Initial orbit determination from one position vector and a very short arc of optical observations

Erica Scantamburlo[1[0000-0003-1556-1102]], Giovanni F. Gronchi[1[0000-0003-1294-0633]], and Giulio Baù[1[0000-0002-9857-0866]]

Dipartimento di Matematica, University of Pisa, Largo B. Pontecorvo 5, 56127, Italy
erica.scantamburlo.93@gmail.com
giovanni.federico.gronchi@unipi.it
giulio.bau@unipi.it

Abstract. The number of asteroids observations performed by modern telescopes is very large. Usually they can be grouped into very short arcs (VSAs) of detections, however it is not easy to determine if the data of VSAs referring to different nights belong to the same observed objects.
In general, from a VSA we can not compute a reliable preliminary orbit, however we can try to put together the information of different VSAs to perform this task. Assuming that two VSAs belong to the same asteroid, we can write polynomial equations to compute a preliminary orbit using the conservation laws of the two-body dynamics. Recently, different ways to combine these integrals of motion have been tested for this purpose (see [1–4]).
We address the problem of computing an asteroid orbit (OD problem) from one topocentric position vector $\mathcal{P}_1 = (\rho_1, \alpha_1, \delta_1)$, where ρ_1, α_1 and δ_1 denote respectively the topocentric distance, the right ascension and the declination at epoch t_1, and a VSA of optical observations, giving an attributable $\mathcal{A}_2 = (\alpha_2, \delta_2, \dot{\alpha}_2, \dot{\delta}_2)$ at the mean epoch $\bar{t}_2$ of the VSA.
Using the algebraic conservation laws of the angular momentum, the Laplace-Lenz vector and the energy in the two-body dynamics, we can write the OD problem as a system of polynomial equations in the unknowns $\dot{\rho}_1$,$\dot{\alpha}_1$,$\dot{\delta}_1$,ρ_2, $\dot{\rho}_2$,z_2, where z_2 is an auxiliary variable.
We prove that the system is consistent (i.e. it generically admits solutions, at least in the complex field), and we can obtain a univariate polynomial $\mathfrak{u}$ of degree eight in the unknown range ρ_2 at epoch $\bar{t}_2$ to solve the OD problem. Through a symbolic manipulator, we are also able to show that the degree of $\mathfrak{u}$ is minimum among the degrees of all the univariate polynomials in ρ_2 solving this problem.
The proposed method allows us to deal with the orbit determination problem for an asteroid having a close encounter, if this can be modelled with an instantaneous change of direction of the velocity vector. In particular, this method may help to recover a lost asteroid, with a badly determined orbit, that had a close encounter with the Earth. For this case, we show a numerical test with simulated observations.

Keywords: Orbit determination · Very short arcs · two-body dynamics

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