

# Secular Lunisolar Resonances in the MEO Region and the Eccentricity Growth Phenomenon <sup>★</sup>

Edoardo Legnaro<sup>1,2</sup> and Christos Efthymiopoulos<sup>3</sup>

<sup>1</sup> Department of Physics, Aristotle University of Thessaloniki, Greece

<sup>2</sup> Research Center for Astronomy and Applied Mathematics, Academy of Athens  
Greece. [legnaro@academyofathens.gr](mailto:legnaro@academyofathens.gr)

<sup>3</sup> Department of Mathematics, University of Padova, Italy  
[christos.efthymiopoulos@math.unipd.it](mailto:christos.efthymiopoulos@math.unipd.it)

**Abstract.** We discuss the role of inclination-only dependent lunisolar resonances in the dynamics of a MEO (Medium Earth Orbit) object over secular time scales (i.e. several decades). We will focus in particular on their role in the "eccentricity growth" phenomenon. Indeed, the effect of a lunisolar resonance is to increase an object's eccentricity, possibly up to a value where the orbit's perigee meets the earth's atmosphere. In this way, our results provide a framework useful to understand which initial conditions will lead to the re-entry of a MEO object. This allows to design low-cost end-of-life disposal strategies for navigation satellites and mitigation procedures for space debris. This presentation will be based on the results presented in [1] and [2].

**Keywords:** Lunisolar resonances · Eccentricity growth · MEO secular dynamics.

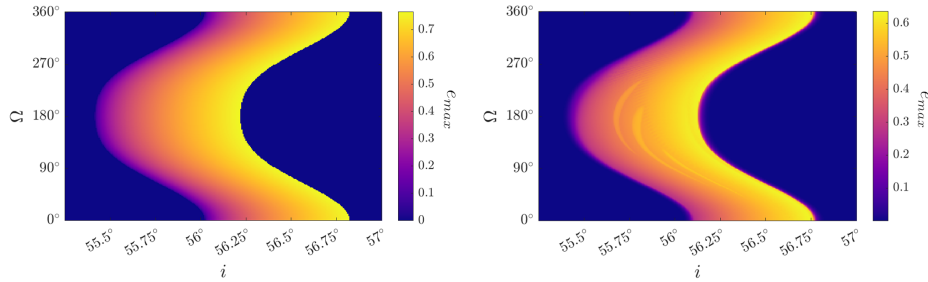
## 1 Contributions

The MEO region is home to various artificial satellites, whose most common applications include navigation, communication, and geodetic/space environment science. The dynamics of these satellites is mainly influenced by either the  $2g+h$  or  $2g$  resonance (close to  $56^\circ$  and  $64^\circ$ ) respectively.

Our main contributions can be summarized as follows. We provide an analytic framework useful to understand the structure of all inclination-only dependent lunisolar resonances and to compute the correct form of their separatrices in the  $(e, i)$  space. We show the correspondence between the theoretical phase portraits at each resonance and the numerical stability maps computed by the "Fast Lyapunov Indicator". We provide quantitative estimates for the maximum eccentricity, as a function of the initial inclination, which can be reached via the

---

<sup>★</sup> We acknowledge the support of the Marie Curie Initial Training Network Stardust-R, grant agreement Number 813644 under the H2020 research and innovation program: <https://doi.org/10.3030/813644>.



**Fig. 1.** Computation of the maximum eccentricity reachable along the  $2g+h$  resonance for  $a = 20000$  km in the  $(i, \Omega)$  plane. Left: analytical computation, using the procedure described in [1]. Right: numerical computation up to 1000 years on a  $300 \times 300$  grid in the  $(i, \Omega)$ .

”eccentricity growth” mechanism, separately for each resonance, providing also the limits in inclination within which the mechanism is active, as well as the dependence of these limits on the initial phases  $\Omega, \Omega_L$ . This allows to understand numerical results found in previous works. We identify these limits by the inclination values that mark the transition of circular orbits ( $e = 0$ ) from stable to unstable via an analogue of the Konzai mechanism.

## 2 Conclusions

The integrable model we derived allows to predict the shapes found in FLI cartography and predicts the limiting values  $i_1$  and  $i_2$  within which circular orbits are unstable. This permits to find limiting values for  $i$  within which the eccentricity growth mechanism is active. Moreover, our model explains the dependence on the phases of the angles  $\omega, \Omega$  and  $\Omega_L$  of FLI cartography, which allows to understand the dependence on the phases of the maximum eccentricity reachable along the resonance found in previous numerical works (see Figure 1). Finally, this analytic theory provides a framework useful to understand the eccentricity growth phenomenon in the MEO region, allowing to predict which initial conditions will lead to the re-entry of an object.

## References

1. Legnaro, E., Efthymiopoulos, C.: A detailed dynamical model for inclination-only dependent lunisolar resonances. Effect on the “eccentricity growth” mechanism. *Advances in Space Research* (2022). <https://doi.org/10.1016/j.asr.2022.07.057>
2. Jérôme, D., Legnaro, E., Efthymiopoulos, C., Gkolias, I.: A deep dive into the  $2g + h$  resonance: separatrices, manifolds and phase space structure of navigation satellites. *Celestial Mechanics and Dynamical Astronomy* 134.1 (2022): 1-31. <https://doi.org/10.1007/s10569-021-10060-6>