The equation between 3-body mean motion resonances and Yarkovsky drift speeds on eccentricities higher than 0.1*

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Abstract. We studied the motion of asteroids across the 3-body mean motion resonances (MMRs) with Jupiter and Saturn and with the Yarkovsky drift speed in the semimajor axis of the asteroids. The research was conducted using numerical integrations performed using the Orbit9 integrator with 72 000 test asteroids. We calculated time delays, $\mathrm{d}tr$, caused by the six 3-body MMRs on the mobility of test asteroids with 10 positive and 10 negative Yarkovsky drift speeds, which are reliable for Main Belt asteroids. Our final results considered only test asteroids that successfully crossed over the MMRs without close approaches to the planets. We devised equations that approximately describe the functional relation between the average time $\langle \mathrm{d}tr \rangle$ spent in the resonance, the strength of the resonance SR, and the semimajor axis drift speed $\mathrm{d}a/\mathrm{d}t$ (positive and negative) with the orbital eccentricities of asteroids in the range (0.1, 0.2). Comparing the values of $\langle \mathrm{d}tr \rangle$ obtained from the numerical integrations and from the derived functional relations, we analysed average values of $\langle \mathrm{d}tr \rangle$ in all 3-body MMRs for every $\mathrm{d}a/\mathrm{d}t$. The main conclusion is that the analytical and numerical estimates of the average time $\langle \mathrm{d}tr \rangle$ are in very good agreement, for both positive and negative $\mathrm{d}a/\mathrm{d}t$. Finally, this study shows that the functional relation we obtain for 3-body MMRs for orbital eccentricities of asteroids in the range (0.1, 0.2) is analogous to that previously obtained for orbital eccentricities of asteroids in the range (0, 0.1) in [1].

Keywords: methods: numerical · methods: analytical · celestial mechanics – minor planets, asteroids: general.

1 Results

In the first column in Table 1 there are names of 3-body MMRs. In the second column it is showed their nominal semimajor axis. The widths of the 6 selected 3-body resonances for $e = 0.2$ are in the third column, propagated by the numerical method with the Orbit9 [2, 3]. The last two columns contain their strengths SR, calculated with the numerical method given in [4], in cases of negative and positive $\mathrm{d}a/\mathrm{d}t$.

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Table 1. The properties of the 6 selected 3-body resonances.

MMR	a_{res} [au]	width($e = 0.2$) [au]	SR $\mathrm{d}a/\mathrm{d}t < 0$	SR $\mathrm{d}a/\mathrm{d}t > 0$
1:-3J:1S	2.75180	0.00350	0.00137283	0.00130213
2:-7J:3S	2.55896	0.00338	0.00099704	0.00086637
2:-7J:2S	2.44715	0.00200	0.00006268	0.00009325
1:-5J:6S	2.75761	0.00158	0.00000826	0.00000679
3:-6J:-1S	3.13787	0.00156	0.00001396	0.00003654
3:-8J:1S	2.79909	0.00092	0.00000877	0.00001229

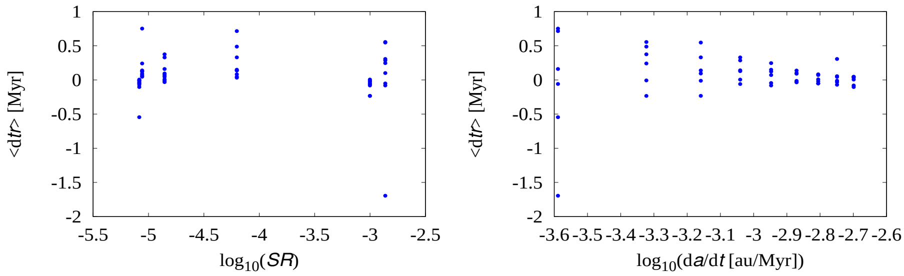

Fig. 1. Relation between $\langle \mathrm{d}tr \rangle$ and $\log_{10}(SR)$ (left panel), and $\log_{10}(\mathrm{d}a/\mathrm{d}t)$ (right panel) without the lowest $\mathrm{d}a/\mathrm{d}t$ for negative Yarkovsky drift speeds. Similar graph we got for positive Yarkovsky drift speeds.

Considering the outcomes presented in Figure 1, we derived the functional relation that describes the correlation between $\langle \mathrm{d}tr \rangle$, SR and $\mathrm{d}a/\mathrm{d}t$, in case of positive and negative $\mathrm{d}a/\mathrm{d}t$, for e in the observed interval (0.1, 0.2):

$$\langle \mathrm{d}tr \rangle = (0.5 + \mathrm{d}a)\log_{10}(SR) + (\mathrm{d}b - 1.0)\log_{10}(\frac{\mathrm{d}a}{\mathrm{d}t}) + (\mathrm{d}c + 5.0). \quad (1)$$

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