res}/d\lambda}{dA_0/d\lambda} \tag{1}$$

where $A_{res}$ is the area of the libration region, $A_0$ is the area from which the trajectory crosses the separatrix and $\lambda$ is the parameter related to how slow the dynamical system changes. The first fundamental resonance model was studied in depth, defining analytically the area variation of the libration region with respect to the slowly changing parameter and the probability of capture into resonance.

## 3 Results

The descent from HAMO to LAMO is performed nominally from a circular and polar orbit. For this case, the PoC into 1:1 GTR is estimated and a sensitivity analysis is carried out considering different spacecraft properties, such as mass and thrust magnitude. The following characteristics have been considered: the change in the PoC value, the randomness of the event, the strength of the capture, and the sensitivity of the trajectory to other resonances. It is found that: by increasing the thrust magnitude the randomness of the phenomena decreases, and the probability of permanent capture decreases but the probability of temporary capture increases; by increasing the spacecraft's mass the PoC into 1:1 GTR decreases, but the PoC into 2:3 GTR increases. Finally, using the AIT, the PoC into 1:1 GTR is estimated as shown in Table 1.

**Table 1.** Estimation of the probability of capture into resonance of Dawn around Vesta using the AIT compared to the numerical estimate of about 8.6%

| Resonance crossing | PoC | Relative error |
|---|---|---|
| $t^*_{min} = 25.3$ days | 7.78% | 9.5% |
| $t^*_{mean} = 26.2$ days | 7.98% | 7.2% |
| $t^*_{max} = 27.4$ days | 8.24% | 4.1% |

The relative error between the numerical and analytical estimation is about 4%. In this way, the methodology shows its potential to be used in astrodynamics.

This research contributes to the state of the art in the field of astrodynamics by systematically and efficiently analyzing the probability of low-thrust spacecraft's capture into resonance around asteroids.