Chebyshev-based Polynomial Algebra Monte Carlo Propagation with Generalised Equinoctial Orbital Elements

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Monte Carlo (MC) methods are the traditional choice for uncertainty propagation in astrodynamics, where a large number of samples from a Probability Density Function (PDF) are propagated individually to analyse the evolution of the PDF with time. One of the fundamental limitations of these methods, however, is their computational time which typically scales poorly as the size of the sample set increases.

Polynomial Algebra Monte Carlo (PAMC) is a method which propagates the entire set at once to yield improvements in computational performance. Under this method, discrete sets of states are represented instead with continuous polynomials. Mathematical operators, such as addition and multiplication, are defined for this polynomials, enabling calculations with entire sets of states. Following propagation of the states in polynomial form, the polynomials are sampled to recover the discrete propagated states. More advanced operations, such as trigonometric functions, can be implemented in two main ways: either through their Taylor expansions, or through a Chebyshev interpolation. This results in two implementations, either Taylor-based or Chebyshev-based [1].

Concurrently, further improvements in computational performance can be made by using alternative state formulations. Generalised Equinoctial Orbital Elements (GEqOEs) are an alternative state formulation, proposed by Baú, Hernando-Ayuso, and Bombardelli [2], which are an improved version of classical Equinoctial Orbital Elements (EqOEs). By embedding the perturbing potential into the definition of the elements, their performance is improved through more linear orbital propagation, even when under the influence of perturbations. This was demonstrated through linear methods, using the State Transition Matrix (STM), which showed that covariance realism was preserved for a longer period when using GEqOEs [3].

It has been shown that GEqOE-based PAMC can be implemented to take advantage of the formulation's more linear evolution with time [4, 5]. Compared to a PAMC implementation with Cowell's method, this resulted in a tenfold reduction in the number of integration steps required for a given solution accuracy, and an up to 80% reduction in computational time. However, until now this has been limited to a Taylor-based implementation. It has been shown previously that improvements in solution accuracy are available by using a Chebyshev-based implementation, for a small increase in computational overhead [6–8].

In this presentation, a Chebyshev-based implementation for set propagation with PAMC will be presented, with both Cowell's method and GEqOEs, and compared with the equivalent Taylor-base implementation. It will be demonstrated that the Chebyshev-based implementation has a slightly higher computational overhead, however with the advantage of a more consistent accuracy-performance trade-off, for both Keplerian and perturbed orbit cases.

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