Physics-Informed Neural Network Gravity Models for Small-Body Exploration

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Gravity field modeling is a problem of key importance for small-body exploration. Due to the irregular geometries and unknown density profiles of objects like asteroids and comets, the gravitational fields produced by these bodies can yield highly non-keplerian spacecraft motion. As such, dynamicists require models which can accurate characterize these fields to ensure that spacecraft remain on safe and fuel efficient trajectories.

Among the set of current gravity models, one of the most widely used is the spherical harmonic gravity model (Kaula, 1966). This model is a particularly convenient way to represent the gravity fields of large celestial bodies like the Earth and the Moon due to its efficient modeling of planetary oblateness. In small body settings however, the model grows considerably less reliable — diverging within the Brillouin sphere and requiring many terms to capture discontinuity and irregular geometries (Werner and Scheeres, 1997; Martin and Schaub, 2022). Alternative gravity models exist which aim to avoid these problems including the polyhedral model and the mascon model (Werner and Scheeres, 1997; Wittick and Russell, 2018; Muller and Sjogren, 1968). These models avoid the diverging numerics of the spherical harmonic model within the bounding sphere, however they come with their own unique caveats. The polyhedral gravity model makes assumptions about the density profile of the body, and it is particularly expensive to evaluate when leveraging high-fidelity shape models. The mascon representation can avoid the computational overhead of the polyhedral model, but it becomes less accurate near the surface of a body where the discrete nature of the mascons grows more apparent (Tardivel, 2016).

These caveats are becoming increasingly difficult to ignore. With spacecraft like OSIRIS-REx, Rosetta, and Hayabusa2 attempting landing or touch-and-go manuevers, the need for models that are efficient and accurate across all operational domains is paramount. This pressure has fueled the development of a second generation of small-body gravity models which shift attention away from analytic models and towards learned solutions. In particular, machine learning has demonstrated the ability to bypass many of the challenges facing past gravity models. By using tools like extreme learning machines, gaussian processes, and neural networks, dynamicists are able to learn efficient bases rather than prescribing them analytically (Furfaro et al., 2020; Gao and Liao, 2019; Cheng, Wang, and Jiang, 2019). This simple change can yield computationally efficient and accurate gravity models without assumptions or operational no-fly zones.

While learned gravity models provide a compelling solution to the problems plaguing their analytic counterparts, work remains to bring these models into the mainstream. There exist relatively large gaps in the literature which fail

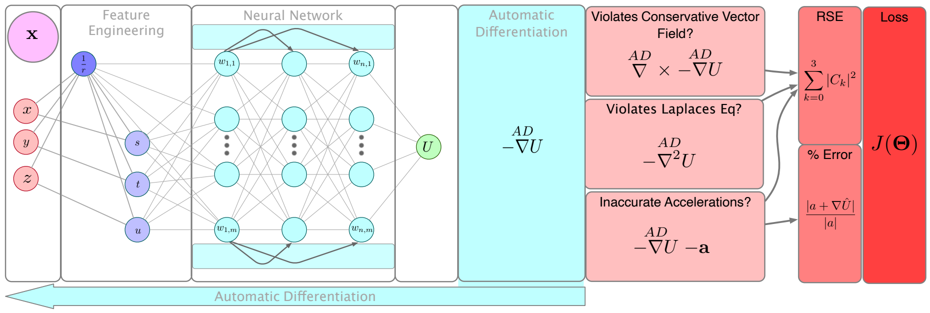

Fig. 1. Physics Informed Neural Network Gravity Model Generation III (PINN-GM-III)

to carefully investigate how these models can be designed to ensure robustness across a wide variety of settings. For example, are there ways in which researchers can ensure that the gravity model learned will satisfy relevant physics? How might a model be designed to minimize the effect of extrapolation error beyond the bounds of the training data? How do these models respond to error in the training data, or amount of training data? How can these models be leveraged in other facets of astrodynamics like orbit determination or trajectory design? These questions require answers before these models can be deployed.

This presentation aims to address many of these questions through the introduction of the latest generation of the Physics-Informed Neural Network Gravity Model (PINN-GM-III). The PINN-GM differs from the other learned gravity models due to its physics-informed loss function. By augmenting the traditional network loss with differential physics constraints (Laplace's Equation, conservative vector field properties, relationships between scalar potentials and accelerations, etc.), the PINN-GM enforces that the model it learns is intrinsically compliant with the underlying differential equations of the system (Martin and Schaub, 2022). This provides significant benefits in model accuracy, compactness, robustness to erroneous training data, and more — all the while retraining computational efficiency.

Over the past two years, considerable effort has been put forth to carefully design, implement, and characterize this new gravity model in a variety of astrodynamics settings and problems. This presentation will summarize this work and will introduce various use cases of the PINN-GM within the fields of reinforcement learning, orbit discovery, and estimation.

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